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An Analysis of the Transformation Geometry of the Primary School Mathematics Curriculum According to Levels

***Gülsüm Demir, **Aytaç Kurtuluş**

Abstract. In this study, it is aimed to examine the Primary Mathematics Curriculum (1-8th grade) according to the transformation geometry levels. In this context, the transformation geometry in primary and secondary school levels or the acquisitions related to this subject were determined. Then, the acquisitions were examined by considering the transformation geometry thinking levels defined by Soon. It has been researched that the achievements in the curriculum are related to which level and whether this level provides the qualifications. As a method, document analysis was carried out. In the research, it was determined that some gains were given in a way that was not very suitable for the hierarchical structure of the Transformation Geometry Levels. The program is examined, it is seen that the most striking shortcomings are that there is no gain at any level regarding the rotational transformation. The fact that there is no rotational transformation outcome in the mathematics curriculum at any level can be a challenging situation for the student in future learning.

Keywords. Transformation geometry, translation, reflection, rotational transformation, transformation geometry level.

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Geometry instruction is an essential and integral part of mathematics education. In mathematics instruction, geometry can be associated with many learning areas such as algebra and numbers, leading to more effective teaching (Özpınar, 2012). In this context, special attention should be given to geometry instruction and the development of geometric thinking in children. Geometric thinking in individuals can be said to begin at an early age with experiences such as getting to know their surroundings, observing objects, and examining them (Baykul, 2009). The development of geometric thinking in individuals continues as they start formal education in schools, where geometry concepts covered in mathematics lessons are introduced in the curriculum. Therefore, an increase in geometric thinking levels is observed based on these experiences (Zeybek, 2019).

Van Hiele (1986) introduced the levels of geometric thinking, which provide a hierarchy of the knowledge to be given to students in geometry learning. Van Hiele's model outlines five levels of development in geometric thinking for individuals (Paksu, 2016). Van Hiele categorizes geometric thinking in individuals into five levels: visualization, analysis, informal deduction (experience-based deduction), formal deduction, and the most advanced level (Fidan, Türnüklü, 2010). These levels are general and can be used in teaching or planning instruction in the targeted topics in geometry. Furthermore, transformation geometry, which is an important learning area in geometry instruction, deals with transformations such as translation, reflection, and rotation. These transformations are not only encountered frequently in everyday life but also serve as fundamental concepts required in other learning areas in mathematics education. In this context, when reviewing the literature, Soon (1989) examined the Van Hiele levels in teaching transformation geometry to middle school students. In this research, soon determined the levels of Transformation Geometry Thinking within the scope of her doctoral thesis, taking into account the Van Hiele levels. In accordance with this, the first three levels to be addressed in the research and their characteristics are presented in Table 1 below.

Table 1.

The characteristics of Van Hiele levels

Van Hiele Levels	The Characteristics
Basic Level	• Recognizes transformations in shapes or movements. • Describes transformations in simple drawings and images in daily life situations. • Defines transformations through actual movements. • Describes transformations using standard or non-standard names and definitions such as rotation and translation.
Level 1	o Defines transformations using the properties of transformations. o Draws the image or pre-image of a given transformation using the properties of transformations. o Discovers properties of transformations through applied transformations. o Uses appropriate terminology for transformations and properties. o Identifies the symmetry axis of reflection and the direction of translation. o Relates transformations using coordinates. o Solves problems using the known properties of transformations. o Composes simple transformations. o Creates patterns using transformations.
Level 2	• Relates properties of transformations to each other. • Modifies situations after creating transformations. • Describes shapes using coordinates. • Relates properties of transformations to each other. • Defines transformations as combinations of basic transformations.

When Table 1 is examined, it can be observed that students at the basic level generally have a simple understanding of transformations between geometric shapes, which can be recognized in everyday life. Students at Level 1 are seen to understand what changes are made to geometric shapes and what characteristics they possess. At Level 2, they are able to perform compound transformations.

When the Van Hiele geometric levels are organized according to transformation geometry, the characteristics of the levels are defined as follows. The topics covered by transformation geometry include reflection, translation, and rotation, which are fundamental concepts within geometry. To teach these concepts, the National Ministry of Education has included specific learning objectives in the Mathematics Curriculum. These objectives aim to provide students with an understanding of these transformations and their intended outcomes. The curriculum is a government-mandated program that clearly outlines the objectives, goals, content, methods, and assessment elements, serving as a guide for instructional planning. These learning objectives specified in the Mathematics Curriculum are delivered to students in primary and middle school according to the prescribed plan. Additionally, as learning tools prepared by considering the curriculum, textbooks are made available to teachers and students for free. Additionally textbooks are fundamental learning tools for both teachers and students, serving as resources that are freely accessible (Seguin, 1989) and have important functions in structuring learning, providing guidance, and supporting self-learning (Erkılıç, Can, 2018). Moreover, textbooks are considered reliable because they are prepared based on the curriculum, which is approved by the Ministry of National Education and includes the outcomes aimed to be achieved by students.

In Buyruk-Akıl's (2020) study, the relationship between the achievement in transformation geometry and the Van Hiele geometric thinking levels of 8th-grade students was examined. According to the research results, the Van Hiele geometric thinking levels of the participating students were found to be below the expected level. Additionally, a significant relationship was found between the Van Hiele geometry test and the transformation geometry test, and this relationship did not vary when considering school and gender factors.

Demir and Kurtuluş' study (2019) was conducted with the aim of determining the impact of 7th-grade students on Van Hiele transformation geometry thinking levels. The research was carried out with a 7th-grade class consisting of 28 students. The topic of transformation geometry was taught over a period of 4 weeks using action plans prepared in accordance with the 5E model. At the end of the implementation, the results showed that the action plans conducted using the 5E model increased the students' Van Hiele transformation geometry thinking levels.

In this study, the achievements in transformation geometry within the Primary School (1-8th grade) mathematics curriculum, which was published by the Ministry of National Education in 2018 and used in schools in Turkey, were examined, taking into account the Van Hiele transformation geometry thinking levels. It was investigated whether the transformation geometry levels and the expected characteristics at these levels are addressed by the curriculum. Subsequently, examples and activities through which these achievements are presented in textbooks were provided. In the literature review, there are studies that combine transformation geometry and Van Hiele levels; however, these studies are conducted with a focus on specific grade levels. These studies generally concentrate on academic achievement of students. This study, on the other hand, is a comprehensive examination of the primary school mathematics curriculum in line with these levels. It aims to identify the areas where the curriculum falls short at these levels and intends to bridge this gap in the curriculum by highlighting the deficiencies to practitioners.

Method

Research Model

In this research, a qualitative research approach was used to examine the outcomes of transformation geometry covered in the primary education (grades 1-8) mathematics curriculum, taking into account Van Hiele's levels of geometric thinking. Qualitative research can be defined as a research process that utilizes qualitative data collection techniques such as observation, interviews, and document analysis to present perceptions and events in a natural and comprehensive manner in their natural environment (Yıldırım and Şimşek, 2008, p. 39). Document analysis was used as the research model in this study. Document analysis is a scientific research method that involves collecting, examining, questioning, and analyzing various documents as the primary source of research data (Sak et al., 2021).

Data Collection Tools

In this research, the updated 2018 Primary Education (Grades 1-8) Mathematics Curriculum and the 1-8 Grade textbooks used in the year 2022, published by the Ministry of National Education (MoNE), were used as the primary sources.

Process

During this study, the first step involved an examination of the Primary and Elementary School Mathematics Teaching Program published by the Ministry of National Education. In the course of this examination, evaluation criteria were initially established, taking into consideration the Geometric Levels. When observing the topics taught to elementary school students, it was evident that they should possess the first three levels (Basic level, Level 1, Level 2) of Geometric Levels. Since this study primarily focuses on transformation geometry, only the achievements related to this topic were examined.

After determining the achievements related to transformation geometry at the elementary school level, criteria established by Soon (1989) were utilized. Soon had assessed the Van Hiele levels of students' geometric thinking in middle school while determining the Transformation Geometry Thinking Levels as part of his doctoral thesis, taking Van Hiele levels into account.

Following the examination of the mathematics curriculum, textbooks, which represent the practical implementation of the curriculum, were analyzed using the same criteria. Initially, the textbooks were examined, and examples related to transformation geometry were identified. These examples were further evaluated based on the criteria set forth by Soon. As a result of these analyses, it was determined which grade levels provided examples at which levels. These findings were presented to the readers in the findings section and were subject to analysis.

Data Analysis

In this research aimed at examining the outcomes of transformation geometry in the primary education (grades 1-8) mathematics curriculum, taking into account Van Hiele's levels of geometric thinking, the indicators progressing hierarchically at these levels were first determined. These indicators were created based on the Transformation Geometry Thinking levels defined by Soon (1989), as identified in the literature review. Accordingly, characteristics defined for Basic Level, Level 1, and Level 2 were determined as indicators. These defined indicators were used as criteria in the comparisons made during the examination of the mathematics curriculum and mathematics textbooks. In the second stage, all outcomes specified in the mathematics education program under transformation geometry were classified according to their compatibility with the levels. In this context, transformation geometry outcomes were categorized according to the levels covering translation, reflection-symmetry, and rotation transformations. The mathematical tasks determined in the curriculum and textbook were examined separately by the researchers, and the Van Hiele transformation geometry thinking levels of the textbook contents were determined by considering the indicators presented in Table 1. After these determinations were made by the researcher, the results were discussed with the field education expert and the discussion continued until a consensus was reached among the researchers for differing decisions. In the analysis of the data, Miles & Huberman (1994) reliability formula Reliability = Consensus / (Agreement + Disagreement) was used and the agreement rate between the coders was found to be 90%. Additionally, the textbooks, which represent the implemented version of the education programs, were presented with examples of mathematical tasks that address these outcomes.

The numbering system is used to simplify the presentation of the findings in the table and explanations. It's based on the first letter indicating which subject the achievement belongs to. For example, if it's a mathematics achievement, it starts with the letter "M". The first number next to the letter indicates which grade level the achievement belongs to. The second number shows which learning area it belongs to. The third number indicates the sub-learning area, and the last number represents the achievement number. For example, the schema M.6.3.3.2. means: It represents the first achievement in the "Circles" sub-learning area in the "Geometry and Measurement" learning area for the 6th grade. The descriptions for these learning areas and sub-learning areas can be found in the curriculum. The number written in the 6th section represents the description of that particular achievement.

This study only includes primary education, and therefore, the examination was conducted only for the first 3 levels of geometric thinking.

Results

In the study that examined the outcomes of transformation geometry in the primary education (grades 1-8) mathematics curriculum, taking into account Van Hiele's levels of geometric thinking, the findings were categorized into Basic Level, Level 1, and Level 2. The findings are presented under the following headings:

Findings on the Distribution of Mathematics Curriculum Transformation Geometry Outcomes to Levels

The examination of the outcomes covered under the title of transformation geometry in the mathematics education program from grades 1 to 8, including translation, reflection-symmetry, and rotation transformations, and the determination of the levels of transformation geometry resulted in data presented in Table 2, organized according to outcome numbers

Table 2.

Distribution of Mathematics Curriculum Transformation Geometry Attainments to Levels

Transformatio	**Levels of Transformation Geometry**		
	Basic Level	**Level 1**	**Level 2**
Translational	M.1.2.2.1. M.2.2.2.1. M.8.3.2.1.	M.8.3.2.3.	It is not directly stated as an outcome to be achieved on the coordinate plane. However, in the explanation sections, it is required to perform translation on the coordinate plane.
Reflection- Symme	M.2.2.2.2. M.3.2.2.1. M.4.2.2.1.	M.8.3.2.2. M.8.3.2.3.	There is no direct attainment to be achieved on the coordinate plane. However, in the explanation sections, performing a translation on the coordinate plane is required."
Rotation	There is no specific attainment.	There is no specific attainment.	There is no specific attainment.

According to Table 2, the topic of translation is first introduced in the description section of a attainment under the heading 'Position' in Grade 1. However, conceptually, learning for all three levels takes place in Grade 8. Symmetry/reflection transformation is hierarchically presented up to the 4th grade of primary school. In Grade 8, there are outcomes provided for other levels, but there is no direct attainment for Level 2. Instead, the provided outcomes are given in the description sections. As for rotation transformation, it is observed that there is no attainment related to it.

Findings on Mathematics Textbooks Transformation Geometry Outcome Examples

Transformations for Basic Level Thinkers

Attainments aligned with this level in the primary school mathematics curriculum and their implementation in textbooks:

When examining the 1st-grade mathematics curriculum and the way it is implemented in textbooks, it is observed that there is no direct attainment explicitly related to transformation geometry. However, indirectly, there are outcomes related to position that can be considered fundamental for transformation geometry.

M.1.2.2.1. Expresses spatial (position, place, direction) relationships.

a) Conducts activities related to the use of expressions indicating place and direction (under, over, around, left, right, between, in front of, behind, high, low, far, near, inside, outside) in daily life situations.

b) Pays attention to determining the reference point when expressing relationships.

c) In addition to daily life examples, activities can also be carried out on models.

The explanation in part b of this outcome is related to position but is of a nature that could serve as a foundation for translation. There is no attainment related to reflection and rotation in the curriculum.

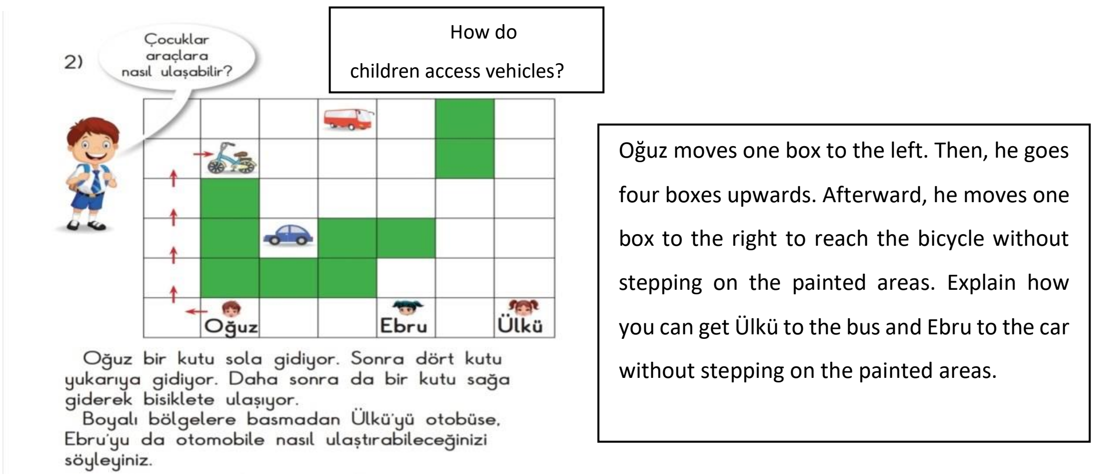


Figure 1. Implementation of Translation Transformation in the 1st Grade Textbook.

In Figure 1, an example from the textbook under the heading of 'Position' has been taken. Upon examination of this example, it is evident that students are instructed in a basic-level translation movement (The English translations of the descriptions in Figure 1 are provided on the side).

The transformation geometry attainment given in the 2nd grade is as follows:

M.2.2.2.1. Mathematical language to specify location, direction, and movement.

a) Mathematical language is used to describe location, direction, and movement along a straight line.

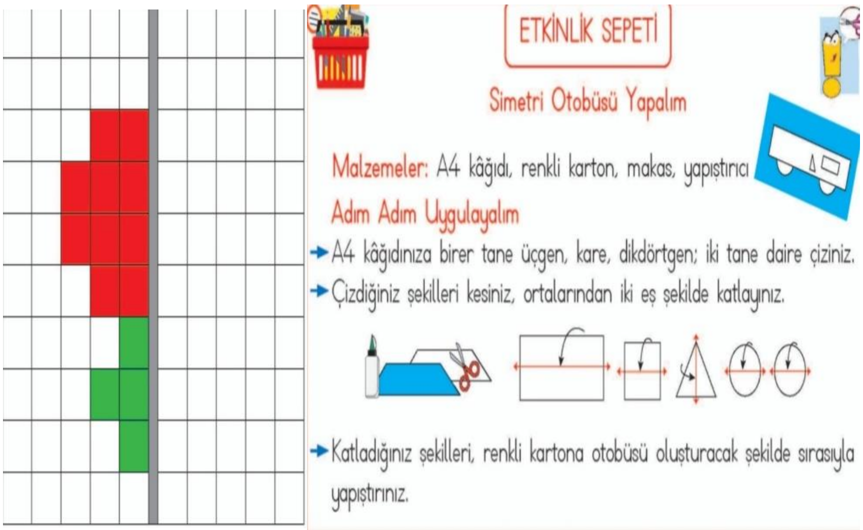
b) Interactive activities involving appropriate information and communication technologies may be included.

M.2.2.2.2. Recognizes symmetrical shapes in the surroundings.

a) The mathematical definition of symmetry is not introduced.

b) Square, triangle, rectangle, and circle are once folded appropriately to divide them into two equal parts, and it is emphasized that there are shapes that cannot be divided into two equal parts.

The concept of symmetry is first introduced at this grade level. Here, students are expected to understand conceptually what symmetry is. When encountering any symmetrical object, they are expected to recognize it as symmetrical. The provided examples are suitable for this level, including symmetry examples with irregular shapes (Figure 2). However, it can be considered a limitation that the symmetries given this level are limited to regular polygons.



Activity Basket
Let's Make a Symmetry Bus
Materials: A4 paper, colored cardboard, scissors, glue
Step by Step Application:
-Draw one triangle, square, and rectangle each on your A4 paper, and two circles.
-Cut out the shapes you drew, and fold them in half symmetrically.
-Glue the folded shapes onto colored cardboard to create the bus in order.

Figure 2. Symmetry Example of Irregular and Regular Shapes.

In Figure 2, two visuals are provided, each representing an example of the two types of reflection required by the attainment. The concept of translation is not introduced in the 2nd grade. However, under the topic of 'position,' students are actually performing translations (The English translations of the descriptions in Figure 2 are provided on the side).

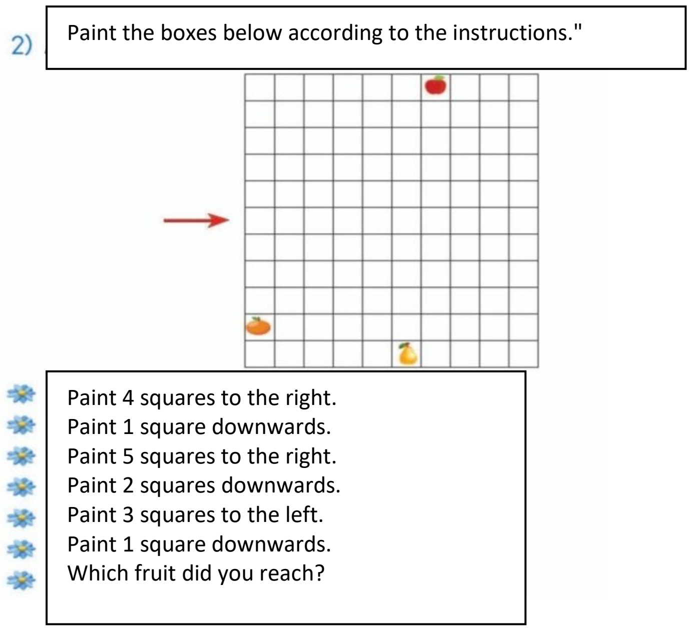


Figure 3. An Example Given in the Textbook Suitable for Position Attainments.

The example provided in Figure 3 can be related to translation. It is observed that the student is performing a translation using the concepts of place and direction. There is no attainment related to rotation at this grade level. Only one activity related to rotation was encountered when examining the textbook:

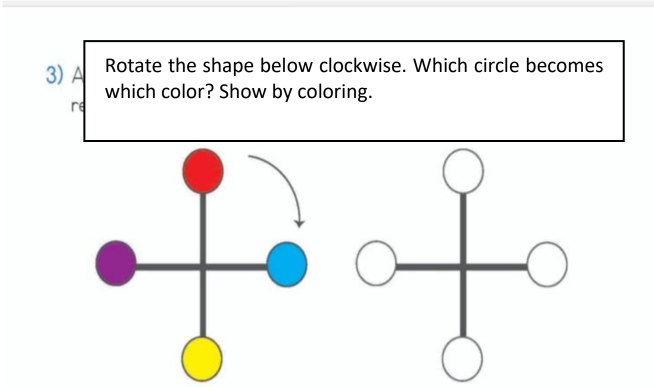


Figure 4. Rotation Activity Included in the Textbook.

Figure 4 features an example provided under the title 'Fractions.' Here, the student is required to rearrange the given colors through a rotation movement as many times as desired. Although the topic is directly related to fractions, the student is asked to use a rotation movement to achieve the desired transformation. However, it is observed that there is no prior explanation related to rotation.

The transformation geometry attainments for the 3rd grade are provided below. The symmetry topic is further elaborated, and the concept of the symmetry axis is introduced in this grade level.

Terms or concepts: axis of symmetry

M.3.2.2.1. Determines that shapes have multiple axes of symmetry by folding the shape.

a) Limited to square, rectangle, and circle.

b) Emphasizes that the diagonal is not an axis of symmetry in the rectangle.

M.3.2.2.2. Completes a symmetric shape given a part of it with respect to a vertical or horizontal axis of symmetry. Examines, correlates, and notices the properties of symmetric shape's matching parts.

At this level, students are expected to use the axis of symmetry both horizontally and vertically. It is important for correct or complete learning that the provided examples include not only regular polygons. Our textbook pays attention to this aspect and asks students to find the symmetries of both regular and irregular shapes.

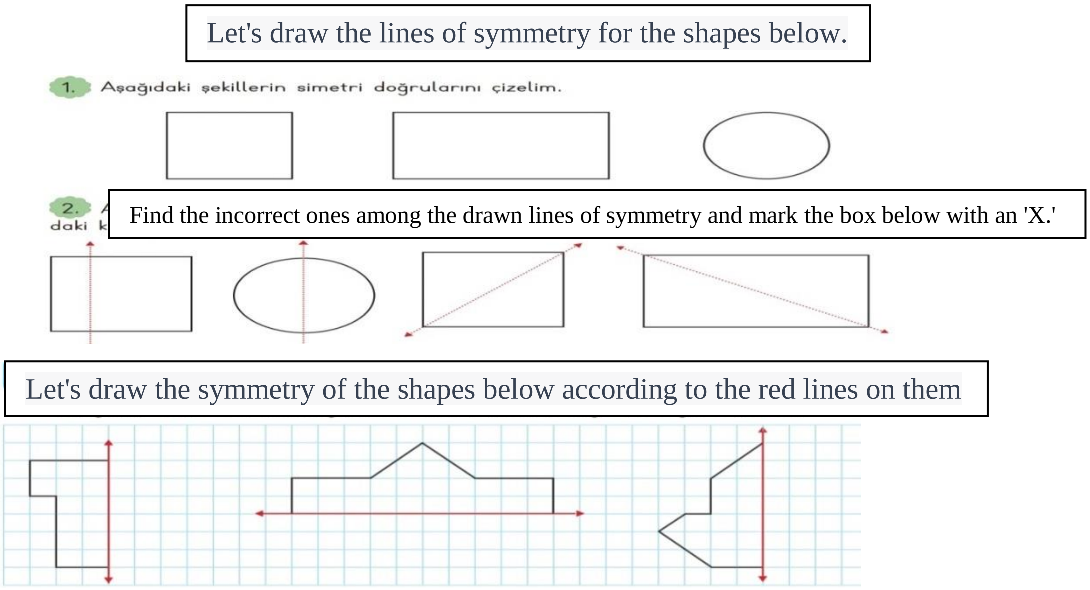


Figure 5. Activity for Finding the Symmetry of Symmetric Polygons and Irregular Shapes.

In Figure 5, two different examples are provided from the textbook. The aim is to find the symmetry of the shapes in accordance with the attainments of this grade. It is observed that selecting shapes that have symmetry in multiple directions and not providing only one-sided shapes as examples help prevent potential learning errors. In the 3rd grade, there is no coverage of translation and rotation. The 4th grade represents a level slightly higher than the previous grades, and the achievements related to transformations in this grade can be considered as the end of the basic level and the beginning of level 1.

Terms or concepts: Mirror symmetry

M.4.2.2.1. Explains mirror symmetry by drawing the symmetry line on geometric shapes and models. It emphasizes that objects such as butterfly wings, flowers, leaves, fabrics, rug patterns, letters, etc., can be observed to have symmetry by placing a mirror in suitable positions and observing identical parts. It highlights that this type of symmetry is called "mirror symmetry" or "symmetry with respect to the mirror" or "symmetry with respect to the line."

M.4.2.2.2. Draws the symmetry of the given shape with respect to a line.

The Van Hiele levels progress hierarchically, and this grade level allows us to see that it includes examples that could be considered as the end of the basic level and the beginning of level 1.

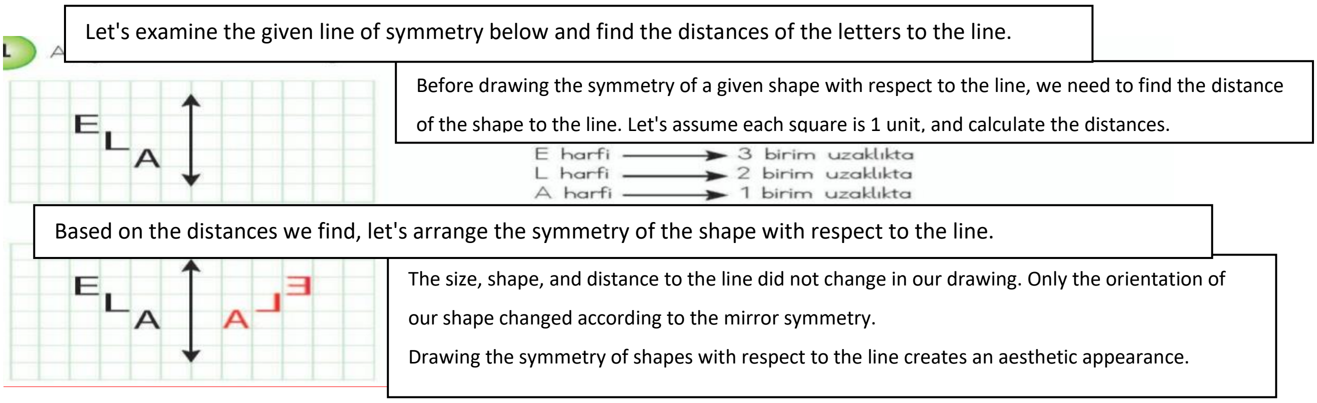


Figure 6. An Example at the End of the Basic Level and the Beginning of Level 1 in Symmetry.

Figure 6 illustrates that in 4th grade, which is the final stage of elementary school, there is a transition from the Basic Level to Level 1 in reflection transformation.

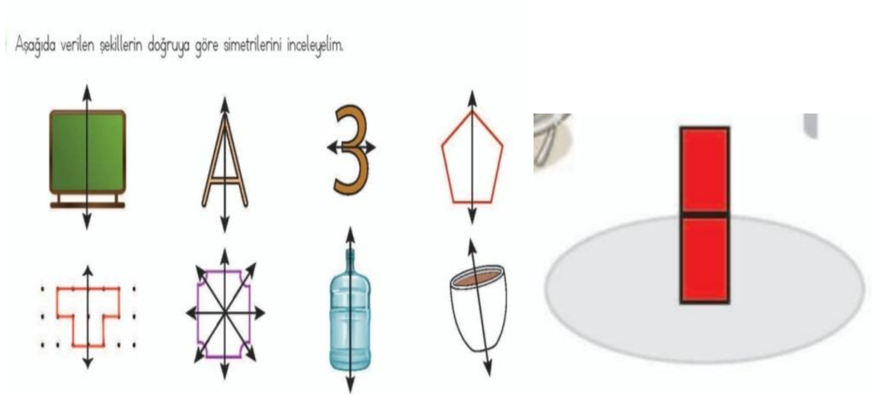


Figure 7. Examples Given for the Symmetry Line and Symmetry Mirror.

In Figure 7, when examining the examples provided, it can be observed that the shapes given are somewhat different from other levels, and the symmetry axis is presented with slopes and also in horizontal shapes. The transitions between levels are not sharply delineated. The end of the Basic Level can form the beginning of Level 1. This can be seen in the symmetry section, but in the other two sections (translation and rotation), the information required at the levels is quite lacking. It does not meet the competencies of the levels.

Transformations for level 1 thinkers

The achievements in transformation geometry topics given at the primary school level are suitable for the Basic Level. When examining the achievements in secondary school, there are no achievements related to transformation geometry in grades 5, 6, and 7. After primary school, students encounter their first geometric transformation topics in the 8th grade.

8th Grade:

The achievements for this grade level are provided below. When these achievements and their explanations are examined, it can be seen that there are achievements suitable for the level in the translation and reflection sections of transformation geometry. However, there is no achievement related to rotation.

M.8.3.2. Transformation Geometry Terms or concepts: reflection, translation, image, symmetry axis

M.8.3.2.1. Draws the images of points, line segments, and other shapes after translation.

a) Work is done on graph paper or dotted paper, and coordinate system activities are performed.

b) Interactive geometry software may also be used for activities.

c) In translation, it is emphasized that each point on the shape moves in the same direction, and the shape and its image are congruent.

Let's examine the symmetries of the shapes given below with respect to the line.

Let's draw the shape given on the checkered background by shifting it 9 units to the left and 2 units up.

Let's shift the polygon on the checkered background in the figure 9 units to the left and 2 units up. By shifting all the corner points of the shape 9 units to the left and 2 units up, let's connect the corner points and obtain the image resulting from the translation. The shape and image are congruent.

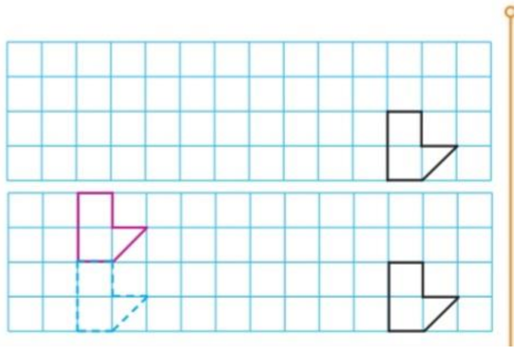


Figure 8. Example Provided in the Textbook Regarding Translation Transformation.

Figure 8 shows a shape that has been translated. In other grade levels, examples of displacement were provided, but they were presented under the "position" heading. It is observed that for the first time, displacement is performed under the "translation" heading.

M.8.3.2.2. The student creates the image resulting from reflection for points, line segments, and other shapes.

a) Work is carried out on squared or dotted paper, and on coordinate systems.

b) Interactive geometry software may also be used for these activities.

c) In reflection, the student is made aware that points corresponding to each other on the shape and its image are perpendicular to the line of symmetry, and the distances between them are equal, which is why the shape and its image are congruent.

d) Activities with shapes on the lines of symmetry are also conducted.

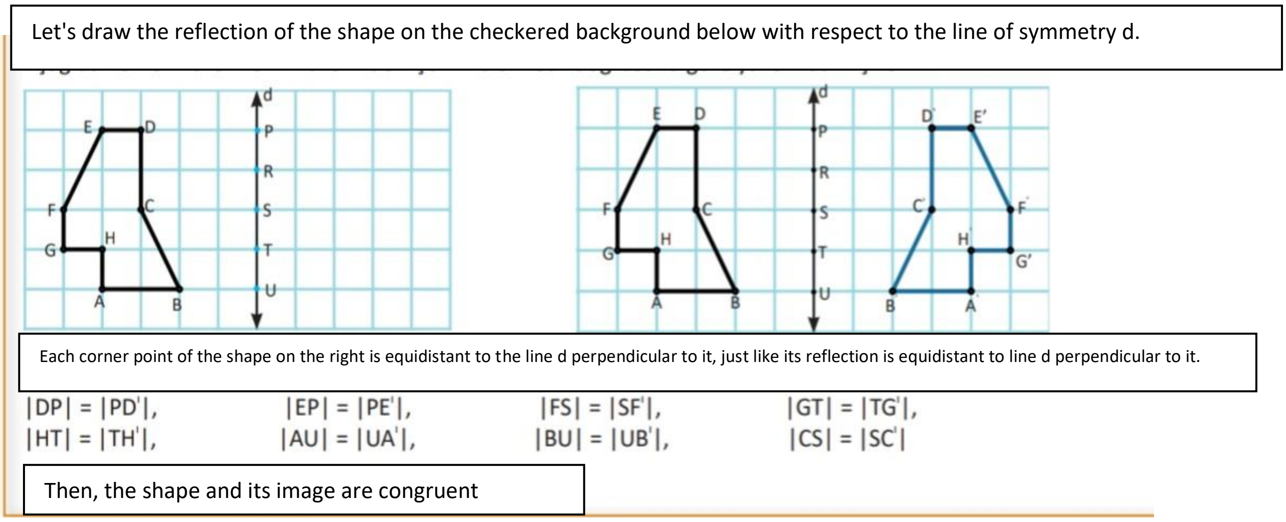


Figure 9. An Example of Reflection in the Textbook.

Figure 9 shows a reflection example in the textbook, and upon examination, it is noticeable that it differs from examples in other levels. While examples in other levels are more suitable for the Fundamental Level and consist of everyday life examples, the example in this level utilizes analytical representations and the properties of symmetry.

M.8.3.2.3. It creates the images resulting from translations and reflections of polygons.

a) Up to two consecutive translations or reflections are included.

b) Work related to determining translations or reflection transformations in patterns, motifs, and similar visuals is included.

c) Examples from our traditional arts (ceramics, weaving, etc.) are also taken into account.

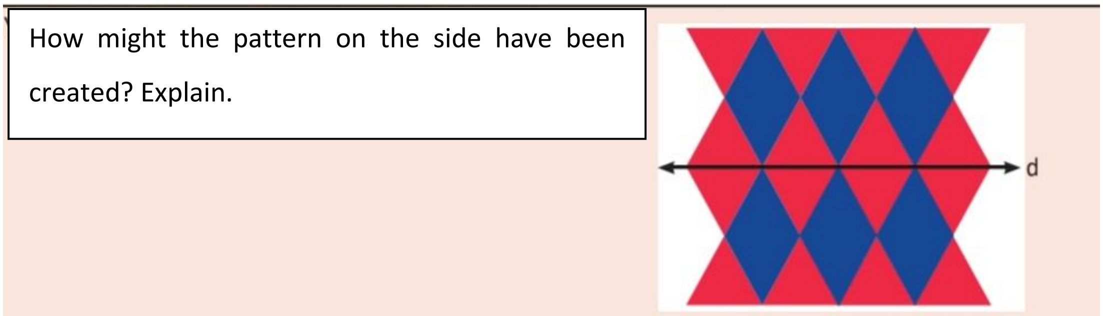


Figure 10. Example of Multiple Transformation Provided in the Textbook.

In Figure 10, a motif is created using reflection and translation. The use of composite transformations is a desired feature at Level 1. However, it also contains the requirement of Level 2, which is to "identify which transformations the given shape underwent to reach its initial and final states." When looking at the provided example, it can be said that it serves as an example bridging Level 1 and Level 2.

When examining the examples provided in the textbooks, it is observed that the selected objects are not limited to regular shapes. The chosen decorations are selected in a way that allows the student to see that only the position or location of the shape changes when these transformations are applied. Selecting decorations from ceramics that students can encounter in their daily lives is seen as a facilitator of learning.

Transformations for level 2 thinkers

At this level, students are expected to use the knowledge they have learned in a higher-level coordinate system. The curriculum for this level is provided only in the 8th grade.

8th Grade:

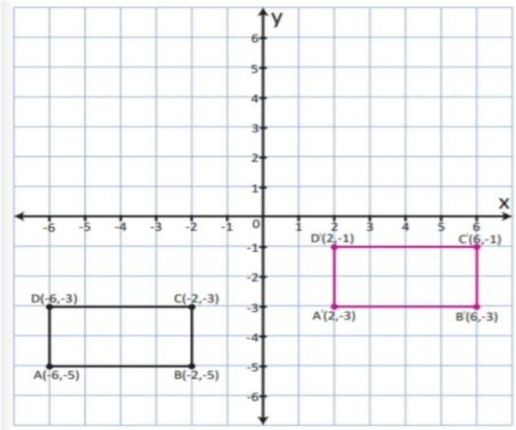
In the 8th grade curriculum, the provided learning outcomes are not limited to a single level. A learning outcome can encompass both Level 1 and Level 2. Below are the parts suitable for Level 2:

M.8.3.2.1. Draw the images resulting from the translation of points, line segments, and other shapes.

a) Work on squared or dotted paper, and perform activities on a coordinate system.

b) Interactive geometry software can also be used for these activities.

c) Emphasize that in translation, every point on the shape moves in the same direction, and that the shape and its image are congruent.



Let's draw the rectangle ABCD with corner coordinates A (-6, -5), B (-2, -5), C(-2, -3), and D(-6,-3), by shifting it 8 units to the right, parallel to the x-axis, and 2 units upward, parallel to the y-axis.

When we translate the ABCD rectangle in the figure 8 units to the right along the x-axis, the first term of the ordered pair increases by 8 units. When we translate it 2 units upward along the y-axis, the second term of the ordered pair increases by 2 units.

(x, y)	(x + 8, y + 2)
A(-6, -5)	A'(2, -3)
B(-2, -5)	B'(6, -3)
C(-2, -3)	C'(6, -1)
D(-6, -3)	D'(2, -1)

In translation, every point on the shape moves in the same direction. The shape is congruent to its image."

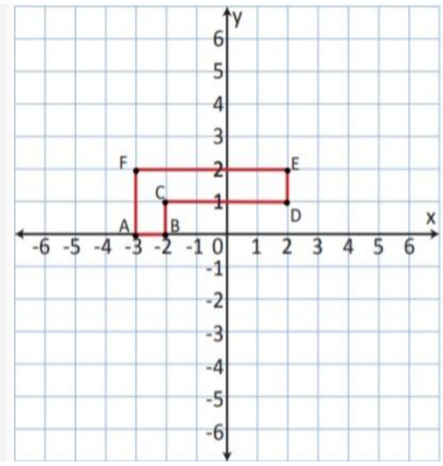
Figure 11. Translation Movement on the Coordinate Plane.

M.8.3.2.2. It creates the image resulting from the reflection of points, line segments, and other shapes.

a) Work is done on graph paper or a dotted grid on the coordinate system.

b) Interactive geometry software may also be used for these studies.

c) In reflection, it is emphasized that there are points on the shape and its image that are perpendicular to the line of symmetry and their distances are equal, thus showing that the shape and its image are congruent.



"Let's find the coordinates of the corner points of the shape obtained as a result of its reflection over the x-axis.

We can determine the image point resulting from the reflection over the x-axis of a point A (x, y) using the rule A (x, y)-> A'(x, -y).

A(-3, 0) → A'(-3, 0)
B(-2, 0) → B'(-2, 0)
C(-2, 1) → C'(-2, -1)
D(2, 1) → D'(2, -1)
E(2, 2) → E'(2, -2)
F(-3, 2) → F'(-3, -2)

For points on the x-axis and their reflections over the x-axis, the coordinates of the points remain unchanged.

Figure 12. Reflection Movement on the Coordinate Plane.

In Figure 12, an example of reflection transformation suitable for Level 2 is provided. When examining the achievements in the Primary School Mathematics Curriculum, it is observed that a direct achievement suitable for Level 2 is not provided. When the explanations under the given achievements are examined, it is seen that some of them are suitable for this level.

In this study, the findings were obtained by examining the Mathematics Curriculum (1st-8th grade) and Mathematics Textbooks for all levels (1st-8th grade) published by the Ministry of National Education (MoNE). The data obtained from the analyses are presented under two headings below.

Discussion and Conclusion

In this study were obtained by examining the Mathematics Curriculum (1-8th grade) and Mathematics Textbooks of all levels (1-8th grade) published by the Ministry of National Education (MoNE). The data obtained from the analyses conducted are presented below under these two headings.

Examination Based on Attainments

The achievements related to transformation geometry in the 1st to 8th-grade mathematics curriculum and their contents have been examined, taking into account the levels of transformation geometry. As a result of this examination, it is observed that the achievement related to translation appears for the first time as a content under the "Position" heading in the 1st grade, but it is not conceptually explained here. Conceptual explanation of translation under the title of "Translation" is first given to students in the 8th grade. The achievements in the 8th grade are in line with Level 3. It is not in line with the Van Hiele philosophy for translation to be given to students for the first time in the 1st grade, not to be given in other grades, and to be directly given at Level 3. It can be argued that translation is not hierarchically provided to students. In the studies conducted by Fidan and Türnüklü (2010) with 5th-grade students, it was found that 47.9% of students were at Level 0, meaning they could not be assigned to any level, 29.3% at Level 1, 16.7% at Level 2, and 6.1% at Level 3 in terms of their geometric thinking levels. This indicates that a significant portion of students' geometric thinking levels is at Level 0, meaning they cannot be assigned to any level. As seen in this study, even though students are in the 5th grade, they have not reached the desired level. Distributing these levels hierarchically and equally between classes is an important issue.

When the achievements up to the 4th grade of primary school are examined, it is determined that the hierarchy of symmetry/reflection transformation is given appropriately. In other grades (except for the 8th grade), there is no achievement related to transformation geometry. In the 8th grade, it is inferred that achievements in line with Basic Level and Level 1 are provided, but there is no direct achievement related to Level 2. When all grade levels are examined, it is seen that there is no achievement related to rotation transformation. Without a hierarchical, well-planned education program, students, including future teachers, may not achieve the desired Geometric Levels. In Erdogan's (2020) study with prospective middle school mathematics teachers in the 3rd grade, it was concluded that their geometric thinking levels and problem-posing abilities were low.

In the study conducted by Kılıç (2015), a significant difference was found in the academic achievements of students between the experimental group, where geometry instruction was conducted according to Van Hiele levels, and the control group, where geometry instruction was not based on Van Hiele levels. This difference favored the group where geometry instruction was carried out according to Van Hiele levels. Therefore, it can be concluded that geometry instruction based on Van Hiele levels in the 5th-grade mathematics class of primary education improves students' academic achievements and is more effective than traditional instruction. As seen in the results of this study, education that takes into account geometric levels leads to higher student success. To support and guide this type of education, instructional programs should be prepared accordingly.

Analysis of Textbooks

In the examination of textbooks across all grade levels for transformation geometry, it was observed that reflection transformation includes activities at the Fundamental Level for primary school (grades 1-4). However, for those thinking at Level 1 and Level 2, it is noted that the achievements are presented with numerous items in the 8th grade. The Van Hiele philosophy advocates the hierarchical presentation of levels. In this section, examples for primary school are provided in accordance with the hierarchy. However, what is provided for middle school level is not in line with the hierarchy. It would be more appropriate to distribute Level 1 and Level 2 to other classes instead of giving them only in one class.

Regarding the translation transformation, it was found that it is only included in the mathematics curriculum for the 8th grade. Upon examining the textbooks, it was found that some activities under the "position" heading involve translation transformation. However, it is observed that the main purpose of these activities is not to teach translation transformation conceptually. Therefore, students learn translation transformation as a change in position without understanding its conceptual basis. In this context, the conceptual learning of translation transformation, which should be learned at the Fundamental Level, has not been directly addressed until the 8th grade. When examining the mathematics curriculum, it is seen that translation transformation is taught with characteristics of Fundamental Level, Level 1, and Level 2, all together with numerous achievements.

This can be considered as an inappropriate approach according to the Van Hiele levels, as students are expected to progress hierarchically, and presenting all levels at once may result in a lack of deep understanding.

The final part of transformation geometry is rotational transformation. Upon examination of the mathematics curriculum, it is observed that there are no achievements related to rotational transformation. When analyzing the textbooks, it is noted that rotation is used under different headings but is not presented as a separate topic. In a study conducted by Temur and Tertemiz (2012), it was observed that teachers did not use activities related to establishing relationships between shapes in the 1st, 2nd, and 3rd grades. In the 4th grade, however, teachers were observed solving problems related to geometric shapes, and students asked questions to both their peers and teachers about the properties of shapes and their relationships, trying to discover these relationships. When examples/activities that support hierarchical geometric learning are not provided to students in previous grades, they may encounter difficulties in later stages and higher levels. Likewise, in the 5th-grade study, it was observed that the teacher had difficulty in having students establish relationships between shapes. Overcoming this problem is easier when learning is done in accordance with the levels.

When examining the Van Hiele levels, it is expected that a student who completes secondary education should be at Level 2. However, when considering the levels of transformation geometry, it is observed that there is no content related to rotational transformation in the textbooks and the Mathematics Curriculum. When reviewing the transformation levels, it is recommended that translation, reflection, and rotational transformations should be taught simultaneously in the first three levels. In the mathematics curriculum, while translation transformation is only introduced in the 8th grade, there is no mention of rotational transformation. In this context, considering the significant practical applications of rotational transformation in daily life and other subjects, it would be appropriate to add achievements suitable for grades 1-8, taking into account the levels of geometric thinking.

Recommendations

- As evidenced by the results of this study, educational approaches that consider students' geometric thinking levels lead to increased student success. To promote and guide this type of education effectively, it is advisable to develop instructional programs that align with these levels.

- When students are not exposed to examples and activities that support hierarchical geometric learning in earlier grades, they may encounter challenges as they progress to higher levels. For instance, during the 5th-grade study, it was observed that students had difficulty establishing relationships between geometric shapes. Addressing this issue becomes more feasible when learning is structured in accordance with the levels of geometric thinking.

- In the mathematics curriculum, while translation transformation is introduced only in the 8th grade, there is no mention of rotational transformation. Considering the practical applications of rotational transformation in daily life and across various subjects, it would be advisable to incorporate achievements suitable for grades 1-8, while also aligning them with the levels of geometric thinking.

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