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Math Teaching Practices

Van Hiele model level 5 teaching activity example for mathematically gifted students: expand polygon

Tuba Erben [1]

Elazığ Science and Art Cener, Elazığ, Türkiye

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Abstract

Geometry is one of the sub-learning areas of mathematics and meaningful geometry learning includes reasoning methods such as generalizing, classifying, and inferring. Therefore, it improves students' mathematical thinking skills, increases their ability to support their thoughts with mathematical arguments, and positively affects their mathematics achievement in general. The aim of this study is to prepare an instructional activity for teaching geometry to gifted students in accordance with the 5th level they have reached in Van Hiele geometry learning levels. While preparing this teaching activity, the decomposition technique in the Pythagorean Proof Application in the study "Application of the Pythagorean Relation Expressed for Square to Other Regular Polygons and Circle" by Aslaner and İlhan (2018) was used. In the activity, the table is filled in by fulfilling the steps of disassembly and assembly in accordance with the instructions given, and a mathematical relationship is asked to be found between the polygons formed. The Expand Polygon activity can be used as an activity in the support education mathematics activities of gifted students and in the mathematics applications courses of middle school and high school students from the 7th grade onwards to improve students' reasoning and spatial skills.

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Introduction

Instructional activities are structured activities that aim to provide students with the targeted skills and knowledge in a planned, organized and supervised manner (Bransford et al., 2000; Fidan, 1993). Through instructional activities, students' active participation in the process, reasoning and generalization skills on basic mathematical concepts are developed, and their ability to make mathematical abstractions and calculations are supported (Henningsen & Stein, 1997; Ministry of National Education-MoNE, 2009; Olkun & Uçar, 2007). This requires students to play an active role mentally and physically and to make an intrinsic effort through learning activities. For this reason, it is extremely important to develop activities that are appropriate for the goals set in the learning-teaching processes and to implement them successfully in the classroom environment (Özgen & Alkan, 2014). The more sensory organs the teaching activity appeals to, the better and more permanent the learning event will be, and the later the forgetting will be. Students should actively participate in the lesson because students who participate in the learning environment by using all their sensory organs can learn more easily (Demirel et al., 2002). In this context, activities in geometry teaching are important tools that strengthen the learning process by enabling students to experience abstract geometric concepts in a concrete way.

Geometry is one of the sub-learning areas of mathematics and meaningful geometry learning involves reasoning methods such as generalizing, classifying, and inferring. Therefore, it improves students' mathematical thinking skills, increases

[1] Mathematics Teacher of Gifted, Elazığ Science and Art Cener, Elazığ, Türkiye. E-mail: tubaerben@outlook.com ORCID: 0000-0003-4532-8328

their ability to support their thoughts with mathematical arguments, and positively affects their mathematics achievement in general (National Council of Teachers of Mathematics-NCTM, 1989). Spatial visualization, which is an important part of geometric thinking, is defined as the ability to perceive objects from different angles, to create and apply the appearances of two and three dimensional objects in the mind (Çetin & Dane, 2004). The Van Hiele model, which is one of the geometric thinking models, is based on an understanding that geometric thinking skills in students pass through five hierarchically ordered levels. These levels are named as visual period (Level 1), analytical period (Level 2), spontaneous inference period (Level 3), inference period (Level 4) and the most advanced period (Level 5). The factor for individuals to pass these levels is their experiences; as the skills provided by these levels increase, the level of geometric thinking also increases (Demir & Kurtuluş, 2019).

For the smooth implementation of the teaching activity, the cognitive, affective, social and physiological characteristics of the target group of students and their needs based on these characteristics should be taken into consideration (Kuzgun & Deryakulu, 2004). Each student has unique qualities and differs from others. One of the most important determinants of this diversity among students is their level of intelligence. The concept of intelligence has been defined in various ways by different researchers in the literature. Generally defined as the ability to adapt to one's environment and learn through experiences, intelligence can be considered from many different perspectives (Sternberg & Detterman, 1986, as cited in Sternberg, 2005).

The concept of giftedness and talent, which is defined differently in different cultures, has an important place among the different intellectual potentials of students at the center of our education system. This concept was first used for children with abnormally rapid development or high IQ scores in intelligence tests. While special education programs for gifted children are called "gifted education", the children included in these programs are referred to as "gifted children". All these terms define giftedness by emphasizing genetic and hereditary characteristics (Feldhusen, 2005).
Giftedness in mathematics and geometry refers to a high level of ability in understanding mathematical ideas and comprehending mathematical logic, rather than simply showing a high level of ability in arithmetic calculations (Miller, 1990, as cited in Dağlıoğlu, 2004). Mathematically gifted students need different instructional designs than their peers with normal development (Erdogan & Gul, 2023). Sağır-Gürlevik (2017) evaluated the geometry levels of gifted students in terms of some variables. As a result of the study, when the Van Hiele Geometry Levels of gifted students were examined, it was seen that the Level 5 group had the highest scores in spatial and critical thinking skills. In this context, when preparing teaching activities for gifted students, these levels should be taken into consideration. For example, it is known that there are different proofs of Pythagoras theorem in geometry teaching. The most well-known proof is the visual proof that the sum of the areas of squares placed on perpendicular sides is equal to the area of the square placed on the hypotenuse length. However, Aslaner and İlhan (2018) tried to express and prove the Pythagorean theorem for other regular polygons to be drawn on the sides of the right triangle after the construction of the right triangle in Cabri II Plus program. These proofs are given as Pythagorean theorem for equilateral triangles, Pythagorean theorem for square, Pythagorean theorem for regular pentagons, Pythagorean theorem for regular hexagons and Pythagorean relation for circle. Thus, the proof was questioned and proved for other polygons and the geometry learning level was raised to a higher level.

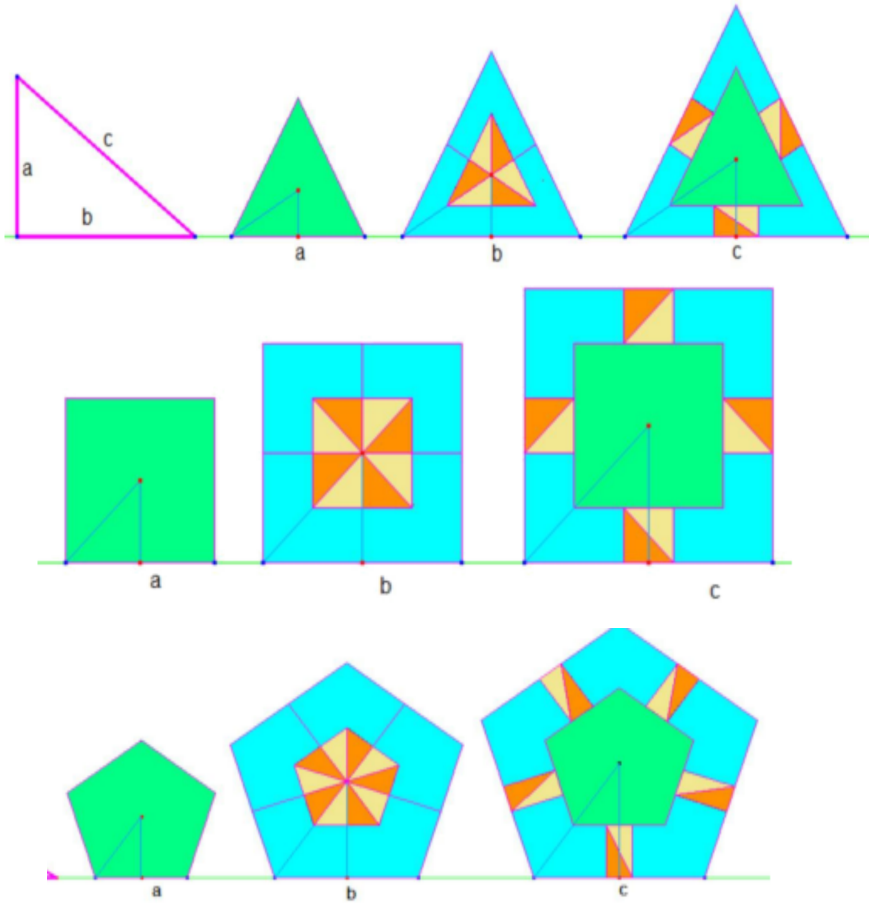


Figure 1. Application of the proof of pythagorean theorem to different polygons (Aslaner & İlhan, 2018)

Purpose of the Instructional Activity

In teaching geometry to gifted students, the aim of this study was to prepare a teaching activity suitable for the 5th level of Van Hiele geometry learning levels. While preparing this teaching activity, the decomposition technique in the Pythagorean Proof Application in the study of Aslaner and İlhan (2018) on the Application of the Pythagorean Relation Expressed for the Square to Other Regular Polygons and the Circle was used.

Information on the Teaching Activity

The related instructional outcomes of the instructional activity in the mathematics curriculum are as follows;

- Explains the side and angle properties of regular polygons.
- Draws the bisector and bisector auxiliary elements of a regular polygon.
- Uses ratio to compare lengths.
- Obtains the whole by using polygon parts.

Implementation of the Instructional Activity

Students are asked how fraction numbers are expanded. If we can expand fractions, how can we expand polygons? With the answers received, the teacher repeats the properties of polygons, the auxiliary elements of the triangle, the concepts of bisector and bisector by question and answer.

The worksheet with the prepared instruction is distributed to the student group one by one.

Note: If the physical conditions of the classroom are suitable, it is recommended to use the Geogebra program. Alternatively, A4 paper, ruler, protractor and scissors can be used to complete the instruction.

Working Paper

Method for Expanding Regular Polygons

Do the algorithm given below step by step.

Partition

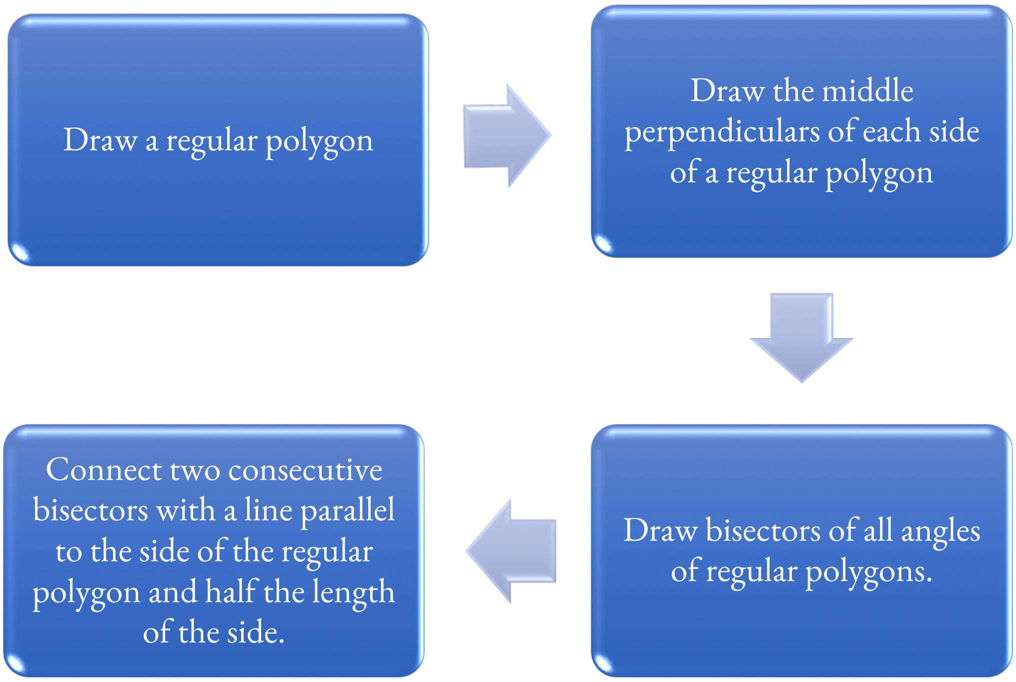


Figure 2. Shredding steps

Combination

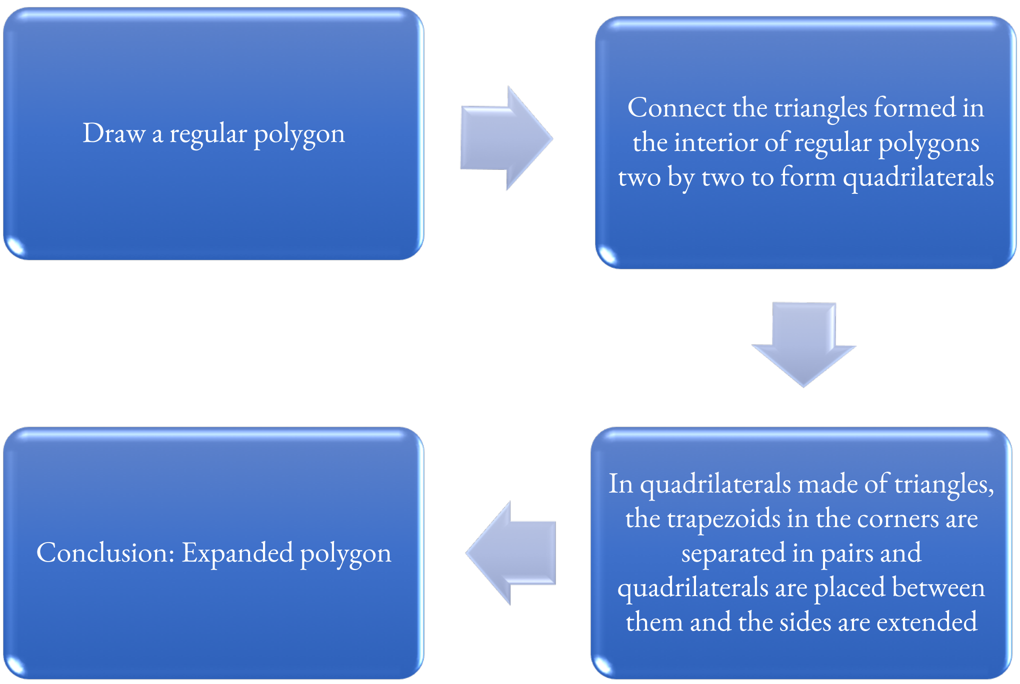


Figure 3. Combining steps

- Assessment: Find a mathematical relationship between the polygons you created.
- All the results are entered in the table and a mathematical relationship is found.

Table 2. The relationship between the side lengths of the resulting polygons

Polygon (segmented) Side Length	Polygon (formed in the inner region) Side Length	Polygon (Combined) Side Length

Examples of Structures to Be Obtained after the Algorithm

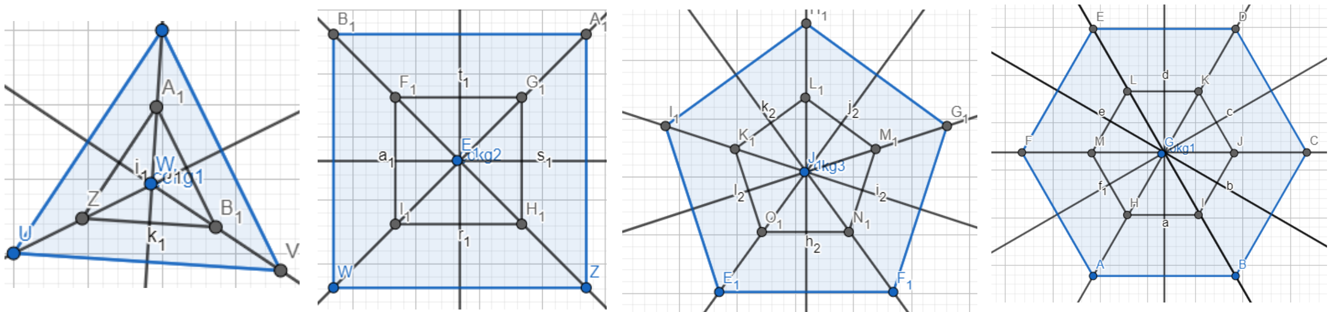


Figure 4. Connecting two points taken on two consecutive bisectors

Combining Technique

The triangle pieces formed in the inner region are removed and joined together as two-by-two quadrilaterals. The quadrilaterals are placed between the two trapezoids on the sides.

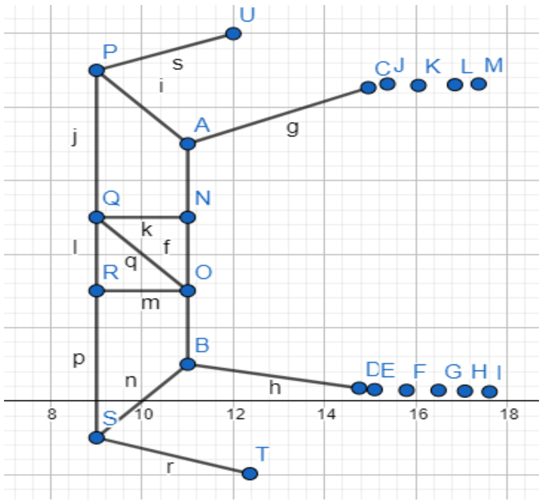
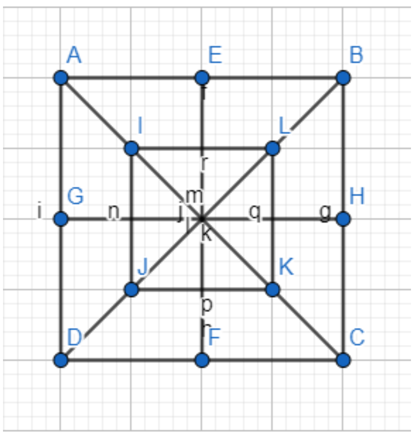


Figure 5. Section through one side of an expanded polygon

Placement of Triangles between Trapezoids on the Edge



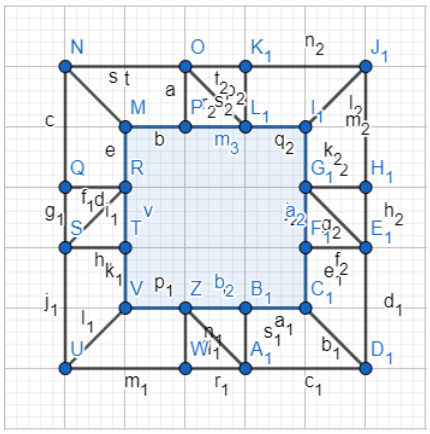


Figure 6. The Method of Segmenting Regular Polygons **Figure 7.** Expanded polygon

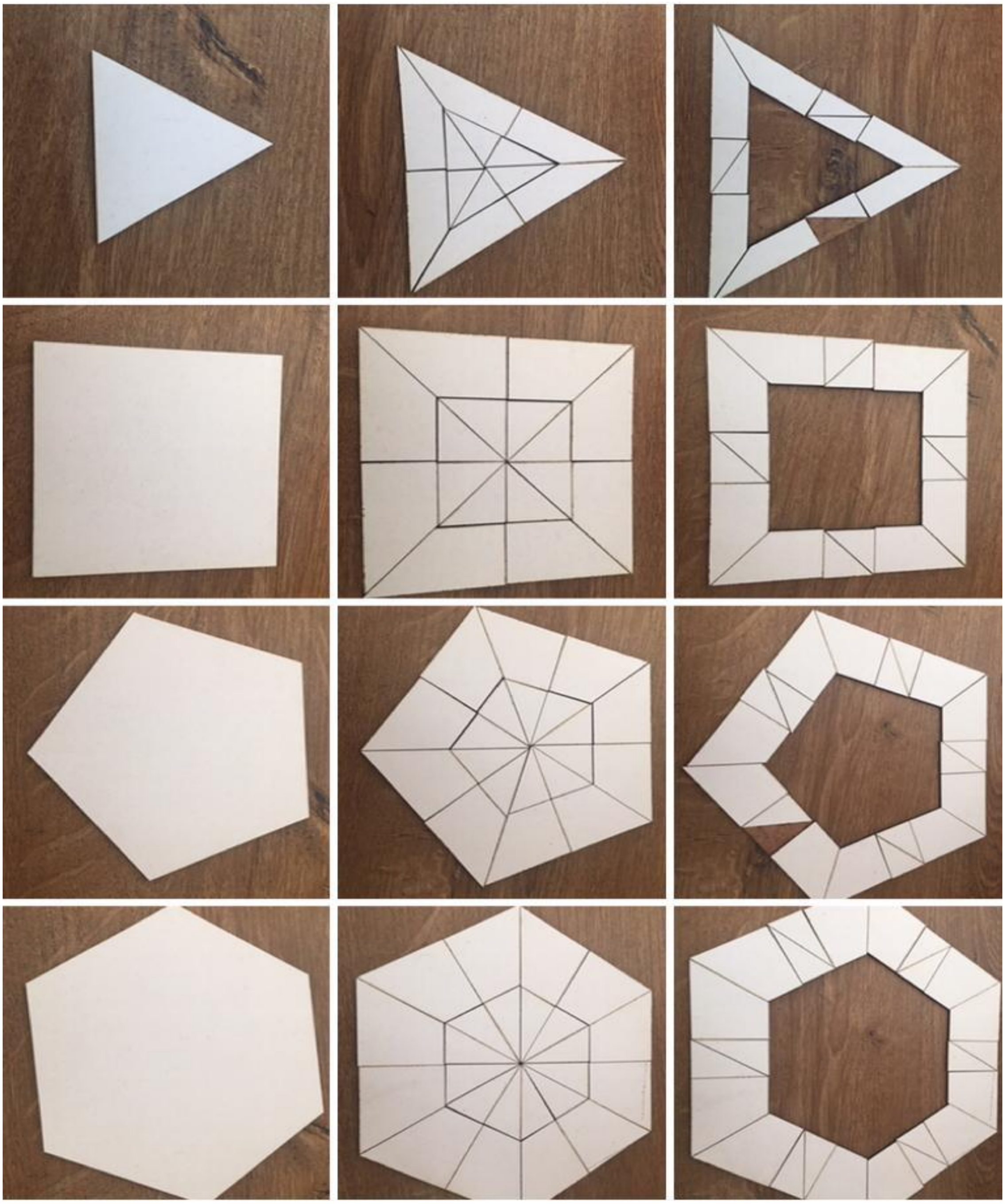


Photo 1. Structures formed when the algorithm steps are applied to papers

Conclusion and Recommendations

In geometric thinking, spatial visualization, perceiving objects from different angles, creating and applying the appearances of two and three dimensional objects in the mind are important. In this context, mathematics and geometry lessons should include activities to develop these skills. The Expand Polygon activity is an activity that will contribute to students' geometric thinking and help them discover new formulas by capturing mathematical relationships, as it is an activity that will discuss how it can be applied to polygons based on the application of expanding fraction numbers.

Expand polygon activity can be used as an activity in the support education mathematics activities of gifted students and in the mathematics applications courses of middle school and high school students from the 7th grade onwards to improve students' reasoning and spatial skills. It can also be designed as a material and used in educational environments.

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