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Research Article

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Investigation of Secondary School Mathematics Teachers' Processes of Preparing Mathematical Modelling Activities

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Abstract

In this study, it was aimed to investigate the mathematical modelling activity preparation processes of secondary school mathematics teachers who participated in mathematical modelling training. The study was conducted with six mathematics teachers working in secondary schools. In the study designed as action research, two action plans were applied to the teachers. The first activity forms collected from the teachers before the application, the second activity forms collected as a result of the theoretical training about modelling and the activity forms collected at the end of the active participation of the teachers in the modelling activities constitute the data source of this study. An activity evaluation form was used in data analysis. The results of the study showed that teachers initially had deficiencies in preparing modelling activities in accordance with all criteria. After the theoretical training given, it was determined that teachers' competencies in preparing modelling activities improved very little. In addition to theoretical training, it was seen that the learning environment prepared to ensure teachers' active participation in modelling activities affected their activity preparation competencies more positively. The results show that teachers' theoretical knowledge deficiencies should be eliminated in modelling teaching and learning environments should be prepared for their active participation in mathematical modelling activities.

Key Words

Mathematical modelling activity • Mathematics teacher • Principles of modelling task

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Mathematics curricula focus on the development of students' mathematical knowledge, thinking, skills and attitudes to solve the situations they may encounter in daily life. In this context, the importance of associating mathematics with real-life events and other disciplines is also emphasised (MoNE, 2011). In the 2017 curriculum for secondary school mathematics, as outlined by the Ministry of National Education, there was a notable shift in focus towards emphasizing the acquisition of fundamental skills in mathematical modeling. Within this context, a mathematical model is conventionally characterized as a mathematical construct that serves as a representation of real-world phenomena (Greefrath & Vorhölter, 2016). Blum and Borromeo Ferri (2009), on the other hand, elucidate mathematical modeling as a systematic process characterized by the reciprocal transformation of both the mathematical realm and the physical reality it seeks to capture. In this context, the importance of teachers' pedagogical content knowledge specific to mathematical modelling is emphasised in the literature (Borromeo Ferri, 2018; Wess et al., 2021).

According to the studies, it has been revealed that teachers' knowledge and skills about mathematical modelling are insufficient (Güder, 2013; Sağıroğlu, 2018; Sarı, 2019), they have difficulties in preparing and applying mathematical modelling activities (Çiltaş, 2015; Deniz & Akgün, 2017), and they do not tend to use mathematical modelling activities in mathematics lessons (Akgün et al., 2013; Işık & Mercan, 2015). The reasons for this situation include the fact that mathematical modelling is more widely used at the secondary level and above (Deniz & Akgün, 2017; Erbaş et al., 2013; Tekin Dede & Bukova Güzel, 2013; Urhan & Dost, 2016), it is newly included in the secondary school curriculum (MoNE, 2017), secondary school teachers' knowledge and skill levels are insufficient in this framework (Çiltaş, 2015; Güder, 2013; Sarı, 2019) and there are deficiencies in terms of resources (Şahin, 2019). This study focuses specifically on teachers' processes of developing mathematical modelling activities because developing mathematical modelling activities is an important practice for internalising and better understanding modelling (Borromeo Ferri, 2018). Similarly, the cultivation of effective modeling proficiencies may be promoted through the judicious application of well-chosen or purposefully crafted modeling exercises within the educational setting (Wess & Greefrath, 2020). This study is motivated, in part, by the dearth of modeling activities tailored to the secondary school level, a lacuna evident in both the official curriculum and extant scholarly literature (Şahin, 2019). An additional impetus for this investigation arises from the challenges faced by educators in the formulation of modeling activities, as documented in the works of Çiltaş (2015) and Deniz & Akgün (2017).

In the literature, studies that reflect the situation of mathematics teachers regarding their competencies in preparing mathematical modelling activities were examined. In the studies, mathematical modelling problems prepared by teachers or candidates were examined. Most of the studies show that teachers have deficiencies in preparing modelling activities (Bilgili et al., 2020; Dede et al., 2017; Deniz & Akgün, 2016; Stohlmann et al., 2017; Yu & Chang, 2009). Bilgili et al. (2020) first informed teachers about the theoretical framework of mathematical modelling and then asked them to create activities. As a result of this study, it was determined that very few of the activities prepared were in accordance with the principles for these mathematical modelling problems. In the study conducted by Bilgili and Çiltaş (2022), participants were asked to prepare mathematical modelling activities after the training in which theoretical information about mathematical modelling was provided. In the activities evaluated in line with the principles, it was observed that the most considered principles were realism and self-evaluation, while the most ignored principle was the principle of generalising the model. Similar results were also found in the study conducted by Deniz and Akgün (2016) with mathematics teachers. Within the purview of this research, akin to prior inquiries, educators were tasked with crafting mathematical modeling exercises subsequent to attending informative sessions elucidating the tenets of mathematical modeling. The findings revealed that teachers encountered challenges in formulating activities that adhered comprehensively to the requisite criteria for developing modeling tasks, with several criteria remaining unfulfilled in numerous instances. In a related study conducted by Yu and Chang (2009), pedagogical activities generated by instructors following specialized training for mathematics educators were assessed against the backdrop of foundational modeling principles. When the activities were analysed, it was observed that they did not carry these principles sufficiently. The same findings were also found in the study conducted by Chamberlin and Moon (2008). In a study conducted by Stohlmann et al. (2017), the activities prepared by three mathematics teachers were evaluated within the framework of modelling principles. While two of the prepared activities could meet these principles, the other activity did not meet the principles of model generalisation and self-evaluation. Sağıroğlu (2018) also examined the competencies of secondary school mathematics teachers to create activities suitable for mathematical modelling and apply them in the classroom. During the four-week training process, necessary information about mathematical modelling, the characteristics of modelling activities, the creation of mathematical modelling activities and their application in the classroom was provided. The process of generating modeling activities posed significant challenges for the educators, with scarcely any of them demonstrating an ability to formulate activities that adhered to the prescribed principles. In Şahin's investigation (2019), an assessment was made of the competencies of mathematics teachers concerning the development of mathematical modeling problems. This evaluation was accompanied by structured training modules focusing on the introduction of mathematical modeling, the cognitive analysis of mathematical modeling problems, and considerations pertinent to problem formulation. After the training, teachers were asked to prepare activities. The results showed that although the teachers performed successfully in the process of preparing mathematical modelling activities, they experienced some difficulties. In these studies, teachers were given theoretical information about the concepts of mathematical models and modelling and the characteristics of modelling problems and their skills in preparing modeling problems were measured. As a result of the findings, it was concluded that teachers had difficulty in designing modelling activities. It can be concluded that teachers' level of knowledge about mathematical modelling increased in the training given to prepare mathematical modelling problems, but they had difficulties in preparing their own problems. This study is different from other studies in that it was carried out by providing mathematical modelling training to secondary school mathematics teachers in the process of preparing mathematical modelling activities as well as their active participation in modelling activities. Since teachers' participation in modeling activities and solving modeling problems will contribute to their modeling competencies, it is thought that it will also make a difference in their competencies in preparing modeling problems. In this respect, this study conducted with secondary school mathematics teachers is not only contains theoretical information provided teachers but also provides a learning environment in which teachers can improve their modeling competencies. So, this study will support previous studies.

Purpose of the Study

Thus, this study aims to examine the changes in the mathematical modelling activity preparation competencies of secondary school mathematics teachers who participated in mathematical modelling training. The sub-problems based on this purpose are as follows.

1. What are the competencies of secondary school mathematics teachers in preparing mathematical modeling activities before receiving mathematical modeling training?
2. What are the competencies of secondary school mathematics teachers in preparing mathematical modeling activities after receiving theoretical training on mathematical modelling?
3. What are the competencies of secondary school mathematics teachers in preparing mathematical modeling activities after receiving practical training on modelling?

Method

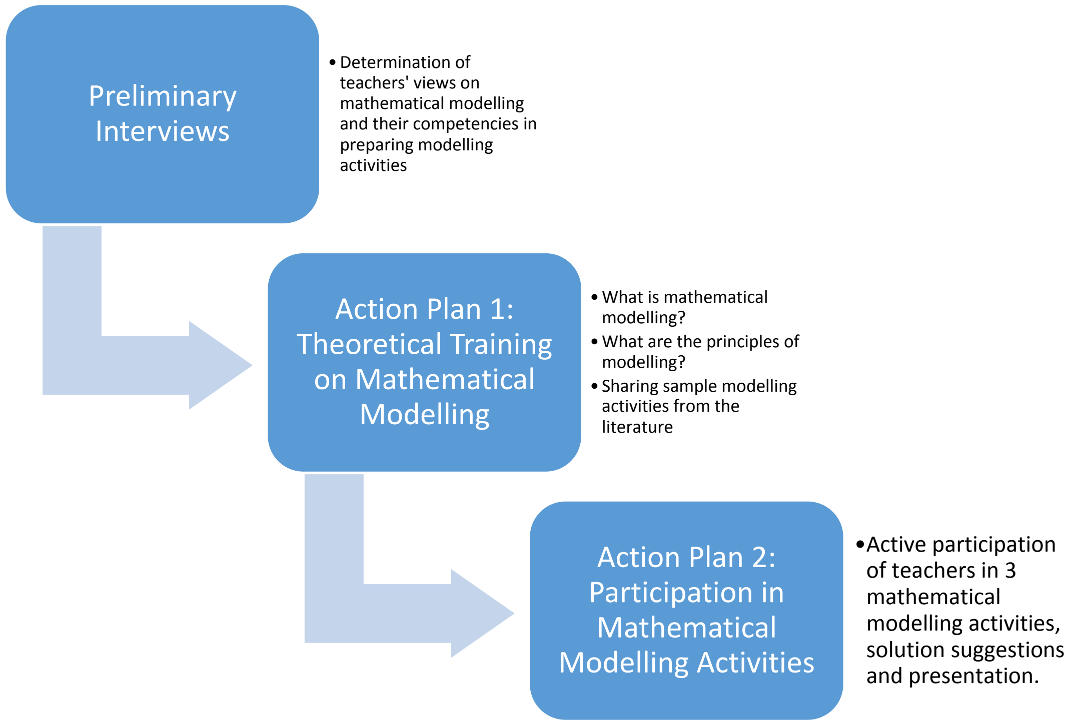
Research Design

In this study, the action research method, one of the qualitative research methods, was used. Action research is a research process carried out in a classroom or school environment to determine and improve the level of actions and teaching (Johnson, 2012; McKernan, 2008; McTaggart, 1997). The aim of action research is not only to collect and make sense of information about the relevant environment but also to develop information about the practices of a particular environment (McKernan, 2008). From this point of view, researchers should make various interventions in the environment. In this study, rather than investigating the competencies of secondary school mathematics teachers to prepare mathematical modelling activities, the development of their competencies to prepare modelling activities was monitored by designing learning environments in line with two action plans.

Action plans. First of all, a semi-structured knowledge scale about mathematical models and modeling was administered to teachers. Then, they were asked to prepare a modeling problem. According to the data, it was determined that teachers' modeling definitions were incomplete and they had problems in preparing modeling problems. In the endeavor to devise a suitable modeling problem, the foundational prerequisite entails a comprehensive internalization of the concept of modeling. Consequently, the initial action plan was set into motion, entailing the provision of instructional sessions that imparted theoretical insights into the domain of mathematical modeling. Instances of analogous pedagogical interventions are documented in extant scholarly literature. Subsequent to this training, an evaluative reassessment was conducted to gauge the teachers' proficiencies in crafting modeling problems. It was concluded that these competencies improved but were still not at a sufficient level. The next action plan is to design a learning environment to support teachers' active participation in mathematical modelling activities. In this practice, teachers solve modelling activities as a group and share their results. At the end of the application, teachers' competencies in preparing modelling activities were measured and the action plans were finalized. Information about the application process is presented in Figure 1.

Figure 1.

Application process of the study



The application process of the study is summarised in Figure 1. Action research stages were taken into consideration in the realisation of these stages. As a result of the analyses made at the end of each application, the next action plan was decided.

Study Group

In this study, the convenience sampling method was used. The researcher turns to the easiest items he/she can reach to select the participants in this method (Yıldırım & Şimşek, 2013). It is known that researchers conducting qualitative studies often prefer situations that are easy and not expensive to study. The study group consisted of 6 secondary school mathematics teachers working in secondary schools in the centre of a medium-sized province in the 2020-2021 academic year. Before the study, the teachers were informed about the study and their voluntary participation was ensured. Since the study involves a long-term application, it is important that the participants are both accessible and voluntary. In the application, teachers were coded as T1, T2, ... and the demographic information of the teachers is presented in Table 1.

Table 1.

Demographic information of the teachers

Teachers	Gender	Professional Experience	Education Status
T1	F	16	MD
T 2	F	19	MD
T 3	M	8	MD
T 4	F	9	MD
T 5	F	16	MD
T 6	F	19	BD

According to Table 1, the study group consists of 6 secondary school mathematics teachers, 5 female and 1 male, working in secondary schools. Two of the secondary school mathematics teachers have 5-10 years of professional experience and the others have 15-20 years of professional experience. In the study group, there is 1 teacher with a bachelor's degree and 5 teachers with a master's degree.

Research Instruments and Processes

During the data collection process, teachers were asked to prepare mathematical modelling activities before the training, after the theoretical training and after their participation in the learning environment. The modelling activities prepared after each activity were examined using the mathematical modelling activity evaluation form. This form is presented in the data analysis section. Written documents were collected for the mathematical modelling problems prepared by the teachers. Throughout this procedure, the instructional sessions were meticulously recorded as a potential resource for future reference; however, it is noteworthy that these recorded meetings were not employed as a primary data source. These training sessions were elucidated with the aim of acquainting individuals with the educational milieu in question.

Training meetings. In training meetings, theoretical information about mathematical modelling is given and mathematical modelling problems are solved in group dynamics. In the first stage of the meetings, training on mathematical model and modelling, the mathematical modeling cycles, the characteristics of modelling activities, and the principles of mathematical modeling problems was given to the teachers. In addition, mathematical modelling examples in the literature were shared. In the second stage, mathematical modelling examples were solved with group work. The mathematical modelling problems solved are 1st Water sprayers (Bukova Güzel, 2016), 2nd Let's Build Environmentally Friendly Buildings with Pet Bottles (Gürbüz & Doğan, 2018), 3rd Water Purifier or Carboy? (Ural, 2018). In the training meetings, while solving the examples with group interaction, the opportunity was provided for the internalisation of mathematical modelling and the solved example was compared with the prepared example and evaluated. At the end of the activities, teachers were asked whether they would revise the modeling problems. All trainings were carried out on the online platform. The researchers guided these learning environments.

Data Analysis

In this study, which was conducted to examine the competencies of secondary school mathematics teachers in preparing mathematical modelling activities, the descriptive analysis method was used to analyse the data obtained from interviews and written documents (Yıldırım & Şimşek, 2013). Yıldırım and Şimşek (2013) defined descriptive analysis as the classification and interpretation of the data collected according to the themes determined before the study. In descriptive analysis, the data are classified according to the pre-specified themes and the findings related to these data are summarised and interpreted by the researcher. In this study, the activities prepared by the teachers were coded and analysed. For example, the first activity of T1 is shown as T1.1, the second activity of T1 is shown as T1.2 and the third activity of T1 is shown as T1.3. In the first stage of the descriptive analysis, the conceptual framework of the study was established and the categories under which the obtained data would be analysed were determined. In this context, data analysis was carried out according to Wess et al. (2021), principles of preparing mathematical modelling activities. The activity evaluation form prepared in accordance with these criteria is explained below:

Mathematical modelling activity evaluation form

This form was used in the study to determine the qualities of mathematical modelling activities created by secondary school mathematics teachers. To fulfill this objective, specific criteria derived from the foundational principles governing the preparation of mathematical modeling problems were adopted, as delineated by Wess et al. (2021). These criteria encompass reference to reality, relevance, authenticity, openness, and sub-competencies associated with modeling. A comprehensive elaboration of these criteria, along with their corresponding indicators, can be found in Table 2.

Table 2.

Criteria and indicators for modelling problems according to Wess et al. (2021)

Criteria	Indicators
Reference to reality	The mathematical modelling activity has a non-mathematical, realistic, and factual starting point.
Relevance	Mathematical modelling activity is closely related to the student's environment or real life.
Authenticity	Authenticity in the sense that the mathematical modelling activity is a real problem of individuals, and the results are used in concrete situations.
Openness	Mathematical modelling activity has different solutions and allows different levels of approach.
Modelling sub-competencies	The mathematical modelling activity provides the cognitive competencies in the steps of the mathematical modelling cycle.

Considering the criteria and indicators in Table 2, the principles can be explained as follows (Wess et al., 2021): The reference to reality criterion states that the problem situation should be a situation that exists in real life. The problem situation has a non-mathematical factual reference. The relevance criterion reveals that the problem situation should be closely related to the students' experiences. This closeness does not require students to be directly related to the problem situation. The problem situation can be directly, indirectly or in the future related to the students. Authenticity criterion refers to both the inclusion of a non-mathematical context in the problem situation and the application of mathematics in the given situation. The non-mathematical context should be real and not specifically designed for a particular traditional problem. The application of the results obtained in this context should also be realistic and should not be used only in mathematics lessons. Authentic modelling problems should belong to a subject that really exists, and their results should be acceptable to people working in these fields (Niss, 1992). The openness criterion is that the problem situation allows more than one approach or solution. Not giving all the data related to the problem in the problem statement enables students to exhibit different approaches during the solution. In the criterion of Modelling sub-competencies, it is important that the problem situation develops the sub-competencies of modelling, and this problem should encourage the development of modelling competencies in the modelling cycle. It is found that most of the mathematical modelling activities prepared in parallel studies in the literature are examined in line with the principles put forward by Lesh et al. (2000). However, the mathematical modelling activities prepared in this study were examined in accordance with the criteria prepared by Wess et al. (2021). One of the original aspects of this study is the introduction of these criteria.

The activity prepared by the teacher coded T1 for sample analysis is presented in Figure 2.

Figure 2.
The activity prepared by the teacher coded T1.

> Due to the epidemic disease, schools had to conduct their education remotely. Zeynep cannot attend distance education classes because she does not have internet at home. Her family decided to connect the internet so that Zeynep could attend live lessons and her education would not be disrupted. In order for Zeynep to attend her classes, 16 GB of internet is required.
>
> Zeynep's family has two options for internet connection:
>
> 1st Option: 1 GB internet with GSM operators is 7TL.
>
> 2nd Option: 1 GB internet with a fixed telephone line is 5,8 TL. However, in order to establish an internet connection with a fixed telephone line, they need to buy a telephone device for their home.
>
> In your opinion, which option would be logical for Zeynep's family to choose, explain the reasons.

The analysis of the activity prepared by the teacher coded T1 presented in Figure 2 according to the criteria given in the evaluation form is given in Table 3.

Table 3.

Evaluation of the mathematical modelling activity prepared by teacher T1 according to the criteria

Criteria	Suitable	Partly Suitable	Not Suitable
Reference to reality	√		
Relevance	√		
Authenticity			√
Openness		√	
Modelling sub-competencies		√	

When Table 3 is analysed, the reference to reality criterion is that the problem situation has a non-mathematical factual reference. In this context, the problem situation is deemed entirely congruent with the reality criterion, as it can feasibly manifest in real-life scenarios. Similarly, the problem situation's ability to establish a meaningful connection to the students' personal experiences renders it in alignment with the relevance criterion. Authenticity refers to the application of both the non-mathematical context in the problem situation and the application of mathematics in this particular situation. The context of the problem situation is realistic but not authentic because it is expressed in terms of fictionalised numbers. Openness means that the problem situation allows more than one approach or solution. From this point of view, the fact that some values are given in the problem situation shows that it partially complies with the principle of openness since it will limit the solutions. From the same point of view, since the options are limited, it does not direct the students to the model creation step. Rather, it encourages students to obtain a mathematical result. Thus, it is seen that the prepared activities are partially suitable for the sub-competencies criteria.

Validity and Reliability Studies

One of the ways to increase internal validity in the study is to benefit from expert suggestions regarding different stages of the study (Yıldırım & Şimşek, 2013). In the process of selecting the mathematical modelling activities included in the study, the modelling problems were checked structurally by an instructor and necessary changes were made in line with the expert opinion. Thus, it was tested whether all modelling problems provided the modelling principle. At the same time, expert opinion was consulted in the evaluations regarding the Mathematical Modelling Activity Evaluation Forms prepared for the teachers' activities. The researcher's long-term interaction with the study is another method used to increase internal validity. What is meant by long-term interaction is to spread the interaction between the researcher and the data source over a long period of time in order to further increase the credibility of the data obtained. Since this study lasted 8 weeks, it can be considered as a long-term study. External validity (transferability) is related to the extent to which certain findings obtained from a study can be adapted to similar situations provided that the meaning and inferences are preserved (Arastaman et al., 2018). Transferability is ensured by the detailed description method used in qualitative studies (Yıldırım & Şimşek, 2013). In order to strengthen the transferability of this research, data collection tools and the entire data collection process were explained in detail. In this study, direct quotations were included in the activities prepared by the teachers in order to be as faithful as possible to the nature of the data and to set an example.

One of the ways to increase internal reliability is the role of the researcher. In the study, the researcher plays a role as the person who plans the teaching environment and manages the process with the practitioner. The researcher participated in the teaching environment one-to-one and supported the teachers in developing their competencies in preparing mathematical modelling activities by making necessary interventions both in the information meetings and in the training meetings where the mathematical modelling activities were solved. For internal consistency, the mathematical modelling activities prepared by the research group were evaluated by the researcher and an expert in accordance with the indicators.

Results

Table 4 shows the information about the conformity of the activities designed by the teachers before receiving any training on mathematical modelling with the principles in the evaluation form.

Table 4.

Examination of modelling problems created by teachers before the application according to the criteria

Teacher	Criteria	Reference to reality	Relevance	Authenticity	Openness	Modelling sub-competencies	Completely Suitable
	Suitable	√	√				
T1.1	Partly Suitable				√	√	X
	Not Suitable			√			
	Suitable						
T2.1	Partly Suitable						X
	Not Suitable	√	√	√	√	√	
	Suitable						
T3.1	Partly Suitable	√					X
	Not Suitable		√	√	√	√	
	Suitable	√					
T4.1	Partly Suitable						X
	Not Suitable		√	√	√	√	
	Suitable	√		√	√	√	
T5.1	Partly Suitable						X
	Not Suitable		√				
	Suitable						
T6.1	Partly Suitable						X
	Not Suitable	√	√	√	√	√	

As can be seen in Table 4, three of the six teachers presented activities in accordance with the reference to reality principle. The teachers tried to associate the activities with real life, but they did not pay attention to the students' experiences. In this context, it was observed that no activity was prepared in accordance with the relevance principle except for one activity. Correspondingly, the empirical observations revealed a notable scarcity of activities that adhered to the principle of authenticity, with only a solitary activity found to align with this particular criterion.

Again, except for one teacher, there was no modelling activity that adequately met the criteria of promoting openness and sub-competencies. As a result, when all criteria were evaluated together, it was seen that a modelling activity that met all of them was not developed. For example, the activity prepared by the teacher coded T2 is given in Figure 3.

Figure 3.

The first activity prepared by teacher T2.

Find the result of the operation $\frac{3}{4} x \frac{2}{5}$ *by modelling.*

While it was determined that the problem prepared in Figure 3 was not suitable for any of the mathematical modelling criteria, it is noteworthy that this problem was prepared in the context of modelling mathematics. It was observed that three of the other five activities were classical verbal problems that were tried to be associated with daily life.

The mathematical modelling activities prepared by secondary school mathematics teachers after receiving training in mathematical modelling were examined to answer the question “What is the level of competencies of secondary school mathematics teachers in preparing mathematical modelling activities after receiving training on mathematical modelling?”. When the process after the information meetings was examined, it was seen that teachers coded T1 and T3 did not revise the mathematical modelling activities they had prepared, while other teachers prepared new activities. The findings related to this sub-problem are presented in Table 5.

Table 5.

Examination of the modelling problems created by the teachers after the information meetings according to the criteria

Teacher	Criteria	Reference to reality	Relevance	Authenticity	Openness	Modelling sub-competencies	Completely Suitable
T1.2	Suitable	√	√				
	Partly Suitable				√	√	X
	Not Suitable			√			
T2.2	Suitable						
	Partly Suitable						X
	Not Suitable	√	√	√	√	√	
T3.2	Suitable						
	Partly Suitable	√					X
	Not Suitable		√	√	√	√	
T4.2	Suitable	√					
	Partly Suitable						X
	Not Suitable		√	√	√	√	
T5.2	Suitable	√	√	√	√	√	
	Partly Suitable						√
	Not Suitable						

T6.2	Suitable						
	Partly Suitable						X
	Not Suitable	√	√	√	√	√	

As can be seen in Table 5, after the theoretical training, only T5.2 is the only activity that is completely suitable for the criteria. The other five activities were partially suitable or not suitable with the criteria. From this point of view, it was determined that the information meetings alone were not sufficient to improve the preparation of mathematical modelling problems.

The mathematical modelling activities prepared by the teachers after their active participation in the mathematical modelling activities carried out following the information meetings were examined in order to answer the question "What is the level of secondary school mathematics teachers' competencies in preparing mathematical modelling activities after receiving training on mathematical modelling?". Throughout this process, it was discerned that all educators, with the exception of T1 and T5, engaged in the process of revising and enhancing their originally conceived activities. Detailed insights into the conformity of these post-training activities with the established principles are presented in Table 6.

Table 6.

Examination of mathematical modelling activities prepared by teachers after participation in modelling activities according to criteria

Teacher	**Criteria**	**Reference to reality**	**Relevance**	**Authenticity**	**Openness**	**Modelling sub-competencies**	**Completely Suitable**
T1.2	Suitable	√	√				
	Partly Suitable				√	√	X
	Not Suitable			√			
T2.2	Suitable	√	√	√	√	√	
	Partly Suitable						√
	Not Suitable						
T3.2	Suitable	√	√	√	√	√	
	Partly Suitable						√
	Not Suitable						
T4.2	Suitable	√	√	√	√	√	
	Partly Suitable						√
	Not Suitable						
T5.2	Suitable	√	√	√	√	√	
	Partly Suitable						√
	Not Suitable						
T6.2	Suitable	√	√	√	√	√	
	Partly Suitable						√
	Not Suitable						

As can be seen in Table 6, after the activity solutions, the activities that fully fulfil the criteria are T2.3, T3.3, T4.3, T5.3 and T6.3. Only the T1.3 activity partially fulfils the criteria of authenticity, openness, and modelling sub-competencies. Based on the findings, it was determined that the learning environment designed to improve mathematical modelling problem preparation was significantly more effective than the information meetings. An example of the modelling activity prepared by the teacher coded T3 after the activity solutions is given in Figure 4.

Figure 4.

The third activity prepared by teacher T3.

Residents of Fatih neighbourhood in Erzincan have observed that there are accidents and chaos on the street they often use because there is no traffic light. A group of students who applied to the Traffic Branch for a traffic light at the junction where Terzibaba Street and 700th Street meet, submitted a document to the Traffic Branch about what they should observe until they made this application and why they wanted a traffic light. Explain how you would help these students by sending a letter.

This activity prepared by the teacher coded T3 in Figure 4 was prepared in accordance with all modelling criteria. It was determined that the activity reflected a problem in the province where the teachers were located, and the information given was realistic and authentic. In addition, it is an open-ended activity related to the close environment of the students. It was also seen that it encouraged modelling competencies. Similarly, the example of the modelling activity prepared by the teacher coded T4 after the activity solutions is given in Figure 5.

Figure 5.
The third activity prepared by teacher T4.

CONCRETE ROAD OR ASPHALT ROAD?

Roads, which constitute an important part of the infrastructure elements of a nation, are one of the important indicators that the nation has a civilisation. To the extent that transport is easy and convenient in a country, science and art can develop to the same extent. Therefore, roads should be given importance in order to ensure easy transport and the development of social welfare.

Concrete roads are more durable and environmentally friendly than asphalt roads. On the other hand, when the slipping of vehicles in snowy weather conditions is considered, asphalt roads have a safer structure. Considering that asphalt is a petroleum derivative, the fact that concrete is produced with domestic raw materials in relation to increasing oil prices makes concrete roads advantageous compared to asphalt in the initial construction cost. However, since asphalt is not a reactive substance like concrete, asphalt roads require less maintenance.

In line with this information, we are asked to determine whether the construction of concrete roads or asphalt roads is more advantageous and to explain what we have taken into account in reaching this conclusion.

It is seen that the activity given in Figure 5 complies with all principles. It can be claimed that the activity given above is suitable for the reference to reality principle because it reflects the situations that exist in real life. It is seen that it is suitable for the principle of relevance because it is directly or indirectly related to the student's life; it is suitable for the principle of openness because it allows different approaches, and all data are not given in the problem. Likewise, the fact that it expresses mathematical applications in both non-mathematical contexts and special cases shows that the activity complies with the principle of authenticity. Since the options are not limited, it is seen that they encourage modelling sub-competencies.

Discussion, Conclusion & Suggestions

In this study, which aimed to examine secondary school mathematics teachers' processes of preparing mathematical modelling problems, the following results were obtained:

It was observed that the activities designed by half of the teachers before the training meetings were suitable with the reference to reality principle. However, while preparing the activities, teachers did not pay attention to the fact that they were related to students' experiences. This finding of the study is similar to the results obtained by examining the mathematical modelling activities prepared in the studies conducted by Dede et al. (2017) with prospective mathematics teachers and Şahin (2019) with mathematics teachers. This result is also consistent with the studies of Borromeo Ferri and Lesh (2013). It was evident that the activities formulated based on pre-existing knowledge exhibited significant incongruities with respect to the principles of openness, relevance, authenticity, and modeling sub-competencies. In light of these findings, it can be claimed that teachers' initial competencies in preparing mathematical modelling problems are low. This result coincides with the results of many similar studies in the literature (Bilgili et al., 2020; Dede et al., 2017; Deniz & Akgün, 2016; Stohlmann et al., 2017; Yu & Chang, 2009). Since mathematical modeling skills have just entered the curriculum (MoNE, 2017), many of the teachers may not have received training or knowledge on this subject.

The teachers' modeling problems after their mathematical modelling training were examined. When the activities prepared at the beginning of the process were compared with the activities prepared during the process, it was observed that the number of activities prepared suitable with the principles gradually increased. However, it was still observed that they could not prepare activities suitable with all criteria. From this point of view, it is concluded that there are deficiencies in preparing activities in accordance with all criteria based on prior knowledge or theoretical training about mathematical modelling. This finding overlaps with the results of many similar studies (Bilgili et al., 2020; Dede et al., 2017; Stohlmann et al., 2017). Similarly, in the studies conducted by Sağıroğlu (2018), Deniz (2014) and Şahin (2019), it was observed that teachers had difficulties in the process of creating modelling activities even after the training. This may be due to the fact that teachers have not encountered modelling activities before (Korkmaz, 2010; Urhan & Dost, 2016). In this study, teachers were exposed to mathematical modelling activities after the information meetings. Teachers' active participation in modelling activities positively affected their competencies in preparing mathematical modelling activities and it was determined that they were generally able to prepare activities in accordance with the principles. In conclusion, it can be posited that the teachers' competencies in the development of mathematical modeling activities have shown discernible enhancement, attributable to their active involvement in mathematical modeling tasks and their attendance at informational sessions dedicated to mathematical modeling. Previous studies show that after participation in mathematical modelling activities, there are positive differences in the understanding of the activities (Korkmaz, 2010; Shahbari, 2017).

When the activities prepared during the process are analysed in principle, it is seen that they were first tried to be prepared in accordance with the reference to reality principle and to some extent they could be prepared in accordance with this criterion. These findings are in line with the studies conducted by Deniz and Akgün (2016), Bilgili and Çiltaş (2022), Dede et al. (2017), Deniz (2014), Şen (2020), Şahin et al. (2023). In fact, it can be expressed that the criterion that teachers pay the most attention to when preparing mathematical modelling problems is the real-life situation because the prepared problems are suitable for real life even if they are not mathematical modelling activities. As in the studies of Deniz and Akgün (2016), Bilgili and Çiltaş (2022), Dede et al. (2017), Şahin (2019) and Şen (2020), it is found that the reference to reality principle plays a binding role in modeling problems. In fact, teachers are used to real-life problems in their classes. For this reason, they may have taken this criterion into consideration first. The reality criterion is important but not sufficient for modeling problems.

One of the most prominent features of mathematical modelling problems is that they are based on assumptions and preferences (Lesh & Doerr, 2003). This feature of the problem is that it allows more than one approach or solution for the solution. This feature, which is considered as the principle of openness in this study, can also be considered as the problem being open to interpretation. Not giving all the data in the problem statement allows students to exhibit different approaches during the solution. For this reason, it is important that the problem sentence is open-ended. The findings obtained show that most of the teachers were not sufficient in this regard before receiving training. However, when the problems prepared during the training process were evaluated, the most important criteria for all teachers after the reference to reality principle were the relevance and the openness principle. Deniz (2014) and Şen (2020) reached the same conclusion in their study. However, it was observed that some teachers gave some numerical data required for the solution of the problem not as a single value but as an interval. From this point of view, it can be expressed that some teachers perceive the ability to use the desired numerical values within a certain range as using different variables, and the fact that the numerical results of the solutions are different as different models. These results are similar to Şahin (2019)'s study. This situation may stem from the existing education system's habits.

Another criterion for mathematical modelling problems is their authenticity. Authentic modelling problems belong to an existing subject or problem area and are accepted by people working in these areas (Niss, 1992). When the problems prepared in the process are analysed, it is seen that the authenticity principle is provided at least in the problems prepared at the beginning of the process. In the problems prepared at the beginning of the process, the non-mathematical context is not real and is specially designed for a specific arithmetic problem. In the problems prepared as the process progresses, it is seen that the principle of authenticity is largely fulfilled.

Another important feature of mathematical modelling problems is that they promote the sub-competencies of mathematical modelling. It is important that the problem situation develops the sub-competencies of modelling, and this problem should encourage the development of modelling competencies in the modelling cycle. When the prepared problems were analysed, it can be expressed that activities were prepared in accordance with this principle, especially as the process progressed. This result coincides with the results of Deniz and Akgün (2016), Dede et al. (2017), Deniz (2014) and Şen (2020). It contradicts the result of Bilgili and Çiltaş (2022) and Şahin et al. (2023), according to their study this principle is the most ignored one. Although only model creation and model generalization criteria are taken into account in these studies, teachers do not develop problems in accordance with these criteria. This difference may be due to the different modeling problem criteria taken into consideration in the studies. In this study, the new criteria proposed by Wess et al (2021) were taken into account.

In the study, it was determined that teachers' competencies in preparing modelling problems was low. In order to complete these deficiencies, it is recommended to design environments for the development of teachers' modelling competencies as well as theoretical training. In this study, the learning environment includes both theoretical knowledge and the application of modelling activities to teachers. By designing different learning environments, teachers' competencies to develop activities for modelling can be examined. New ideas about which environment will be used in practice can be put forward. In addition, the new criteria for modelling problems expressed in this study can guide teachers in developing or selecting modelling problems. This study is limited to four modeling problems applied during training. The results can be tested by applying different and more problems. In the study, training meetings were applied in an online environment. The outcomes derived from this investigation are circumscribed within the confines of online training. Thus, it is required to consider the potential variance and commonalities that may emerge in a face-to-face learning environment. Furthermore, the study uncovered that one of six teachers continued to encounter challenges in formulating modeling problems despite the training. This prompts further examination into the underlying reasons for the resistance demonstrated by certain teachers in the context of mathematical modeling.

Ethic

According to the decision of Erzincan Binali Yıldırım University Educational Sciences Ethics Committee, dated 26/02/2021 numbered 66253, this study received ethical approval.

Author Contributions

This research is derived from the first author's master dissertation conducted under the supervision of the second author.

Conflict of Interest

The authors declare that they have no conflict of interest.

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