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L_2 GAIN VIBRATION CONTROL of AIRCRAFT LANDING GEAR HAVING INPUT DELAY

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ABSTRACT

In this paper a state feedback delay dependent L_2 gain controller is designed in order to control of aircraft landing gear vibration. Based on the selection of suitable Lyapunov-Krasovskii (L-K) functional, first a Bounded Real Lemma (BRL) is obtained which enables defining stability criteria in terms of Linear Matrix Inequalities (LMIs). Extending BRL, sufficient delay-dependent criteria is developed for a stabilizing L_2 gain controller synthesis involving a matrix inequality. Bilinear Matrix Inequality (BMI) problem is solved by utilizing cone complementary algorithm. To show the effectiveness of proposed controller on aircraft landing gear vibration, simulation studies are given. Time responses of system show that the controller guarantees stability of system with delay and has sufficient disturbance attenuation performance.

Keywords: Aircraft landing gear, Vibration, İnput delay.

1. INTRODUCTION

Aircraft landing gear is the leading component since runway induced vibrations directly affects the safety, ground operations and passenger-crew comfort. It is well known that most of the nonfatal accidents occur during landing [1]. The landing gear is intermediate element between the aircraft fuselage and the runway [2]. Landing gear is not only designed to absorb the energy of landing impact but also carry the aircraft mass during all ground operations [3]. Landing loads are recognized as a significant factor in causing damage [4]. Additionally, landing gear provides stability and maneuvering capability during ground operations [5]. As it is well known that traditional landing gear is equipped with tires and passive damping elements. Passive damping elements cannot always work efficiently due to various operational conditions. That causes to fatigue damage and increase the risk of accidents during landing and taking off. Therefore, attenuation of vibration on landing gear system is a requirement to improve safety and comfort.

In order to overcome the disadvantages of passive dampers, active and semi-active damping applications are being studied in the literature. Li et al. have designed a fault-tolerant controller for semi-active autonomous damper [6]. They guaranteed stability with fault tolerant H_∞ controller in case of an actuator fail that can occur cause of high landing impact as well as regular operational conditions. Zapateiro et al. have used an adaptive backstepping H_∞ control for suspension system of landing gear. Lyapunov stability of system is guaranteed and comparison of active, semi-active and passive controller time responses under random and bump type disturbance is given in the paper. Effectiveness of active and semi-active systems against to passive system in the sense of disturbance attenuation is demonstrated [7]. Hua-Lin et al. have designed fuzzy PID controller for landing gear based on MR damper [8]. Nonlinear factors of landing gear were taken into consideration and in simulation study using two-degree of freedom model effectiveness of active control is shown. Ghiringhelli et al. have designed a hybrid semi-active controller that combines a nonlinear PID term to mitigate the ground induced vibrations [9]. Sateesh et al. have developed a torsional MR damper to enhance the stability of the nose landing gear

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system due to the ground induced lateral excitation [10]. Ross et al. have designed and active landing gear system to mitigate the landing loads and ground induced vibrations under various runway irregularities in their brief study [11]. Yazici and Sever have designed an observer based optimal state feedback controller having pole location constraint for landing gear system with biodynamic pilot model [12]. Li et al. have presented a new magneto-rheological buffer structure for impact load [13]. A semi-active model predictive control method is used to adjust damping force. Toloei et al. have presented PID controller based on Bees Intelligent Algorithm [14]. Yazici and Sever have designed LMI based robust LQR controller having pole location constraints for non-linear landing gear system equipped with oleo pneumatic shock absorber [15]. Sivakumar et al. have proposed PID controller and done vibration analysis of a full aircraft [16]. In the study performance reductions caused by actuator delay is mentioned and actuator delay problem in landing gear is suggested as future work.

Due to the long transmission lines used for remote control and finite processing rate of computers, there exists time delay. Time delay is often a source of instability and poor performance [17]. From the stabilization point of view delay dependent controller becomes a requirement for systems having state and/or input delay. Zhao et al. designed robust delay dependent controller with actuator saturation for a semi-active suspension system with human body model [18]. Du et al. applied delay dependent H_∞ controller and demonstrated that delay dependent controller has more sufficient results than delay independent, by simulation results under random and bump type disturbance for active suspension system [19]. However, a considerable amount of attention has been paid to time delay problem, studies about time delay problem in aircraft landing gear is very limited.

In this study, LMI based delay dependent L_2 gain controller is designed and applied to two-degree-of-freedom landing gear in for vibration attenuation. Sufficient delay dependent stability criteria is derived by choosing L-K functional candidate. This sufficient condition for designing such controller is given by delay dependent BMIs .Then, a cone complementary linearization method is adopted to the problem in terms of LMIs [20, 21]. With the use of proposed method, suboptimal state-feedback controller, maximum allowable delay upper bound and minimum allowable disturbance attenuation level are obtained simultaneously. Simulation results show the efficiency of proposed method.

Rest of the paper is organized as follows: Section 2 describes the problem formulation. The design of delay-dependent L_2 gain controller and main results are presented in Section 3. Section 4 includes nominal state feedback L_2 gain controller design. Simulation results with discussion are given in Section 5. Finally, Section 6 concludes the paper.

Notation: A fairly standard notation is used throughout the paper. $\Re$ stands for the set of real numbers. $\Re^{n\times n}$ is the set of $n\times n$ dimensional real matrices. diag denotes the diagonal matrices. Trace stands for the standard trace operator, and $\Re_+$ symbolizes the set of positive real numbers. The identity and null matrices are denoted by I and 0, respectively. $X>0$ ($\geq,<0$) denotes that X is a positive definite (positive semidefinite, negative definite) matrix. Finally, the notation ‘‘$*$’’ denotes off-diagonal blocks in a symmetric matrix.

2. PROBLEM FORMULATION

Consider a class of time-delay system with time-varying input delays given as

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B_h x\big(t-h(t)\big) + B_w w(t)\\ z(t) &= Cx(t) + Du(t)\ , x(t) = 0, t \in [-\bar{h}, 0]\end{aligned} \quad (1)$$

where $x(t) \in \Re^{n}$ is the state vector, $u(t) \in \Re^{m_u}$ is the control input, $w(t) \in \Re^{m_w}$ is disturbance input affecting on the system and $z(t) = \Re^{p}$ is the controlled output A, B_h, B_w, C and D are state-space matrices which are known, real, constant. Also, the delay $h(t)$ is assumed to be a continuous, time varying function which satisfies.

$$0 \leq h(t) < \bar{h}, |\dot{h}(t)| < \mu, \ \forall t \geq 0 \tag{2}$$

In this expression, $\bar{h}$ and μ are known positive constants which represent upper bound of delay and its derivation. The assumption that the disturbance signal affecting on system has bounded energy is made in this work, i.e.,

$$W_\delta := \{w: \Re_+ \to \Re^{m_w} : \int_0^\infty w^T(t)w(t)dt < \infty\} \tag{3}$$

In that case, we aim to find a suitable state-feedback control law in the form of $u(t) = Kx(t)$, such that globally asymptotically stability of the closed-loop system is guaranteed and minimum L_2 gain from $w(t)$ to $z(t)$ where disturbance distribute from set W_δ. By the use of this control law, the closed-loop system can be obtained as follows

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_h Kx(t - h(t)) + B_w w(t) \\ z(t) &= (C + DK)x(t), \ x(t) = 0, t \in [-\bar{h}, 0] \end{aligned} \tag{4}$$

3. MAIN RESULTS

So as to investigate the L_2 stability, let us consider a nominal time delay system given by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + H_k x(t - h(t)) + B_w w(t) \\ z(t) &= \bar{C}x(t), \ x(t) = 0, \ t \in [-\bar{h}, 0] \end{aligned} \tag{5}$$

The following theorem presents a Bounded Real Lemma(BRL).

Lemma 1 [22]. Given positive scalar constant, $\bar{h} > 0, \mu > 0$ *and* $\gamma > 0$ the nominal time-delay system (5) is globally asymptotically stable with a disturbance attenuation level of γ and for any time varying delay $h(t)$ satisfying Eq.(2), if there exist positive definite symmetric matrices P, Q, W, Z and matrices N_1, N_2, S_1, S_2 with appropriate dimensions such that

$$\bar{\Sigma} := \begin{bmatrix} \bar{\Sigma}_{11} & \bar{\Sigma}_{12} & -S_1 & PB_w & \bar{h}A^TZ & \bar{h}N_1 & \bar{h}S_1 & \bar{C}^T \\ * & \bar{\Sigma}_{22} & -S_2 & 0 & \bar{h}H_k^TZ & \bar{h}N_2 & \bar{h}S_2 & 0 \\ * & * & -W & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & -\bar{h}B_w^TZ & 0 & 0 & 0 \\ * & * & * & * & -\bar{h}Z & 0 & 0 & 0 \\ * & * & * & * & * & -Z & 0 & 0 \\ * & * & * & * & * & * & -Z & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{6}$$

where

$$\begin{aligned} \bar{\Sigma}_{11} &:= PA + A^T P + N_1 + N_1^T + Q + W, \\ \bar{\Sigma}_{12} &:= PH_k - N_1 + N_2^T + S_1 \\ \bar{\Sigma}_{22} &:= -(1-\mu)Q - N_2 - N_2^T + S_2 + S_2^T \end{aligned}$$

In the following sequel, Lemma 1 is extended to design of a state-feedback L_2 gain controller in the form of $u(t) = Kx(t)$ synthesis for system (5) by replacing H_k with $B_h K$ and $\bar{C}$ with C+DK.

Theorem 1 [22]. Given positive scalar constants $\bar{h} > 0$, $\mu > 0$ and $\gamma > 0$, the closed-loop system (4) is globally asymptotically stable for every $h(t)$ satisfying Eq.(2) with disturbance attenuation level γ if there exist positive definite symmetric matrices X,$\bar{Q}, \bar{W}, \bar{Z}, \bar{T}$ matrices $\bar{N}_1, \bar{N}_2, \bar{S}_1, \bar{S}_2, \bar{L}$ with appropriate dimensions such that

$$\Xi := \begin{bmatrix} \Xi_{11} & \Xi_{12} & -\bar{S}_1 & B_w & 0 & \bar{h}\bar{N}_1 & \bar{h}\bar{S}_1 & \Xi_{18} & 0 & XC^T + \bar{L}^T D^T \\ * & \Xi_{22} & -\bar{S}_2 & 0 & 0 & \bar{h}\bar{N}_2 & \bar{h}\bar{S}_2 & \Xi_{28} & 0 & 0 \\ * & * & -\bar{W} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 & 0 & 0 & \bar{h}B_w^T & 0 & 0 \\ * & * & * & * & -\overline{hZ} & 0 & 0 & 0 & \bar{Z} & 0 \\ * & * & * & * & * & -\bar{Z} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\bar{Z} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -T & 0 & 0 \\ * & * & * & * & * & * & * & * & -XT^{-1}X & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{7}$$

where

$\Xi_{11} = AX + XA^T + \bar{N}_1 + \bar{N}_1^T + \bar{Q} + \bar{W}$, $\Xi_{12} = B_h\bar{L} - \bar{N}_1 + \bar{N}_2^T + \bar{S}_1$, $\Xi_{18} = \bar{h}XA^T$,
$\Xi_{22} = -(1-\mu)\bar{Q} + \bar{S}_2 + \bar{S}_2^T - \bar{N}_2 - \bar{N}_2^T$, $\Xi_{28} = \bar{h}\bar{L}^T$

Then, γ is an L_2 upper bound of the resulting closed-loop system from $w(t)$ to $z(t)$ for all $t \geq 0$ and the control law $u(t) = \bar{L}X^{-1}x(t)$ is a L_2 gain controller associate with γ.

In the light of Theorem 1, in order to achieve a minimum L_2 norm from $w(t)$ to $z(t)$ for all $t \geq 0$, one can solve the following nonconvex optimization problem for symmetric positive-definite matrix variables $X, \bar{Q}, \bar{W}, \bar{Z}, T$ and matrices $\overline{N_1}, \overline{N_2}, \overline{S_1}, \overline{S_2}, \bar{L}$ and positive scalar γ:

$$\min_{s.t.(7)} \gamma$$

If the above problem has a feasible solution, we can say that the controller constructed by the control law $u(t) = \bar{L}X^{-1}x(t)$ is defined to be the suboptimal controller for the given problem. Note that the matrix inequality condition (7) is not in the form of an LMI due to the existence of nonlinear term $-XT^{-1}X$. Hence, we cannot find a global minimum for the above optimization problem using convex optimization problem. However, if one affords more computational techniques such as cone complementary algorithm, one may still obtain a suboptimal controller for the problem definition in Sec. 2 using an iterative algorithm presented next. First, we define a new variable $R = R^T > 0$ such that

$R \leq XT^{-1}X$ and replace the condition (7) with

$$\bar{\Xi} := \begin{bmatrix} \overline{\Xi_{11}} & \overline{\Xi_{12}} & -\bar{S}_1 & B_w & 0 & \bar{h}\bar{N}_1 & \bar{h}\bar{S}_1 & \overline{\Xi_{18}} & 0 & XC^T + \bar{L}^T D^T \\ * & \overline{\Xi_{22}} & -\bar{S}_2 & 0 & 0 & \bar{h}\bar{N}_2 & \bar{h}\bar{S}_2 & \overline{\Xi_{28}} & 0 & 0 \\ * & * & -\bar{W} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 & 0 & 0 & \bar{h}B_w^T & 0 & 0 \\ * & * & * & * & -\overline{hZ} & 0 & 0 & 0 & \bar{Z} & 0 \\ * & * & * & * & * & -\bar{Z} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\bar{Z} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -T & 0 & 0 \\ * & * & * & * & * & * & * & * & -R & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{8}$$

where

$\overline{\Xi_{11}} = AX + XA^T + \bar{N}_1 + \bar{N}_1^T + \bar{Q} + \bar{W}$, $\overline{\Xi_{12}} = B_h\bar{L} - \bar{N}_1 + \bar{N}_2^T + \bar{S}_1$, $\overline{\Xi_{18}} = \bar{h}XA^T$,

$\overline{\Xi_{22}} = -(1-\mu)\bar{Q} + \bar{S}_2 + \bar{S}_2^T - \bar{N}_2 - \bar{N}_2^T$, $\overline{\Xi_{28}} = \bar{h}\bar{L}^T B_h^T$

and

$$\begin{bmatrix} \bar{R} & \bar{X} \\ \bar{X} & \bar{T} \end{bmatrix} \geq 0, \begin{bmatrix} \bar{R} & I \\ I & T \end{bmatrix} \geq 0, \begin{bmatrix} \bar{X} & I \\ I & X \end{bmatrix} \geq 0, \begin{bmatrix} \bar{T} & I \\ I & T \end{bmatrix} \geq 0 \qquad (9)$$

which can be justified following the inequality $R^{-1} - X^{-1}TX^{-1} \geq 0$, using Schur complement formula and defining $\bar{R} := \bar{R}^{-1}, \bar{X} := X^{-1}, \bar{T} := T^{-1}$ allow to obtain the inequalities in Eq. (9). Hence, in order to achieve a minimum L_2 norm for the closed-loop system (4), one may use the following linearized optimization problem

$$\min_{s.t.(8),(9)} Trace(\bar{R}R + \bar{X}X + \bar{T}T)$$

Finally, to achieve the suboptimal controller that provides minimum L_2 gain, say γ_0, and maximum allowable delay bound, say $\bar{h}_0$, one can solve the following linearized algorithm [20].

Algorithm:

1. Choose a sufficiently large initial γ and small $\bar{h}$ such that there exists a feasible solution set $\{\bar{X}_0, X_0, \bar{R}_0, R_0, \bar{T}_0, T_0\}$ to the LMI conditions in Eqs. (8), (9) and set $k = 0$.

2. Solve the following LMI optimization problem for the variables:

$$\{\bar{X}, X, \bar{R}, R, \bar{T}, T\}$$

$$\min_{s.t:(8),(9)} Trace\,(\bar{R}_k R + \bar{X}_k X + \bar{T}_k T + \bar{R}R_k + \bar{X}X_k + \bar{T}T_k)$$

and set: $\bar{X}_{k+1} \coloneqq \bar{X}$, $X_{k+1} \coloneqq X$, $\bar{R}_{k+1} \coloneqq \bar{R}$, $R_{k+1} \coloneqq R$, $\bar{T}_{k+1} \coloneqq \bar{T}$, $T_{k+1} \coloneqq T$

3. If $R \leq XT^{-1}X$ is feasible for the above solution, set $\gamma_0 = \gamma$, $\bar{h}_0 = \bar{h}$ and return to Step 1 by modifying $\gamma = \gamma - \Delta\gamma$,$\bar{h} = \bar{h} + \Delta\bar{h}$ where, $\Delta\gamma, \Delta\bar{h}$ are predefined step sizes. Otherwise, set $k = k + 1$ and go to Step 2 and repeat the optimization for a prespecified number of iterations, say k_{max}, until finding a feasible solution satisfying $R \leq XT^{-1}X$. If such a solution does not exist, then exit.
If one finds a feasible solution set with this algorithm, then the minimum achievable γ is said to be the suboptimal L_2 norm for this system. Moreover, the suboptimal state-feedback L_2 gain controller can be constructed as $u(t) = \bar{L}X^{-1}x(t)$.

4. NOMINAL STATE-FEEDBACK L_2 GAIN CONTROLLER DESIGN

In order to compare the performance of designed delay-dependent L_2 gain controller with nominal L_2 gain state-feedback controller, in the following sequel, the design equations for a static state-feedback L_2 gain controller has been obtained [23].

Let us consider a nominal linear time invariant system given by

$$\dot{x}(t) = Ax(t) + B_1w(t) + B_2u(t)$$

$$z_1(t) = C_1x(t) + D_{11}w(t) + D_{12}u(t) \quad (10)$$

where $x(t) \in \Re^n$ is the state vector. $u(t) \in \Re^{m_u}$ is the control input, $w(t) \in \Re^{m_w}$ is the disturbance input, $z_1(t) \in \Re^p$ is the controlled output vector. Suppose that the control input is linear function of the state, *i.e.*,

$$u(t) = Kx(t) \quad (11)$$

where $K \in \Re^{m_u \times n}$ is the state feedback gain. The closed-loop system is given by

$$\dot{x}(t) = (A + B_2K)x(t) + B_1w(t)$$

$$z_1(t) = (C_1 + D_{12}K)x(t) + D_{11}w(t) \quad (12)$$

The optimal nominal L_2 gain state-feedback controller can be obtaind by searching minimum allowable γ , which satisfies the following LMI for $X = X^T > 0$ and any matrix L.

$$\begin{bmatrix} \mathrm{AX} + \mathrm{XA}^\mathrm{T} + B_2L + L^TB_2^T & B_1 & XC_1^T + L^TD_{12}^T \\ B_1^T & -\gamma I & D_{11}^T \\ C_1X + D_{12}L & D_{11} & -\gamma I \end{bmatrix} < 0 \quad (13)$$

If there exists a feasible solution to the optimization problem (13), the optimal L_2 gain state-feedback controller can be constructed as $u(t) = LX^{-1}x(t)$.

5. SIMULTION STUDY

In order to illustrate the effectiveness of the proposed controller a two-degree-of-freedom landing gear model is used. For simplicity the landing gear is modeled as quarter car model as shown in Figure 1.

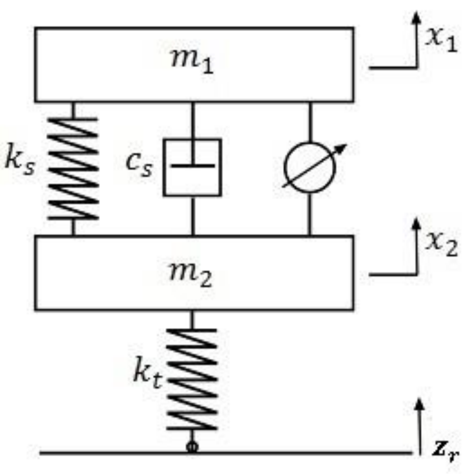

Figure 1. Landing gear model.

Here, $\mathbf{m_1}$ is represents aircraft body mass and $\mathbf{m_2}$ represents wheel mass. The mass, damping and stiffness coefficients are given below [7].

$$\mathbf{m_1 = 11739\ kg},\ \mathbf{m_2 = 300\ kg}$$

$$\mathbf{k_t = 300000\ ^{N}/_{m}, k_s = 252000\ ^{N}/_{m}}$$
$$\mathbf{c_s = 10000\ ^{Ns}/_{m}}$$

The equation of motion of the system with actuator delay can be expressed as

$$\mathbf{M\ddot{y}(t) + C\dot{y}(t) + Ky(t) = Fu(t-h) + Ew(t)} \tag{14}$$

Here, $\mathbf{y(t)} = [\mathbf{x_1} \quad \mathbf{x_2}]^{\mathbf{T}}$ is the displacement state vector, $\mathbf{u(t)}$ is the control force, $\mathbf{F} \in \mathbf{R^{n\times m_u}}$ gives the location of the controller, $\mathbf{w(t)} \in \mathbf{L_2}$ and $\mathbf{w(t)} = \mathbf{z_r}$ is the disturbance input and $\mathbf{E} \in \mathbf{R^{m_w}}$ is a weight matrix that weights the disturbances.$\mathbf{M_s, C_s, K_s} \in \mathbf{R^{n\times n}}$ are mass, damping and stiffness matrices.

Using Eq. (30) and definition $\mathbf{x^T(t)} = [\mathbf{x_1(t)} \quad \mathbf{x_2(t)} \quad \mathbf{\dot{x}_1(t)} \quad \mathbf{\dot{x}_2(t)}]^{\mathbf{T}}$ state space of system can be written as follows

$$\mathbf{\dot{x}(t) = Ax(t) + B_h u(t-h(t)) + B_w w(t)} \tag{15}$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ k_s/m_1 & -k_s/m_1 & -c_s/m_1 & c_s/m_1 \\ -(k_s+k_t)/m_2 & k_s/m_2 & c_s/m_2 & -c_s/m_2 \end{bmatrix},$$

$$\mathbf{B_w} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{kt/m_2} \end{bmatrix}, \mathbf{B_u} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{-1/m_1} \\ \mathbf{1/m_2} \end{bmatrix}$$

All simulations and computations are accomplished using MATLAB with Simulink. For the solution of resulting LMIs, YALMIP parser and SEDUMI solver are used [24, 25].

The runway disturbance is generated by a shaping filter method which is described in [16].The profile can be approximated by Power Spectral Density (PSD) distribution considered as vibration and it is typically specified as random process with a ground displacement PSD of

$$S(\omega) = \frac{2\alpha V\sigma^2}{\omega^2 + \alpha^2 V^2} \tag{16}$$

where, σ^2 denotes the runway roughness variance (m^2), V is the aircraft longitudinal speed (m/s), α depends on the type of runway surface (rad/m). Hence, if the aircraft runs with the constant velocity, the PSD and the random runway profile signal may be obtained as the output of a first order linear filter expressed as,

$$z_r(t) = -\alpha V z_r(t) + \omega(t) \tag{17}$$

where $\omega(t)$ is a white noise process with the spectral density $S(\omega)$. The road roughness standard deviations for various runway types are given in Table 1 [16].

Table 1. Road roughness and standard deviation [16].

Road Class	$\sigma(10^{-3}m)$	$\phi(\Omega_0)(10^{-6}m^3)$, $\Omega_0=1$	α (rad/m)
A (very good)	2	1	0.127
B (good)	4	4	0.127

C (average)	8	16	0.127
D (poor)	16	64	0.127
E (very poor)	32	256	0.127

In both case runway excitation with D grade poor random road profile has been considered for the value of longitudinal velocity V is 60 m/s. The modelled bump and very poor random runway disturbances are shown in Figure 2.

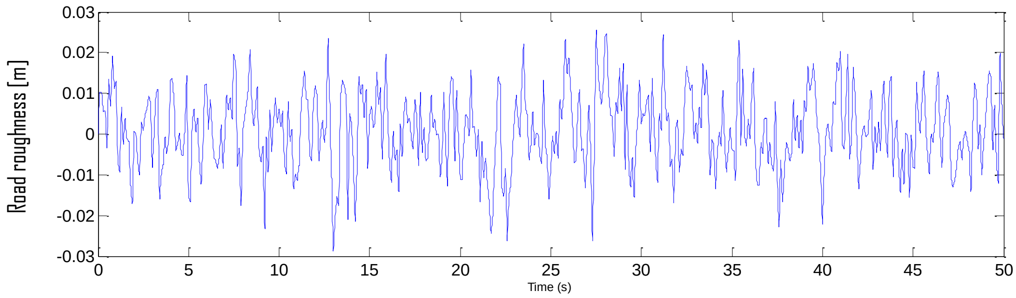

Figure 2. Random road disturbance input

To show the effects of actuator delay on system, first a nominal non-delayed $\mathbf{L_2}$ gain controller is designed. The control law is selected to be a full state feedback controller and the control gain is obtained as

$$\mathbf{K = 10^5 \times [1.7879 \quad -2.9801 \quad -0.0245 \quad 0.0262]}$$

Simulation studies are given in Figure 3. As it can be seen from figure satisfactory vibration suppression is achieved in case of assuming the input signal has no time delay.

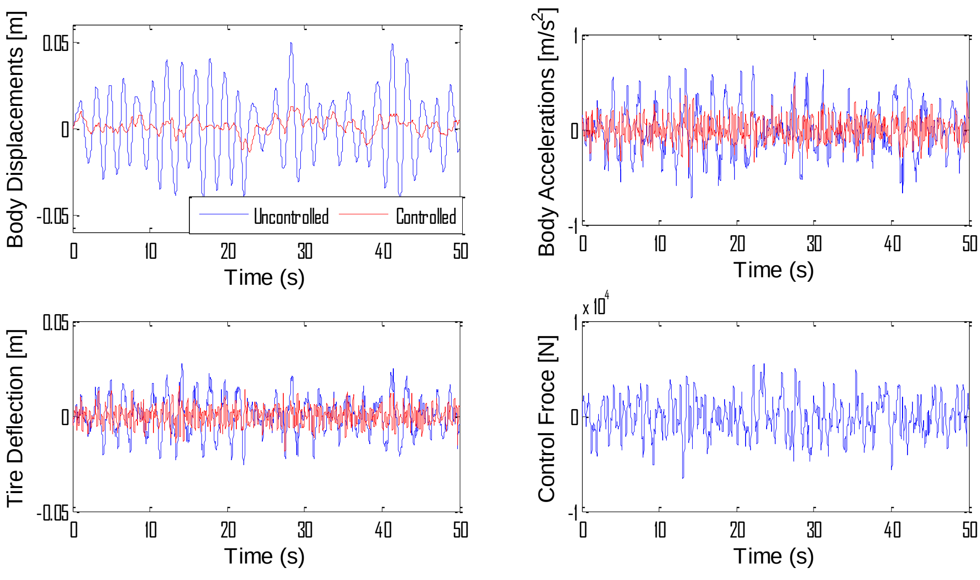

Figure 3. Controlled and uncontrolled time responses of nominal system

Then we added the time delay to input signal. When the actuator delay $\mathbf{\bar{h} = 0.0081}$ is introduced for the control input, the body displacement is plotted in Figure 4. As it can be seen from Figure 4 the stability and performance problems occurred in the closed loop system for nominal nondelayed controller even though there is a very small delay on control input.

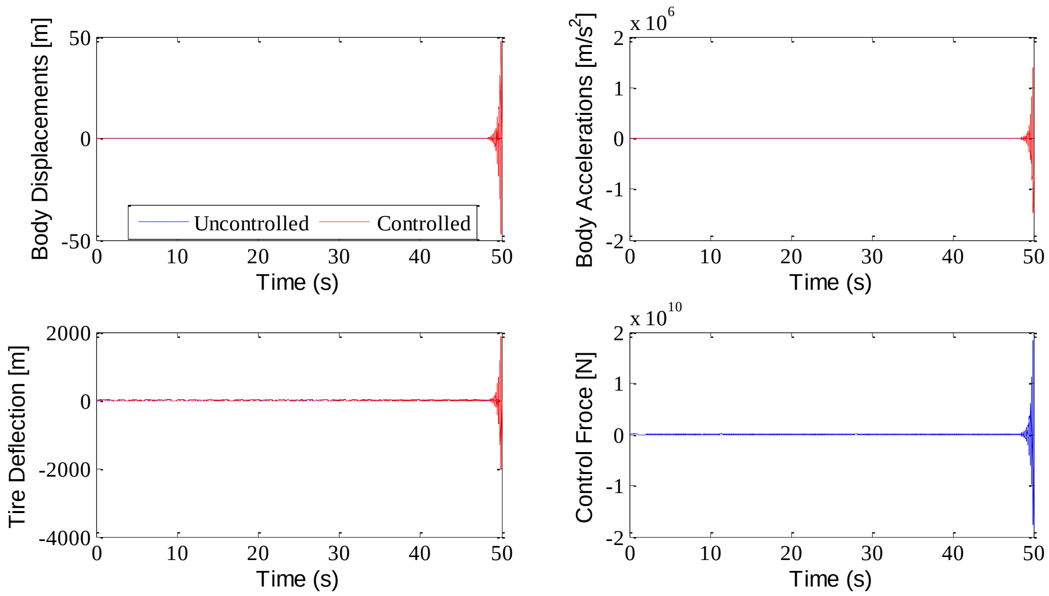

Figure 4. Controlled and uncontrolled time responses of the system

Hence, in order to overcome this problem, a delay dependent control algorithm should be designed. The control law is selected to be a full state feedback controller. The system is considered as a fixed time-delay system. Hence, $\boldsymbol{\mu}$ is assumed to be equal to $\mathbf{0}$. Applying Theorem 1 to the system, we obtain the minimum allowable disturbance attenuation level $\boldsymbol{\gamma}$ as 540, the maximum allowable actuator delay bound $\bar{\mathbf{h}}$ as 0.45 s by the use of the proposed Algorithm. Maximum iteration number $\mathbf{k_{max}}$ is selected 900, this result is obtained in 809 iteration and the controller gain is calculated as

$$K = \mathbf{10^5}[\mathbf{0.1502} \quad \mathbf{-2.0195} \quad \mathbf{0.0071} \quad \mathbf{0.6440}].$$

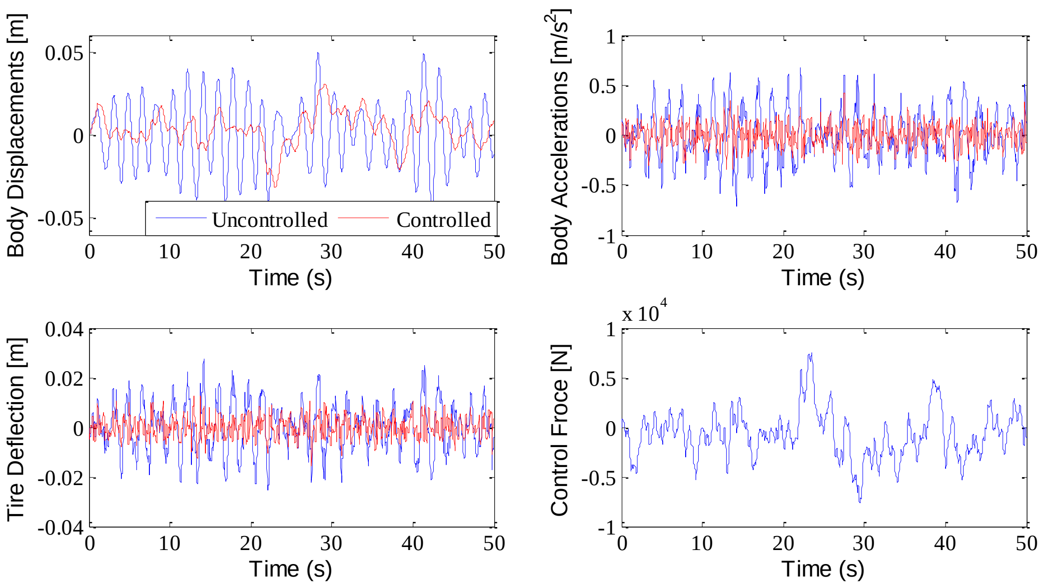

Figure 5. Controlled and uncontrolled time responses of system having actuator delay

The simulation result is given in Figure 5. It shows displacement and acceleration of time response of aircraft mass having actuator delay. As it can be obtained from Figure 5 designed controller is all effective in reducing vibration amplitudes and has guaranteed asymptotic stability at maximum allowable actuator delay. The consideration of actuator delay within the controller design process provides more realistic realization for vibration control. The simulation results show that proposed controller can stabilize the closed loop system regardless actuator delay and achieve high vibration mitigation.

6. CONCLUSIONS

In this paper delay dependent $\mathbf{L_2}$ gain controller is designed for the aircraft landing gear. A class of time-delay control system with time-varying actuator delay is taken into consideration. First, choosing a suitable Lyapunov-Krasovskii functional a delay-dependent BRL is derived in terms of an LMI, which is then extended a stabilizing $\mathbf{L_2}$ gain controller is designed for the closed-loop, actuator delayed system. Then, a cone-complementary algorithm is introduced to solve matrix inequality form of the synthesis criterion. To show effectiveness of the approach, performance of the proposed controller is examined in disturbance attenuation of vibration, in two-degree-of-freedom landing gear system having actuator delay. Simulation results obtained by using random road profile, the proposed control technique is all effective in reducing vibration amplitudes of aircraft body and guarantees stability at the maximum allowable actuator delay bound. That demonstrates necessity of delay dependent controller. Expanding the proposed method with robust delay dependent $\mathbf{L_2}$ gain controller design under consideration of actuator saturation might be direction for future work.

Appendix

Proof of Lemma 1 [22]: Let us choose a Lyapunov-Krasovskii functional candidate as $V(x(t),t) = \sum_{i=1}^{4} V_i(t)$ where

$$
\begin{aligned}
V_1(t) &= x^T(t)Px(t) \\
V_2(t) &= \int_{-\bar{h}}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)Z\dot{x}(s)ds d\beta \\
V_3(t) &= \int_{t-h(t)}^{t} x^T(s)Qx(s)ds \\
V_4(t) &= \int_{t-\bar{h}}^{t} x^T(s)Wx(s)ds
\end{aligned}
\tag{18}
$$

Taking the time derivative of $V(x(t),t)$ with respect to t along the trajectory of the system (5) and subject to Eq.(2) yields

$$
\dot{V}(x(t),t) = \sum_{i=1}^{4} \dot{V}_i(t)
$$

Defining the extended state vector as $\chi(t) \triangleq [x^T(t) \quad x^T(t-h(t)) \quad x^T(t-\bar{h}) \quad w^T(t)]^T$ and

$$
\Gamma_1 \triangleq [I \quad 0 \quad 0 \quad 0]
$$

Allows to write $x(t) = \Gamma_1\chi(t)$. In a similar manner, we define

$$
\Gamma_2 \triangleq [A \quad H_k \quad 0 \quad B_w]
$$

To obtain $\dot{x}(t) = \Gamma_2\chi(t)$. Then we obtain

$$\dot{V}_1(t) = 2x^T(t)P\dot{x}(t) = \chi^T(t)\Omega_1\chi(t) \tag{19}$$

where $\Omega_1 \triangleq \Gamma_1^T P\Gamma_2 + \Gamma_2^T P\Gamma_1$. In addition to this

$$\dot{V}_2(t) = \bar{h}\left(Ax(t) + H_k x\big(t-h(t)\big) + B_w w(t)\right)^T Z\left(Ax(t) + H_k x\big(t-h(t)\big) + B_w w(t)\right)$$

$$-\int_{t-\bar{h}}^{t} \dot{x}^T(s)Z\dot{x}(s)ds \tag{20}$$

note that

$$-\int_{t-\bar{h}}^{t} \dot{x}^T(s)Z\dot{x}(s)ds = -\int_{t-h(t)}^{t} \dot{x}^T(s)Z\dot{x}(s)ds - \int_{t-\bar{h}}^{t-h(t)} \dot{x}^T(s)Z\dot{x}(s)ds \tag{21}$$

and for any matrices N_1, N_2, S_1 and S_2 in appropriate dimensions, using Newton-Leibnitz relation allows us to write the following null equations:

$$2\big(x^T(t)N_1 + x^T\big(t-h(t)\big)N_2\big) \times (x(t) - x\big(t-h(t)\big) - \int_{t-h(t)}^{t} \dot{x}(s)ds) = 0$$
$$2\big(x^T(t)S_1 + x^T\big(t-h(t)\big)S_2\big) \times (x(t) - x\big(t-\bar{h}\big) - \int_{t-\bar{h}}^{t-h(t)} \dot{x}(s)ds) = 0 \tag{22}$$

Then adding the null-equations given in Eq. (22) to the right-hand side of Eq. (20), utilizing Eqs. (21) and (2) allows us to write

$$\begin{aligned}\dot{V}_2 \le\ & \bar{h}x^T(t)A^TZAx(t) + \bar{h}x^T(t)A^TZH_kx\big(t-h(t)\big) + \bar{h}x^T(t)A^TZB_ww(t) + \bar{h}x^T\big(t-h(t)\big)H_k^TZAz(t) + \\ & \bar{h}x^T\big(t-h(t)\big)H_k^TZH_kx\big(t-h(t)\big) + \bar{h}x^T\big(t-h(t)\big)H_k^TZB_ww(t) + \bar{h}w^T(t)B_w^TZAx(t) + \\ & \bar{h}w^T(t)B_w^TZH_kx\big(t-h(t)\big) + \bar{h}w^T(t)B_w^Tw(t) - \int_{t-\bar{h}}^{t} \dot{x}^T(s)Z\dot{x}(s)ds - \int_{t-\bar{h}}^{t-h(t)} \dot{x}^T(s)Z\dot{x}(s)ds + \\ & 2\big(x^T(t)N_1 + x^T\big(t-h(t)\big)N_2\big) \times (x(t) - x\big(t-h(t)\big) - \int_{t-h(t)}^{0} \dot{x}^T(s)ds) + 2(x^T(t)S_1 + \\ & x^T\big(t-h(t)\big)S_2) \times (x\big(t-h(t)\big) - x\big(t-\bar{h}\big) - \int_{t-\bar{h}}^{t-h(t)} \dot{x}(s)ds)\end{aligned} \tag{23}$$

It follows from completing to squares that one can always construct the following nonnegative terms:

$$\begin{aligned}0 \le -\int_{t-h(t)}^{t} & (x^T(s)N_1 + x^T\big(s-h(s)\big)N_2 \\ & + \dot{x}^T(s)Z^{-1}\left(N_1^Tx(s) + N_2^Tx(s) + N_2^Tx\big(s-h(s)\big) + Z\dot{x}(s)\right)ds \\ & + \bar{h}x^T(t)N_1Z^{-1}N_1^Tx(t) + 2\bar{h}x^T(t)N_1Z^{-1}N_2^Tx\big(t-h(t)\big) \\ & + \bar{h}x^T\big(t-h(t)\big)N_2Z^{-1}N_2^Tx(t-h(t))\end{aligned} \tag{24}$$

and similarly,

$$\begin{aligned}0 \le & -\int_{t-\bar{h}}^{t-h(t)} (x^T(s)S_1 + x^T\big(s-h(s)\big)S_2 + \dot{x}^T(s)Z^{-1}\left(S_1^Tx(s) + S_2^Tx(s) + S_2^Tx\big(s-h(s)\big) + \right. \\ & \left. Z\dot{x}(s)\right)ds + \bar{h}x^T(t)S_1Z^{-1}S_1^Tx(t) + 2\bar{h}x^T(t)S_1Z^{-1}S_2^Tx\big(t-h(t)\big) + \bar{h}x^T\big(t-h(t)\big)S_2Z^{-1}S_2^Tx(t- \\ & h(t))\end{aligned} \tag{25}$$

then

$$\dot{V}_2(t) \leq \chi^T(t)\Omega_2\chi(t). \quad (26)$$

We can compute $\dot{V}_3(t)$ as given

$$\begin{aligned}\dot{V}_3(t) &= x^T(t)Qx(t) - \left(1-\dot{h}(t)\right)x^T\big(t-h(t)\big)Qx\big(t-h(t)\big)\\ &\leq x^T(t)Qx(t) - (1-\mu)x^T\big(t-h(t)\big)Qx\big(t-h(t)\big)\end{aligned} \quad (27)$$

Then describing

$$\Gamma_3 \triangleq [0 \quad I \quad 0 \quad 0]$$

and using definition $\Omega_3 \triangleq \Gamma_1^T Q\Gamma_1 - (1-\mu)\Gamma_3^T Q\Gamma_3$ leads to write

$$\dot{V}_3 \leq \chi^T(t)\Omega_3\chi(t). \quad (28)$$

Besides, $\dot{V}_4(t)$ can be written as

$$\dot{V}_4(t) = x^T(t)Wx(t) - x^T\big(t-\bar{h}\big)Wx\big(t-\bar{h}\big) \quad (29)$$

Describing $\Gamma_4 \triangleq [0 \quad 0 \quad I \quad 0]$ we can obtain

$$\dot{V}_4 \leq \chi^T(t)\Omega_4\chi(t) \quad (30)$$

where $\Omega_4 = \Gamma_1^T W\Gamma_1 - \Gamma_4^T W\Gamma_4$.

As a result, substituting $\dot{V}_1 = 1 \ldots 4$ into $\dot{V}(x(t),t)$ allows to calculate

$$\dot{V}(x(t),t) + z^T(t)z(t) - \gamma^2 w^T(t)w(t) \leq \chi^T(t)\Omega\chi(t)$$

where

$$\Omega \triangleq \sum_{i=1}^4 \Omega_i + \Gamma_1^T \bar{C}^T \bar{C}\Gamma_1 - \gamma^2\Gamma_5^T\Gamma_5 \quad (31)$$

and

$$\Gamma_5 = [0 \quad 0 \quad 0 \quad I]$$

It is obviously seen that when $w(t) \equiv 0, \forall t \geq 0, \dot{V}(x(t),t) < 0$ is ensured guaranteeing that system (5) under zero disturbance is globally asymptotically stable. Moreover, integrating both sides of $\dot{V}(x(t),t) + z^T(t)z(t) - \gamma^2 w^T(t)w(t) \leq 0$ from 0 to infinity allows to get $\lim_{t\to\infty} V(x(t),t) - V(x(0),0) + \int_0^\infty z^T(t)z(t) - \gamma^2\int_0^\infty w^T(t)w(t) < 0$.Since $V(x(0),0) = 0$ and $\lim_{t\to\infty} V(x(t),t) > 0$, we obtain $\int_0^\infty z^T(t)z(t)dt - \gamma^2\int_0^\infty w^T(t)w(t)dt < 0$, which implies $\|z\|_2 < \gamma\|w\|_2$, $\forall w(t) \in L_2[0,\infty)$.Finally applying Schur complement formula to Eq. (31) allows to get Eq.(6) [23]. This completes proof.

□

Proof of Theorem 1 [22]: Applying Schur's complement formula on Eq.(6) and replacing H_k with $B_h K$ and $\bar{C}$ with $C + DK$, respectively

$$\tilde{\Omega} = \begin{bmatrix} \tilde{\Omega}_{11} & \tilde{\Omega}_{12} & -S_1 & PB_W & \bar{h}A^TZ & \bar{h}N_1 & \bar{h}S_1 & C^T + K^TD^T \\ * & \tilde{\Omega}_{22} & -S_2 & 0 & \bar{h}K^TB_H^TZ & \bar{h}N_2 & \bar{h}S_2 & 0 \\ * & * & -W & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & \bar{h}B_w^TZ & 0 & 0 & 0 \\ * & * & * & * & -\bar{h}Z & 0 & 0 & 0 \\ * & * & * & * & * & -Z & 0 & 0 \\ * & * & * & * & * & * & -Z & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{32}$$

where

$\tilde{\Omega}_{11} = PA + A^TP + N_1 + N_1^T + Q + W$, $\tilde{\Omega}_{12} = PB_hK - N_1 + N_2^T + S_1$,

$\tilde{\Omega}_{22} = -(1-\mu)Q + S_2 + S_2^T - N_2 - N_2^T$

Pre- and post multiplying Eq. (32) by $diag\{X, X, X, I, X, X, X, I\}$ and applying the variable changes $\bar{N}_1 \coloneqq XN_1X$, $\bar{N}_2 \coloneqq XN_2X$, $\bar{S}_1 \coloneqq XS_1X$, $\bar{S}_2 \coloneqq XS_2X$, $\bar{W} \coloneqq XWX$, $\bar{Z} \coloneqq XZX$, $\bar{Q} \coloneqq XQX$ inequalities (32) is congruent to

$$\hat{\phi} \coloneqq \begin{bmatrix} \hat{\phi}_{11} & \hat{\phi}_{12} & -\bar{S}_1 & B_W & \bar{h}XA^TX^{-1}\bar{Z} & \bar{h}\bar{N}_1 & \bar{h}\bar{S}_1 & XC^T + XK^TD^T \\ * & \hat{\phi}_{22} & -\overline{S_2} & 0 & \bar{h}XK^TB_H^TX^{-1}\bar{Z} & \bar{h}\overline{N_2} & \bar{h}\overline{S_2} & 0 \\ * & * & -\bar{W} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & \bar{h}B_w^TX^{-1}\bar{Z} & 0 & 0 & 0 \\ * & * & * & * & -\bar{h}\bar{Z} & 0 & 0 & 0 \\ * & * & * & * & * & -\bar{Z} & 0 & 0 \\ * & * & * & * & * & * & -\bar{Z} & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{33}$$

where

$\hat{\phi}_{11} = AX + XA^T + \bar{N}_1 + \bar{N}_1^T + \bar{Q} + \bar{W}$, $\hat{\phi}_{12} = B_hKX - \bar{N}_1 + \bar{N}_2^T + \bar{S}_1$,

$\hat{\phi}_{22} = -(1-\mu)\bar{Q} + \bar{S}_2 + \bar{S}_2^T - \bar{N}_2 - \bar{N}_2^T$,

Note that $\hat{\phi}$ can be decomposed as $\hat{\phi} = \hat{\phi}_0 + \hat{\phi}_1 + \hat{\phi}^T$, where

$$\widehat{\phi_0} \coloneqq \begin{bmatrix} \hat{\Phi}_{11} & \hat{\Phi}_{12} & -\bar{S}_1 & B_W & 0 & \bar{h}\bar{N}_1 & \bar{h}\bar{S}_1 & XC^T + XK^TD^T \\ * & \hat{\Phi}_{22} & -\overline{S_2} & 0 & 0 & \bar{h}\overline{N_2} & \bar{h}\overline{S_2} & 0 \\ * & * & -\bar{W} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 & 0 & 0 & 0 \\ * & * & * & * & -\bar{h}\bar{Z} & 0 & 0 & 0 \\ * & * & * & * & * & -\bar{Z} & 0 & 0 \\ * & * & * & * & * & * & -\bar{Z} & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0$$

$$\widehat{\phi_1} := \begin{bmatrix} 0 & 0 & 0 & 0 & \bar{h}XA^TX^{-1}\bar{Z} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{h}XK^TB_H^TX^{-1}\bar{Z} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{h}B_w^TX^{-1}\bar{Z} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0$$

Besides, $\hat{\phi}_1$, can be written as $\hat{\phi}_1 = \Pi_1^T X^{-1} \Pi_2$ where

$$\Pi_1 = [\bar{h}AX \quad \bar{h}B_hKX \quad 0 \quad \bar{h}B_w \quad 0 \quad 0 \quad 0 \quad 0],$$

$$\Pi_2 = [0 \quad 0 \quad 0 \quad 0 \quad \bar{Z} \quad 0 \quad 0 \quad 0]$$

Also note that for any symmetric positive definite matrix T

$$\Pi_1^T X^{-1} \Pi_2 + (\Pi_1^T X^{-1} \Pi_2)^T \leq \Pi_1^T T^{-1} \Pi_1 + \Pi_2^T (XT^{-1}X)^{-1} \Pi_2$$

Hence, $\widehat{\phi_0} + \Pi_1^T T^{-1} \Pi_1 + \Pi_2^T (XT^{-1}X)^{-1} \Pi_2 < 0$ implies that $\hat{\phi} < 0$ .Then applying Schur complement formula on $\widehat{\phi_0} + \Pi_1^T T^{-1} \Pi_1 + \Pi_2^T (XT^{-1}X)^{-1} \Pi_2 < 0$ and defining $\bar{L} := KX$ we obtain the matrix inequality condition (7).

□

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