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Fekete-Sezegö problem for certain subclass of analytic and univalent functions associated with cosine and sine functions of complex order

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Abstract

The main focus of this paper is to give some coefficient estimates for the analytic and univalent functions on the open unit disk in the complex plane that are subordinated to cosine and sine functions of complex order. For the defined subclass of analytic and univalent functions $C(\tau)_{cos,sin}$, $\tau \in \mathbb{C} - \{0\}$ with the quantity

$$1 + \frac{1}{\tau}\left[\frac{\left(zf^{'}(z)\right)^{'}}{f^{'}(z)} - 1\right]$$

subordinated to $\cos z + \sin z$ in the study, we obtain the coefficient estimates for the initial two coefficients and examine the Fekete-Szegö problem.

Keywords: Starlike function, convex function, cosine function, sine function, coefficient estimate, Fekete-Szegö problem, complex order.

1. *Introduction*

In this section, we give some basic information that we will use in proof of the main results and to discuss the studies known in the literature related to our subject.

Let $U = \{z \in \mathbb{C}: |z| < 1\}$ be open an unit disk in the complex plane $\mathbb{C}$ and $H(U)$ denote the class of all analytic functions in U.

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By A, we will denote the class of the functions $f \in H(U)$ given by the following series expansion, which satisfying the conditions $f(0) = 0$ and $f^{'}(0) - 1 = 0$

$$\begin{aligned} f(z) &= z + a_2 z^2 + a_3 z^3 + \cdots \\ &= z + \sum_{n=2}^{\infty} a_n z^n, a_n \in \mathbb{C}. \end{aligned} \tag{1.1}$$

As it is known that the subclass of univalent functions of A is denoted by S, in the literature. This class was first introduced by Köebe (Köebe 1909) and has become the core ingredient of advanced research in this field. Bieberbach (Bieberbach 1916) published a paper in which the famous coefficient hypothesis was proposed. This conjecture states that if $f \in S$ and has the series form (1.1), then $|a_n| \le n$ for all $n \ge 2$. Many researchers worked hard to solve this problem. But for the first time this long-lasting conjecture was solved by De-Branges (De-Branges 1985) in 1985.

It is well-known that a univalent function $f \in S$ is called a convex function, if this function maps open unit disk U onto the convex shaped domain of the complex plane. The set of all convex functions in U, which satisfies the following condition is denoted by C

$$Re\left(\frac{\left(zf^{'}(z)\right)^{'}}{f^{'}(z)}\right) > 0, z \in U;$$

that is,

$$C = \left\{f \in S:\ Re\left(\frac{\left(zf^{'}(z)\right)^{'}}{f^{'}(z)}\right) > 0, z \in U\right\}.$$

Some of the important and well-investigated subclass of S include the class $C(\alpha)$ given below, which is called of the convex functions of order $\alpha\left(\alpha \in 0,1)\right)$

$$C(\alpha) = \left\{f \in S:\ Re\left(\frac{\left(zf^{'}(z)\right)^{'}}{f^{'}(z)}\right) > \alpha, z \in U\right\}.$$

It is well-known that an analytical function ω satisfying the conditions $\omega(0) = 0$ and $|\omega(z)| < 1$ is called Schwartz function. Let $f, g \in H(U)$, then it is said that f is subordinate to g and denoted by $f \prec g$, if there exists a Schwartz function ω , such that $f(z) = g(\omega(z))$.

In 1992, Ma and Minda (Ma and Minda 1994) using subordination terminology presented the class $C(\varphi)$ as follows

$$C(\varphi) = \left\{ f \in S : \frac{\left(zf^{'}(z)\right)^{'}}{f^{'}(z)} \prec \varphi(z), z \in U \right\},$$

$\beta \in [0,1]$,

where $\varphi(z)$ is a univalent function with $\varphi(0) = 1$, $\varphi^{'}(0) > 0$ and the region $\varphi(U)$ is star-shaped relative to the point $\varphi(0) = 1$ and symmetric with respect to real axis. Such a function has a series expansion of the following form

$$\varphi(z) = 1 + b_1 z + b_2 z^2 + b_3 z^3 + \cdots = 1 + \sum_{n=1}^{\infty} b_n z^n,$$

$b_1 > 0$.

In the past few years, numerous subclasses of the collection S have been introduced as special choices of the class $C(\varphi)$ (see for example (Sokol 2011, Janowski 1970, Arif, et al 2019, Brannan 1969, Sokol and Stankiewcz et al. 2021, Sharma et al. 2016, Kumar and Arora 2020, Mendiratta et a.l 2015, Shi et al 2019, Bano and Raza 2020, Alotaibi et al. 2020, Ullah et al. 2021, Cho et al. 2019, Mustafa et al.2023a, Mustafa et al.2023b, Mustafa et al.2023c, Mustafa et al.2023d, Mustafa et al.2023e, Mustafa et al.2023f, Mustafa et al.2023g)).

Finding bounds for the function coefficients in a given collection is one of the most fundamental problems in geometric function theory, since it impacts geometric features.

The first order of Hankel determinant of the function $f \in S$ defined by

$$H_{2,1}(f) = \begin{vmatrix} 1 & a_2 \\ a_2 & a_3 \end{vmatrix} = a_3 - a_2^2$$

and is known as the Fekete-Szegö functional. The functional $H_{2,1}(f,\mu) = a_3 - \mu a_2^2$ is known as the generalized Fekete-Szegö functional, where μ is a complex or real number (Duren 1983). Estimating the upper bound of $|a_3 - \mu a_2^2|$ is known as the Fekete-Szegö problem in the theory of analytic functions.

In this paper, we introduce a new subclass of analytic and univalent functions on the open unit disk in the complex plane that are subordinated to cosine and sine functions of complex order. For the defined here subclass $C(\tau)_{cos,sin}$, we obtain the coefficient estimates for the initial two coefficients and examine the Fekete-Szegö problem. A similar study for the class $S^{*(\tau)}_{cos,sin}$ was done in (Mustafa et al. 2023g).

2. *Materials and Methods*

By using the definition of subordination, we introduce a new subclass of analytic and univalent functions associated with cosine and sine functions of complex order.

Definition 2.1. For $\tau \in \mathbb{C} - \{0\}$ a function $f \in S$ is said to be in the class $C(\tau)_{cos,sin}$, if the following condition is satisfied

$$1 + \frac{1}{\tau}\left[\frac{\left(zf^{'}(z)\right)^{'}}{f^{'}(z)} - 1\right] \prec \cos z + \sin z\, , z \in U;$$

that is,

$$C(\tau)_{cos,sin} = \left\{ f \in S \left| 1 + \frac{1}{\tau}\left[\frac{\left(zf^{'}(z)\right)^{'}}{f^{'}(z)} - 1\right] \prec \cos z + \sin z\, , z \in U \right.\right\}$$

Remark 2.2. Taking $\tau = 1$ in the Definition 2.1, we have the class $C_{cos,sin}$given as follows

$$C_{cos,sin} = \left\{ f \in S : \frac{\left(zf^{'}(z)\right)^{'}}{f^{'}(z)} \prec \cos z + \sinh z\, , z \in U \right\}.$$

Remark 2.3. In the case $\tau = 1$, we have the class $C_{cos,sin}$ which reviewed in (Mustafa et al. 2023d).

Let P be the class of analytic functions in U satisfied the conditions $p(0) = 1$ and $Re\big(p(z)\big) > 0$, $z \in U$, which from the subordination principle easily can written

$$P = \left\{ p \in A : p(z) \prec \frac{1+z}{1-z}, z \in U \right\},$$

where $p(z)$ has the series expansion of the form

$$\begin{aligned} p(z) &= 1 + p_1 z + p_2 z^2 + p_3 z^3 + \cdots \\ &= 1 + \sum_{n=1}^{\infty} p_n z^n, z \in U. \end{aligned} \qquad (1.2)$$

The class P defined above is known as the class Caratheodory functions (Miller 1975).

Now, let us present some necessary lemmas known in the literature for the proof of our main results.

Lemma 2.4 (Duren 1983). Let the function $p(z)$ belong in the class P. Then,

$$|p_n| \le 2 \text{ for each } n \in \mathbb{N} \text{ and } |p_n - \lambda p_k p_{n-k}| \le 2 \text{ for } n,k \in \mathbb{N}, n > k \text{ and } \lambda \in [0,1].$$

The equalities hold for the function

$$p(z) = \frac{1+z}{1-z}.$$

Lemma 2.5 (Duren 1983). Let analytic function $p(z)$ be of the form (1.2), then

$$2p_2 = p_1^2 + (4 - p_1^2)x,$$
$$4p_3 = p_1^3 + 2(4 - p_1^2)p_1x - (4 - p_1^2)p_1x^2 + 2(4 - p_1^2)(1 - |x|^2)y$$

for $x, y \in \mathbb{C}$ with $|x| \leq 1$ and $|y| \leq 1$.

3. *Main Results*

In this section, we give upper bound estimates for the initial two coefficients of the function belonging to the class $C(\tau)_{cos,sin}$ and examine the Fekete-Szegö problem for the class $C(\tau)_{cos,sin}$.
First of all, let us give the following theorem on coefficient estimates.

Theorem 3.1. Let the function $f \in A$ given by (1.1) belong to the class $C(\tau)_{cos,sin}$. Then,

$$|a_2| \leq \frac{|\tau|}{2} \text{ and } |a_3| \leq \frac{|\tau|}{6}\begin{cases} 1 & \text{if } |2\tau - 1| \leq 2, \\ \frac{|2\tau-1|}{2} & \text{if } |2\tau - 1| \geq 2. \end{cases}$$

Proof. Let $f \in C(\tau)_{cos,sin}$, $\tau \in \mathbb{C} - \{0\}$, then there exists a Schwartz function ω, such that

$$1 + \frac{1}{\tau}\left[\frac{\left(zf'(z)\right)'}{f'(z)} - 1\right] = cos\,\omega\,(z) + sin\,\omega\,(z).$$

We express the Schwartz function ω in terms of the Caratheodory function $p \in P$ as follows

$$p(z) = \frac{1+\omega(z)}{1-\omega(z)} = 1 + p_1z + p_2z^2 + \cdots.$$

It follows from that

$$\omega(z) = \frac{p(z) - 1}{p(z) + 1} = \frac{1}{2}p_1z + \frac{1}{2}\left(p_2 - \frac{p_1^2}{2}\right)z^2 + \cdots \quad . \tag{3.1}$$

From the series expansion (1.1) of the function $f(z)$, we can write

$$1 + \frac{1}{\tau}\left[\frac{\left(zf'(z)\right)'}{f'(z)} - 1\right] = 1 + \frac{1}{\tau}[2a_2z + (6a_3 - 4a_2^2)z^2 + \cdots]. \tag{3.2}$$

Since

$$cos\,z + sin\,z = 1 + z - \frac{z^2}{2!} + \cdots \quad , \tag{3.3}$$

using equality (3.1), we have

$$sin\,\omega\,(z) + cos\,\omega\,(z) = 1 + \frac{1}{2}p_1z + \frac{1}{2}\left(p_2 - \frac{3}{4}p_1^2\right)z^2 + \cdots \quad . \tag{3.4}$$

By equalizing (3.2) and (3.4), then comparing the coefficients of the same degree terms on the right and left sides, we obtain the following equalities for two initial coefficients of the function $f(z)$

$$a_2 = \frac{\tau p_1}{4}, \tag{3.5}$$

$$a_3 = \frac{\tau}{48}[4p_2 + (2\tau - 3)p_1^2]. \tag{3.6}$$

Using Lemma 2.4, from the equalities (3.5) we can easily see that

$$|a_2| \leq \frac{|\tau|}{2}.$$

Using Lemma 2.5, from the equality of (3.6) we can write

$$a_3 = \frac{\tau}{48}[(2\tau - 1)p_1^2 + 2(4 - p_1^2)x],$$

for $x \in \mathbb{C}$ with $|x| \leq 1$. Applying triangle inequality, from this equality we obtain

$$|a_3| \leq \frac{|\tau|}{48}[|2\tau - 1|t^2 + 2(4 - t^2)\xi],$$

where $\xi = |x|$ and $t = |p_1|$.
If we maximize the function $\varphi: [0,1] \to \mathbb{R}$ defined as

$$\varphi(\xi) = |2\tau - 1|t^2 + 2(4 - t^2)\xi, \xi \in [0,1],$$

we write

$$|a_3| \leq \frac{|\tau|}{48}[(|2\tau - 1| - 2)t^2 + 8], t \in [0,2].$$

From this, immediately obtained the desired estimate for $|a_3|$.
Thus, the proof of theorem is completed.

In the case $\tau \in \mathbb{R} - \{0\}$, Theorem 3.1 is given as follows.
Theorem 3.2. Let the function $f \in A$ given by (1.1) belong to the class $C(\tau)_{cos,sin}$,$\tau \in \mathbb{R} - \{0\}$. Then,

$$|a_2| \leq \begin{cases} \frac{-\tau}{2} & \text{if } \tau \in (-\infty, 0), \\ \frac{\tau}{2} & \text{if } \tau \in (0, +\infty). \end{cases}$$

and

$$|a_3| \le \frac{\tau}{6}\begin{cases} \frac{2\tau-1}{2} & \text{if} \tau \in -\infty, -\frac{1}{2}, \\ -1 & \text{if } \tau \in -\frac{1}{2}, 0\Big), \\ 1 & \text{if } \tau \in 0, \frac{3}{2}, \\ \frac{2\tau-1}{2} & \text{if } \tau \in \frac{3}{2}, +\infty\Big). \end{cases}$$

Taking $\tau = 1$ in Theorem 3.1, we obtain the following estimates for the initial two coefficients for the function belonging to the class $C_{cos,sin}$.

Theorem 3.3. (Mustafa et al. 2023d) Let the function $f \in A$ given by (1.1) belong to the class $C_{cos,sin}$. Then,

$$|a_2| \le \tfrac{1}{2} \text{ and } |a_3| \le \tfrac{1}{6}.$$

Now, let us give the following theorem on the Fekete-Szegö inequality for the class $C(\tau)_{cos,sin}$.

Theorem 3.4. Let the function $f \in A$ given by (1.1) belong to the class $C(\tau)_{cos,sin}$, $\tau \in \mathbb{C} - \{0\}$ and $\mu \in \mathbb{C}$ or $\mu \in \mathbb{R}$. Then,

$$|a_3 - \mu a_2^2| \le \frac{|\tau|}{12}\begin{cases} 2 & \text{if } |\tau(2-3\mu)-1| \le 2, \\ |\tau(2-3\mu)-1| & \text{if } |\tau(2-3\mu)-1| \ge 2, \end{cases}$$

where

$$\gamma(\tau,\mu) = \tfrac{1}{12}|\tau(2-3\mu)-1|.$$

Proof. Let $f \in C(\tau)_{cos,sin}$,$\tau \in \mathbb{C} - \{0\}$ and $\mu \in \mathbb{C}$ or $\mu \in \mathbb{R}$. Then, from the equalities (3.5) and (3.6), we can write

$$a_3 - \mu a_2^2 = \tfrac{\tau}{48}\{4p_2 + [\tau(2-3\mu)-3]p_1^2\}.$$

Using Lemma 2.5, the last equality we can write as follows

$$a_3 - \mu a_2^2 = \tfrac{\tau}{48}\{[\tau(2-3\mu)-1]p_1^2 + 2(4-p_1^2)x\} \quad (3.7)$$

for $x \in \mathbb{C}$with $|x| \le 1$. Applying triangle inequality to last equality, we have

$$|a_3 - \mu a_2^2| \le \frac{|\tau|}{48}\{|\tau(2-3\mu)-1|t^2 + 2(4-p_1^2)\xi\}$$

with $t = |p_1| \in [0,2]$ and $\xi = |x|$.

By maximizing the function

$$\psi(\xi) = |\tau(2-3\mu)-1|t^2 + 2(4-p_1^2)\xi,\ \xi \in [0,1],$$

we obtain the following inequality

$$|a_3 - \mu a_2^2| \le \frac{|\tau|}{48}\{[|\tau(2-3\mu)-1|-2]t^2 + 8\}, \quad t \in [0,2]. \ (3.8)$$

From this, we obtain the desired result of theorem. Thus, the proof of theorem is completed.

In the cases $\tau \in \mathbb{R} - \{0\}$ and $\mu \in \mathbb{R}$, Theorem 3.4 is given as follows.

Theorem 3.5. Let the function $f \in A$ given by (1.1) belong to the class $C(\tau)_{cos,sin}$, $\tau \in \mathbb{R} - \{0\}$ and $\mu \in \mathbb{R}$. Then,

$$|a_3 - \mu a_2^2| \le \frac{|\tau|}{6}\begin{cases} -6\gamma(\tau,\mu) & \text{if } \tau < 0 \text{ and } \mu \le \mu_1(\tau), \\ 1 & \text{if } \tau < 0 \text{ and } \mu \in [\mu_1(\tau), \mu_2(\tau)], \\ 6\gamma(\tau,\mu) & \text{if } \tau < 0 \text{ and } \mu_2(\tau) \le \mu, \\ 6\gamma(\tau,\mu) & \text{if } \tau > 0 \text{ and } \mu \le \mu_2(\tau), \\ 1 & \text{if } \tau > 0 \text{ and } \mu \in [\mu_2(\tau), \mu_1(\tau)], \\ -6\gamma(\tau,\mu) & \text{if } \tau > 0 \text{ and } \mu_1(\tau) \le \mu, \end{cases}$$

where

$$\gamma(\tau,\mu) = \tfrac{1}{12}|\tau(2-3\mu)-1|,\ \mu_1(\tau) = \tfrac{2\tau+1}{3\tau}, \quad \mu_2(\tau) = \tfrac{2\tau-3}{3\tau}.$$

Taking $\tau = 1$ in Theorem 3.4, we obtain the following estimates for the Fekete-Szegö functional for the function belonging to the class $C_{cos,sin}$.

Theorem 3.6. (Mustafa et al. 2023d) Let the function $f \in A$ given by (1.1) belong to the class $C_{cos,sin}$ and $\mu \in \mathbb{C}$ or $\mu \in \mathbb{R}$. Then,

$$|a_3 - \mu a_2^2| \le \frac{1}{12}\begin{cases} 2 & \text{if } |1-3\mu| \le 2, \\ |1-3\mu| & \text{if } |1-3\mu| \ge 2. \end{cases}$$

Taking $\mu = 0$ and $\mu = 1$ in Theorem 3.4, we get the second result of Theorem 3.1 and the following result for the first order Hankel determinant, respectively.

Corollary 3.1. If the function $f \in A$ given by (1.1) belong to the class $C(\tau)_{cos,sin}$, then

$$|a_3 - a_2^2| \le \frac{|\tau|}{12}\begin{cases} 2 & \text{if } |1+\tau| \le 2, \\ |1+\tau| & \text{if } |1+\tau| \ge 2. \end{cases}$$

Setting $\tau \in \mathbb{R} - \{0\}$ in Theorem 3.4, we obtain the following result for the first order Hankel determinant.

Corollary 3.2. If the function $f \in A$ given by (1.1) belong to the class $C(\tau)_{cos,sin}$, then

$$|a_3 - a_2^2| \leq \frac{|\tau|}{12}\begin{cases} \tau + 1 \text{ if} & \tau \in -\infty, -3, \\ 2 \quad \text{if } \tau \in -3,0) \cup 0,1, \\ 2 \quad \text{if} & \tau \in 0,1, \\ \tau + 1 \text{ if} & \tau \in 1, +\infty). \end{cases}$$

Setting $\tau = 1$ in Corollary 3.2, we obtain the following result obtained in (Mustafa et al. 2023d).

Corollary 3.3. If the function $f \in A$ given by (1.1) belong to the class $C_{cos,sin}$, then,

$$|a_3 - a_2^2| \leq \frac{1}{6}.$$

4. Discussion

In the study, we defined a new subclass of analytic and univalent functions and give coefficient estimates for this class. Also, Fekete-Szegö problem is solved for defined class. In addition, the results obtained here found were compared with the results available in the literature.

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