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RESEARCH ARTICLE

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Numerical Analysis for Investigation of Hydraulic Fracturing Potential of the Rockfill Dam

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Abstract

Embankment dams may collapse because of internal erosion that develops in the crack developed in the upstream-downstream direction by hydraulic fracturing. It is known in the literature that many dams collapsed due to hydraulic fracturing. The hydraulic fracturing mechanism is defined as the propagation of an existing crack on the upstream face of the clay core under hydrostatic stresses or the formation of a new crack in low-stress zones by hydrostatic stress. The variety of materials and materials' mechanical properties generally affect the hydraulic fracturing potential. This study examined the effect of the deformation parameters (Elasticity modulus and Poisson ratio) of the impermeable curtain-function clay core material on the hydraulic fracturing potential. Normal Stress and Mohr-Coulomb methods were used to determine the hydraulic fracturing potential. The principal stress values required for these two methods were determined for the maximum cross-section of the clay-core rockfill type Çınarcık Dam by the finite element method. While the hydraulic fracturing potential is negligibly affected by the change of deformation parameters in the Normal Stress method, this effect is clearly seen in the Mohr-Coulomb method.

Keywords: Embankment dam; rockfill dam; hydraulic fracturing; Elasticity modulus; Poisson ratio.

1. Introduction

The concept of hydraulic fracturing is a complex problem that arises in embankment dams and a helpful process used to extract underground energy sources. Hydraulic fracturing (hydrofracking or hydrofracturing) is the technique

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of injecting water and chemicals into the bedrock formation at high pressure through a typical well to create new cracks or to increase the size and connectivity of existing cracks to extract gas and oil from the formation [1]. The most general description of hydraulic fracturing for embankment dams is the growth of existing cracks in low-stress zones on the upstream surface with the impact of the hydrostatic pressure in the reservoir or the formation of a new crack with the same effect [2,3]. Hydraulic fracturing can theoretically occur in homogeneous earth-fill dams but is more likely to occur in zoned earth-fill dams when materials have different deformation and permeability characteristics [4]. Accordingly, it is an issue that threatens dam safety by causing internal erosion with the formation of transverse and horizontal cracks in the upstream-downstream direction, especially in clay-core rockfill dams [5,6]. Balderhead, Hyttejuvet, Viddalstavn, Teton, and Yard's Creek Dams are world-renowned for observing hydraulic fracturing [7].

As mentioned above, stress conditions in the environment where the failure occurs are crucial for hydraulic fracturing. Hydraulic fracturing potential also increases where transverse, longitudinal, and local arching occurs in embankment dams [8]. Because the elasticity modulus of the clay core in clay core rockfill dams is less than that of the rock utilized in the shell, the clay core will want to settle more due to the loads coming from above. For this case, vertical loads are transmitted to the shell, and transverse arching occurs. This leads to a vertical stress reduction, particularly in the portion of the clay core close to the shell. Longitudinal arching is another type caused by stress transfer to the abutments in dams constructed in narrow valleys. On the other hand, local arching occurs at the contact surface of the outlet conduit and spillway with the embankment. In addition, low-stress zones may occur due to irregularities, discontinuities, and differential settlements that will develop from surfaces with a stepped profile on the abutment [9,10]. If the dam foundation has compressible alluvial soil, transverse cracks may develop due to low-stress zones resulting from differential settlements [11]. Tension zones may occur when arching and differential settlements occur, and tension cracks may develop [12].

There are different failure approaches in the literature for the mechanism of hydraulic fracturing, defined as failure due to hydrostatic pressure in dams. The cracks expanded under shear failure [13-15] and under tensile failure [3,16-19], as well as cases where both shear and tensile failure occur together [20-22], are also available. There are also conflicting approaches regarding the presence or absence of pre-existing fractures in the embankments for hydraulic fracturing development. Therefore, two critical conditions for hydraulic fracturing initiation come to the fore. These are the low-stress zones formed due to arching and differential settlements that facilitate the development of hydraulic fracturing and the state of the existing cracks occurring due to the same factors [23]. According to Wang [24], hydraulic fracturing development requires rapid impounding, unsaturated core soil with low permeability and an upstream-downstream crack.

The method for determining the hydrostatic pressure that causes hydraulic fracturing is divided into three categories: empirical methods based on field or laboratory experiments [3,14,25], a theoretical model based on the progression of the circular cylindrical cavity under triaxial stress conditions [26], and the conceptual model based on fracture mechanics theories [27]. In addition, the hydraulic fracturing potential can be estimated by Nobari et al., [17] known as the normal stress criterion. Knowing the stress conditions developed in embankment dams for the normal stress criterion approach and empirical and theoretical models in estimating the hydraulic fracturing potential is necessary. For this reason, 2D or 3D numerical analyses must be performed for embankment dams when evaluating the hydraulic failure potential.

In recent years, hydraulic fracturing has been widely evaluated through studies based on finite element method numerical analysis [28,29]. For dams where hydraulic fracturing is observed, hydraulic fracturing is provided by numerical analysis [30-32].

In this paper, finite element analyses were performed to obtain the effect of the clay cores' deformation parameters on the behaviour of hydraulic fracturing of a clay-core rockfill dam. Numerical analyses containing 18 different combinations of Elasticity modulus and Poisson ratio of clay core were used to calculate the principal stress values required to determine hydraulic fracturing potential. After the analyses, it was seen that the result based on the Mohr-Coulomb criterion was affected by the clay core deformation parameters; in the normal stress criterion, this effect was determined to be negligible.

2. Material and method

In this paper, the cross section of the Çınarcık Dam, located in Bursa, the fourth largest city in Turkey, was used to investigate the influence of the deformation parameters (E,ν) of the clay material used as an impervious core on the hydraulic fracturing potential. Çınarcık Dam was built on the Orhaneli stream and has a height of 125. m from the thalweg. It is located south of the Marmara Region at the coordinates of 40.01611°N and 28.77389°E (Fig. 1). It was operated in 2002 as a clay-core rockfill dam for drinking water, agricultural irrigation, and energy production. In the granular filter design of Çınarcık Dam, a crack-stopper filter arranged in the form of sand-gravel-rock rubble from fine to coarse material was used. Crack-stopper filters are recommended to prevent collapse due to internal erosion, especially in cracks formed in the upstream-downstream direction that may develop due to seismic ground movements [33].

Fig. 1. Location of Çınarcık Dam.

Normal stress and the Mohr-Coulomb criterion were used to determine the hydraulic fracturing potential within the scope of the study. The maximum cross-section of Çınarcık Dam is illustrated in Fig. 2.

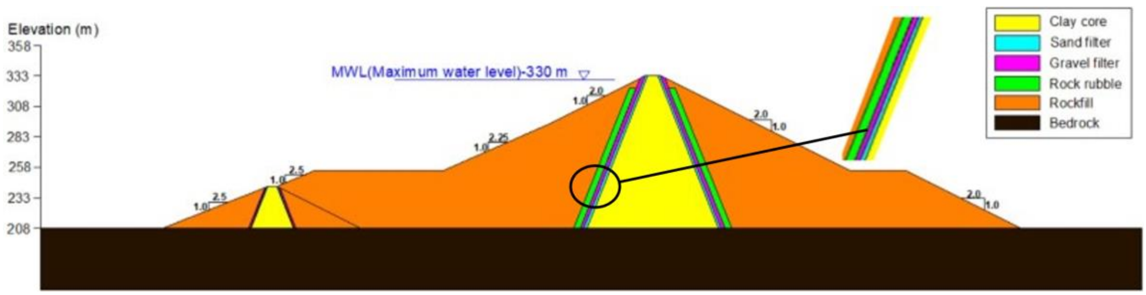

Fig. 2. Maximum cross-section of Çınarcık Dam.

2.1. The Normal Stress Criterion (Nobari et al. [17])

The normal stress criterion (NSC) compares the major principal stresses occurring on the upstream face of the clay core with the hydrostatic stresses acting on this face. When calculating the hydrostatic stresses, the maximum water level is considered the origin point. If the factor of safety (FS) is calculated when the maximum principal stresses (σ_1) at any level are in proportion to the hydrostatic stress (P_h) resulting from the reservoir water level is less than 1, hydraulic fracturing potential can be mentioned (Eq.1).

$$\mathrm{FS} = \frac{\sigma_1}{P_h} \tag{1}$$

In the normal stress criterion, the walls of the cracks formed by the action of hydrostatic stresses are assumed to be in the horizontal plane. In this method, based on the principle of tensile failure, the tensile strength of clay soils is conservatively considered to be 0.

2.2. Mohr-Coulomb Criterion (Yanagisawa and Panah [15])

According to the Mohr-Coulomb criterion (MCC), it is stated that the planes where hydraulic fracturing occurs are shear failure planes based on total stress. In this method, hydrostatic stresses (P_f) that cause hydraulic fracturing, undrained shear strength parameters of clay soils (ϕ_u: undrained internal friction angle, c_u: undrained interception of cohesion), and major (σ_1) and minor (σ_3) principal stresses are considered. It is calculated as shown in Eq.2. When the calculated hydrostatic stresses (P_f) are in proportion to the reservoir-derived hydrostatic stresses (P_h) acting on the upstream face, the factor of safety obtained with FS= P_f/P_h is less than 1. Then, it is accepted that hydraulic fracturing will occur in the shear plane.

$$P_f = (1.5\sigma_3 - 0.5\sigma_1) \times \left(1 + \sin\phi_u\right) + 2c_u . \cos\phi_u \tag{2}$$

Undrained shear strength parameters must be considered realistically for this method to determine the hydraulic fracturing potential accurately.

3. Numerical Analysis

Geostudio 2018/R2 program was used for the principal stress values required to determine hydraulic fracturing potential using the methods described above. The analyses were carried out under two-dimensional plain strain conditions. The finite element method was applied as the calculation method. Analyzes were carried out in 18 different combinations to investigate the change in hydraulic fracturing potential if the clay core material had different deformation parameters (Table 1). These combinations include six different elasticity modulus (15000, 20000, 25000, 30000, 35000 and 40000 kPa) and three Poisson ratio (0.35, 0.40 and 0.45) values of core material. Material properties of foundation, shell, and transition zones were kept constant.

Table 1. Deformation parameters considered for clay core material.

Model No	Elasticity Modulus, E (kPa)	Poisson ratio, ν	Model No	Elasticity Modulus, E (kPa)	Poisson ratio, ν
1	15000	0.35	10	30000	0.35
2	15000	0.40	11	30000	0.40

3	15000	0.45	12	30000	0.45
4	20000	0.35	13	35000	0.35
5	20000	0.40	14	35000	0.40
6	20000	0.45	15	35000	0.45
7	25000	0.35	16	40000	0.35
8	25000	0.40	17	40000	0.40
9	25000	0.45	18	40000	0.45

The design parameters of the materials in the foundation, shell, and transition zones in the embankment dam cross-section were taken from the literature to reflect their characteristic values. These material zones are defined by the linear-elastic material model (Table 2). The Mohr-Coulomb material model was chosen for the clay core. Literature was adhered to for undrained shear strength parameters, and reasonable values were taken (c_u=75 kPa; ϕ_u=10^o).

Table 2. Design parameters of materials in foundation, shell, and transition zones.

Material	Elasticity Modulus, E (kPa)	Poisson ratio, ν	Unit weight (kN/m^3)
Bedrock	20x10^6	0.15	16.0
Rockfill	20x10^4	0.25	20.0
Rock rubble	50x10^3	0.28	19.5
Gravel Filter	40x10^3	0.30	19.0
Sand Filter	30x10^3	0.30	19.0

The maximum cross section of Çınarcık Dam was transformed into a geometric model consisting of 1498 nodes and 1440 quads&triangles elements in numerical analysis (Fig. 3). The geometric model defines boundary conditions around the bedrock, excluding the reservoir and downstream sides. While the base of the bedrock is limited to displacement in both directions, the sides of the bedrock are limited to displacement only in the horizontal direction.

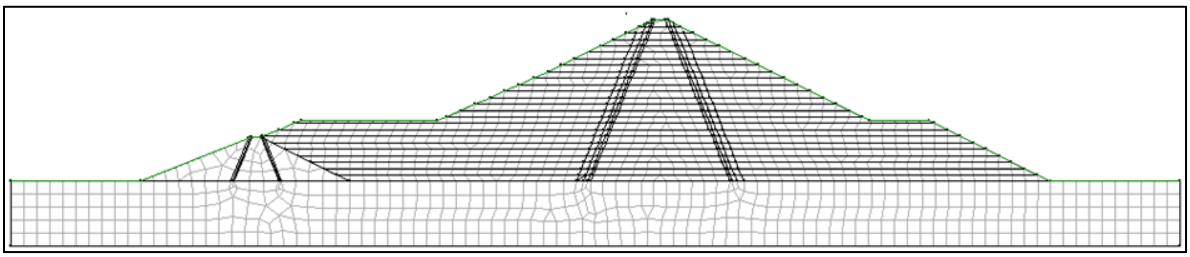

Fig. 3. Geometric model for maximum cross-section of Çınarcık Dam.

The numerical analysis considered the stress-deformation behaviour observed during the embankment dam construction. For this purpose, the analysis stages were carried out as follows:

• Modeling the stress conditions in the underlying bedrock.
• Complete modeling of the upstream cofferdam.
• Creation of stresses by modeling the main dam fill in 25 layers step by step.
• Water retention in the reservoir in 3 stages and modeling of hydrostatic stresses.

In addition, at each reservoir impounding stage, buoyancy forces are applied to the materials in the shell and transition zones below the reservoir water level by calculating the submerged unit weights. At each stage, the analysis is continued by reducing the elasticity modules by half for the shell and transition zone materials.

4. Analyses Results

4.1. Stress and Deformation

Several analyses, including 18 combinations given in Table 1, were carried out for the determination of stresses and deformations in the body of Çınarcık Dam. To not exceed the page limit, all combinations' stress and deformation contours are not given; the contours of Model 18 are provided as an example in Fig. 4-7. As seen in Fig. 4, the maximum total stress after the water impoundment occurred at the bottom of the shell and adjacent to the corners of the core. Also, the reduction of maximum total stress in the clay core can be seen in Fig. 4.

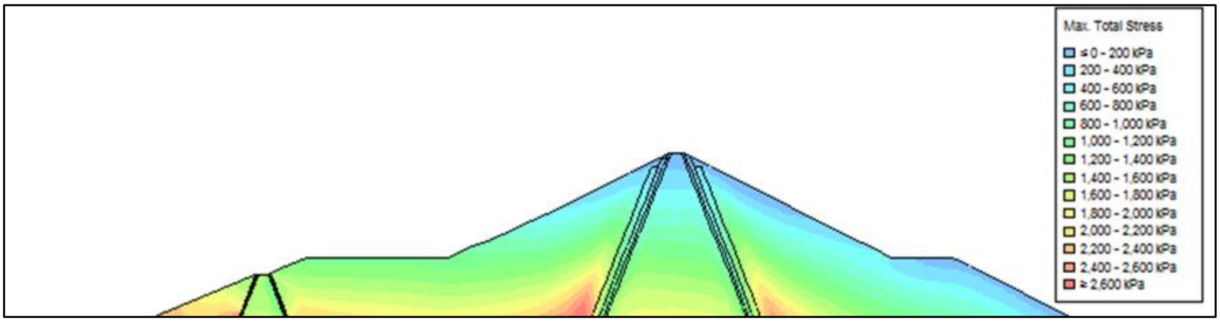

Fig. 4. Contour of the maximum total stress.

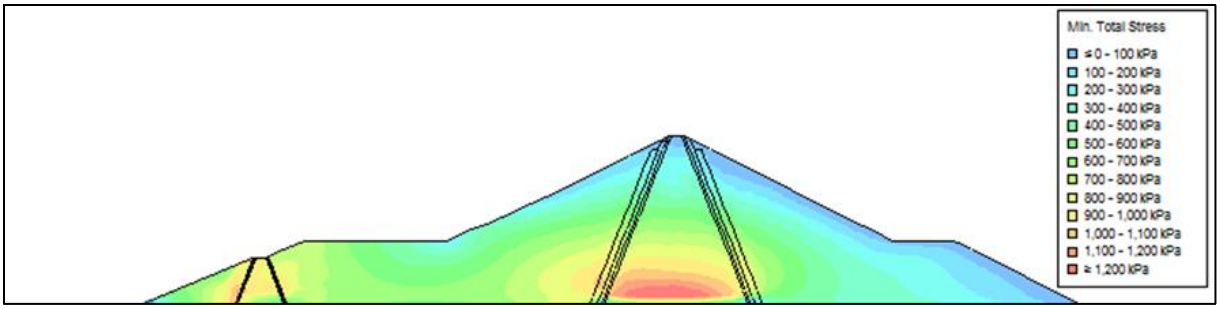

Fig. 5. Contour of the minimum total stress.

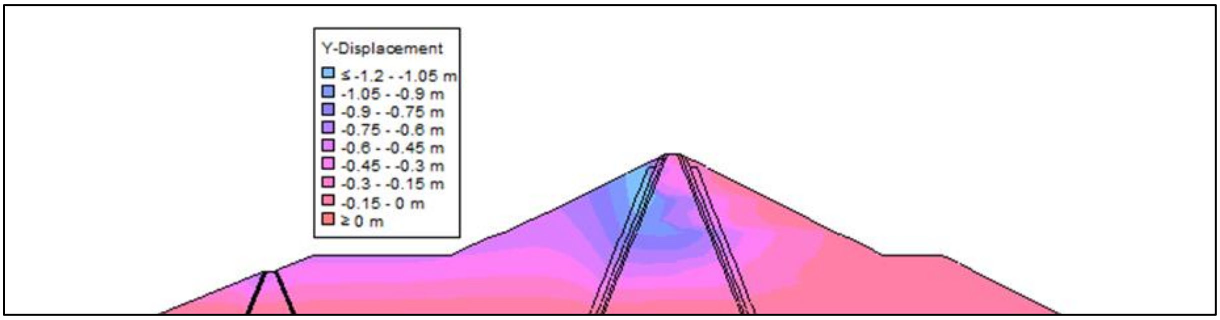

Fig. 6. Contour of the vertical deformations.

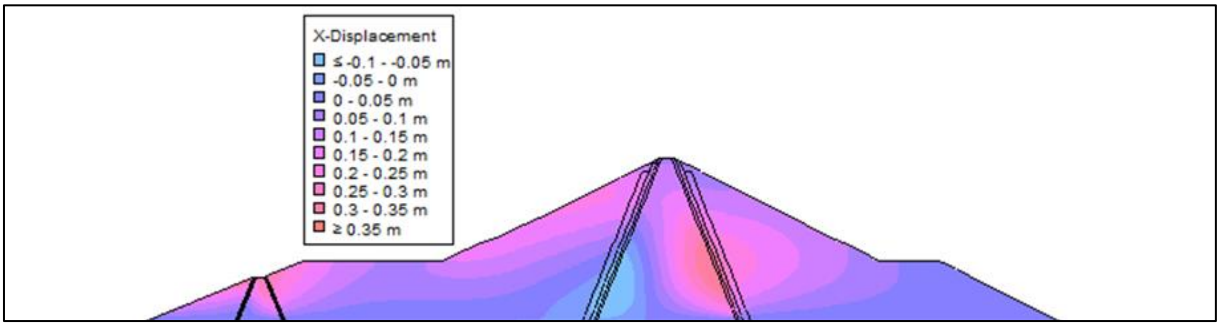

Fig. 7. Contour of the horizontal deformations.

4.2. Hydraulic Fracturing

To determine the factor of safety against hydraulic fracturing to occur on the upstream face of the clay core, major and minor principles stress values obtained from finite element analyses discussed above were used. Calculated variations of the factor of safety against hydraulic fracturing of the Çınarcık dam with depth are illustrated in Fig. 8. The graphs in the upper part of Fig. 8. were obtained according to the MCC, and the graphs in the lower part were obtained by using NSC.

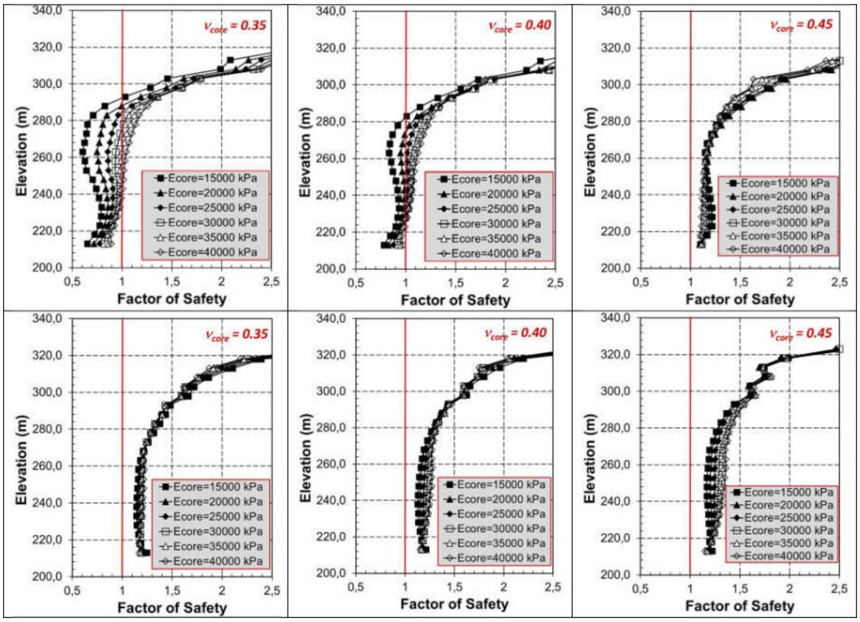

Fig. 8. Variation of a factor of safety against hydraulic fracturing with depth for constant poisson ratio values.

As can be seen in Fig. 8, there is no hydraulic potential for the selected Elasticity Modulus and Poisson ratio values for the clay core based on NSC. Contrary to NSC, in the MCC, it is understood that the factor of safety values is affected by the deformation parameter values of the clay core, especially at the middle and lower levels of the dam. In the MCC, the factor of safety values also increases with the increment of the elasticity modulus and Poisson ratio. But this increase is not noteworthy for the case where the Poisson ratio is 0.45.

To more clearly interpret the variation of the factor of safety values based on the MCC and NSC method, Fig. 9 and Fig. 10 were illustrated, respectively. According to Fig. 9, in cases where the elasticity modulus is 15000 kPa, 20000 kPa, and 25000 kPa, rising the Poisson ratio from 0.35 to 0.45 increases the F.S. values. In comparison, this increase remains limited when the elasticity modulus is 30000 kPa, 35000 kPa, and 40000 kPa. Another important point in Fig. 9 is that with the increase in the elasticity modulus, the zone of the hydraulic fracturing potential becomes smaller, and hydraulic fracturing is expected to occur, especially in the lower part of the dam. When the factor of safety values calculated based on NSC was investigated, it was seen that there was no hydraulic fracturing potential.

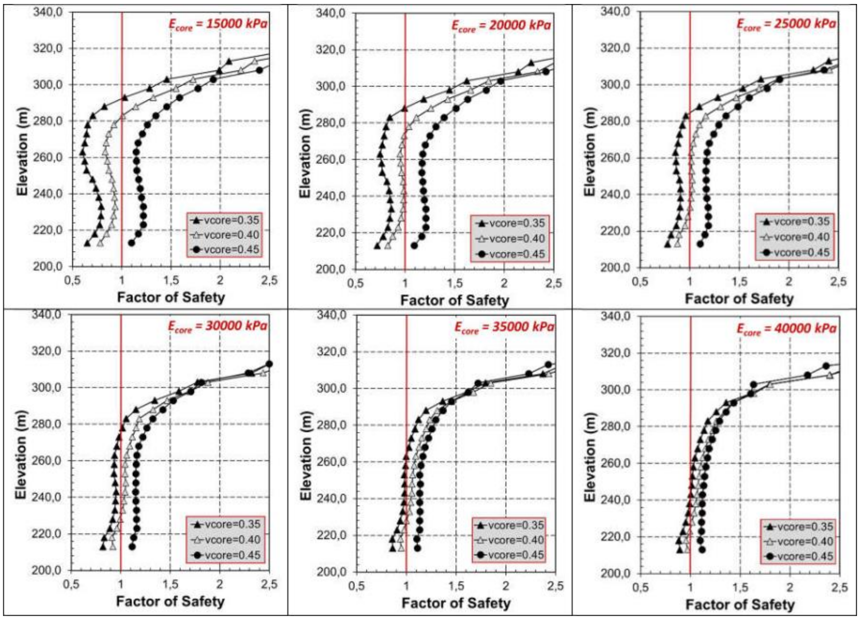

Fig. 9. Variation of the factor of safety against hydraulic fracturing with depth for constant elasticity modulus values by using MCC.

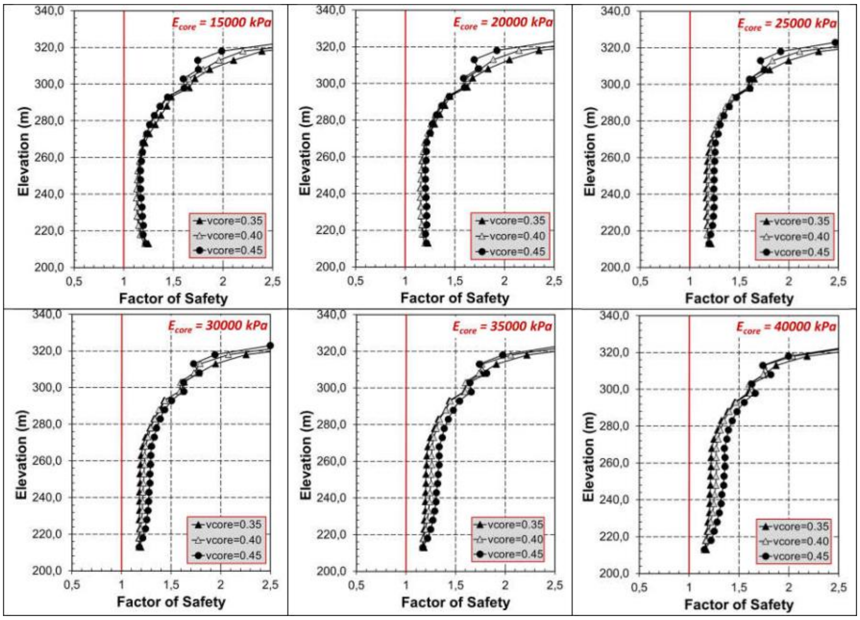

Fig. 10. Variation of the factor of safety against hydraulic fracturing with depth for constant poisson ratio values by using NSC.

Minimum values of the factor of safety for 18 different combinations and the location of the point corresponding to this value are given in Table 3. According to the MCC method, the minimum factor of safety values varies between 0.603 and 1.113. In the same method, the points having minimum factor of safety values are located between 0.040 and 0.080 of the z/H value except for Model 1 (E=15000 kPa and ν=0.30). This difference can be explained by the fact that Model 1 has the smallest Elasticity Modulus and Poisson ratio in the clay-core among 18 different combinations. It should be stated that minimum value of the factor of safety are also obtained for Model 1. For the NSC method, the factor of safety values is greater than 1.0 in 18 different combinations.

Table 3. Minimum factor of safety values and corresponding location as depth (z/H).

Model No	MCC*		NSC**		Model No	MCC		NSC	
	FS_{min}	z/H***	FS_{min}	z/H		FS_{min}	z/H	FS_{min}	z/H
1	0.603	0.440	1.143	0.240	10	0.821	0.040	1.176	0.080
2	0.780	0.040	1.128	0.240	11	0.911	0.080	1.174	0.040
3	1.098	0.040	1.153	0.040	12	1.113	0.040	1.178	0.040
4	0.719	0.040	1.154	0.240	13	0.857	0.080	1.173	0.080
5	0.827	0.040	1.153	0.240	14	0.939	0.080	1.163	0.040
6	1.092	0.040	1.176	0.040	15	1.105	0.080	1.167	0.040
7	0.775	0.040	1.167	0.280	16	0.885	0.080	1.167	0.080
8	0.877	0.040	1.176	0.080	17	0.956	0.080	1.151	0.040
9	1.106	0.040	1.193	0.040	18	1.097	0.080	1.152	0.040

* MCC: Mohr-Coulomb Criterion

** NSC: Normal Stress Criterion

*** z: Height from dam foundation (m)   H: Dam height (m)

5. Conclusions

This paper investigated the influence of deformation characteristics of clay core on the behaviour of hydraulic fracturing of a rockfill dam. For this purpose, eighteen combinations including six elasticity modulus values and three Poisson ratio values were established. It should be stated the geometry and other zones of the dam was kept just to see the effect of clay core material in the series of numerical analysis. Numerical analysis shows that:

- According to the MCC method, the minimum factor of safety values varies between 0.603 and 1.113.
- The factor of safety values is greater than 1.0 in the NSC method for all combinations.
- The factor of safety values is affected by the Elasticity Modulus and Poisson ratio values of the clay core, especially at the middle and lower levels of the dam for the MCC method.
- The factor of safety values also increases with the increment of the Poisson ratio and elasticity modulus. But, this increase is not noteworthy for the case where the Poisson ratio is 0.45.
- The zone of the hydraulic fracturing potential becomes smaller and hydraulic fracturing is expected to occur especially in the lower part of the dam with the increase in the elasticity modulus in the MCC method.

Results given above show that, hydraulic fracturing potential is closely related deformation characteristics of clay core. It is not possible to claim that this is the only effect. In the future, when more detailed studies including variations in the properties of the transition zone and shell material with the variations in the clay core parameters will be performed, results will be guiding the professionals working on dam engineering.

Acknowledgements

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