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ARAŞTIRMA MAKALESİ /**RESEARCH ARTICLE**

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KONSOL I KESİTLİ KİRİŞLERİN ELASTİK YANAL BURULMALI BURKULMA MOMENTİNİN HESAPLANMASINDA SONLU FARKLAR YAKLAŞIMI

ÖZ

Çelik yapılarda kirişler, yapı malzemesinin ekonomik kullanılması amacı ile, genellikle kuvvetli asal eksenlerine göre eğilme etkisinde olacak şekilde yerleştirilirler. Söz edilen çelik kirişlerin tasarımında iki ana problem vardır. Bunlardan ilki, kesitin en dışındaki lifte akma gerilmesinin aşılmasıdır. Herhangi bir yükleme hali için, çelik kirişte akma gerilmesinin aşılmasına yol açan dış yük kolaylıkla hesaplanabilir. İkinci problem ise stabilite kaybıdır. Kuvvetli asal eksenlerine göre eğilme etkisinde olacak şekilde yüklenen kirişlerde, belirli bir yük şiddeti için, elemanın burulduğu ve zayıf eksenine göre burkulduğu bir denge hali de mümkündür. Bu durum yanal burulmalı burkulma olarak adlandırılır. Yükleme durumuna, yapı malzemesinin akma gerilmesine ve kirişin narinliğine (kesit özelliklerine ve kiriş uzunluğuna bağlıdır) bağlı olarak, en dıştaki lif akma gerilmesine ulaşmadan yanal burulmalı burkulma meydana gelebilir. Bu durumda kirişin tasarımında, ilk akma momenti yerine yanal burulmalı burkulma momenti göz önünde bulundurulmalıdır. Bu çalışmada, kayma merkezinden yüklenmiş konsol I kesitli kirişlerin elastik kritik yanal burulmalı burkulma momentinin bulunmasında sonlu farklar yöntemi uygulaması sunulmuştur. Sonuçlar ABAQUS yazılımı ile karşılaştırılmış ve sunulan 1 boyutlu model ile ABAQUS yazılımından elde edilen sonuçların örtüştüğü görülmüştür.

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FINITE DIFFERENCES APPROACH FOR CALCULATING ELASTIC LATERAL TORSIONAL BUCKLING MOMENT OF CANTILEVER I SECTIONS

ABSTRACT

In steel structures, beams are usually assembled so as to be under bending about their major axis in order to use the structural material economically. Two major problems take place in design of these steel beams. First problem is exceeding of yield stress at the extreme fiber of the section. External force magnitude for any loading case which causes exceeding of yield stress on a steel beam can be easily calculated. Second problem is loss of stability. In beams which are loaded so as to be under bending about their major axis an equilibrium state is also possible for a certain load magnitude, in which the beam is twisted and buckled about its weak axis. This case is called lateral torsional

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buckling. Depending on the loading case, yield stress of the structural material and slenderness of the beam, which is related to section properties and length of the element, lateral torsional buckling may occur before the extreme fiber of the section reaches to yield stress. In this case, instead of first yield moment, lateral torsional buckling moment should be considered in design of the beam. In this paper, application of finite differences method for determining elastic critical lateral torsional buckling moment of cantilever I sections which are loaded from shear center is presented. The results are compared with ABAQUS software and it is seen that results obtained from the presented 1D model and ABAQUS software coincide.

Keywords: Lateral torsional buckling, Finite differences method, Cantilever beam, I section, Stability

1. INTRODUCTION

Lateral torsional buckling is a serious stability loss in cantilever beams. When the load, which produces bending moment about strong axis of the beam, reaches to a critical value, the beam experiences non-uniform twisting and laterally buckles (Fig. 1.1).

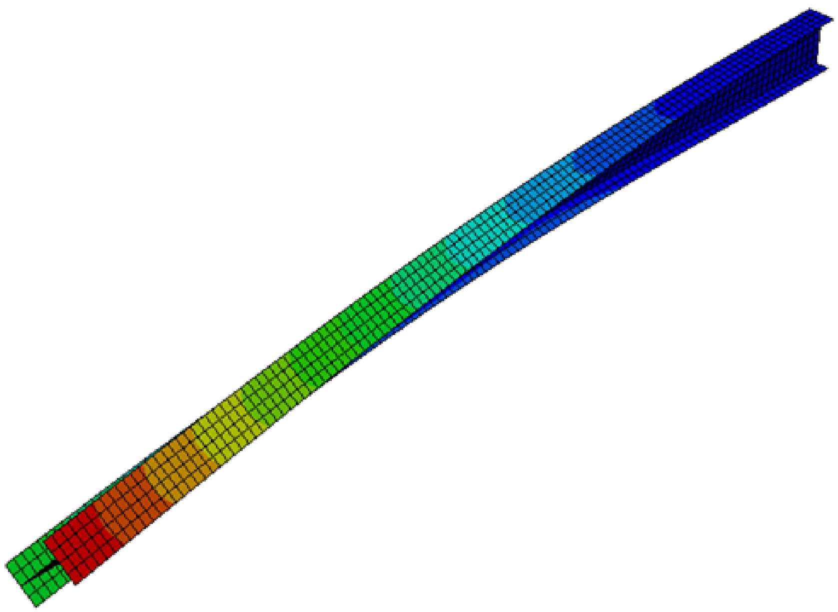

Figure 1.1. Lateral torsional buckling of a cantilever I beam

The smallest load which causes lateral torsional buckling of the beam is called critical lateral torsional buckling load and form of the twisting angle function of the buckled beam is called critical lateral torsional buckling mode. If lateral torsional buckling occurs before the extreme fiber of the section reaches to yield stress, this case is called elastic lateral torsional buckling. If lateral torsional buckling occurs after the extreme fiber of the section reaches to yield stress this case is called inelastic lateral torsional buckling. However, some beams fail without experiencing lateral torsional buckling depending on the cantilever length, section properties, elasticity modulus and yield stress of the structural material. Lateral torsional buckling load is not considered in design of these beams.

Many studies are conducted on lateral torsional buckling of beams. Andrade, Camotim and Providência e Costa included singly and doubly symmetric cantilever I sections with warping free or restrained on fixed end in the application field of 3-factor formula (C.E.N., 1992), which is a commonly used method for determining lateral torsional buckling moment of steel beams (Andrade, et al., 2007). Zhang and Tong compared traditional theory (Timoshenko and Gere, 1961) and Lu's theory (Lu, et al., 1983), which are slightly different from each other, used for determining lateral torsional buckling loads of thin-walled elements (Zhang and Tong, 2008). Samanta and Kumar studied singly symmetric I sections, of which top and bottom flanges are laterally restrained, under different loading cases (Samanta and Kumar, 2008). Qiao, Zou and Davalos conducted experimental and analytical studies on lateral torsional buckling of fiber-reinforced plastic cantilever I sections and presented simplified formulas (Qiao, et al., 2003). Eryiğit, Zor and Arman studied the effect of hole diameter and location to lateral torsional buckling strength of composite cantilever beams by experimental and analytical methods (Eryiğit, et al., 2009).

It is possible to classify the methods for determining lateral torsional buckling load of beams into 3 main classes. First is finite element perturbation analysis. This method is more accurate compared to other alternative methods. Since calculation procedure is very complex, application of this method requires use of a finite element analysis software.

Second is energy method. Energy method is based on the principle that critical lateral torsional buckling load of a system can be found by equalizing external work and internal work done at the limit state. To apply energy method, a twisting angle function which is in the form of critical lateral torsional buckling mode should be chosen. Also, selected function should satisfy the boundary conditions for the mentioned problem. However, for cantilever beams, form of the critical lateral torsional buckling mode changes due to loading case, cantilever length, torsional rigidity and warping rigidity of the section. Existence of a function which takes the form of critical lateral torsional buckling mode of any cantilever beam with loading case, cantilever length, torsional rigidity and warping rigidity parameters does not seem possible. Such function has not been encountered in literature as it is expected.

The last is solution of differential equation of equilibrium, which is the main concern of this study and introduced in the next section for various loading cases. A closed form solution of the equilibrium equation for pure bending case (Timoshenko and Gere, 1961) is recalled in many studies. On the contrary, for other loading cases, general solutions for lateral torsional buckling of cantilever beams are not known by the author. Use of numerical methods are mentioned in existing studies, however presentation of a detailed application on lateral torsional buckling of cantilever I beams has not been encountered.

In this paper, application of finite differences method for calculating lateral torsional buckling moment of cantilever I sections is presented for concentrated load at free end, uniformly distributed load, combination of concentrated load at free end and uniformly distributed load and constant moment cases.

2. EQUATIONS OF EQUILIBRIUM

Equation of equilibrium for lateral torsional buckling is a 4th order differential equation. For doubly symmetric sections, equation for concentrated load at free end acting from shear center is given below (Timoshenko and Gere, 1961).

$$\frac{d^4\phi}{ds^4}-\frac{C}{C_1}\frac{d^2\phi}{ds^2}-\frac{P^2s^2}{EI_\eta C_1}\phi=0 \qquad (2.1)$$

In Eq. 2.1, ϕ is twisting angle, s is the distance from free end, C is torsional rigidity calculated by multiplying shear modulus by torsional constant, C_1 is warping rigidity calculated by multiplying elasticity modulus by warping coefficient, E is elasticity modulus, I_η is moment of inertia about weak axis and P is the magnitude of the concentrated load at free end. Equation for uniformly distributed load is given below (Özbaşaran, 2013).

$$\frac{d^4\phi}{ds^4}-\frac{C}{C_1}\frac{d^2\phi}{ds^2}-\frac{q^2s^4}{4EI_\eta C_1}\phi=0 \qquad (2.2)$$

In Eq. 2.2, q is the magnitude of uniformly distributed load. Equation for combination of concentrated load at free end and uniformly distributed load is given below (Özbaşaran, 2013).

$$\frac{d^4\phi}{ds^4}-\frac{C}{C_1}\frac{d^2\phi}{ds^2}-\frac{q^2s^2(2\lambda L+s)^2}{4EI_\eta C_1}\phi=0 \qquad (2.3)$$

In Eq. 2.3, λ is the concentrated load multiplier. In Eq 2.3, in addition to uniformly distributed load q, a concentrated load with a magnitude of λqL at free end is acting to the system. Finally, equation for constant moment is given below (Özbaşaran, 2013).

$$\frac{d^4\phi}{ds^4} - \frac{C}{C_1}\frac{d^2\phi}{ds^2} - \frac{M^2\phi}{EI_\eta C_1} = 0 \qquad (2.4)$$

In Eq. 2.4, M is the magnitude of bending moment. Equations 2.1, 2.2 and 2.3 are valid only if cantilever beam is loaded from its shear center. For singly symmetric sections, torsional rigidity C should be reduced by $M_x\beta_x$. Here, M_x is the bending moment about strong axis. β_x is the Wagner's coefficient, which can be calculated as follows (Galambos, 1998).

$$\beta_x = \frac{\int_A y(x^2+y^2)dA}{I_x} - 2y_0 \qquad (2.5)$$

In Eq. 2.5, A is area of the cross section, x and y are the coordinates of the infinitesimal area along strong and weak axes with respect to gravity center of the section, respectively. dA is infinitesimal area, I_x is the moment of inertia about strong axis and y_0 is the coordinate of shear center along weak axis with respect to gravity center of the section.

Instead of searching for a closed form solution, it is easier to investigate the solutions of equations 2.1, 2.2, 2.3 and 2.4 by numerical methods. In this study, finite differences method is used. First, equilibrium equation of lateral torsional buckling is written by finite differences method for considered loading case. Then, boundary conditions are added to equation set. Smallest load value, which satisfies the obtained homogenous equation system, is assumed as elastic critical lateral torsional buckling load of the cantilever beam.

3. APPLICATION OF FINITE DIFFERENCES METHOD

The mentioned beam should be divided into finite elements to apply finite differences method. Accuracy of the results increase as the number of finite elements increases. Also, by increasing finite element number, the problem becomes harder to solve. Therefore, cantilever beam should be divided into optimum number of finite elements for easy calculation. In this study, cantilever beams are divided into 100 finite elements to determine elastic critical lateral torsional buckling loads precisely. Finite differences model of a cantilever I beam is given in Fig. 3.1.

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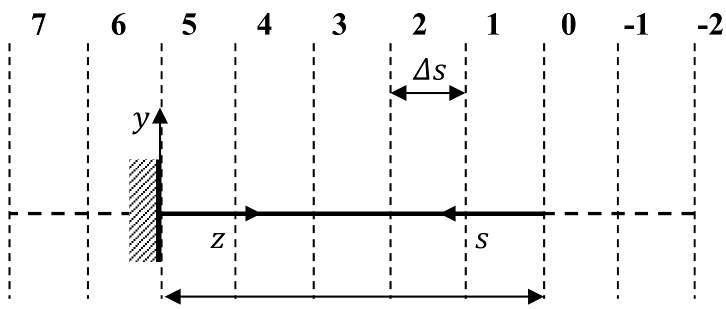

Figure 3.1. 1D model of a cantilever beam

In Fig. 3.1, L is cantilever length and Δs is finite element length. In Fig. 3.1, it is seen that cantilever beam is divided into 5 sub elements. As told above, this number is insufficient to determine elastic critical lateral torsional buckling moment of the element precisely. However, finite element number is chosen 5 to simplify the explanation of the method. In Equations 2.1, 2.2, 2.3 and 2.4, 2nd and 4th derivative of the ϕ angle at point i can be written as given below (Özbaşaran, 2013).

$$\frac{d^2\phi}{ds^2} = \frac{1}{\Delta s^2}(\phi_{i-1} - 2\phi_i + \phi_{i+1}) \tag{3.1}$$

$$\frac{d^4\phi}{ds^4} = \frac{1}{\Delta s^4}(\phi_{i-2} - 4\phi_{i-1} + 6\phi_i - 4\phi_{i+1} + \phi_{i+2}) \tag{3.2}$$

In this case, following equation should be satisfied on every i point on cantilever beam.

$$k_1(\phi_{i-2} - 4\phi_{i-1} + 6\phi_i - 4\phi_{i+1} + \phi_{i+2}) - k_2(\phi_{i-1} - 2\phi_i + \phi_{i+1}) - Z_1\phi_i = 0 \tag{3.3}$$

In Eq. 3,k_1, $k_2$and Z_1parameters are used for simplifying the equation. These parameters are given in Table 3.1 for different loading cases.

Table 3.1. k_1, $k_2$and Z_1 parameters

Loading Case	$\boldsymbol{Z_1}$	$\boldsymbol{k_1}$	$\boldsymbol{k_2}$
Concentrated load at free end	$\frac{P^2s^2}{EI_\eta C_1}$	$\frac{1}{\Delta s^4}$	$\frac{C}{C_1\Delta s^2}$
Uniformly distributed load	$\frac{q^2s^4}{4EI_\eta C_1}$		
Combination of concentrated load at free end and uniformly distributed load	$\frac{q^2s^2(2\lambda L + s)^2}{4EI_\eta C_1}$		
Constant moment	$\frac{M^2}{EI_\eta C_1}$		

For the beam given in Fig. 3.1, equations given in Table 3.2 can be written. These equations indicate the equilibrium at every node on the beam

Table 3.2. Equilibrium equations for nodes

Node	**Equation**	
0	$k_1(\phi_{-2} - 4\phi_{-1} + 6\phi_0 - 4\phi_1 + \phi_2) - k_2(\phi_{-1} - 2\phi_0 + \phi_1) - Z_1\phi_0 = 0$	(3.4)
1	$k_1(\phi_{-1} - 4\phi_0 + 6\phi_1 - 4\phi_2 + \phi_3) - k_2(\phi_0 - 2\phi_1 + \phi_2) - Z_1\phi_1 = 0$	(3.5)
2	$k_1(\phi_0 - 4\phi_1 + 6\phi_2 - 4\phi_3 + \phi_4) - k_2(\phi_1 - 2\phi_2 + \phi_3) - Z_1\phi_2 = 0$	(3.6)
3	$k_1(\phi_1 - 4\phi_2 + 6\phi_3 - 4\phi_4 + \phi_5) - k_2(\phi_2 - 2\phi_3 + \phi_4) - Z_1\phi_3 = 0$	(3.7)
4	$k_1(\phi_2 - 4\phi_3 + 6\phi_4 - 4\phi_5 + \phi_6) - k_2(\phi_3 - 2\phi_4 + \phi_5) - Z_1\phi_4 = 0$	(3.8)
5	$k_1(\phi_3 - 4\phi_4 + 6\phi_5 - 4\phi_6 + \phi_7) - k_2(\phi_4 - 2\phi_5 + \phi_6) - Z_1\phi_5 = 0$	(3.9)

In Table 3.2, twisting angles for virtual nodes -2, -1, 6 and 7 can be seen in equations. These virtual nodes do not exist on the cantilever beam. However, these nodes should be implemented into the homogenous equation system for applying finite differences method. Equilibrium equation number is 6. On the contrary, unknown twisting angle number is 10. Other required 4 equations for the solution of this homogenous equation system are obtained by boundary conditions of the beam. Mathematical expressions of the boundary conditions of the system are given in Table 3.3.

Table 3.3. Boundary conditions

Node	Equation	
0	$\frac{d^2\phi_0}{ds^2} = 0$	(3.10)
0	$C\frac{d\phi_0}{ds} - C_1\frac{d^3\phi_0}{ds^3} = 0$	(3.11)
5	$\frac{d\phi_5}{ds} = 0$	(3.12)
5	$\phi_5 = 0$	(3.13)

In Table 3.3, it is mentioned that bending moment in flanges, which is produced by warping, is 0 at free end. Therefore, 2nd derivative of the twisting angle should be 0. Also, there is no torsion at free end. At fixed end, there is no deformation caused by warping, therefore, 1st derivative of the twisting angle should be 0. Obviously, twisting is restrained at fixed end (Deren, et al., 2003). First and third derivative of the twisting angle at node i can be written by finite differences method as given below.

$$\frac{d\phi}{ds} = \frac{1}{2\Delta s}\left(-\phi_{i-1} + \phi_{i+1}\right) \tag{3.14}$$

$$\frac{d^3\phi}{ds^3} = \frac{1}{2\Delta s^3}\left(-\phi_{i-2} + 2\phi_{i-1} - 2\phi_{i+1} + \phi_{i+2}\right) \tag{3.15}$$

By substituting Equations 3.14 and 3.15 in the equations given in Table 3.3, boundary conditions can be written with finite differences method for the cantilever beam given in Fig. 3.1 (Table 3.4).

Table 3.4. Finite differences expressions of boundary conditions

Node	Equation	
0	$j_1(\phi_{-1} - 2\phi_0 + \phi_1) = 0$	(3.14)
0	$Cj_2(-\phi_{-1} + \phi_1) - C_1 j_3(-\phi_{-2} + 2\phi_{-1} - 2\phi_1 + \phi_2) = 0$	(3.15)
5	$j_2(-\phi_4 + \phi_6) = 0$	(3.16)
5	$\phi_5 = 0$	(3.17)

In Table 3.4, j_1, j_2 and j_3 parameters, which are given in Table 3.5, are independent from loading case.

Table 3.5. j_1, j_2 and j_3 parameters

Parameter	Value
j_1	$\frac{1}{\Delta s^2}$
j_2	$\frac{1}{2\Delta s}$
j_3	$\frac{1}{2\Delta s^3}$

With given equations in Table 3.2 and 3.4, a homogenous equation set with 10 unknown ϕ variables is obtained. Therefore, determining lateral torsional buckling load of the beam becomes an eigenvalue problem. Determinant of the coefficient matrix for the equations given in Table 3.2 and 3.4 should be 0 if the homogenous equation system has a solution other than 0 for every ϕi value. This eigenvalue problem can be easily solved by a mathematical software.

4. COMPARISON OF RESULTS

Finite element models of the compared sections are generated by ABAQUS software. S8R5 shell elements are used. This shell element has 8 nodes and 5 constraints in every node. Models are restrained from one end for displacement and rotation about all 3 axes. Loads are divided into 2 parts and applied to top flange and bottom flange of the section for preventing local perturbation problems. Dimensions of sections are given in Fig. 4.1, which are used for comparison of the results obtained by finite differences method and ABAQUS software.

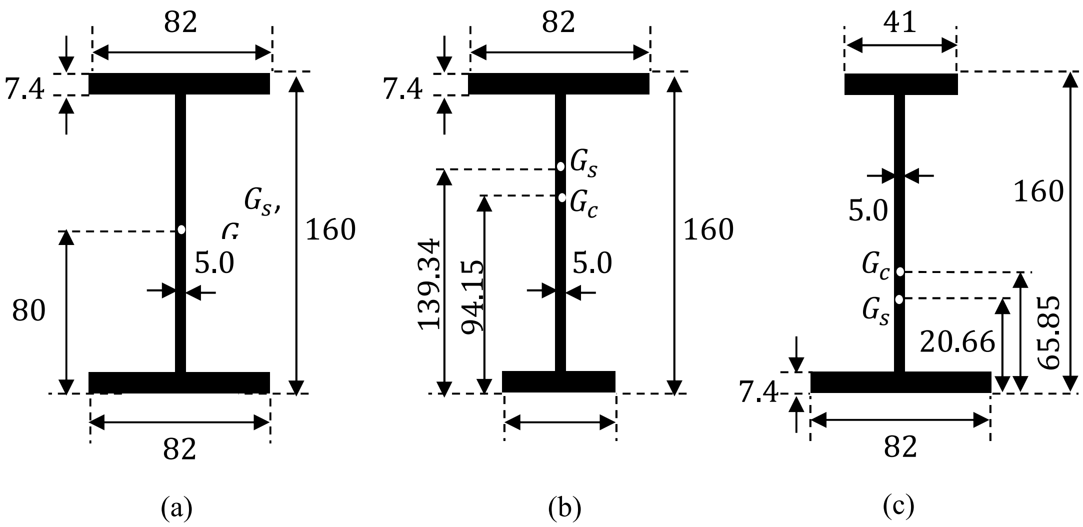

Fig. 4.1. Dimensions of selected sections in mm a) Section I, b) Section II, c) Section III

In Fig. 4.1, G_s indicates the shear center of the section. G_c is center of gravity. Properties of these sections are given in Table 4.1.

Table 4.1. Section properties

	Section I	**Section II**	**Section III**
$\boldsymbol{E}$ (**MPa**)	200000	200000	200000
$\boldsymbol{G}$ (**MPa**)	76923	76923	76923
$\boldsymbol{I_\eta}$ ($\mathbf{mm^4}$)	68.16*10^4	38.40*10^4	38.40*10^4
$\boldsymbol{I_t}$ ($\mathbf{mm^4}$)	28.20*10^3	22.66*10^3	22.66*10^3
$\boldsymbol{C_w}$ ($\mathbf{mm^6}$)	395.89*10^7	87.98*10^7	87.98*10^7
$\boldsymbol{G_c}$ (**mm**)	80.00	94.15	65.85
$\boldsymbol{G_s}$ (**mm**)	80.00	139.34	20.66
$\boldsymbol{\beta_x}$ (**mm**)	0	-111.97	111.97

In Table 4.1, G is shear modulus and I_t is torsional constant. In Table 4.2, elastic critical lateral torsional buckling moments are given for different cantilever lengths of Section I.

Table 4.2. Elastic critical lateral torsional buckling moments for Section I (kNm)

$\boldsymbol{L}$ (**m**)	**Method**	$\boldsymbol{M}$	$\boldsymbol{P}$	$\boldsymbol{q+P}$ (**λ=1.0**)	$\boldsymbol{q}$
1.5	FDM	28.34	98.93	120.18	198.19
	ABQ	28.03	98.06	118.50	192.79
	FDM /ABQ	**1.01**	**1.01**	**1.01**	**1.03**
2.0	FDM	19.25	63.96	77.28	124.72
	ABQ	18.93	65.28	77.88	122.10
	FDM /ABQ	**1.02**	**0.98**	**0.99**	**1.02**
3.0	FDM	11.47	35.61	42.66	66.83
	ABQ	11.19	36.06	42.12	65.70
	FDM /ABQ	**1.03**	**0.99**	**1.01**	**1.02**
4.0	FDM	8.08	24.08	28.80	44.00
	ABQ	7.89	24.32	28.56	44.00
	FDM /ABQ	**1.02**	**0.99**	**1.01**	**1.00**

FDM: Finite Differences Method
ABQ: ABAQUS software

In Table 4.2, M, P, $q+P$ (λ=1.0) and q columns indicate constant moment, concentrated load at free end, combination of uniform distributed load and concentrated load at free end (λ=1.0) and uniform distributed load cases respectively. Elastic critical lateral torsional buckling moments are given for Section II in Table 4.3

Table 4.3. Elastic critical lateral torsional buckling moments for Section II (kNm)

L **(m)**	**Method**	***M***	***P***	***q + P*** **(*λ*=1.0)**	***q***
1.5	FDM	10.79	27.75	32.23	45.73
	ABQ	10.57	27.63	32.06	45.05
	FDM /ABQ	**1.02**	**1.00**	**1.00**	**1.02**
2.0	FDM	8.31	20.72	23.94	33.42
	ABQ	8.03	20.76	24.00	33.34
	FDM /ABQ	**1.04**	**1.00**	**1.00**	**1.00**
3.0	FDM	5.67	13.89	16.07	22.01
	ABQ	5.57	13.89	16.07	22.01
	FDM /ABQ	**1.02**	**1.00**	**1.00**	**1.00**
4.0	FDM	4.31	10.56	12.24	16.56
	ABQ	4.18	10.52	12.24	16.56
	FDM /ABQ	**1.03**	**1.00**	**1.00**	**1.00**

FDM: Finite Differences Method
ABQ: ABAQUS software

Finally, for Section III, elastic critical lateral torsional buckling moments are given in Table 4.4

Table 4.4. Elastic critical lateral torsional buckling moments for Section III (kNm)

L **(m)**	**Method**	***M***	***P***	***q + P*** **(*λ*=1.0)**	***q***
1.5	FDM	22.52	86.25	106.99	185.99
	ABQ	21.63	83.94	103.07	180.57
	FDM /ABQ	**1.04**	**1.03**	**1.04**	**1.03**
2.0	FDM	14.54	52.44	64.62	109.58
	ABQ	14.08	52.04	63.72	105.28
	FDM /ABQ	**1.03**	**1.01**	**1.01**	**1.04**
3.0	FDM	8.29	27.54	33.48	54.72
	ABQ	7.98	27.54	33.48	54.05
	FDM /ABQ	**1.04**	**1.00**	**1.00**	**1.01**
4.0	FDM	5.75	18.12	21.84	34.72
	ABQ	5.63	18.12	21.84	34.48
	FDM /ABQ	**1.02**	**1.00**	**1.00**	**1.01**

FDM: Finite Differences Method
ABQ: ABAQUS software

As can be seen from Table 4.2, 4.3 and 4.4, results obtained by solution of differential equation of lateral torsional buckling and ABAQUS are nearly identical. As a result of the considered finite element number and ABAQUS mesh size, slight differences are seen some examples.

4.CONCLUSIONS

In the study, application of finite differences method in solution of differential equation of lateral torsional buckling is summarized by a 1D cantilever I section beam model, which is divided into 5 finite elements. One doubly symmetric and two singly symmetric I sections are used for comparison of obtained results by finite differences solutions of differential equation of lateral torsional buckling with ABAQUS software. Length of the cantilever beams which are considered for comparison of results vary from 1.5 to 4.0 m. Elastic critical lateral torsional buckling moments of these cantilever beams are calculated for constant moment, concentrated load at free end, combination of uniformly distributed load and concentrated load at free end and uniformly distributed load cases. It is seen from the calculations that results obtained by finite differences method and ABAQUS software coincides for the examined loading cases.

It is concluded that finite differences solution of differential equation of lateral torsional buckling is an efficient method for determining elastic critical lateral torsional buckling moment of cantilever beams. It should be noted that as in every numerical method, more accurate results can be obtained by dividing the beam into more parts, however, calculation time increases by increasing finite element number.

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