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Meanings of Logical Connectives in the Light of Alethic Relations

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Abstract

One of the central questions in 20th-century discussions within logic and philosophy of language is where logical constants, specifically propositional connectives, get their meaning from. What kind of determination/justification bond is found between the inference rules peculiar to a given connective and *the meaning of* that connective? Are these rules justified by it, or rather do they contribute to its construction? An observation made by G. Gentzen, who founded in the 1930s the proof-theoretical approach at large, triggered a view (called by some *logical inferentialism*) that gives a remarkable answer to the above question: the meaning of a logical constant in a logical language is provided, not by some sort of *representational* content, but by the inferential norms that govern its overall use. In 1960 A. N. Prior fictionalized as a counter-instance the connective *tonk* solely using a couple of inference rules, a connective capable of overthrowing the system of deduction; N. Belnap's 1962 reply in the form of an analysis of the *tonk* problem opens the way to discussions in logic-cum-philosophy of language with important outcomes. The present little study can be read as some further deflation of the *tonk* problem with a relatively unconstrained inferentialistic view of the matter. The two main theses of the study are (i) that the problem posed by *tonk*-like connectives can be captured, more simply than in Belnap's (otherwise correct) analysis, through inferential relations of a certain type which will be dubbed *alethic relations*; and (ii) that Prior's challenge, brought to completion in whichever way, cannot give any result against the inferentialist conception.

Keywords: Propositional Connectives, *Tonk*, Meaning, Alethic Relations, Inference Rules, Logical Inferentialism.

Aletik Bağıntılar Işığında Mantık Eklemlerinin Anlamları

Öz

Mantıksal sabitlerin, özellikle de doğruluk fonksiyonları mantığındaki önerme eklemlerinin, anlamlarını nereden aldıkları 20. yüzyıl mantık ve dil felsefesi tartışmalarında merkezde yer alan bir sorudur. Bir ekleme özgü temel çıkarım kurallarıyla o eklem*in anlamı* arasında nasıl bir belirleme/haklılaştırma bağıntısı bulunur? Bu kurallar ilgili eklemin anlamı yoluyla mı haklılaştırılır, yoksa o anlamın belirlenişine mi katılırlar? 1930'larda modern kanıt kuramsal yaklaşımın temellerini atan G. Genzten'in yaptığı bir saptama, anılan soruya dikkat çekici bir yanıt oluşturan (bazılarının *mantıksal çıkarımsalcılık* olarak andığı) şu görüşü tetiklemiştir: Mantık dilindeki bir mantıksal sabitin anlamı, bir çeşit *temsil* içeriği tarafından değil, onun kullanımını idare eden çıkarım normları tarafından sağlanır. Arthur N. Prior 1960 tarihli ünlü yazısında bu yaklaşıma bir karşı-örnek olarak, salt çıkarım kuralları yoluyla, türetim sistemini altüst eden *tonk* eklemini kurgular; Nuel Belnap'ın Prior'a da yanıt oluşturan *tonk* sorunu çözümlemesi ise kapıyı, önemli sonuçları olan mantık-dil felsefesi tartışmalarına açar. Elinizdeki küçük çalışma, tonk sorununu, daha serbest bir çıkarımsalcı bakışla biraz daha söndürme girişimi olarak düşünülebilir. Çalışmanın iki ana savı, (i) tonk-vari eklemlerin yol açtığı sorunun, Belnap'ın (haklı) çözümlemesindekinden belki daha yalın bir şekilde, *aletik bağıntılar* olarak adlandıracağımız, belli tipte çıkarımsal bağıntılar yoluyla yakalanabileceği; ve (ii) Prior'ın itirazının, hangi şekilde tamamlanırsa tamamlansın, çıkarımsalcı anlayış aleyhine bir sonuç üretemediğidir.

Anahtar Kelimeler: Önerme Eklemleri, *Tonk*, Anlam, Aletik Bağıntılar, Çıkarım Kuralları, Mantıksal Çıkarımsalcılık.

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Introduction

Gerhard Gentzen, the primary figure in the emergence and development of the modern proof-theoretical approach to logic in the 20th century, made in his 1934 essay[2] a certain remark about the inference rules given for logical connectives (in any sort of natural deductive system), which opened the way to a striking view about the meaning of logical expressions, called (by some) *logical inferentialism*.[3] Gentzen's remark was roughly that inference rules (of a certain type) peculiar to a given logical connective could be seen as *defining* the meaning of that connective. What makes this remark important is the fact that it is conveniently taken as the proposal of a wholly different kind of analysis than the dominant one based on a *representationalist* conception of meaning[4] – and, less clearly, on some elements from G. Frege's theory of meaning[5] – since this dominant analysis holds that propositional connectives get their meaning from the truth-functions that they denote or represent. So the novel idea goes: inference rules governing the use of propositional connectives (or generally, any kind of logical constants) within a natural deduction calculus (or any other kind of proof-theoretical context) *constitute*, instead of *being justified by*, the meanings of these connectives (or logical constants); thus, proof-theoretic tools and units/items that have been considered syntactical (in the sense, of course, of logical syntax) in nature should rather play the leading role in the analysis of meaning, at least for formal languages.[6]

In close contact with philosophical views about linguistic meaning that are centred around the belief that linguistic meaning is provided and determined essentially by *use* or *norms of use*, the above raw idea first evolved into the above-mentioned alternative conception of meaning for logical languages, namely logical inferentialism, but then through a different route and with a much wider scope of interests it takes the shape of a whole philosophy of language (and of mind) at large, namely *inferentialism* proper.[7] The present little study deals with the problem that lies at the centre of a relatively old discussion concerning narrowly the logical inferentialist thesis, from a relatively abstract perspective and by means of some novel but simple notions.

Because the problem apparently turns around a specific fictional propositional connective, *tonk*, which was put forward by A. N. Prior in his short 1960 Analysis paper[8] as a counter-instance to the (then developing) logical inferentialist approach to the meaning of logical constants, dub it *the tonk problem*. What this paper

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[2] G. Gentzen, "Untersuchungen über das logische Schliessen," *Mathematische Zeitschrift* 39, (1934-35): 176-210 and 405-431. An English translation, with the title "Investigations into Logical Deduction", can be found in M. E. Szabo (ed.), *The Collected Papers of Gerhard Gentzen* (Amsterdam: North Holland Pub. Co., 1969), 68-131.

[3] This label, which is not that widely accepted, is suggested by and employed mainly in Jaroslaw Peregrin, *Inferentialism: Why Rules Matter* (London: Palgrave-Macmillan, 2014). I employ the label for ease with the same specific sense (given in the following) throughout this paper.

[4] For a quick review of the inferentialist-representationalist (or -denotationalist) divide in semantics (and its relevance to the present topic) see Peter Schroeder-Heister, "Proof-Theoretic Semantics," *The Stanford Encyclopedia of Philosophy* (Fall 2023 Edition), Edward N. Zalta & Uri Nodelman (eds.), https://plato.stanford.edu/archives/fall2023/entries/proof-theoretic-semantics/ , especially §1.2.

[5] Primarily, the idea of assigning functions to certain subsentential expression types as their denotations (referents).

[6] For (an instance of) the historical-cum-philosophical development of this idea, see the content and references of Peter Schroeder-Heister, "Proof-Theoretic Semantics"; see also Paolo Mancosu, Sergio Galvan and Richard Zach, *An Introduction to Proof Theory: Normalization, Cut-Elimination, and Consistency Proofs* (Oxford: Oxford University Press, 2021).

[7] Inferentialism – alternatively called *semantic inferentialism* – is Robert Brandom's philosophy of language, where the idea of inferentiality is taken widely enough to explain meaning in natural languages, but also meaning in activities *not wholly* linguistic, such as perceptual judgment-giving and rational action. The original work is Robert Brandom, *Making It Explicit: Reasoning, Representing, and Discursive Commitment* (Cambridge (Mass.): Harvard University Press, 1994).; see also Robert Brandom, *Articulating Reasons: An Introduction to Inferentialism* (Cambridge (Mass.): Harvard University Press, 2000).

[8] Arthur N. Prior, "The Runabout Inference-Ticket," *Analysis* 21, no. 2 (Dec. 1960): 38-39.

tries to do is, then, put the *tonk* problem in place anew in a considerably simpler form, by taking a relatively liberal inferentialistic perspective (which permits incomplete meanings, see below) and by employing the useful idea of *alethic relation*. The paper aims to show more clearly that the problem (if any) is not even about a particular type of connective, and stated on an adequate level of generality, it clearly does not pose a threat to the inferentialist approach.

In the following three short sections, (i) the *tonk* problem and the context of its discussion is quickly presented; (ii) Nuel Belnap's powerful analysis of the *tonk* problem is briefly outlined and revaluated in the light of the idea of alethic relation; finally, (iii) J. T. Stevenson's development of Prior's objection into a full-fledged argument in favour of a representational (or denotational, or model-theoretical) conception of meaning, more specifically his comparison of inference rules with truth-functions in terms of explanatory priority, is criticized.

1. Inferential Meaning and *Tonk*

Prior's famous 1960 paper is directed against the non-standard view that the meanings of logical constants, especially the truth-functional connectives, are not an independent source of justification for the inference rules specific to connectives, but that on the contrary they are defined by means of these very rules. So the *tonk* problem is to find, by the defender of this view, an answer to Prior's challenge. The view, logical inferentialism, says that the meaning of a given logical constant is defined by means of few inference rules peculiar to it, which govern the possible inferential moves from/to formulae where the connective is the main operator to/from others (where it does not necessarily occur). Clearly, the roots of this view is in a remark by Gentzen on the explanatory significance of such inference rules which take the shape of *introduction* and *elimination* rules within systems of deduction: one can consider the introduction rules for the connective which exhibit the minimal sufficient conditions of introducing the connective into the proof discourse as defining the meaning of the connective, and the elimination rules which exhibit the minimal sufficient conditions of taking it out from the discourse – i.e. of passing from a formula with the connective as the main operator to another where it is not – as exhibiting the main logical outcomes of that meaning. Departing from this apparently moderate remark and placing itself in the line of thought characterized as *meaning-as-use*, logical inferentialism holds that the meanings of sub-sentential logical constants (such as connectives, quantifiers etc.) are specified by their inferential relations, governed by norms of use of the basic sentence types in which they occur, so that meaning should not be analysed in representational or denotational terms, but in terms of inference rules, at least for the expressions in logical languages.

The meaning, for instance, of the conjunction operator *&* is to be defined, not by means of some representational content – such as the relevant truth-function assigned to it – which is somehow capable of justifying the basic &-introduction and &-elimination rules, but by means of these rules themselves. Thus the meaning of & is:

&-I: From *A*, *B* infer *A&B*.

&-E: From *A&B* infer *A*; from *A&B* infer *B*.

Now the key operation in a context of proof based on natural deduction is the *discharge of assumptions*, and accordingly the key connective is the conditional (or material implication) operator. The meaning of this connective (for the classical logic of truth-functions) is:

⊃-I: From *[A]...B* infer *A⊃B*.

⊃-E: From *A⊃B*, *A* infer *B*.

The introduction rule of the conditional (as it is presented here) is based on the operation of discharging the assumption under which the relevant statement is put forward, by *beating the assumption into the statement* as an antecedent. The philosophical import of this operation gets a clarification in the philosophy of logic of Robert Brandom, the founder and leading defender of inferentialism proper: the conditional is not merely some tool for forming a novel composite formula from given formulae within a specified language; it is essentially a tool for expressing a certain type of semantic relation between these sentences without rising above the level of object-language. Viewed the other way round, it thus provides the object-language with the capability, as it were, of self-reflection; indeed, logic *in generatim*, for Brandom, is a medium for expressing *semantic self-consciousness.*[9]

Arthur N. Prior puts forward in his 1960 Analysis paper a novel, fictional connective *tonk*, against the rule-based (or inferentialist) conception of meaning of logical constants. The novel connective is fictional, but it is defined by a pair of inference rules of the introduction-elimination type, hence in a way that should be welcomed by the inferentialist:

tonk-I: From *A* infer *A-tonk-B*.

tonk-E: From *A-tonk-B* infer *B*.

It is immediately clear that a connective of this sort will warrant the proof-theoretic transition from an arbitrarily chosen *A* to an arbitrarily chosen *B* – and in particular, from any formula to one that contradicts it; and this will lead to the annihilation of the internal boundaries, hence the whole sense, of deducibility. So Prior's general point can be summarized in the form of the following simple argument: if a logical connective can be appropriately defined solely in terms of inference rules, then one should be able to define connectives like *tonk* appropriately; but as we clearly see there is nothing appropriate about *tonk* and *tonk*-like connectives; therefore, logical connectives cannot be defined solely in terms of inference rules.

Note that this argument cannot by itself take the shape of a satisfactory refutation, simply because Prior does not show us the 'semantic' (i.e. representational/denotational) ground for connectives which is independent from the introduction-elimination rules, and which, unlike inference rules, is still capable of ruling out *tonk* and *tonk*-like connectives as undefinable or meaningless. Before going around J. T. Stevenson's 1961 paper,[10] which tries to bring Prior's argument to completion by means of a particular comparison of inference

9 This final point, a crucial element in Brandom's inferentialism, belongs to the layer or aspect of this theory which he calls *logical expressivism*. See especially Brandom, *Articulating Reasons*, ch. 1 for the details.

10 J. T. Stevenson, "Roundabout the Runabout Inference-Ticket," *Analysis* 21, no. 6 (Jun. 1961): 124-128.

rules and truth-tables, I now turn to the most effective and productive response from the inferentialist (or contextualist[11]) side, namely N. Belnap's analysis of the *tonk* problem.

2. Deducibility Claims and Alethic Relations

Belnap's 1962 paper[12] is the first extensive response to Prior's challenge; but it also displays more clearly the philosophical ground of some key discussions in the theory of natural deduction and in proof-theory at large.[13] Belnap thinks that the core of the problem lies at the interaction between the newly added *tonk*-like connective and the original, assumed framework of deducibility to which the addition is made. Naturally, logical systems are not raised solely on an assignment of meanings (of the inferential or representational type) to the logical constants; certain criteria could also be defined/captured, which concern, not any particular (type of) logical constant or compound form, but the notion of deducibility (or logical consequence) itself.

So according to Belnap, logical constants, especially logical connectives are always defined within a given *deducibility context*; this context is provided by certain (meta-level) assumptions which are embodied in rules such as the transitivity of deducibility; thus the problem with connectives like *tonk* that are defined solely by means of I-E rules is not that they are so defined, but that they are inconsistent with the context in which the unproblematic ones are settled: "In short, we can distinguish between the admissibility of the definition of *and* and the inadmissibility of *tonk* on the grounds of consistency – i.e. consistency with antecedent assumptions."[14]

Belnap draws out this criterion of *consistency with a deducibility* context in three steps. First, Gentzen's *structural rules* about deducibility are put forward, in the form of rules about connective-free deducibility statements (or schemata), along with a single axiom:[15]

[11] Actually, Belnap draws the relevant distinction among the two rival conceptions of meaning mentioned frequently above by means of the traditional (pre-Kantian) notions of analyticity and syntheticity. (He calls the approach which begins with the sentential contexts in which an expression is used in order to explain its meaning the analytic approach, which he is trying to keep tenable in view of the *tonk* problem.) However, I will continue to use the label "(logical) inferentialism" to name the common idea/thesis presented above.

[12] Nuel Belnap, "Tonk, Plonk and Plink," *Analysis* 22, no. 6 (1962): 130-34.

[13] Specifically: discussions on the Harmony criterion for I-E rule couples and on Normalization, and the whole field of *substructural* logics, i.e. logical systems which preclude at least one of Gentzen's structural rules. See the relevant sections and bibliographical entries in Francis J. Pelletier and Allen Hazen, "Natural Deduction Systems in Logic," *The Stanford Encyclopedia of Philosophy* (Spring 2023 Edition), Edward N. Zalta & Uri Nodelman (eds.), https://plato.stanford.edu/archives/spr2023/entries/natural-deduction/ for the discussions on Harmony and Normalization. For substructural logics, see Greg Restall, *An Introduction to Substructural Logics* (New York: Routledge, 1999); for the connection between the idea of a substructural logic and semantic inferentialism, see Brandom's works cited above.

[14] Belnap, "Tonk, Plonk and Plink," 131.

[15] Hence in a higher-order language? Not necessarily so: the above deducibility statements can be taken as object-level formulae of *a single-conclusion sequent calculus*, where structural rules are ranked as rules or meta-statements about these formulae; see in this connection P. Suppes's famous neat presentation style of natural deductive reasoning/proof: Patrick Suppes, *Introduction to Logic* (Princeton (NJ): Van Nostrand/Reinhold Press, 1957). Because my point in this paper does not specifically concern proofs, I will not follow any particular type of deductive system or any presentation style. Note, however, the presence of some (not-so-standard) inferentialist approaches to the matter that makes the choice among these types and/or styles semantically significant; see especially James W. Garson, *What Logics Mean: From Proof Theory to Model-Theoretic Semantics* (New York: Cambridge University Press, 2013).

Axiom: A⊢A.

Rules:

- **Weakening:** From $A_1, \ldots, A_n \vdash C$ infer $A_1, \ldots, A_n, B \vdash C$.
- **Permutation:** From $A_1, \ldots, A_i, A_{i+1}, \ldots, A_n \vdash B$ infer $A_1, \ldots, A_{i+1}, A_i, \ldots, A_n \vdash B$.
- **Contraction:** From $A_1, \ldots, A_n, A_n \vdash B$ infer $A_1, \ldots, A_n \vdash B$.
- **Cut (or Transitivity):** From $A_1, \ldots, A_m \vdash B$ and $C_1, \ldots, C_n, B \vdash D$ infer $A_1, \ldots, A_m, C_1, \ldots, C_n \vdash D$.

The second step is the formulation of the proposal for an *extension* of the relevant formal system with the addition of inference rules that define an arbitrary connective *plonk* (and, of course, of the sentence form 'A-plonk-B' to the syntax). The third, final step is the definition of an appropriate criterion of conservativeness, namely: although there may appear (as expected) novel deducibility statements in the system after the extension, all of them should contain the novel connective. In other words, a deducibility statement $A_1, \ldots, A_n \vdash B$ *which is free from 'plonk'* and which is not a member of the system before the extension, should not become a member after the extension.

Now why the criterion of consistency (with the context of deducibility) is expressed in terms of the idea of *conservation*? The answer is the covert assumption that all the valid deducibility statements which do not contain and concern any particular connective or molecular sentence form are already at hand; and being consistent with the context defined by *these* non-specific deducibilities will naturally mean not bringing forth any more non-specific deducibility – hence conservation of the 'structure'. Prior's *tonk*, Belnap shows, is simply not conservative in this sense; for this reason, it is not a genuine counter-instance for a rule-based or inferentialist conception of meaning.[16]

Tonk's problematic interaction with the deducibility context of the target system can be captured in simpler terms in the following way. Now the core problem posed by *tonk*, at an attentive first glance, is that it validates the inference from an *arbitrary* formula to another; so the fact that it validates the inference from any formula to its contradictory (Prior's own example[17]) remains an instance of this core problem. How is this idea of arbitrariness projected onto the object-language level? Primarily as the *inferential isolation* assumed between any two *atomic* formulae, say *p* and *q*. But one can then argue in principle that the distinction between any two atoms, or the whole notion of atomicity in logic itself, can be explained in terms of this inferential isolation which they express.

First of all, this state of inferential isolation which we assume to be found between any two atoms, but which can also be found among molecular formulae, is a special relation defined (determined) by a number of deducibility claims of a certain type: the relation between (arbitrary) A and B such that, *neither from the truth nor from the falsity of A can be inferred the truth or the falsity of B (vice versa)*. So this special relation can be characterized by means of the following four indeducibility claims: A⊬B; A⊬~B; ~A⊬B; and ~A⊬~B. (We can obtain the converse of each form by means of the contrapositives and a double negation rule, so add: B⊬A;

[16] Belnap, "Tonk, Plonk and Plink," 132.

[17] Prior, "The Runabout Inference-Ticket," 39.

B⊬~A; … But of course this requires the introduction/justification of a contraposition rule for the (meta-) connective of indeducibility, '⊬'.) Actually, this is only one of a series of relations, all of which are defined by means of the same types of (in)deducibility. The common abstract form of these types of (in)deducibility can be represented in the following manner:

(Given that A^{+}: A; A^{-}: ~A, $\vdash^{+}$: ⊢, and $\vdash^{-}$: ⊬)

$$A^{\pm} \vdash^{\pm} B^{\pm}$$

There are three polarities in this schema: the two *aletheia* (i.e. truth-falsity) polarities of *A* and *B*, and the deducibility-indeducibility polarity inbetween. If we decide to neutralize the last polarity by means of assigning '1' or '0' to each of the four $\vdash^{+}$-statements obtained from this schema, we can easily tabulate the 16 relations which are specified by means of such assingments. Now note, particularly, the following seven of them:

	VII	VIII	X	XII	XIV	**XV**	**XVI**
A⊢B	1	1	0	0	0	0	0
A⊢~B	0	0	1	1	0	0	0
~A⊢B	0	0	1	0	1	0	0
~A⊢~B	1	0	0	0	0	1	0

It is immediately clear that relations VII and X here correspond respectively to *logical equivalence* and *contradiction*. Four of the remaining five relations correspond to the 'sides' of the traditional *square of opposition*: VIII corresponds to *subalternation*, XII to *contrariety*, XIV to *subcontrariety*, and XV to *superalternation*.[18] However, relation XVI is the most significant one here, for it underlines a fact we can easily disregard: even the state of inferential isolation which we assume to be found between any two logical (i.e. inferential) atoms *is* a kind of inferential relation. Whenever one employs two or more atomic letters, say 'p' and 'q', in the context of a proof, one covertly accepts a range of (in)deducibility claims which together fix this relation between the relevant formulae, namely: p⊬q; p⊬~q; ~p⊬q; and ~p⊬~q. (One might even argue that expressing this relation is the primary function of logical atoms.)

Thus, from an abstracting perspective, the problem posed by *tonk* for the deducibility context reduces to the presence of two deducibility claims of the form A⊢B and A ⊬B in the system after the extension. What made *tonk* a striking counter-instance was that it baptized proof-theoretical transition from a logical atom to another, thereby from anywhere to anywhere; but what makes, in turn, this kind of transition unacceptable is nothing other than the fact that it will lead to the acceptance of the two poles of the $p\vdash^{\pm}q$ polarity together to the system. The negative pole, p⊬q, was already there, albeit covertly, thanks to the assumption of inferential isolation written into the notion of *logical atom*; the positive pole, is then accepted via the inference rules that define *tonk*, along with the structural rule of transitivity. Nothing more than this simple fact is required for a connective to be inconsistent with a deducibility context. Therefore, the problem, if any, is not really posed by *tonk* in particular.

[18] These correspondences with traditional relations of opposition can be established, of course, only if we decide to abstract from the categorical propositional forms, primarily for which these relations are defined in traditional logic.

The 16 relations seven of which are tabulated above can be dubbed *alethic inferential relations* – or *alethic relations* in short – since the four type of deducibilities (or proof-theoretic consequences) which define them vary with respect to each other in terms of the two *aletheia* polarities of the followed and following formulae.[19] The problem supposedly posed by *tonk* is simplified into the clash of polar deducibility claims, one of which is introduced into the system by the definition of a special alethic relation covertly assumed to hold between logical atoms, the other by the 'meaning defining' inference rules of a novel connective (plus the transitivity of deducibility). The clash of these poles, which is the only problem here, seems to have nothing to do with the question *how* these poles, i.e. the clashing deducibility claims, are provided: by means of an antecedently approved alethic relation, or by means of the I-E rules attached (with a defining role or not) to a novel connective etc. So the difference between extending the system with the meaning defining I-E rules of a *tonk*-like connective on the one hand, and extending it by simply adding to it a deducibility claim, say, $p \vee q \vdash p \& q$, which will lead to the same kind of clash, will not have any relevance to the discussion about the inferentialist conception of meaning.

3. Final Note on Truth-Tables and Representationality

Thus, the fact that novel (inferential) meanings can be introduced to a given logical system, which are inconsistent with the present ones or (more directly) with antecedently approved (in)deducibility claims, is not a real threat to a rule-based or inferentialistic conception of meaning. However, J. T. Stevenson, in order to complete Prior's challenge to a sound argument against the inferentialist conception, reads this inconsistency in the following way: each one of our unproblematic connectives correspond to a distinct truth-function that can be exhibited by a unique truth-table (more precisely, by a unique column in a truth-table); but *no* truth-table *could* exhibit the so-called inferential meaning, i.e. the I-E rules together, of *tonk*. This shows according to Stevenson that truth-functions as they are exhibited in tables, and not inference rules, constitute the real semantic ground for connectives, because unlike rules, function tables are able to rule out *tonk*-like 'connectives' as meaningless in advance.

In order to aim at inference rules in particular, Stevenson notes that while the introduction and elimination rules that are to 'define' a connective can be obtained from distinct sources and brought together by fiat (as is the case with *tonk*), the same cannot always be done with these sources themselves, namely the truth-functions that validate those rules. For instance, the introduction and elimination rules of *tonk* can be validated, respectively, by A*B and A%B, which, however, cannot be synthesized into a single truth-function represented by a single column:

[19] Here, the appeal to the 'semantic' (i.e. model-theoretical) notion of truth-falsity in my inferentialism-based simple analysis should not give the reader an impression of *impurity*, for this notion, just like the algebraic assignments of '1' and '0's on the tables, is employed solely for expressive purposes; this is in line with certain 20th century deflationary conceptions of truth – such as *minimalism*, *prosententialism* etc. – which underline the expressive roles played by alethic locutions (e.g. the truth predicate, the truth operator etc.) in language, and which can remain neutral to some degree with respect to the inferentialism-representationalism divide. However, see, for instance, the minimalist Paul Horwich's works, for how deflationism about truth and a use-conception of meaning can interact in favour of both: Paul Horwich, *Reflections on Meaning* (New York: Oxford University Press, 2005).

A	**B**	**A*B**	**A%B**	
1	1	1	1	1
1	0	1	0	?
0	1	0	1	?
0	0	0	0	0
		A ⊢ A*B	***A%B ⊢ B***	***A ⊢ A*%B*** ***A*%B ⊢ B***

Thus, concludes the argument, the ground of meaning for logical constants, particularly for logical connectives, is ultimately representational/denotational: the connective gathers its meaning from the truth-function it represents (if any), period. An obvious weakness in this argument is that it employs two truth-functions that merely validate (or justify, or even more loosely, suggest) the rules that 'define' *tonk*. But if we wish to represent these rules and their inferentialistic synthesis by means of columns that *correspond* to them *exactly*, we would have instead something like:

A	**B**	**A*B**	**A%B**	**A*%B**
1	1	1	?	1
1	0	1	0	?
0	1	?	?	?
0	0	?	0	0
		*A ⊢ A*B*	*A%B ⊢ B*	*A ⊢ A*%B* *A*%B ⊢ B*

If the representational ground of meaning is to be the truth-table (in the abstract), which is designed to exhibit *all and only* truth-functions, then any expression to which one and only one of the above two rules is attached should also be ruled out as meaningless; however, neither of these rules in isolation leads to a *tonk*-like result for the system. Now some of the question marks on the above table signify a truth-value *gap* while others a truth-value *glut*. For instance, because no truth-value for A*B can be calculated when A takes the value 0, the last two cells of the relevant column are gaps; but the second cell on the column of A*oB is a glut, since both the truth and the falsity of A*oB can be obtained in the second row, by means, respectively, of A's truth and tonk-I, and of B's falsity and tonk-E (and of course contraposition).

In order for the truth-table to be genuinely superior (as a semantic ground) to I-E rules, it should be able to rule out, without the aid of these rules, all and *only* tonk-like connectives as meaningless. Then maybe the criterion should be modified so as to rule out only columns that contain at least one glut, not widely columns that contain at least one glut *or gap*. But why? What does meaningfulness have to do with the impermissibility of placing both '1' and '0' in one and the same cell? Put in another way: what is that semantically significant difference between columns that contain gaps but not any gluts, and columns that contain gluts, which render the latter but not necessarily the former meaningless, a difference which cannot in principle be captured by inference rules?

The key claim in Stevenson's argument was that one could not (in principle) construct an appropriate truth-table column for the I-E rule couple that 'define' tonk; it is now clear that this is too coarse a criterion unless columns like A*B and AoB, columns that contain gaps but not any gluts, can be saved for some reason – so the representationalist should answer the above question. Now if the representationalist chooses to give this answer by reference to the structure of the truth-table in the abstract, then he/she would be calling non-representational grounds to aid again: the definition of the table-drawing practice (for classical logic) contains the *rule* that at most one truth-value could be written into a cell – simply another rule. Alternatively, the representationalist might choose to continue his/her argument with the claim that gluts are ruled out as killers of meaning on the grounds of the meanings of *true* and *false*, and that these latter can be given (ultimately) a representational explanation or analysis. In this case, however, he will be expected as before to show how the (representationally explicable) *classical* meanings of true and false rule out, not truth-value gaps, but only gluts – i.e. how these meanings will permit a proposition to be not true and not false at once – which does not look like a straightforward task.

Conclusion

From the abstract perspective taken above, the problem posed by *tonk* for the deductive system which it extends reduces to a simple clash of two polar deducibility claims found in the system after the extension. In the *tonk* instance, one of the poles is provided by a special alethic relation covertly assumed to hold between any two logical atoms, the other by the introduction-elimination rules for *tonk*; so the clash is not necessarily the clash of inferentially definable but representationally ruled-out meanings on the one hand with a stable deductive system on the other – any kind of addition, *via inferential meanings or not*, to any system that will lead to the validation of polar $A \vdash^{\pm} B$s at once will achieve the exact same result. For this reason, Prior's challenge by itself does not pose any threat to the inferentialist conception of meaning, supported in whichever way by arguments from truth-functions and/or truth-tables. The above discussion was meant to show that the problem did not particularly concern the logical inferentialist conception of meaning – but maybe it does not concern *meaning* at all.

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