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Lacunary statistical delta 2 quasi Cauchy sequences

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ABSTRACT

The notion of a lacunary statistical δ^2-quasi-Cauchyness of sequence of real numbers is introduce and investigated. In this work, we present interesting theorems related to lacunary statistically δ^2-ward continuity. A function f, whose domain is included in R, and whose range included in R is called lacunary statistical δ^2 ward continuous if it preserves lacunary statistical δ^2 quasi-Cauchy sequences, i.e. $(f(x_k))$ is a lacunary statistically δ^2 quasi-Cauchy sequence whenever (x_k) is a lacunary statistically δ^2 quasi-Cauchy sequence, where a sequence (x_k) is called lacunary statistically δ^2 quasi-Cauchy if $(\Delta^2 x_k)$ is a lacunary statistically quasi-Cauchy sequence. We find out that the set of lacunary statistical δ^2 ward continuous functions is closed as a subset of the set of continuous functions.

Keywords: summability, quasi Cauchy sequence, lacunary statistical convergence, continuity

İstatistiksel boşluklu delta 2 quasi Cauchy dizileri

ÖZ

Bu makalede istatistiksel boşluklu δ^2-quasi-Cauchy dizisi kavramı tanımlanmış ve araştırılmıştır. Bu araştırmada istatistiksel boşluklu δ^2-süreklilik ile ilgili ilgi çekici teoremler ispatlanmıştır. $(\Delta^2 x_k)$ istatistiksel boşluklu quasi Cauchy dizisi olduğunda (x_k) dizisine istatistiksel boşluklu δ^2-quasi-Cauchy dizisi dendiğine göre, reel sayılar kümesinin bir alt kümesi üzerinde tanımlı reel değerli bir f fonksiyonuna eğer terimleri A da olan istsatistiksel boşluklu δ^2-quasi-Cauchy dizilerini koruyor ise, yani (x_k) dizisi terimleri A da olan istatistiksel boşluklu δ^2-quasi-Cauchy dizisi olduğunda $(f(x_k))$ dizisi de istatistiksel boşluklu δ^2-quasi-Cauchy dizisi oluyor ise istatistiksel boşluklu δ^2-ward süreklidir denir. İstatistiksel boşluklu δ^2-ward sürekli fonksiyonların kümesinin sürekli fonksiyonlar uzayının kapalı bir alt kümesi olduğu ortaya çıkarılmıştır.

Anahtar Kelimeler: toplanabilme, quasi Cauchy dizisi, istatistiksel boşluklu yakınsaklık, süreklilik

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1. INTRODUCTION

Cakalli introduced a generalization of compactness, a generalization of connectedness via a sequential method in [2] and [3], respectively. In [6] Fridy and Orhan introduced the notion of lacunary statistical convergence in the manner that a sequence (x_k) of points in R is called lacunary statistically convergent, or S_θ-convergent, to an element L of R if

$$\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r: |x_k-L|\geq\varepsilon\}|=0$$

for every positive real number ε where

$$I_r=(k_{r-1},k_r]$$

and $k_0=0$, $h_r: k_r-k_{r-1}\to\infty$ as $r\to\infty$ and $\theta=(k_r)$ is an increasing sequence of positive integers (see also [1], and [7]). In this case we write S_θ-$\lim x_k=L$. The set of lacunary statistically convergent sequences of points in R is denoted by S_θ. In this paper, we will always assume that $\lim\inf_r q_r>1$. A sequence (x_k) in R is called lacunary statistically quasi-Cauchy if S_θ - $\lim\Delta x_k=0$, where $\Delta x_k=x_{k+1}-x_k$ for each positive integer k. The set of lacunary statistically quasi-Cauchy sequences will be denoted by ΔS_θ.
The aim of this paper is to investigate the notion of lacunary statistical δ^2 ward continuity.

2. MAIN RESULTS

A sequence (x_k) in R is called lacunary statistically δ quasi-Cauchy if
S_θ - $\lim\Delta^2x_k=0$, where $\Delta^2x_k=x_{k+2}-2x_{k+1}+x_k$ for each positive integer k. The set of lacunary statistically δ quasi-Cauchy sequences of points in R is denoted by Δ^2S_θ. If we put $|\Delta^3x_k|$ instead of $|\Delta^2x_k|$ in the above definition given in [5] we have:
Definition 1. A sequence (x_k) in R is called lacunary statistically δ^2 quasi-Cauchy, or S_θ - δ^2 quasi Cauchy if the sequence (Δ^2x_k) is lacunary statistically quasi-Cauchy, i.e.

$$\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r: |\Delta^3x_k|\geq\varepsilon\}|=0$$

for every positive real number ε, where $\Delta^3x_k=x_{k+3}-3x_{k+2}+3x_{k+1}-x_k$ for each positive integer k.
We note that any S_θ-quasi Cauchy sequence is also S_θ - δ^2-quasi Cauchy, so is a slowly oscillating sequence, so is a Cauchy sequence, so is a convergent sequence, but the converses are not always true. Thus the inclusions
$C\subset\Delta S_\theta\subset\Delta^3 S_\theta$ hold strictly, where $\Delta^3 S_\theta$ denotes the set of S_θ - δ^2-quasi-Cauchy equences, and C denotes the set of Cauchy sequences of points in R.
Proposition 1. If (x_k) and (y_k) are lacunary statistically δ^2 quasi-Cauchy sequences, then (x_k+y_k) is a lacunary statistically δ^2 quasi-Cauchy sequence.
Proof. Let (x_k) and (y_k) be lacunary statistically δ^2 quasi-Cauchy sequences. To prove that (x_k+y_k) is a lacunary statistically δ^2 quasi-Cauchy sequence, take any ε > 0. Then we have

$$\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r: |\Delta^3x_k|\geq\frac{\varepsilon}{2}\}|=0$$

and

$$\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r: |\Delta^3y_k|\geq\frac{\varepsilon}{2}\}|=0.$$

Hence

$$\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r: |\Delta^3(x_k+y_k)|\geq\varepsilon\}|=0.$$

This completes the proof.
Definition 2. A real valued function f defined on a subset A of R is called lacunary statistically δ^2 ward continuous, or S_θ-δ^2 ward continuous on A if it preserves lacunary statistically δ^2 quasi-Cauchy sequences in A.
The set of lacunary statistical δ^2 ward continuous functions on A will be denoted
by $\Delta^3 CS_\theta(A)$.
Proposition 2. If $f\in\Delta^3 CS_\theta(A)$, $g\in\Delta^3 CS_\theta(A)$, then $f+g\in\Delta^3 CS_\theta(A)$.
Proof. Let $f\in\Delta^3 CS_\theta(A)$, $g\in\Delta^3 CS_\theta(A)$. To prove that the sum f + g is lacunary statistically δ^2 ward continuous on A, take any $(x_k)\in\Delta^3 S_\theta$. Then $(f(x_k))\in\Delta^3 S_\theta$ and $(g(x_k))\in\Delta^3 S_\theta$. Let ε > 0 be given. Since $(f(x_k))\in\Delta^3 S_\theta$ and $(g(x_k))\in\Delta^3 S_\theta$, we have

$$\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r: |\Delta^3f(x_k)|\geq\frac{\varepsilon}{2}\}|=0$$

and

$$\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r: |\Delta^3g(x_k)|\geq\frac{\varepsilon}{2}\}|=0.$$

Hence

$$\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r: |\Delta^3(f(x_k)+g(x_k))|\geq\varepsilon\}|=0.$$

This completes the proof.
On the other hand, the product of a constant real number and $f\in\Delta^3 CS_\theta$ is an element of $\Delta^3 CS_\theta$ Thus $\Delta^3 S_\theta$ is a vector space.
In connection with lacunary statistically δ^2-quasi-Cauchy sequences and convergent sequences the

problem arises to investigate the following types of continuity of functions on R:

(δ^2 Sθ δ^2): $(x_n) \in \Delta^3 S_\theta \Rightarrow (f(x_n)) \in \Delta^3 S_\theta$

(δ^2 Sθc): $(x_n) \in \Delta^3 S_\theta \Rightarrow (f(x_n)) \in c$

(Sθ): $(x_n) \in S_\theta \Rightarrow (f(x_n)) \in S_\theta$

(ΔSθ_): $(x_n) \in \Delta S_\theta \Rightarrow (f(x_n)) \in \Delta S_\theta$

(c): $(x_n) \in c \Rightarrow (f(x_n)) \in c$

(cSθ δ^2): $(x_n) \in c \Rightarrow (f(x_n)) \in \Delta^3 S_\theta$

We see that (δ^2 Sθ δ^2) is lacunary statistically δ^2-ward continuity of f, (Sθ) is Sθ-sequential continuity of f, and (c) is the ordinary continuity of f. It is easy to see that (δ^2 Sθc) implies (δ^2 Sθ δ^2), and (δ^2 Sθ δ^2) does not imply (δ^2 Sθc); and (δ^2 Sθ δ^2) implies (c Sθ δ^2), and (c Sθ δ^2) does not imply (δ^2 Sθ δ^2); (δ^2 Sθc) implies (c), and (c) does not imply (δ^2 Sθc).

Now we give the implication (δ^2 Sθ δ^2) implies (ΔSθ).

Theorem 1. If $f \in \Delta^3 CS_\theta(A)$, then $f \in \Delta CS_\theta(A)$.

Proof. Suppose that $f \in \Delta^3 CS_\theta(A)$. Let $(x_n) \in \Delta S_\theta(A)$. Then the sequence

$(x_1, x_1, x_1, x_2, x_2, x_2, \ldots, x_{n-1}, x_{n-1}, x_{n-1}, x_n, x_n, x_n \ldots)$

is in $\Delta S_\theta(A)$, so is in $\Delta^2 S_\theta(A)$. Since f is in $\Delta^2 CS_\theta(A)$, the sequence

$(y_n) = (f(x_1), f(x_1), f(x_1), f(x_2), f(x_2), f(x_2), \ldots, f(x_{n-1}), f(x_{n-1}), f(x_{n-1}), f(x_n), f(x_n), f(x_n) \ldots)$

is in $\Delta^2 S_\theta(A)$. Then $(f(x_n)) \in \Delta S_\theta(A)$.

Corollary 1. If $f \in \Delta^3 CS_\theta(A)$, then f is continuous.

Proof. The proof follows immediately from the preceding theorem and [14, Corollary 2], so is omitted.

We note that any lacunary statistically δ^2 ward continuous function is G-sequentially continuous for any regular subsequential sequential method G (see [2]).

Theorem 2. If a real valued function f is uniformly continuous on a subset A of R, then $(f(x_n))$ is lacunary statistically δ^2 quasi-Cauchy whenever (x_n) is a quasi-Cauchy sequence of points in A.

Proof. Let f be uniformly continuous on A. Take any quasi-Cauchy sequence (x_n) of points in A. Let ε be any positive real number. Since f is uniformly continuous, there exists $\delta > 0$ such that $|f(x) - f(y)| < \frac{\varepsilon}{5}$ whenever $|x - y| < \delta$.

As (x_k) is a quasi-Cauchy sequence, for this δ there exists an $n_0 \in N$ such that $|x_{k+1} - x_k| < \delta$ for $k \geq n_0$. Therefore $|f(x_{k+1}) - f(x_k)| < \frac{\varepsilon}{5}$ for $n \geq n_0$, so the number of indices k for which $|f(x_{k+1}) - f(x_k)| \geq \frac{\varepsilon}{5}$ is less than n_0. Hence

$$\lim_{r\to\infty}\frac{1}{h_r}|\{k \in I_r : |\Delta^3 f(x_k)| \geq \varepsilon\}|$$

$$= \lim_{r\to\infty}\frac{1}{h_r}|\{k \in I_r : |f(x_{k+3}) - 3f(x_{k+2}) + 3f(x_{k+1}) - f(x_k)| \geq \varepsilon\}|$$

$$\leq \lim_{r\to\infty}\frac{1}{h_r}\left|\left\{k \in I_r : \left|f(x_{k+3}) - f(x_{k+2})\right| \geq \frac{\varepsilon}{5}\right\}\right|$$

$$+ \lim_{r\to\infty}\frac{1}{h_r}\left|\left\{k \in I_r : \left|2f(x_{k+2}) - 2f(x_{k+1})\right| \geq \frac{\varepsilon}{5}\right\}\right|$$

$$+ \lim_{r\to\infty}\frac{1}{h_r}\left|\left\{k \in I_r : \left|f(x_{k+1}) - f(x_k)\right| \geq \frac{\varepsilon}{5}\right\}\right|$$

$$\leq \lim_{r\to\infty}\frac{n_0}{h_r} + \lim_{r\to\infty}\frac{2n_0}{h_r} + \lim_{r\to\infty}\frac{n_0}{h_r} = 0+0+0=0.$$

This completes the proof of the theorem.

Theorem 3. The uniform limit of sequence of lacunary statistically δ^2 ward continuous functions is lacunary statistically δ^2 ward continuous.

Proof. Let (f_n) be a sequence of lacunary statistically δ^2 ward continuous functions on a subset A of R and (f_n) is uniformly convergent to a function f. To prove that f is lacunary statistically δ^2 ward continuous on A, take a lacunary statistically δ^2 quasi-Cauchy sequence (x_k) of points in A, and let ε be any positive real number. By the uniform convergence of (f_n), there exists a positive integer n_1 such that $|f(x) - f_k(x)| < \frac{\varepsilon}{5}$ for $n \geq n_1$ and every $x \in A$. As f_{n_1} is lacunary statistically δ^2 ward continuous on A, it follows that

$$\lim_{r\to\infty}\frac{1}{h_r}\left|\left\{k \in I_r : \left|f_{n_1}(x_{k+3}) - 3f_{n_1}(x_{k+2}) + 3f_{n_1}(x_{k+1}) - f_{n_1}(x_k)\right| \geq \frac{\varepsilon}{5}\right\}\right| = 0.$$

Now

$$\lim_{r\to\infty}\frac{1}{h_r}|\{k \in I_r : |f(x_{k+3}) - 3f(x_{k+2}) + 3f(x_{k+1}) - f(x_k)| \geq \varepsilon\}|$$

$$= \lim_{r\to\infty}\frac{1}{h_r}\left|\left\{k \in I_r : \left|f(x_{k+3}) - 3f(x_{k+2}) + 3f(x_{k+1}) - f(x_k) - [f_{n_1}(x_{k+3}) - 3f_{n_1}(x_{k+2}) + 3f_{n_1}(x_{k+1}) - f_{n_1}(x_k)] + [f_{n_1}(x_{k+3}) - 3f_{n_1}(x_{k+2}) + 3f_{n_1}(x_{k+1}) - f_{n_1}(x_k)]\right| \geq \frac{\varepsilon}{5}\right\}\right|$$

$$\leq \lim_{r\to\infty}\frac{1}{h_r}\left|\left\{k \in I_r : \left|f(x_{k+3}) - f_{n_1}(x_{k+3})\right| \geq \frac{\varepsilon}{5}\right\}\right|$$

$$+ \lim_{r\to\infty}\frac{1}{h_r}\left|\left\{k \in I_r : \left|-3f(x_{k+2}) + 3f_{n_1}(x_{k+2})\right| \geq \frac{\varepsilon}{5}\right\}\right|$$

$$+ \lim_{r\to\infty}\frac{1}{h_r}\left|\left\{k \in I_r : \left|3f(x_{k+1}) - 3f_{n_1}(x_{k+1})\right| \geq \frac{\varepsilon}{5}\right\}\right|$$

$$+\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r:|f(x_k)-f_{n_1}(x_k)\,|\geq\frac{\varepsilon}{5}\}|$$

$$+\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r:|\,f_{n_1}(x_{k+3})-3f_{n_1}(x_{k+2})+3f_{n_1}(x_{k+1})-f_{n_1}(x_k)\,|\geq\frac{\varepsilon}{5}\}|$$

=0 + 0 + 0 + 0 + 0 = 0.

So f preserves lacunary statistically δ^2 quasi-Cauchy sequences. This completes the proof of the theorem.

Theorem 4. The set of lacunary statistically δ^2 ward continuous functions on a subset A of R is closed as a subset of the set of continuous functions on A.

Proof. Let f be an element in the closure of the set of lacunary statistically δ^2 ward continuous functions on A. Then there exists a sequence (f_n) of points in the set of lacunary statistically δ^2 ward continuous functions such that $\lim_{n\to\infty} f_n=f$. To show that f is lacunary statistically δ^2 ward continuous, consider a lacunary statistically δ^2 quasi Cauchy-sequence (x_k) of points in A. Since (f_k) converges to f, there exists a positive integer N such that for all $x\in$ A and for all $n\geq N$, $|f_k(x)-f(x)|<\frac{\varepsilon}{12}$. As f_N is lacunary statistically δ^2 ward continuous on A, we have that

$$\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r:|\,f_N(x_{k+3})-3f_N(x_{k+2})+3f_N(x_{k+1})-f_N(x_k)\,|\geq\frac{\varepsilon}{5}\}|=0.$$

Now

$$\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r:|\,f(x_{k+3})-3f(x_{k+2})+3f(x_{k+1})-f(x_k)|\geq\varepsilon\}|$$

$$=\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r:|\,f(x_{k+3})-3f(x_{k+2})+3f(x_{k+1})-f(x_k)-[f_N(x_{k+3})-3f_N(x_{k+2})+3f_N(x_{k+1})-f_N(x_k)]+[f_N(x_{k+3})-3f_N(x_{k+2})+3f_N(x_{k+1})-f_N(x_k)]\,|\geq\frac{\varepsilon}{5}\}|$$

$$\leq\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r:|f(x_{k+3})-\ f_N(x_{k+3})\,|\geq\frac{\varepsilon}{5}\}|$$

$$+\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r:|-3f(x_{k+2})+3\ f_N(x_{k+2})\,|\geq\frac{\varepsilon}{5}\}|$$

$$+\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r:|3f(x_{k+1})-3f_N(x_{k+1})\,|\geq\frac{\varepsilon}{5}\}|$$

$$+\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r:|f(x_k)-f_N(x_k)\,|\geq\frac{\varepsilon}{5}\}|$$

$$+\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r:|\,f_N(x_{k+3})-3f_N(x_{k+2})+3f_N(x_{k+1})-f_N(x_k)\,|\geq\frac{\varepsilon}{5}\}|.$$

=0 + 0 + 0 + 0 + 0 = 0.

Thus f preserves lacunary statistically δ^2 quasi-Cauchy sequences. This completes the proof of the theorem.

Corollary 2. The set of lacunary statistically δ^2 ward continuous functions on a subset A of R is complete as a subset of the set of continuous functions on A.

Theorem 5. The set of functions on a subset A of R which map quasi Cauchy sequences to lacunary statistically δ^2 quasi Cauchy sequences is closed as a subset of the set of continuous functions on A.

Proof . It is easy to see that any function which maps quasi Cauchy sequences to lacunary statistically δ^2 quasi Cauchy sequences is continuous. Let f be an element in the closure of the set of functions on A which map quasi Cauchy sequences to lacunary statistically δ^2 quasi Cauchy sequences. Then there exists a sequence (f_n) of points in the set of functions on a subset A of R which map quasi Cauchy sequences to lacunary statistically δ^2 quasi Cauchy sequences such that $\lim_{n\to\infty} f_n=f$. To show that f maps quasi Cauchy sequences to lacunary statistically δ^2 quasi Cauchy sequences, consider a quasi Cauchy-sequence (x_k) of points in A. Since (f_k) converges to f, there exists a positive integer N such that for all $x\in$ A and for all $n\geq N$, $|f_k(x)-f(x)|<\frac{\varepsilon}{5}$. As f_N maps quasi Cauchy sequences to lacunary statistically δ^2 quasi Cauchy sequences, we have that

$$\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r:|\,f_N(x_{k+3})-3f_N(x_{k+2})+3f_N(x_{k+1})-f_N(x_k)\,|\geq\frac{\varepsilon}{5}\}|=0.$$

Now

$$\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r:|\,f(x_{k+3})-3f(x_{k+2})+3f(x_{k+1})-f(x_k)|\geq\varepsilon\}|$$

$$= \lim_{r\to\infty} \frac{1}{h_r} |\{k \in I_r : | f(x_{k+3}) - 3f(x_{k+2}) + 3f(x_{k+1}) - f(x_k) - [f_N(x_{k+3}) - 3f_N(x_{k+2}) + 3f_N(x_{k+1}) - f_N(x_k)] + [f_N(x_{k+3}) - 3f_N(x_{k+2}) + 3f_N(x_{k+1}) - f_N(x_k)] \Big| \geq \frac{\varepsilon}{5}\} |$$

$$\leq \lim_{r\to\infty} \frac{1}{h_r} |\{k \in I_r : |f(x_{k+3}) - f_N(x_{k+3}) \Big| \geq \frac{\varepsilon}{5}\} | + \lim_{r\to\infty} \frac{1}{h_r} |\{k \in I_r : | -3f(x_{k+2}) + 3\, f_N(x_{k+2}) \Big| \geq \frac{\varepsilon}{5}\} |$$
$$+ \lim_{r\to\infty} \frac{1}{h_r} |\{k \in I_r : |3f(x_{k+1}) - 3f_N(x_{k+1}) \Big| \geq \frac{\varepsilon}{5}\} | + \lim_{r\to\infty} \frac{1}{h_r} |\{k \in I_r : |f(x_k) - f_N(x_k) \Big| \geq \frac{\varepsilon}{5}\} |$$
$$+ \lim_{r\to\infty} \frac{1}{h_r} |\{k \in I_r : | f_N(x_{k+3}) - 3f_N(x_{k+2}) + 3f_N(x_{k+1}) - f_N(x_k) \Big| \geq \frac{\varepsilon}{5}\} |.$$

$=0 + 0 + 0 + 0 + 0 = 0$.

Corollary 3. The set of functions that map quasi Cauchy sequences to lacunary statistically δ^2 quasi Cauchy sequences in A is complete in the set of continuous functions on A.

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