Konya Mühendislik Bilimleri Dergisi, c. 11, Özel Sayı, 23-31, 2023
Konya Journal of Engineering Sciences, v. 11, Special Issue, 23-31, 2023
ISSN: 2667-8055 (Electronic)
DOI: 10.36306/konjes.1384835

BENDING ANALYSIS OF A PERFORATED MICROBEAM WITH LAPLACE TRANSFORM

[1,*] **Büşra UZUN**, [2] **Mustafa Özgür YAYLI**

[1,2] *Bursa Uludag University, Engineering Faculty, Civil Engineering Department, Bursa, TÜRKİYE*
[1] buzun@uludag.edu.tr, [2] ozguryayli@uludag.edu.tr

Highlights

- Laplace transform is applied to bending problem of the microbeam.
- Size dependency is considered with the modified couple stress theory.
- Perforation properties are investigated with size effects for the deflections of the microbeam.
- The results presented with this study can be used by designers to model the perforated micro or macro structural elements.

***Corresponding Author:** Büşra UZUN, buzun@uludag.edu.tr
The study was selected among the papers presented at the 23rd National Mechanics Congress of TUMTMK
(04-08 September 2023 Konya, TURKIYE)

Konya Mühendislik Bilimleri Dergisi, c. 11, Özel Sayı, 23-31, 2023
Konya Journal of Engineering Sciences, v. 11, Special Issue, 23-31, 2023
ISSN: 2667-8055 (Electronic)
DOI: 10.36306/konjes.1384835

BENDING ANALYSIS OF A PERFORATED MICROBEAM WITH LAPLACE TRANSFORM

1, * Büşra UZUN, 2 Mustafa Özgür YAYLI

1,2 Bursa Uludag University, Engineering Faculty, Civil Engineering Department, Bursa, TÜRKİYE
1 buzun@uludag.edu.tr, 2 ozguryayli@uludag.edu.tr

(Received: 01.11.2023; Accepted in Revised Form: 23.11.2023)

ABSTRACT: In recent years, the analysis of materials and elements with dimensions at nano/micro levels has gained momentum. While analyzing these small-scale materials and elements, higher-order elasticity theories have started to be used instead of classical elasticity theories (CETs). One of these theories, which includes a small-scale parameter in its constitutive equations, is the modified couple stress theory (MCST). In this study, the bending analysis of a cantilever perforated microbeam is investigated by MCST and Euler-Bernoulli (EB) beam theory. First, the perforation characteristics of the microbeam are described and incorporated into the equation governing the bending problem based on the modified couple stress theory found in the literature. Then, the Laplace transform is applied to the governing equation. The known boundary conditions of the cantilever microbeam are substituted into the equation and the inverse Laplace transform is applied to obtain the deflection equation.

Keywords: *Laplace transform, Perforated microbeam, Modified couple stress theory, Bending*

1. INTRODUCTION

Materials containing regular holes at equal intervals in a sequential manner are called perforated materials. In nuclear power plants, ships, offshore structures, micro-electro-mechanical systems and nano-electro-mechanical systems, a perforated structure is created due to design requirements [1-3]. One-dimensional bending elements made of perforated materials are also called perforated beams. Due to their wide range of applications, the analysis of perforated beams has recently attracted great interest.

Almitani et al. [2] have investigated the perforation effects on the multilayered beam structure. Luschi and Pieri [4] have presented a study investigating the parameters of perforated beams. Abdelrahman et al. [5] have studied the free and forced dynamics of perforated thick and thin beams. For this purpose, Euler-Bernoulli (EB) and Timoshenko beam theories have been considered and also, the rotary inertia effect has been investigated in the study [5]. Assie et al. [6] have examined the vibration response of a perforated Timoshenko beam affected by a moving load by using the Ritz method. Also in the study, the authors have adopted the Newmark average acceleration method to obtain the time response. Almitani et al. [7] have examined the free and forced vibrations of perforated thin beams by using the semi-analytical mixed Galerkin Laplace method. Eltaher et al. [8] have analyzed the vibrations and stresses of perforated rotated beams via a computational finite element model.

As can be seen from Refs. [5-8], free vibration, forced vibration and stress analyses of perforated beams are presented with various solution techniques. Then, the analysis of perforated beams at nano and micro scales attracted attention. Many authors who presented the analyses at these scales have carried out their studies by taking into account the theories of higher-order elasticity. Bourouina et al. [9] have considered the size effect based on the nonlocal elasticity theory in conjunction with EB and shear deformable beam theories for perforated nanobeams. Kerid and Bounnah [10] have investigated the pull-in voltage of perforated cantilever nanobeam (nanoswitch) via nonlocal elasticity theory. Kafkas et al. [11] have investigated the vibrational frequencies of perforated nanobeams affected by thermal loads based on the nonlocal strain gradient theory. Abdelrahman et al. [1,12] have presented the vibration and bending of perforated beams based on the modified couple stress theory (MCST).

***Corresponding Author:** Büşra UZUN, buzun@uludag.edu.tr
The study was selected among the papers presented at the 23rd National Mechanics Congress of TUMTMK
(04-08 September 2023 Konya, TURKIYE)

In this study, the MCST will be used as the size-effective theory and the Laplace transform will be used as the solution method. These two main themes are also adopted by the many researchers. Park and Gao [13] have proposed the modified couple stress solution of bending of the Euler-Bernoulli cantilever beam affected by a concentrated point load. Sobhy and Zenkour [14] have studied the static of viscoelastic nano-scaled beams resting on visco-Pasternak foundations via the MCST and different beam theories. Awrejcewicz et al. [15] have studied the thermoelastic vibrations of a nonlinear thick microbeam by using the MCST. Yaylı [16] has proposed the bending analysis of a homogeneous microbeam in conjunction with the Laplace transform and the MCST. Nazmul and Devnath [17] have explored the bending deflection of bi-directional functionally graded nanobeams based on the nonlocal elasticity theory and Laplace transform. Bian and Qing [18] have adopted the Laplace transform method to investigate the stability and dynamic of the nonlocal strain gradient integral model of the Bernoulli-Euler beam. Ike et al. [19] have applied the Laplace transform to solve the buckling problem of moderately thick beams. Li et al. [20] have studied the size-dependent bending response of the Euler–Bernoulli beam based on the Laplace transform.

In this study, the static equation of a perforated microbeam based on the MCST is solved using the Laplace transform. As a continuation of the work presented by Yaylı [16], this study aims to investigate the parameters describing the properties of a perforated microbeam and also to show the effect of size. The bending equation for the perforated microbeam including the filling ratio, the number of holes and the material length scale parameter is obtained. Then, the static displacement curves of a uniformly distributed loaded perforated microbeam are analyzed to show the effect of the filling ratio, number of holes and material length scale parameter.

2. MATERIAL AND METHODS

In this part of the study, for the first step, the calculation of the properties of a microbeam composed of perforated material will be shown. The properties of a microbeam made of perforated material depend on the number of holes and the filling ratio. Then the bending relations based on the MCST will be presented in a form adapted to the perforated microbeam. Finally, the formula for the deflection of a cantilever microbeam will be calculated by applying the Laplace transform.

2.1. Properties of perforated materials

Figure 1 shows a perforated microbeam with length L, width b and height h. To calculate the bending stiffness ($\overline{EI}$) of this perforated microbeam to be investigated, the following equation is presented [4]:

$$\overline{EI} = EI\frac{\beta(N+1)(N^2+2N+\beta^2)}{(1-\beta^2+\beta^3)N^3+3\beta N^2+\beta^2 N(3+2\beta-3\beta^2+\beta^3)+\beta^3} \tag{1}$$

In Equation (1), E represents the modulus of elasticity, I specifies the moment of inertia and is caculated by $I=bxh^3/12$, β is the filling ratio and N is the number of holes. Similarly, shear stiffness ($\overline{GA}$) is introduced as follows [4]:

$$\overline{GA} = EA\frac{\beta^3(N+1)}{2N} \tag{2}$$

Here, A is the cross-sectional area of the solid beam and is defined by A=bxh.

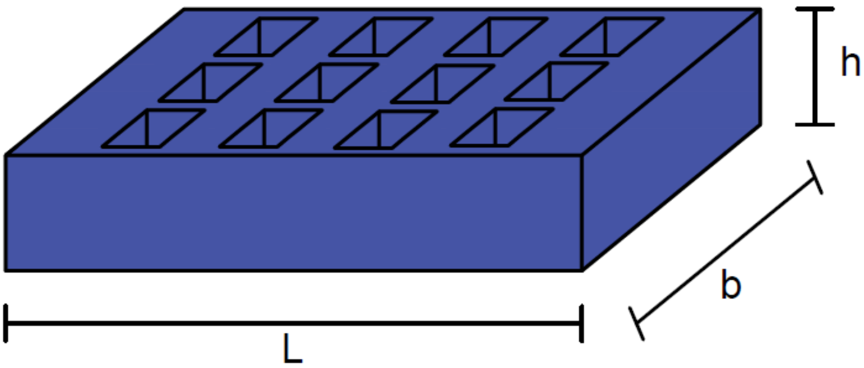

Figure 1. Demonstration of a perforated microbeam

2.2. Bending based on the modified couple stress theory

MCST is a higher-order elasticity theory presented by Yang et al. [21] and introduced to perform size effect-dependent analyses of nano/microelements. This theory defines the strain energy (U) for a linear elastic material as follows [21]:

$$U = \int_V (\boldsymbol{\sigma}:\boldsymbol{\varepsilon} + \boldsymbol{m}:\boldsymbol{\chi})dV \tag{3}$$

here, σ, ε, m and χ are the tensors of stress, strain, deviatoric part of couple stress and symmetric curvature, respectively. By using the defined parameters and displacement fields of Euler-Bernoulli beam, U is re-written as follows [13]:

$$U = -\frac{1}{2}\int_0^L M_y \frac{d^2w(x)}{dx^2}dx - \frac{1}{2}\int_0^L Y_{xy}\frac{d^2w(x)}{dx^2}dx \tag{4}$$

here, $w(x)$ is the transverse deflection, M_y and Y_{xy} are the resultant moment and couple moment, respectively and they are defined by [13]:

$$M_y = \int_A \sigma_{xx} z dA \tag{5}$$

$$Y_{xy} = \int_A m_{xy} dA \tag{6}$$

For a microbeam affected by transversely uniformly distributed loads, the work done by the external forces is defined as follows:

$$W = \int_0^L q(x)w(x)\,dx \tag{7}$$

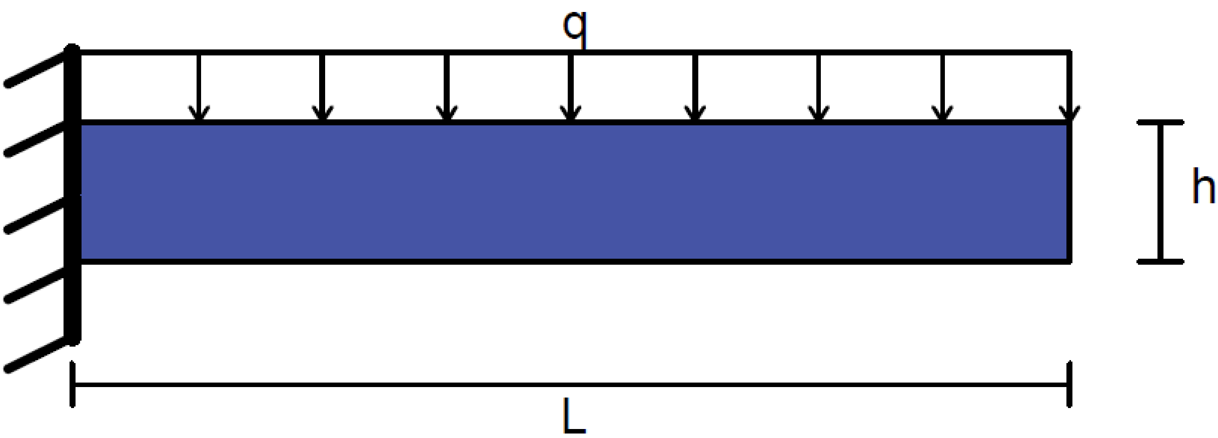

Figure 2. A cantilever perforated microbeam

The total minimum potential energy principle ($\delta\Pi = 0$) can be used to solve this problem and the following equation is established according to this rule:

$$\Pi = U - W = -\frac{1}{2}\int_{0}^{L} M_y \frac{d^2w(x)}{dx^2}dx - \frac{1}{2}\int_{0}^{L} Y_{xy}\frac{d^2w(x)}{dx^2}dx - \int_{0}^{L} q(x)w(x)\,dx \tag{8}$$

With the above equation, the fundamental lemma of calculus of variation is derived as follows [13]:

$$\frac{d^2M_y}{dx^2} + \frac{d^2Y_{xy}}{dx^2} + q(x) = 0 \tag{9}$$

here, M_y and Y_{xy} are defined by:

$$M_y = -\overline{EI}\frac{d^2w(x)}{dx^2} \tag{10}$$

$$Y_{xy} = -\overline{GA}l^2\frac{d^2w(x)}{dx^2} \tag{11}$$

here, l is the material length scale parameter and gives the small-scale effect to the problem. Also, the moment expression (M(x)) based on MCST is obtained as follows:

$$\overline{EI}\frac{d^2w(x)}{dx^2} + \overline{GA}l^2\frac{d^2w(x)}{dx^2} = M(x) \tag{12}$$

If the moment of the beam in Figure 2 is calculated [16]:

$$-\frac{q(x)L^2}{2} - \frac{q(x)x^2}{2} + q(x)Lx = M(x) \tag{13}$$

When this moment expression is substituted in equation (12), the following equation is obtained [16]:

$$\overline{EI}\frac{d^2w(x)}{dx^2} + \overline{GA}l^2\frac{d^2w(x)}{dx^2} = -\frac{q(x)L^2}{2} - \frac{q(x)x^2}{2} + q(x)Lx \tag{14}$$

If the Laplace transform is applied to both sides of equation (14):

$$(\overline{EI} + \overline{GA}l^2)(sw[0] + s^2w[S] - w'[0]) = -\frac{q(x)L}{s^2} - \frac{q(x)}{s^3} + \frac{q(x)L^2}{2s} \tag{15}$$

is found. From here, $w[S]$ is calculated as follows:

$$w[S] = \frac{-q(x)\left(2 + Ls(-2 + Ls)\right) + 2s^3(sw[0] + w'[0])(\overline{EI} + \overline{GA}l^2)}{2s^5(\overline{EI} + \overline{GA}l^2)} \tag{16}$$

When the boundary conditions $w[0]$ and $w'[0]$ are considered for the clamped end of the beam and the inverse Laplace transform is applied, $w(x)$ is found as follows:

$$w(x) = -\frac{q(x)x^2(x^2 + 6L^2 - 4Lx)}{24(\overline{EI} + \overline{GA}l^2)} \tag{17}$$

3. RESULTS AND DISCUSSION

In this section, using the deflection expression obtained in equation (17), the deflection curves of a perforated microbeam are investigated for various parameters. The material properties of the microbeam are chosen as follows: E=1.44 GPa, ν=0.38 [13]. The microbeam properties are as follows: b=17.6 μm, h=2b μm, L=10h μm. Also, l is considered as 17.6 μm. Finally, the following equation is used to calculate the shear modulus:

$$G = \frac{E}{2(1 + \nu)} \tag{18}$$

First, the effect of the filling ratio β on the deflection value of the cantilever perforated microbeam is analyzed by presenting Figure 3. For this example, four different filling ratios with N=4 are compared. It is seen that lower deflection occurs at higher β values. As the stiffness of the perforated microbeam increases as β increases, the deflection decreases.

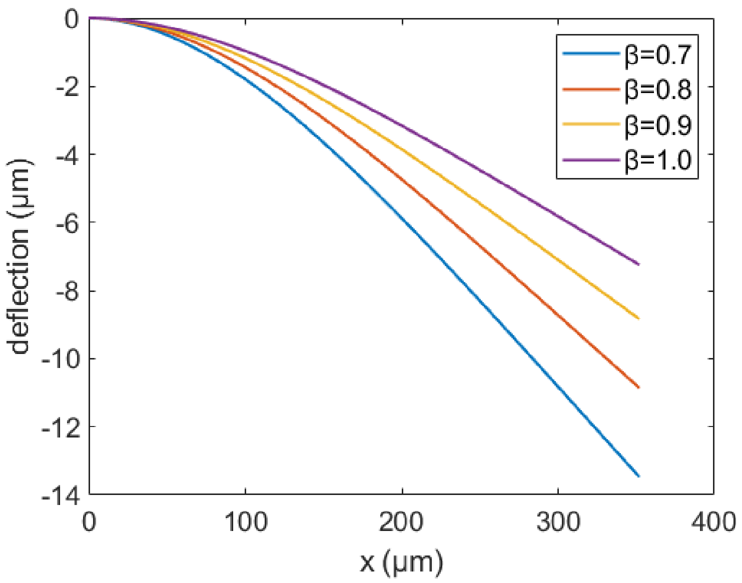

Figure 3. Effect of β on the deflection of a perforated microbeam

In the second example, the effect of the number of holes N on the deflection value of the cantilever perforated microbeam is examined in figure 4. For this example, four different N values with β=0.8 are compared. It is observed that lower deflection occurs at lower N values. Since increasing the number of holes reduces the rigidity of the microbeam, higher deflection values are obtained.

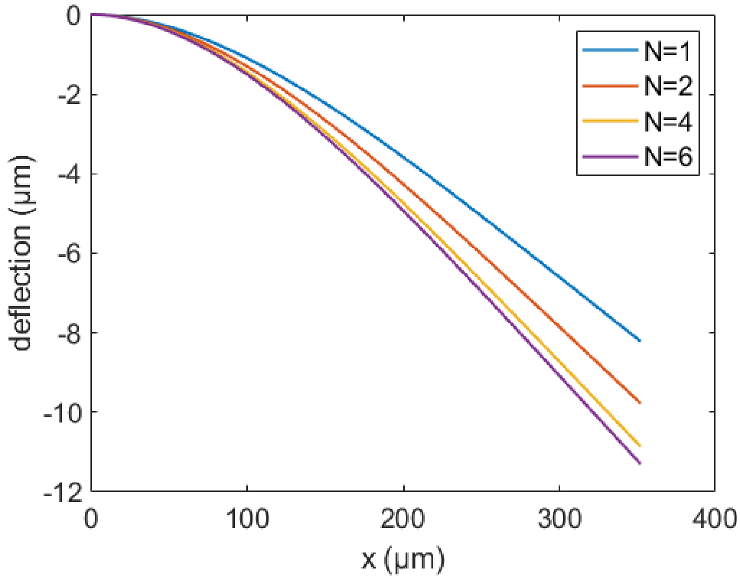

Figure 4. Effect of hole number on the deflection of a perforated microbeam

In the third example, the effect of the MCST on the deflection value of the cantilever perforated microbeam is examined in figure 5. For this example, deflection values based on classical theory and MCST with N=2 and β=0.8 are compared. As can be seen, the deflections obtained with the MCST are smaller than the results of the CET.

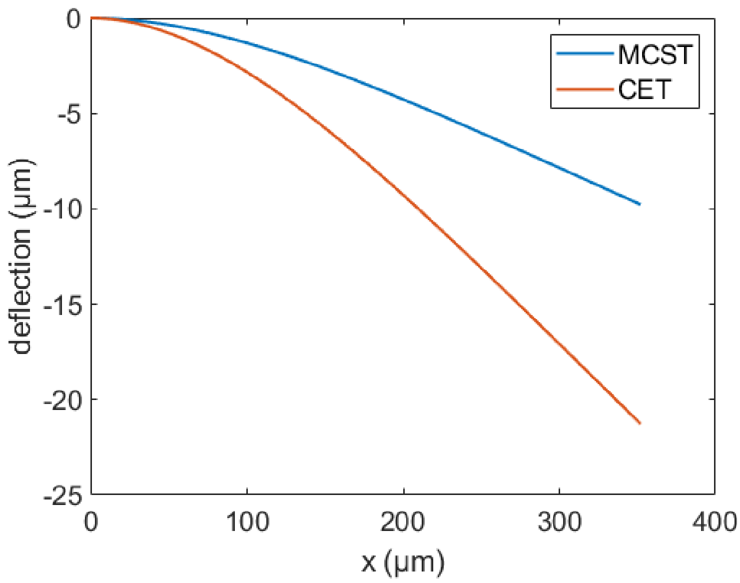

Figure 5. Effect of theory on the deflection of a perforated microbeam

Finally, the effects of l are examined via figure 6. In this example, while the same β and N are used as the previous example, l takes different values from 0 µm to 4b µm. It is understood from the results that as the value of the material length scale parameter increases, the deflection value decreases. It is also worth emphasizing again that at l = 0 µm the results reduce to classical elasticity theory.

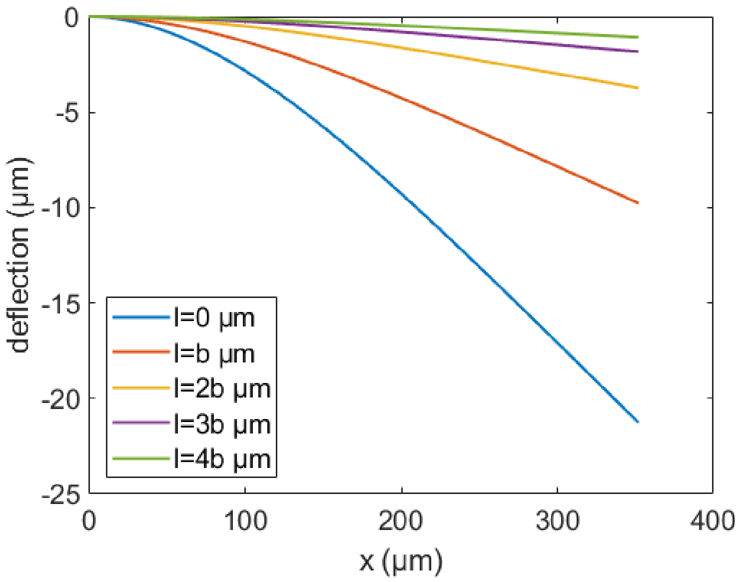

Figure 6. Effect of l on the deflection of a perforated microbeam

4. CONCLUSIONS

In this study, the effect of perforation properties on the deflection of the microbeam is examined with the MCST. The equation governing the bending of the cantilever microbeam under the effect of distributed load and containing the perforation properties is solved by Laplace transform. From the results obtained, it is revealed that the deflection decreased by increasing the filling ratio of the microbeam, and the deflection increased by increasing the number of holes. However, it is revealed that the MCST strengthened the beam and reduced deflection. Taking the material length scale parameter as zero reduces the results to the classical theory, and the deflection values of the microbeam are higher.

Declaration of Ethical Standards

Not applicable.

Credit Authorship Contribution Statement

Büşra UZUN: Conceptualization, Investigation, Methodology, Software, Writing – review, Original draft & editing.

M. Özgür YAYLI: Conceptualization, Investigation, Methodology, Writing - review, Software, Original draft & editing.

Declaration of Competing Interest

Authors declare that there are no declarations of conflict.

The study was selected among the papers presented at the 23rd National Mechanics Congress of TUMTMK
(04-08 September 2023 Konya, TURKIYE)

Funding / Acknowledgements

Not applicable.

Data Availability

This study does not contain usable data.

REFERENCES

[1] A. A. Abdelrahman, İ. Esen, and M. A. Eltaher, "Vibration response of Timoshenko perforated microbeams under accelerating load and thermal environment," *Applied Mathematics and Computation*, vol. 407, pp. 126307–126307, Oct. 2021, doi: https://doi.org/10.1016/j.amc.2021.126307.

[2] K. H. Almitani, A. A. Abdelrahman, and M. A. Eltaher, "Influence of the perforation configuration on dynamic behaviors of multilayered beam structure," *Structures*, vol. 28, pp. 1413–1426, Dec. 2020, doi: https://doi.org/10.1016/j.istruc.2020.09.055.

[3] K. H. Almitani, A. A. Abdelrahman, and M. A. Eltaher, "Stability of perforated nanobeams incorporating surface energy effects," *Steel and Composite Structures*, vol. 35, no. 4, pp. 555–566, Jan. 2020, doi: https://doi.org/10.12989/scs.2020.35.4.555.

[4] L. Luschi and F. Pieri, "An analytical model for the determination of resonance frequencies of perforated beams," *Journal of Micromechanics and Microengineering*, vol. 24, no. 5, p. 055004, Apr. 2014, doi: https://doi.org/10.1088/0960-1317/24/5/055004.

[5] A. Abdelrahman, M. A. Eltaher, A.M. Kabeel, A. M. Abdraboh, and A. A. Hendi, "Free and forced analysis of perforated beams," *Steel and Composite Structures*, vol. 31, no. 5, pp. 489–502, Jan. 2019, doi: https://doi.org/10.12989/scs.2019.31.5.489.

[6] E. Assie, Ş. D. Akbaş, A. H. Bashiri, A. A. Abdelrahman, and M. A. Eltaher, "Vibration response of perforated thick beam under moving load," *The European Physical Journal Plus*, vol. 136, no. 3, Mar. 2021, doi: https://doi.org/10.1140/epjp/s13360-021-01224-2.

[7] K. H. Almitani, A. A. Abdelrahman, and M. A. Eltaher, "On forced and free vibrations of cutout squared beams," *Steel and Composite Structures*, vol. 32, no. 5, pp. 643–655, Jan. 2019, doi: https://doi.org/10.12989/scs.2019.32.5.643.

[8] M. A. Eltaher, H. E. Abdelmoteleb, A. A. Daikh, and A. A. Abdelrahman. "Vibrations and stress analysis of rotating perforated beams by using finite elements method", *Steel and Composite Structures*, vol. 41 no. 4, pp. 505, 2021, doi: https://doi.org/10.12989/scs.2021.41.4.505

[9] H. Bourouina, R. Yahiaoui, R. Kerid, K. Ghoumid, I. Lajoie, F. Picaud and G. Herlem "The influence of hole networks on the adsorption-induced frequency shift of a perforated nanobeam using non-local elasticity theory," *Journal of Physics and Chemistry of Solids*, vol. 136, pp. 109201–109201, Jan. 2020, doi: https://doi.org/10.1016/j.jpcs.2019.109201.

[10] Kerid, R., and Bounnah, Y. (2021). Effects of structure design on electrostatic pull-in voltage of perforated nanoswitch with intermolecular surface forces. Journal of Ultrafine Grained and Nanostructured Materials, 54(2), 219-227.

[11] U. Kafkas, B. Uzun, M. Ö. Yaylı, and G. Güçlü, "Thermal vibration of perforated nanobeams with deformable boundary conditions via nonlocal strain gradient theory," *Zeitschrift für Naturforschung*, vol. 78, no. 8, pp. 681–701, Jun. 2023, doi: https://doi.org/10.1515/zna-2023-0088.

[12] A. A. Abdelrahman, H. E. Abd-El-Mottaleb, and M. A. Eltaher, "On bending analysis of perforated microbeams including the microstructure effects," *Structural Engineering and Mechanics*, vol. 76, no. 6, pp. 765–779, Jan. 2020, doi: https://doi.org/10.12989/sem.2020.76.6.765.

[13] S. K. Park and X. L. Gao, "Bernoulli–Euler beam model based on a modified couple stress theory", *Journal of Micromechanics and Microengineering*, vol. 16, no. 11, 2006, doi: https://doi.org/10.1088/0960-1317/16/11/015

[14] M. Sobhy, and A. M. Zenkour, "The modified couple stress model for bending of normal deformable viscoelastic nanobeams resting on visco-Pasternak foundations," *Mechanics of Advanced Materials and Structures*, vol. 27, no. 7, pp. 525–538, Jul. 2018, doi: https://doi.org/10.1080/15376494.2018.1482579.

[15] J. Awrejcewicz, V.A. Krysko, C. П. Павлов, M. V. Zhigalov, L. A. Kalutsky, and A. V. Krysko, "Thermoelastic vibrations of a Timoshenko microbeam based on the modified couple stress theory," *Nonlinear Dynamics*, vol. 99, no. 2, pp. 919–943, May 2019, doi: https://doi.org/10.1007/s11071-019-04976-w.

[16] M. Ö. Yaylı, "Mikro Ölçekteki Bir Konsol Kirişin Değiştirilmiş Gerilme Çifti Teorisine Göre Laplace Dönüşümüyle Eğilme Analizi," 20. ULUSAL MEKANİK KONGRESİ, 2017, pp. 626-629.

[17] M. Nazmul and I. Devnath, "Closed-form expressions for bending and buckling of functionally graded nanobeams by the Laplace transform," *International Journal of Computational Materials Science and Engineering*, vol. 10, no. 02, pp. 2150012–2150012, Jun. 2021, doi: https://doi.org/10.1142/s2047684121500123.

[18] P. Bian and H. Qing, "Elastic buckling and free vibration of nonlocal strain gradient Euler-Bernoulli beams using Laplace transform," *ZAMM - Journal of Applied Mathematics and Mechanics*, vol. 102, no. 1, Sep. 2021, doi: https://doi.org/10.1002/zamm.202100152.

[19] C.C. Ike, C. U., Nwoji, B. O., Mama, H. N., Onah and M. E. Onyia, "Laplace transform method for the elastic buckling analysis of moderately thick beams", *International Journal of Engineering Research and Technology*, vol. 12, no. 10, pp. 1626-1638, 2019.

[20] C. Li, H. Qing, and C. Gao, "Theoretical analysis for static bending of Euler–Bernoulli using different nonlocal gradient models," *Mechanics of Advanced Materials and Structures*, vol. 28, no. 19, pp. 1965–1977, Jan. 2020, doi: https://doi.org/10.1080/15376494.2020.1716121.

[21] F. Yang, A. C. M. Chong, D. C. C. Lam, and P. Tong, "Couple stress based strain gradient theory for elasticity," *International Journal of Solids and Structures*, vol. 39, no. 10, pp. 2731–2743, May 2002, doi: https://doi.org/10.1016/s0020-7683(02)00152-x.

The study was selected among the papers presented at the 23rd National Mechanics Congress of TUMTMK (04-08 September 2023 Konya, TURKIYE)