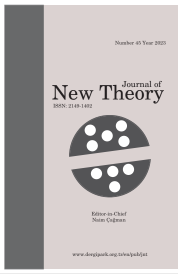
Journal of
New Theory
ISSN: 2149-1402

45 (2023) 120-130
Journal of New Theory
https://dergipark.org.tr/en/pub/jnt
Open Access



Spacelike Ac-Slant Curves with Non-Null Principal Normal in Minkowski 3-Space

Hasan Altınbaş[1]

Article Info
Received: 6 Dec 2023
Accepted: 28 Dec 2023
Published: 31 Dec 2023
doi:10.53570/jnt.1401001
Research Article

Abstract — In this paper, we define a spacelike ac-slant curve whose scalar product of its acceleration vector and a unit non-null fixed direction is a constant in Minkowski 3-space. Furthermore, we give a characterization depending on the curvatures of the spacelike ac-slant curve. After that, we get the relationship between a spacelike ac-slant curve and several distinct types of curves, such as spacelike Lorentzian spherical curves, spacelike helices, spacelike slant helices, and spacelike Salkowski curves, enhancing our understanding of its geometric properties in Minkowski 3-space. Finally, we used Mathematica, a symbolic computation software, to support the notions of an ac-slant curve with attractive images.

Keywords *Acceleration, helices, slant helices, Minkowski 3-spaces*

Mathematics Subject Classification (2020) 53A04, 53A35

1. Introduction

The motion of the object has a route that looks like a curve in space. The position of the object at time t is represented by the position vector of the curve at parameter t in the space. The first, second, and third derivatives of the curve are represented by the object's velocity, acceleration, and jerk vectors at any time t, respectively.

In kinematics and classical mechanics, which deal with the motion of bodies, the physical vector quantities are significant. The magnitude of velocity is known as speed. The rate at which velocity changes is called acceleration. The direction of acceleration is determined by the total force applied to the object. Newton's Second Law was first articulated in the seventeenth century by the English mathematician and scientist Sir Isaac Newton, who also described the magnitude of acceleration. Additionally, the jerk is the acceleration's rate of change [1–3].

In Euclidean 3-space, a regular curve α is said to be a helix if the tangent vector of α makes the fixed angle ϕ with a fixed direction which is the axis of helix where $\phi \in (0,\pi) \setminus \frac{\pi}{2}$. Moreover, the ratio τ/κ is a constant if and only if it is a general helix [4, 5]. A regular curve α is called a slant helix if its principal normal vector of α makes the fixed angle ϕ with a fixed direction which is the axis where ϕ is a constant [6]. If a regular curve α has nonconstant torsion τ but constant curvature κ, then α is called a Salkowski curve [7].

In Minkowski 3-space, A curve is called a helix (resp. slant helix) if the scalar product of its tangent

[1]hasan.altinbas@ahievran.edu.tr (Corresponding Author)
[1]Department of Mathematics, Faculty of Arts and Sciences, Kırşehir Ahi Evran University, Kırşehir, Türkiye

(resp. principal normal) vector and fixed direction is constant [8,9]. Furthermore, Ali [10] modified the definition of spacelike Salkowski curves with spacelike or timelike principle normal in this space with an explicit parametrization. These special curves are studied in different ambient spaces by some authors [11–17].

The plan of this paper is as follows: In section 2, we review the fundamental theory of curves in Minkowski 3-space. In section 3, we define a spacelike ac-slant curve whose scalar product of its acceleration vector and a non-null fixed direction is a constant. First, we provide a characterization based on the torsion and curvature of a spacelike ac-slant curve. Later, we get to the conclusion that when the ac-slant curve is a helix, either the acceleration vector is orthogonal to its axis or the magnitude of its velocity vector is a linear function. Later on, a unit speed curve with constant magnitude acceleration is an ac-slant curve if and only if it is a slant helix. Moreover, a unit speed curve is only a spacelike ac-slant curve if and only if it is a Salkowski curve when the magnitude of the acceleration is equal to one (i.e. $\kappa = 1$).

2. Preliminaries

In this section, we provide basic facts for Minkowski 3-space. For more detail and background, see [8,18,19].

Let $\mathbb{E}_1^3 = \left(\mathbb{R}^3\left(t,x,y\right),g\right)$ be a Minkowski 3-space where $g = -dt^2 + dx^2 + dy^2$ denotes the standard metric and (t,x,y) is the connanical coordinates in 3-dimensional real vector space $\mathbb{R}^3$. A vector u in $\mathbb{E}_1^3$ is called spacelike if $g\left(u,u\right) > 0$ or $u = 0$, timelike if $g\left(u,u\right) < 0$, and null if $g\left(u,u\right) = 0$ and $u \neq 0$, respectively. Moreover, the norm of u is defined by $\|u\| = \sqrt{|g(u,u)|}$. Furthermore, u is a unit vector if $g\left(u,u\right) = \pm 1$.

A curve $\alpha(t)$ is called spacelike, timelike, or null if velocity vector $v = \alpha'(t)$ of $\alpha(t)$ are spacelike, timelike, or null in $\mathbb{E}_1^3$ for each parameter t, respectively. Denote by $\{T,N,B\}$ the moving Frenet-Serret frame along the curve α in $\mathbb{E}_1^3$. Then, T, N, and B are the tangent, the principal normal, and the binormal vector fields, respectively. Besides, Frenet-Serret formulae are provided as follows:

$$T' = \varepsilon\nu\kappa N, \quad N' = -\nu\kappa T - \varepsilon\nu\tau B, \quad B' = -\varepsilon\nu\tau N \tag{1}$$

where κ and τ are curvatures of the curve α and

g	T	N	B
T	1	0	0
N	0	ε	0
B	0	0	$-\varepsilon$

such that $\varepsilon = \pm 1$.

In here, $\nu = g\left(\alpha'\left(t_0\right),\alpha'\left(t_0\right)\right)$ is called speed of α at $t_0 \in I$. Moreover, if $\nu = 1$, for all $t \in I$, then α is a unit speed curve. Lorentzian unit sphere is $S_1^2 = \left\{x \in \mathbb{E}_1^3 : g\left(x,x\right) = 1\right\}$. A curve that lies on the Lorentzian unit sphere is called a Lorentzian spherical curve.

From a physical point of view, the motion of particle P along the curve α at time t is correspond to the position vector of α. Then, it is widely known that the first, second, and third derivatives of α concerning time determine the velocity vector $v(t)$, acceleration vector $\mathbf{a}(t)$, and jerk vector $j(t)$, respectively. These vectors are determined by Equations 1 as follows:

$$\begin{aligned} v &= \alpha' = \nu T \\ \mathbf{a} &= \alpha'' = \nu' T + \varepsilon \nu^2 \kappa N \\ j &= \alpha''' = \left(\nu'' - \varepsilon \nu^3 \kappa^2\right) T + \left(3\varepsilon \nu' \nu \kappa + \varepsilon \nu^2 \kappa'\right) N - \nu^3 \kappa \tau B \end{aligned} \tag{2}$$

3. Spacelike Ac-Slant Curves with Non-Null Principal Normal

This section provides new curves, called spacelike ac-slant curves in Minkowski 3-space. Additionally, we characterize these curves.

Definition 3.1. A spacelike curve α is called a spacelike ac-slant curve whose inner product of a unit non-null fixed direction u, called axis of spacelike ac-slant curve, and acceleration vector $\mathbf{a}$ of the curve is constant, i.e., $g(\mathbf{a}, u) = c$, in Minkowski 3−space.

Remark 3.2. Let α be spacelike curve in $\mathbb{E}_1^3$. Then, α is a spacelike ac-slant helix if and only if the jerk vector of α is orthogonal to its axis u, i.e., $g(j, u) = 0$.

Theorem 3.3. Let α be a spacelike curve with Frenet apparatus $\{T, N, B, \kappa, \tau\}$ in Minkowski 3−space. Then, α is a spacelike ac-slant curve if and only if

$$\eta_1^2 + \varepsilon \eta_2^2 - \varepsilon \eta_3^2 = \epsilon \tag{3}$$

such that

$$\eta_1 = \frac{\frac{1}{\nu}\left(\varepsilon \frac{\tau}{\kappa} - f\right) - \left(\frac{1}{\varepsilon \nu \tau}\left(\frac{1}{\nu^2 \kappa}\right)'\right)' + \left(\frac{1}{\varepsilon \nu \tau}\left(\frac{\nu'}{\nu^3 \kappa}\right)\right)'}{f' + \frac{\nu'}{\nu}\left(\varepsilon \frac{\tau}{\kappa} - f\right)} \tag{4}$$

$$\eta_2 = \frac{1}{\nu^2 \kappa} - \frac{\nu'}{\nu^2 \kappa} \eta_1 \tag{5}$$

and

$$\eta_3 = f \eta_1 + \frac{1}{\varepsilon \nu \tau}\left(\left(\frac{1}{\nu^2 \kappa}\right)' - \frac{\nu'}{\nu^3 \kappa}\right) \tag{6}$$

where c is a nonzero constant, $\epsilon = \pm 1$, and

$$f = \frac{\kappa}{\tau} - \frac{1}{\varepsilon \nu \tau}\left(\left(\frac{\nu'}{\nu^2 \kappa}\right)' - \frac{(\nu')^2}{\nu^3 \kappa}\right)$$

PROOF.
Assume that α is a spacelike ac-slant curve with timelike or spacelike axis u. By Definition 3.1, there exist a constant $c = g(\mathbf{a}, u)$ and differentiable functions λ_i such that

$$u = \lambda_1 T + \lambda_2 N + \lambda_3 B \tag{7}$$

By using Equation 2 and 7,

$$\lambda_2 = \frac{c}{\nu^2 \kappa} - \frac{\nu'}{\nu^2 \kappa} \lambda_1 \tag{8}$$

After differentiating of Equation 7 and using Equation 8,

$$\lambda_1' - \frac{c}{\nu} + \frac{\nu'}{\nu} \lambda_1 = 0 \tag{9}$$

$$\varepsilon \nu \kappa \lambda_1 + c\left(\frac{1}{\nu^2 \kappa}\right)' - \left(\frac{\nu'}{\nu^2 \kappa}\right)' \lambda_1 - \frac{\nu'}{\nu^2 \kappa} {\lambda_1}' - \varepsilon \nu \tau \lambda_3 = 0 \tag{10}$$

and

$${\lambda_3}' - \varepsilon \frac{c\,\tau}{\nu \kappa} + \varepsilon \frac{\nu' \tau}{\nu \kappa} \lambda_1 = 0 \tag{11}$$

By substituting Equation 9 in Equation 10,

$$f\lambda_1 + \frac{c}{\varepsilon\nu\tau}\left(\left(\frac{1}{\nu^2\kappa}\right)' - \frac{\nu'}{\nu^3\kappa}\right) - \lambda_3 = 0 \tag{12}$$

where

$$f = \frac{\kappa}{\tau} - \frac{1}{\varepsilon\nu\tau}\left(\left(\frac{\nu'}{\nu^2\kappa}\right)' - \frac{(\nu')^2}{\nu^3\kappa}\right)$$

After differentiating of Equation 12, by using Equations 9 and 11,

$$\lambda_1 = \frac{\frac{c}{\nu}\left(\varepsilon\frac{\tau}{\kappa} - f\right) - \left(\frac{c}{\varepsilon\nu\tau}\left(\frac{1}{\nu^2\kappa}\right)'\right)' + \left(\frac{c}{\varepsilon\nu\tau}\left(\frac{\nu'}{\nu^3\kappa}\right)\right)'}{f' + \frac{\nu'}{\nu}\left(\varepsilon\frac{\tau}{\kappa} - f\right)} \tag{13}$$

Clearly, from Equation 12,

$$\lambda_3 = f\lambda_1 + \frac{c}{\varepsilon\nu\tau}\left(\left(\frac{1}{\nu^2\kappa}\right)' - \frac{\nu'}{\nu^3\kappa}\right) \tag{14}$$

Hence, by using Equations 8, 13, and 14, it is clear that there exist differentiable functions $\eta_i = \frac{1}{c}\lambda_i$ which is satisfying Equation 3, for $i \in \{1, 2, 3\}$

Conversely, let α be a spacelike curve with Frenet apparatus $\{T, N, B, \kappa, \tau\}$. Assume that there exists a unit non-null fixed direction u provided by Equation 7 where differentiable functions λ_i are presented by Equations 4-6. Then, it is observed that the scalar product of acceleration vector $\mathbf{a}$ is given by Equation 2 of α, and u is equal to a nonzero constant c. Thus, α is an ac-slant curve with the axis u. □

Thus, we conclude the following Corollaries from Theorem 3.3.

Corollary 3.4. Let α be a unit speed spacelike non helix curve with curvatures κ and τ in $\mathbb{E}_1^3$. Then, α is a spacelike ac-slant curve if and only if

$$\left(\frac{\left(\frac{\tau}{\kappa}\right)m}{\left(\frac{\tau}{\kappa}\right)'}\right)^2 + \varepsilon\frac{1}{\kappa^2} - \varepsilon\left(\frac{1}{\varepsilon\tau}\left(\frac{1}{\kappa}\right)' + \frac{m}{\left(\frac{\tau}{\kappa}\right)'}\right)^2 = \epsilon$$

where $m = 1 - \varepsilon\left(\frac{\tau}{\kappa}\right)^2 + \varepsilon\frac{\tau}{\kappa}\left(\frac{1}{\tau}\left(\frac{1}{\kappa}\right)'\right)'$.

Corollary 3.5. Let α be a unit speed spacelike ac-slant curve with curvature $\kappa = 1$ in $\mathbb{E}_1^3$. Then,

$$\tau(t) = \pm\frac{\sqrt{\frac{c^2}{c^2-\epsilon\varepsilon}}\,t}{\sqrt{1 + \varepsilon\frac{c^2}{c^2-\epsilon\varepsilon}t^2}}$$

where c is a nonzero constant.

Example 3.6. The curve

$$\alpha(t) = \left(\frac{1}{4}(t+2)^2, \frac{1}{4}(t+2)^2\sin t, \frac{1}{4}(t+2)^2\cos t\right)$$

is a spacelike curve in $\mathbb{E}_1^3$ since $g(\alpha'(t), \alpha'(t)) = \frac{1}{16}(t+2)^4 > 0$, for $t \in \mathbb{R}$. Moreover, the curve α lies on the surface $y^2 + z^2 = x^2$, and its acceleration vector is

$$\mathbf{a}(t) = \left(\frac{1}{2}, \frac{\sin t}{2} + (t+2)\cos t - \frac{1}{4}(t+2)^2\sin t, \frac{\cos t}{2} - (t+2)\sin t - \frac{1}{4}(t+2)^2\cos t\right)$$

in $\mathbb{E}_1^3$. Furthermore, α is a spacelike ac-slant curve with the timelike axis $u = (1, 0, 0)$ such that $c = -\frac{1}{2}$ (see Figure 1).

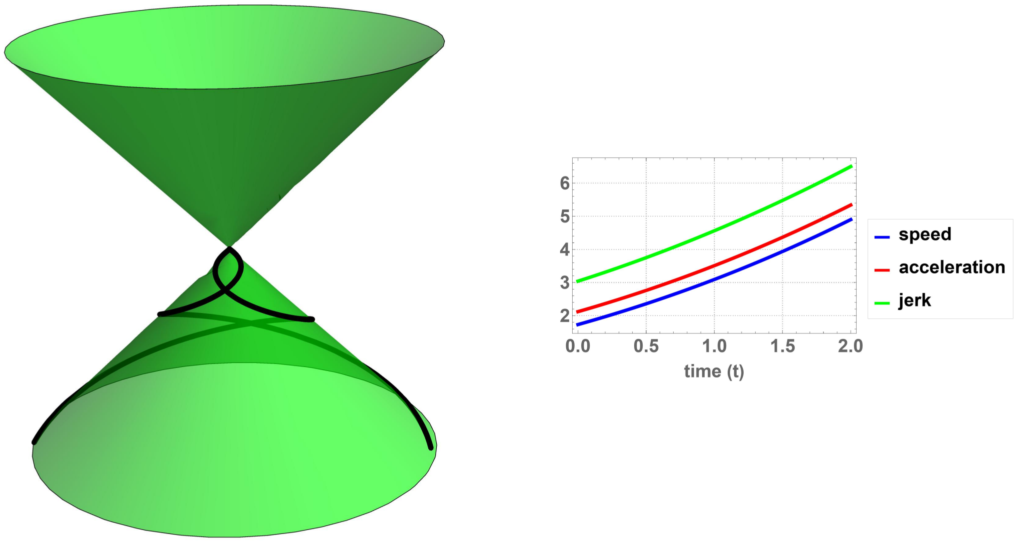


Figure 1. A spacelike ac-slant curve lying on the surface $y^2 + z^2 = x^2$

Example 3.7. The curve

$$\alpha(t) = \left(\log(\cos t), \frac{t^2}{2\sqrt{3}}, \log\left(\sin\frac{t}{2} + \cos\frac{t}{2}\right) - \log\left(\cos\frac{t}{2} - \sin\frac{t}{2}\right)\right)$$

is a spacelike curve in $\mathbb{E}_1^3$ since $g(\alpha'(t), \alpha'(t)) = \frac{1}{3}(t^2+3) > 0$, for $t \in \mathbb{R}$. Moreover, the acceleration vector of α is

$$\mathbf{a}(t) = \left(-\sec^2 t, \frac{1}{\sqrt{3}}, \tan t \sec t\right)$$

in $\mathbb{E}_1^3$. Furthermore, α is a spacelike ac-slant curve with the spacelike axis $u = (0,1,0)$ such that $c = \frac{1}{\sqrt{3}}$ (see Figure 2).

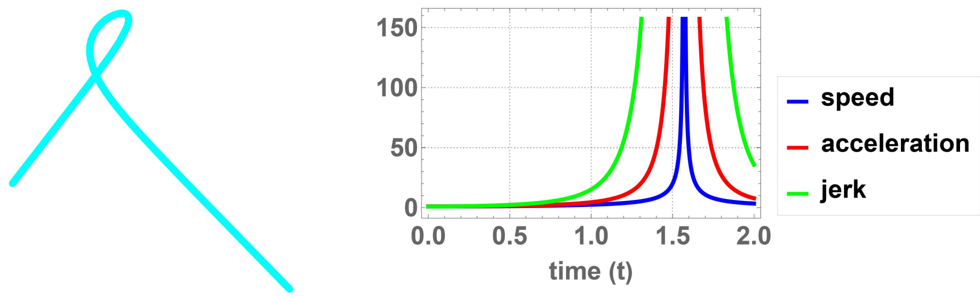


Figure 2. A spacelike ac-slant curve

Lemma 3.8. Let ν be a constant function and $c = 0$. Then, $\boldsymbol{\gamma}$ is a spacelike ac-slant curve if and only if

$$\lambda_1^2 - \varepsilon\lambda_3^2 = \epsilon \quad \text{and} \quad \lambda_2 = 0 \tag{15}$$

where λ_1 and λ_3 are nonzero constants.

PROOF.
Assume that α is a spacelike ac-slant curve with timelike or spacelike axis u. By Definition 3.1, there exist a constant $c = g(\mathbf{a}, u)$ and differentiable functions λ_i such that

$$u = \lambda_1 T + \lambda_2 N + \lambda_3 B \tag{16}$$

By using assumption ν is a constant function and $c=0$, we obtain $\lambda_2=0$ by using Equation 2. After differentiating of Equation 16 and using the fact of $\lambda_2=0$, we have following differential equations

$$\lambda_1'=0$$

$$\varepsilon\nu\kappa\lambda_1-\varepsilon\nu\tau\lambda_3=0$$

and

$$\lambda_3'=0$$

It can be observed that Equation 15 is satisfied since u is a unit fixed direction.

Conversely, let α be a spacelike curve with Frenet apparatus $\{T,N,B,\kappa,\tau\}$. Suppose that there exists a unit fixed direction u presented by Equation 16 where differentiable functions λ_i are provided by Equation 15. Then, it is clear that $g(\mathbf{a},u)=0$ where ν is constant function. □

Remark 3.9. It can be observed that u is not exist if $c=0$ and ν is a nonconstant function.

In the light of [20], we can provide following Corollary.

Corollary 3.10. Let α be a spacelike Lorentzian spherical curve with radius $r\in\mathbb{R}^+$ in Minkowski 3−space. Then, following equations are satisfied:

$$\left(\frac{1}{\kappa}\right)^2-\left(\frac{1}{\nu\tau}\left(\frac{1}{\kappa}\right)'\right)^2=\varepsilon r^2$$

and

$$\frac{\tau}{\kappa}=\frac{1}{\nu}\left(\frac{1}{\nu\tau}\left(\frac{1}{\kappa}\right)'\right)'$$

Theorem 3.11. Let α be a unit speed spacelike spherical curve with radius $r\in\mathbb{R}^+$, not a helix in $\mathbb{S}_1^2$. Then, α is a spacelike ac-slant curve if and only if

$$\left(\frac{1}{\left(\frac{\tau}{\kappa}\right)'}\right)^2\left(\frac{\tau^2}{\kappa^2}-\varepsilon\right)-2\frac{\frac{1}{\tau}\left(\frac{1}{\kappa}\right)'}{\left(\frac{\tau}{\kappa}\right)'}=\frac{1-r^2}{c^2} \tag{17}$$

where c is a nonzero constant.

Proof.
Let α be a unit speed spacelike spherical curve in $\mathbb{S}_1^2$. Then, Corollary 3.10 is satisfied. Assume that α is a spacelike ac-slant curve with a non-zero constant $c=g(\mathbf{a},u)$. Then, by using Corollary 3.10 join with Equations 4-6, there exist differentiable functions λ_i such that

$$u=\lambda_1T+\lambda_2N+\lambda_3B$$

where

$$\lambda_1=c\frac{\frac{\tau}{\kappa}}{\left(\frac{\tau}{\kappa}\right)'}$$

$$\lambda_2=c\frac{1}{\kappa}$$

and

$$\lambda_3=c\left(\frac{1}{\varepsilon\tau}\left(\frac{1}{\kappa}\right)'+\frac{1}{\left(\frac{\tau}{\kappa}\right)'}\right)$$

Using Corollary 3.10, since u is a unit fixed direction, we obtain Equation 17. Conversely, the proof is clear. □

Moreover, we get the following characterization for ac-slant curves from Lemma 3.8.

Remark 3.12. The curve α is a spacelike ac-slant curve with the non-null axis u satisfying Equation 15 if and only if α is a helix with the same axis u.

The proof is straightforward from Equation 15.

Theorem 3.13. Let α be a spacelike helix with axis u in $\mathbb{E}_1^3$. Then, α is a spacelike ac-slant curve with the axis u if and only if the magnitude of the velocity, i.e., $|v| = |\alpha'| = \nu$, is a linear function concerning parameter of α.

Proof.
Let α be a spacelike curve with the Frenet apparatus $\{T, N, B, \kappa, \tau\}$ in $\mathbb{E}_1^3$. We assume that α is a spacelike helix with axis u. Then, there exists a constant c_1 such that

$$g(T, u) = c_1 \tag{18}$$

After differentiating of Equation 18 and using Equation 1,

$$g(N, u) = 0 \tag{19}$$

Suppose that α is a spacelike ac-slant curve with the same axis u which is provided by Equation 16. Then, $\lambda_1 = c_1$ by using Equations 18 and 19. By using Equation 8,

$$\lambda_2 = \frac{1}{\nu^2\kappa}(c - \nu' c_1) = 0$$

Therefore, ν' is a constant.

Conversely, suppose that α is a spacelike helix with axis u in $\mathbb{E}_1^3$ and ν is a linear function with respect to the parameter of α. Then, there exists a constant c_1 such that $u = c_1 T + \sqrt{1 - \varepsilon c_1^2} B$. Thus, by using Equation 2, $g(\mathbf{a}, u) = \nu' c_1 = const$. Hence, α is a spacelike ac-slant curve with the axis u. □

Example 3.14. The curve

$$\alpha(t) = \left(t\cosh t - \sinh t, t\sinh t - \cosh t, \frac{t^2}{2}\right)$$

is a spacelike curve in $\mathbb{E}_1^3$ since $g(\alpha'(t), \alpha'(t)) = 2t^2 > 0$, for $t \in \mathbb{R}$. Moreover, the curve α lies on the surface $y^2 - x^2 + 2z = 1$, and its acceleration vector is

$$\mathbf{a}(t) = (\sinh t + t\cosh t, t\sinh t + \cosh t, 1)$$

in $\mathbb{E}_1^3$. Furthermore, α is an ac-slant curve with the spacelike axis $u = (0, 0, 1)$ such that $c = 1$ (see Figure 3).

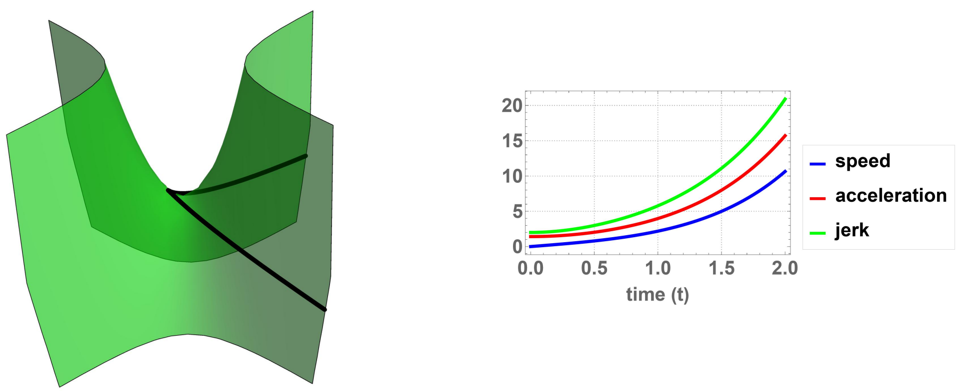


Figure 3. A spacelike ac-slant helix lying on the surface $y^2 - x^2 + 2z = 1$

Moreover, tangent vector field of $\alpha(t)$ is

$$T = \left(\frac{\sinh t}{\sqrt{2}}, \frac{\cosh t}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

Then, $\alpha(t)$ is a spacelike helix since $g(T, u) = \frac{1}{\sqrt{2}}$ and $\frac{\tau}{\kappa} = 1$. Furthermore, Theorem 3.13 is satisfied, i.e., $|v(t)| = \sqrt{2}t$ is a lineer function.

Example 3.15. The curve

$$\alpha(t) = \left(t^2, t^2 \sin\left(\log\left(t^2\right)\right), t^2 \cos\left(\log\left(t^2\right)\right)\right)$$

is a spacelike curve in $\mathbb{E}_1^3$ since $g(\alpha'(t), \alpha'(t)) = 4t^2 > 0$, for $t \in \mathbb{R}$. Moreover, the curve α lies on the surface $y^2 + z^2 = x^2$, and its acceleration vector is

$$\mathbf{a}(t) = (2, 6\cos(2\log(t)) - 2\sin(2\log(t)), -2(3\sin(2\log(t)) + \cos(2\log(t))))$$

in $\mathbb{E}_1^3$. Furthermore, α is an ac-slant curve with the timelike axis $u = (1, 0, 0)$ such that $c = -2$ (see Figure 4).

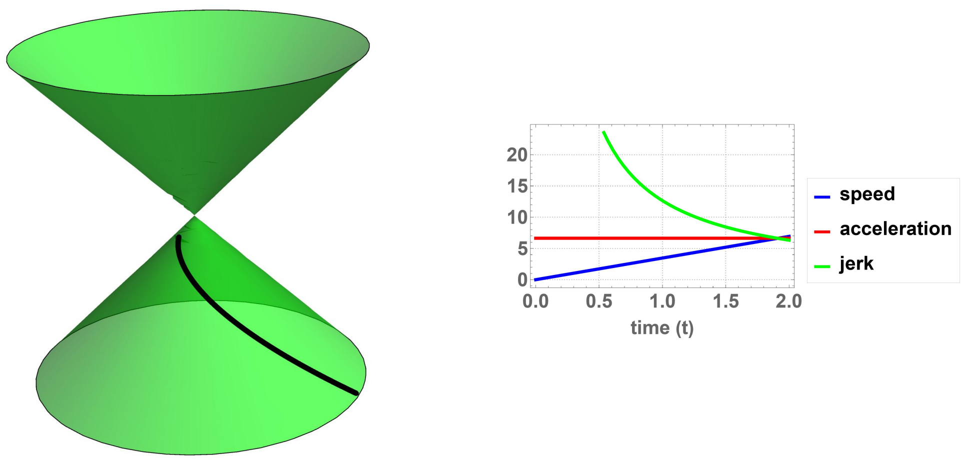


Figure 4. A spacelike ac-slant helix lying on the surface $y^2 - z^2 = x^2$

Moreover, tangent vector field of $\alpha(t)$ is

$$T = (1, \sin(2\log(t)) + \cos(2\log(t)), \cos(2\log(t)) - \sin(2\log(t)))$$

Then, $\alpha(t)$ is a spacelike helix since $g(T, u) = -1$ and $\frac{\tau}{\kappa} = \frac{1}{\sqrt{2}}$. Furthermore, Theorem 3.13 is satisfied, i.e., $|v(t)| = 2t$ is a lineer function.

Corollary 3.16. Let α be a unit speed spacelike curve with a constant magnitude of acceleration, i.e., with constant curvature, in $\mathbb{E}_1^3$. Then, α is a spacelike ac-slant curve if and only if α is a slant helix.

PROOF.
Suppose that α is a unit speed spacelike curve with nonzero constant curvature $\kappa = \kappa_0$. Then, $\mathbf{a} = \varepsilon\kappa_0 N$. Hence, the proof is clear. $\square$

Example 3.17. The curve

$$\alpha(t) = \left(t^2, \sqrt{t^4+1}\cos t, \sqrt{t^4+1}\sin t\right)$$

spacelike curve in $\mathbb{E}_1^3$ since $g(\alpha'(t), \alpha'(t)) > 0$, for $t \in (1, \infty)$. Moreover, the curve α lies on the

Lorentzian sphere $y^2 + z^2 - x^2 = 1$, and its acceleration vector is

$$\mathbf{a}(t) = \begin{pmatrix} 2, \\ \dfrac{-4\left(t^4+1\right)t^3 \sin t - \left(t^8 - 2t^6 + 2t^4 - 6t^2 + 1\right)\cos t}{\left(t^4+1\right)^{3/2}}, \\ \dfrac{4\left(t^7+t^3\right)\cos t - \left(t^8 - 2t^6 + 2t^4 - 6t^2 + 1\right)\sin t}{\left(t^4+1\right)^{3/2}} \end{pmatrix}$$

in $\mathbb{E}_1^3$. Furthermore, α is an ac-slant curve with the timelike axis $u = (1, 0, 0)$ such that $c = -2$ (see Figure 5).

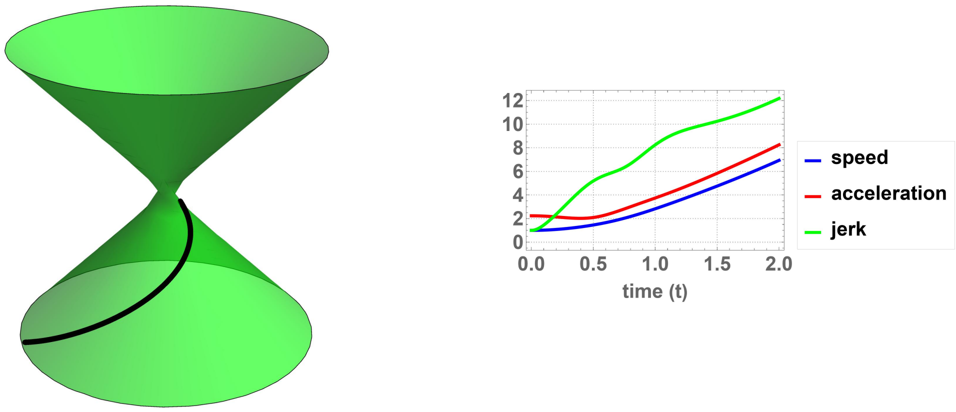


Figure 5. A spacelike Lorentzian spherical ac-slant helix lies on the surface $y^2 + z^2 - x^2 = 1$

Moreover, Corollary 3.10 is satisfied since α is spacelike Lorentzian sphere.

In the light of [10], we can provide the following lemma.

Lemma 3.18. Let α be a unit speed spacelike curve with non-null principal normal vector field with $\kappa = 1$. Its normal vector N makes a constant hyperbolic angle ϕ with a fixed straight line in $\mathbb{E}_1^3$ if and only if $\tau(s) = \pm\frac{s}{\sqrt{\varepsilon(s^2 - \tanh^2\phi)}}$.

Corollary 3.19. Let α be a unit speed spacelike curve with non-null principal normal vector with $\kappa = 1$. Then, α is a spacelike ac-slant curve if and only if α is a Salkowski curve.

PROOF.
Assume that α is a unit speed spacelike ac-slant curve with $\kappa = 1$. Then, α is a Salkowski curve by Corollary 3.16 and Lemma 3.18. Conversely, α is a unit speed spacelike Salkowski curve with $\kappa = 1$. Then, α is a spacelike ac-slant curve by Lemma 3.18 and Corollary 3.4. $\square$

Example 3.20. The curve

$$\alpha(t) = \begin{pmatrix} t^2 - \dfrac{3}{8}, \\ -\dfrac{4t\left(\sqrt{3-2\sqrt{3}t}\sqrt{3-4t^2} + 3t\sqrt{2\sqrt{3}t+3}\right) + 3\sqrt{2\sqrt{3}t+3}}{15\sqrt{2}}, \\ \dfrac{t^2 12\sqrt{3-2\sqrt{3}t} - 4t\sqrt{2\sqrt{3}t+3}\sqrt{3-4t^2} + 3\sqrt{3-2\sqrt{3}t}}{15\sqrt{2}} \end{pmatrix}$$

is a unit speed spacelike curve in $\mathbb{E}_1^3$ since $g\left(\alpha'(t), \alpha'(t)\right) = 1 > 0$, for $t \in \mathbb{R}$. Moreover, the curve α

lies on the surface

$$\left(\frac{x+\frac{9}{8}}{\frac{\sqrt{15}}{2\sqrt{2}}}\right)^2 - \frac{y^2}{2} - \frac{z^2}{2} = \frac{6}{25}$$

and α is a spacelike ac-slant curve with timelike axis $u = (1,0,0)$ since $g(\mathbf{a}, u) = -2$ (see Figure 6).

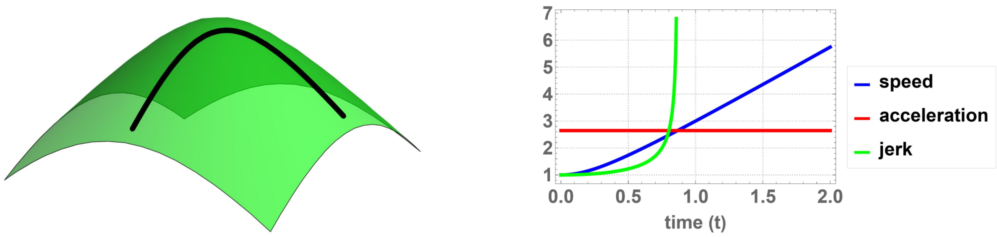


Figure 6. A spacelike ac-slant helix, Salkowski and slant helix lying on the surface $\left(\frac{x+\frac{9}{8}}{\frac{\sqrt{15}}{2\sqrt{2}}}\right)^2 - \frac{y^2}{2} - \frac{z^2}{2} = \frac{6}{25}$

Moreover, $g(N, u) = 2$. Thus, α is also a spacelike slant helix. Furthermore, since $\kappa = 1$, Corollary 3.16 is satisfied. Hence, α is a Salkowski curve. Further, Corollary 3.5 is satisfied and $\tau(t) = -\frac{2t}{\sqrt{3-4t^2}}$

4. Conclusion

Acceleration helps us understand the motion state of an object and aids in controlling that motion. Moreover, acceleration is a fundamental parameter for comprehending object interactions and explaining physical events. This comprehensive study contributes to the theoretical foundation of spacelike ac-slant curves and demonstrates their connections to well-known curves in Minkowski 3-space. We believe further investigation of spacelike ac-slant curves applies to other spaces.

Author Contributions

The author read and approved the final version of the paper.

Conflicts of Interest

The author declares no conflict of interest.

References

[1] S. H. Schot, *Jerk: The Time Rate of Change of Acceleration*, American Journal of Physics 46 (11) (1978) 1090–1094.

[2] H. Bondi, R. J. Seeger, *Relativity and Common Sense: A New Approach to Einstein*, American Journal of Physics 34 (4) (1966) 372–372.

[3] H. Crew, The Principles of Mechanics, BiblioBazaar, Boston, 2008.

[4] M. A. Lancret, *Memoire Sur les Courbes á Double Courbure*, Memoires Presentes a Institut (1806) 416–454.

[5] D. J. Struik, Lectures on Classical Differential Geometry, Courier Corporation, London, 1961.

[6] S. Izumiya and N. Takeuchi, *New Special Curves and Developable Surfaces*, Turkish Journal of Mathematics 28 (2) (2004) 153–164.

[7] E. Salkowski, *Zur Transformation Von Raumkurven*, Mathematische Annalen 66 (4) (1909) 517–557.

[8] R. López, *Differential Geometry of Curves and Surfaces in Lorentz-Minkowski Space*, International Electronic Journal of Geometry 7 (1) (2014) 44–107.

[9] A. T. Ali, R. López, *Slant Helices in Minkowski Space* E_1^3, Journal of the Korean Mathematical Society 48 (1) (2011) 159–167.

[10] A. T. Ali, *Spacelike Salkowski and Anti-Salkowski Curves with a Spacelike Principal Normal in Minkowski 3-Space*, International Journal of Open Problems in Computer Science and Mathematics 2 (3) (2009) 451–460.

[11] E. Nesovic, U. Ozturk, E. B. Koc Ozturk, *On Non-Null Relatively Normal-Slant Helices in Minkowski 3-Space*, Filomat 36 (6) (2022) 2051–2062.

[12] S. Kızıltug, S. Kaya, O. Tarakcı, *The Slant Helices According to Type-2 Bishop Frame in Euclidean 3-Space*, International Journal of Pure and Applied Mathematics 2 (2013) 211-222.

[13] B. Bukcu, M. K. Karacan, *The Slant Helices According to Bishop Frame of the Spacelike Curve in Lorentzian Space*, Journal of Interdisciplinary Mathematics 12 (5) (2009) 691-700.

[14] H. Altinbas, M. Mak, B. Altunkaya, L. Kula, *Mappings That Transform Helices From Euclidean Space to Minkowski Space*, Hacettepe Journal of Mathematics and Statistics 51 (5) (2022) 1333–1347.

[15] K. Ilarslan, *Spacelike Normal Curves in Minkowski Space* E_1^3, Turkish Journal of Mathematics 29 (1) (2005) 53–63.

[16] M. Babaarslan, Y. Yayli, *On Helices and Bertrand Curves in Euclidean 3-Space*, Mathematical and Computational Applications 18 (1) (2013) 1–11.

[17] L. Kula, Y. Yayli, *On Slant Helix and Its Spherical Indicatrix*, Applied Mathematics and Computation 169 (1) (2005) 600–607.

[18] M. P. Do Carmo, Differential Geometry of Curves and Surfaces: Revised and Updated Second Edition, Courier Dover Publications, New York, 2016.

[19] B. O'Neill, Semi-Riemannian Geometry with Applications to Relativity, Academic Press, New York, 1983.

[20] M. Petrovic-Torgasev, E. Sucurovic, *Some Characterizations of Lorentzian Spherical Spacelike Curves with the Timelike and the Null Principal Normal*, Mathematica Moravica (4) (2000) 83–92.