ENTROPİ OPTİMİZASYON ÖLÇÜSÜ İLE OPTİMAL PORTFÖY SEÇİMİ VE BİST ULUSAL-30 ENDEKSİ ÜZERİNE BİR ÇALIŞMA

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Özet

Finansal karar alma yöntemlerinden optimizasyon modelleri gün geçtikçe önemini arttırmaktadır. Günümüzde portföy yöneticileri, portföylerini oluştururken en yaygın modern portföy teoremi olan Markowitz Ortalama-Varyans (MV) yöntemi ile minimum risk ve maksimum getiri düzeyinde portföylerini oluşturmaktadırlar. Bu çalışmanın amacı alışıgelmiş optimal portföy yöntemi olan Markowitz Ortalama-Varyans (MV) modeli dışında minumum entropi ve maksimum entropi ölçüsü ile de optimal portföye ulaşılıp ulaşılamayacağını test etmek ve hesaplanan portföy performans değerlerini karşılaştırmaktır. Çalışmada BİST Ulusal-30 Endeksinde yer alan hisse senetleri ile hem Markowitz Ortalama-Varyans(MV) teoremi hemde Minimum entropi ve Maksimum entropi ölçüsü kullanılarak portföyler seçilmiştir. Öncelikle eşit ağırlıklı portföy oluşturularak portföyün getirisi ve riski hesaplanmıştır. Daha sonra gerekli kısıtlar eklenerek, portföylerin ağırlıkları bulunup minimun risk ve maksimum getiri düzeyinde üç farklı portföy oluşturulmuştur. Oluşturulan portföylerin performansları; Sharp Oranı, Treynor Endeksi ve Jensen Ölçüsü ile hesaplanmıştır.Portföy ağırlıkları ile ilgi geleceğe yönelik tahminler yapılmış ve elde edilen sonuçların nedenleri belirtilmiş olup, en uygun optimal portföy ve en uygun optimizasyon yöntemi belirlenmiştir.

Anahtar Kelimeler: Markowitz Ortalama Varyans(MV), Minumum Entropi, Maksimum Entropi, Optimizasyon

Jel Kodu: G11, C61, C58, C52

A STUDY ON OPTIMAL PORTFOLIO SELECTION VIA ENTROPY OPTIMIZATION METHOD AND APPLICATION WITH BIST NATIONAL-30 INDEX DATA

Abstract

The importance of optimization models on financial decision making process has been increasing over time. Markowitz Mean-Variance Method (MV) is the most widely used method by portfolio managers to estimate the minimum risk and maximum return on their portfolios. The purpose of this study is to determine whether an optimal portfolio can be formed by another method, namely, minimum entropy and maximum entropy optimization. By using BIST National – 30 Index data, three portfolios are generated in this study with minimum entropy, maximum entropy, and Markowitz Mean-Variance (MV) methods. First, risk and return of the equally weighted portfolio is calculated. Subsequently, necessary constraints are added, the weights of the portfolios are determined, and three portfolios are formed at the minimum risk and maximum return condition. The performances of the new portfolios are calculated and compared using Sharp Ratio, Treynor Index and Jensen Scale. Based on the results obtained, the most appropriate optimal portfolio and the optimization method is determined and also portfolio weights made predictions for the future and the results obtained indicated reasons.

Key Words: Markowitz Mean Variance (MV), Minumum Entropy, Maximum Entropy, Optimization,

Jel Classification: G11, C61, C58, C52

1. Introduction

Today, investors are interested in the stock market in order to gain profit. Individual and institutional investors should prefer investing in more than one stock by creating a portfolio, rather than a single stock, in order to maximize profit and avoid possible risks. The portfolios that are created should not be chosen randomly, but rather at the maximum level of return and minimum level of risk. The reason for diversification in portfolio theory is reducing risk. The most common method used for risk calculation is standard deviation. Standard deviation is widely used in investment decisions because its calculation is simple. When we talk about the concept of diversification, the preference is the Markowitz Mean Variance (MV) model, which is the most prevalent modern portfolio theorem today (Birkan, 2006). The Modern Portfolio Theorem is based on the need to maximize returns from stock investments at a certain level of risk. The first study in this field has been conducted by Henry S. Markowitz in 1952. In his 1952 article "Portfolio Selection", Markowitz studied the expected return and return variance limitation, and claimed that the optimal portfolio can be created under these limitations (Markowitz, 1952). Markowitz Mean-Variance (MV) portfolio optimization theory is based on the mean-return, variance and covariance of the assets. However, the criticisms towards the model claim that the weight estimates are based on the first two moments obtained from the sample and therefore statistical estimation error is probable (Altayligil, 2008). When the mean and covariance matrix of the chosen sample is used in portfolio selection, the weights in the portfolio can be very different, in a way that does not correspond to the sample and this can have a negative effect on performance (Bera and Park, 2008). The statistical errors then lead to misevaluation of the weights of the portfolio and this in turn creates a risk of wrong ratios of investment on the different stocks. Various studies have been conducted in order to reduce statistical errors that might arise in the Markowitz Mean-Variance (MV) model. Frost and Savarino in 1986, Jorion in 1986, Ledoit and Wolf in 2003, proposed the use of "Shrinkage Predictors" in order to reduce statistical errors that might arise in the Markowitz Mean-Variance (MV) model. However, since shrinkage predictors are based on empirical Bayesian approach, it has not been easy to determine suitable a priori distributions and there is not an appropriate way to select these distributions. In their 2008 article "Diversification Using Maximum Entropy Principle", Bera and Park proposed the cross entropy method in order to eliminate the possible statistical error (Bera and Park,2008). The main purpose of this study is to test whether the optimal portfolio can be created with the measure of entropy and the validity of the portfolios that are created by applications on stocks from the BIST National-30 Index. Firstly, the study does a portfolio optimization with the modern portfolio theorem, the Markowitz Mean-Variance (MV) method. Then, optimal portfolios were created by the maximum entropy and minimum entropy models, with the necessary limitations included. Performances of the three different portfolios were calculated and the superior model was found. Finally the optimization processes for each of the three models were repeated with the Pearson's skewness limit added. Microsoft Office Excel software was used for the calculations needed for the mathematical models in the study.

2. General Concepts

2.1. Portfolio Analysis

2.1.1 Portfolio

Under current conditions individual and institutional investors invest in stocks in order to increase their profits. The level of return from a stock investment depends on the risk level taken by the investor. Investors aim to invest at the minimum risk level and maximum return level that they can bear during the investment period. The idea that stock investments have higher levels of return, and the fact that the cost are relatively low in comparison to other investments, led individual and institutional investors towards stocks (Halici, 2008). The concept of stock investment gave rise to the portfolio concept. In order to reach their investment goal, or to maximize returns, investors need to invest in stocks that are appropriate for them and that are expected to increase in value. However, there is a risk element that they have to bear in order to collect the expected returns. In order to reduce this risk, the number of stocks needs to be increased. This then leads to the need for investing in multiple stocks and therefore creating and diversifying a portfolio (Halici, 2008). A portfolio is the cumulative value that an investor created by investing in multiple instruments that have similar or different characteristics (such as currency, gold, foreign currency, time deposit or financial deposits such as bond, stock or treasury bill). The portfolio structure and portfolio management of individuals or institutions depend on their tendencies to take risk, preferences of liquidity and return rates of various deposits. In other words, portfolios are collections of various financial instruments, mainly stocks, bonds and their derivatives, held by an individual or group (Korkmaz and Ceylan, 2006: 471). The concept of portfolio selection was brought forward by Henry S. Markowitz's 1952 article titled Portfolio Selection. Markowitz Mean-Variance (MV) theorem, which has been used widely until today, is now criticized and alternative methods are being derived.

2.1.2. Portfolio Management

Portfolio management is the management of multiple investment instruments in one basket at the minimum level of risk and maximum level of return that is possible within the limits determined by the real investor (customer). The person who is managing this portfolio is called the portfolio manager.

2.1.3. Optimization-Diversification

The purpose of portfolio management is to reach the optimal portfolio. Optimal portfolio then, is creating a balance between the expected returns and risks of the selected instruments from a certain number of instruments (Kose, 2001). The purpose of this balance is to provide highest return with minimum risk under market conditions. The investments rates on the selected instruments are determined with this consideration. This situation that the portfolio manager is trying to create is called portfolio optimization. Portfolio managers try to increase the number of instruments that they hold in order to lower the risk of the portfolio. In other words, they can reduce risk by diversifying their portfolio. There are two approaches in portfolio diversification: the traditional approach and the modern approach.

2.1.3.1. Traditional Portfolio Management Approach

The purpose of traditional portfolio management is to maximize the return of the investment. In order to do this, it has been argued that the portfolio needs to contain a lot of instruments and that the risk of the portfolio can be reduced as the number of instruments increase. However, this approach assumes that the risk of the portfolio can be lowered by diversifying the portfolio without taking into consideration the relationships between the returns of the instruments included in the portfolio (Cetindemir, 2006).

2.1.3.2. Modern Portfolio Management Approach

Henry S. Markowitz’s 1952 article titled “Portfolio Selection” has been the basis of modern portfolio theory. Markowitz’s Mean-Variance model invalidated the traditional approach because Markowitz claimed that lowering the risk of the portfolio is more likely via the direction and degree of the relationship between the instruments in the portfolio rather than merely considering diversification. One of the important issues in diversifying the instruments in the portfolio is choosing negative or zero correlation instruments rather than positive ones. Markowitz claims that diversification of opposite direction correlation instruments is more effective in lowering the risk of the portfolio (Cetindemir, 2006).

2.1.4. Basic Concepts Used In Portfolio Analysis

Standard Deviation and Variance

The concepts of Standard Deviation and Variance define risk in portfolio analysis. The lower the standard deviation is the lower the risk. Standard deviation shows how much the return of a stock deviates from the expected return. Therefore the standard deviation of a portfolio shows how much the return of a stock deviates from the expected return and the aim of portfolio management is to minimize standard deviation.

Mean-Return

The mean of a portfolio defines the return of a portfolio. In portfolio management the aim is to maximize return. The expected return in an investment is the mean of the past returns.

Correlation Coefficient

Correlation is a coefficient that indicates the relationship between variants. Correlation value varies between -1 and + 1. When the coefficient is close to -1 it means that there is a strong but opposite relationship between variants while values closer to + 1 indicate strong and same direction (linear) relationships. As the calculated value gets closer to 0, the strength of the relationship is lower and when the coefficient is 0 it means that there is no relationship between the variants.

Covariance Matrix

The matrix calculated in order to create the covariance value is called the covariance matrix. The covariance value indicated the reciprocal changes between two variants, in a sense the relationship itself. The relationship takes a negative or positive value between - ∞ and +∞.

Beta Coefficient

The beta coefficient is calculated as the ratio between the covariance between the statistical return of the stock and the return of the portfolio and the variance of market return. Beta coefficient is a risk measure. It measures the systematic risk, the market risk of an instrument. In other words, it is the relationship between the performance of an instrument and the mean performance of the market (Halici, 2008). It is the coefficient of the change in the instrument in response to one unit of change in the market. The beta coefficient of the market is always considered to be 1. The beta of the portfolio then, is equal to the weighted mean of the betas of the instrument in the portfolio. If the beta of the portfolio is "1", this means that the portfolio is moving parallel to the market. When it is bigger than 1, it means that it will fall or rise more than the market. When it is smaller than “1”, it means that it will fall or rise lower than the market. Portfolios that have betas higher than 1 are more risky in comparison to portfolios that have betas lower than 1 (Halici, 2008).

Pearson’s Skewness Coefficient

Skewness is a measure that gives information about the shape of the distribution and it is an indicator related to symmetry. Skewness determines whether the distribution is positively or negatively skew. If the given skewness value is equal to 0 the distribution is said to be symmetrical, if it is lower than 0 it is positively skew and if it is bigger than 0 it is negatively skew. In positive skewness the right-hand tail of the chart is longer. The mass of the distribution has been concentrated on the left-hand side. This kind of distribution called right skewness. In negative skewness, the left-hand tail of the chart is longer and the mass of the distribution is concentrated on the right-hand side of the chart. This kind of distribution is called left skewness. Skewness is calculated with the formulas below (Birkan, 2006).

Skewness= 3*(Mean-mod)/Standard deviation …(1)

or

Skewness =3*(Mean-median)/ Standard deviation …(2)

2.2. Portfolio Performance Evaluation

2.2.1. Sharpe Ratio

It measures the additional return achieved per to one unit of standard deviation. Therefore we can say that the portfolio that has higher additional return in the same total risk level has better performance. This way it is determined whether the performance of an instrument is lower or higher that the market return. Sharpe Ratio (SR) value is calculated with these equations (Birkan, 2006).

$$SO_m = \frac{(r_m - r_f)}{Stdv_m} \quad \ldots(3)$$

$$SO_p = \frac{(r_p - r_f)}{Stdv_p} \quad \ldots(4)$$

r_m=market return

r_f=riskless interest rate

r_p= portfolio return

$Stdv_m$= standard deviation of the market

$Stdv_p$= standard deviation of the portfolio

2.2.2. Treynor Index

This is the ratio that measures the additional return achieved per one unit of systematic risk taken. Since Treynor index or measure represents the risk premium per systematic risk, we say that the instrument that brings more return at a certain beta level, or the same risk level, has higher Treynor Ratio or relatively higher performance. Treynor Index (TI) value is obtained from the equations below (Halici, 2008).

$$TI_m = \frac{(r_m - r_f)}{\beta_m} \qquad \ldots(5)$$

$$TI_p = \frac{(r_p - r_f)}{\beta_p} \qquad \ldots(6)$$

r_m=market return

r_p=portfolio return

r_f=riskless interest rate

β_m=market beta

β_p=portfolio beta

2.2.3. Jensen’s Measure

It is defined as the difference between the expected return rate of a portfolio and another portfolio that has the same risk level on the instruments market line. “Jensen’s Alfa” which measures the deviation of any portfolio from the instrument market line, can be considered as the difference between the actual return and the expected return. If Jensen alpha is positive, the Financial Asset is above the Market Line. If it is negative, the Financial Asset is below the Market Line. Therefore, a portfolio that has a positive alpha can be considered to show a superior performance. Jensen’s Measure (JM) value is obtained with the equations below (Bekci, 2001).

$$JM_m = r_m - (r_f + \beta_m * (r_m - r_f)) \quad \ldots(7)$$

$$JM_p = r_p - (r_f + \beta_p * (r_p - r_f)) \quad \ldots(8)$$

r_m=market return
r_p=portfolio return
r_f=riskless interest rate
β_m=market beta
β_p=portfolio beta

Entropy Measure

Entropy is a Greek word meaning ambiguity. It is the measure of the disorder that exists statistically in a system (Giriftinoglu, 2005). Entropy has been first defined by Jaynes, who is the pioneer of the probability distribution derivation method. Jaynes' method was named Jaynes' Maximum Entropy Principle and later developed by Shannon as a stochastic process. According to Shannon information can be gathered if a phenomenon involves ambiguity. Accordingly, occurrences of phenomena that are like to occur do not provide very much information while less likely occurrences carry more information (Giriftinoglu, 2005). In this study, Jaynes' maximum entropy principle (MaxEnt) which maximizes Shannon's entropy measure and Kullback's minimum cross entropy method (MinxEnt) has been used in order to use optimal portfolio.

Maximum Entropy

The method that determined the distribution of the system by maximizing Shannon's entropy measure is called MaxEnt and the obtained distribution is called MaxEnt distribution (Jaynes, 1957). IN the MaxEnt method, the concept of ambiguity is studied intensively. The most objective distribution obtained for a physical random variant is the MaxEnt distribution (Jaynes, 1957). Probability distribution with MaxEnt is a distribution where inferences are made based on missing information with existing data (Giriftinoglu, 2005). Since an entropy value that is lower than MaxEnt will not provide reliable additional information, every other distribution will have tendencies towards certain behavior. Therefore it is necessary to have a measurement of a probability distribution ambiguity (Giriftinoglu, 2005). For this ambiguity measurement Shannon defined a single function.

While for each i $p_i \geq 0$ and

$$\sum_{i=1}^{n} p_i = 1$$

$p = (p_1, p_2, p_3, ...p_n)$ is a probability distribution, the Shannon measure of entropy for this distribution is given

$$H_n(p) = -\sum_{i=1}^{n} p_i \ln p_i \qquad \ldots(9)$$

In order to converge portfolio weights to a uniform distribution, it is necessary to maximize Shannon's entropy measure (Kapur and Kesevan, 1992). Because it is thought that the convergence of the portfolio weights to a uniform distribution can reach a well-diversified portfolio when the entropy measure is maximized under limitation of a certain rate of return (Altayligil, 2008). Certain limitations should be provided in order to create a well-diversified portfolio. These limitations are;

while entropy function,

$$H(p) = -\sum_{i=1}^{n} p_i \ln p_i$$

$$\sum_{i=1}^{n} p_i g_r(x_i) = \sum_{i=1}^{n} p_i g_{ri} = \eta_r \qquad \ldots(10)$$

And

$$\sum_{i=1}^{n} p_i = 1 \qquad \ldots(11)$$

and it is thought that under these limitations optimal portfolio can be reached by maximizing Shannon's entropy measure.

Minimum Entropy

Minimum cross entropy method is an optimization method that was pioneered by Kullback and it competes with the MaxEnt method both in theory and in practice. This principle is also known as the minimum rectified deviation principle or minimum information discrimination principle. As in the maximum entropy method, the concept of ambiguity is not utilized in the minimum cross entropy method. However there is a relationship between the two methods. Minimum cross entropy is the cross entropy of probability distribution "p" to another probability distribution "q". In practice the distance is the distance between a determined probability distribution to the uniform distribution "u" (Giriftinoglu, 2005). "u" is the distribution that has the highest ambiguity. The closer a distribution is to "u", the higher the ambiguity. This distance gives us an idea about the size of the ambiguity and "q" is an a priori distribution. Since information is an a priori distribution of moment values derived from physical random variants and these random variants, the method that minimizes Kullback-Leibler measure according to the moment value limitations derived from random variants and the a priori distribution is called MinxEnt method and the distribution obtained here is called MinxEnt distribution (Kullback, 1959). If moment conditions were given before the closest distribution to the a priori distribution is selected. If uniform distribution is selected as a priori distribution the MaxEnt method appears as a special result of the MinxEnt method (Usta, 2006). It minimizes a probability distance from a distribution given in MinxEnt while it maximizes ambiguity in MaxEnt (Kantar, 2006).

Under Kullback- Leiber criteria

$$\sum_{i=1}^{n} p_i = 1$$

And

$$\sum_{i=1}^{n} p_i g_r(x_i) = \eta_r \quad ; \quad r=1,2,\ldots..m \qquad \ldots(12)$$

limitations for each i, $p_i \geq 0$,

$$D(p:q)= \sum_{i=1}^{n} p_i \ln \frac{p_i}{q_i} \qquad \ldots(13)$$

function is minimized.

If q is not given, the distribution that has maximum entropy is selected instead of q. When there are no limitations u is accepted as the a priori distribution and,

$$D(p:q)= \sum_{i=1}^{n} p_i \ln \frac{p_i}{1/n} \qquad \ldots(14)$$

equation is minimized. With this perspective Jaynes' MaxEnt principle is considered to be a special case of Kullback's MinxEnt principle (Giriftinlioglu, 2005).

3. Application

30 stocks from BIST-30 Index have been studied in order to compare the three models that are created in this study. Daily data between February 1, 2013 and February 28, 2013 have been used. Daily corrected closing prices of the stocks were obtained from Osmanli Menkul Company. The optimization process was conducted with the Solver command in Excel. MV, MaxEnt and MinxEnt models were created with the assumption that there is no possibility of selling the stocks openly.

MV Model;

For i=1,2,3…..30,

p_i: weight,

x_i: stock return.

and under limitations $p_i \geq 0$

$$\sum_{i=1}^{n} p_i = 1 \quad \text{ve} \quad p_i \geq 0 \qquad ...(15)$$

the purpose function;

$$\text{Min } \sum_{i=1}^{n} x_i^{\,2} p_i - \{E(x)\}^2 \qquad …(16)$$

MaxEnt;

$$\sum_{i=1}^{n} p_i = 1 \;;\; p_i \geq 0 \qquad ...(17)$$

and

$$\sum_{i=1}^{n} p_i g_r(x_i) = \eta_r \;;\; r=1,2,…..m \qquad …(18)$$

purpose function;

$$\text{Max } \left(- \sum_{i=1}^{n} p_i \ln p_i \right) \qquad ...(19)$$

MinxEnt

$$\sum_{i=1}^{n} p_i = 1 \;;\; p_i \geq 0 \qquad ...(20)$$

and

$$\sum_{i=1}^{n} p_i g_r(x_i) = \eta_r \;;\; r=1,2,…..m \qquad …(21)$$

purpose function;

$$\text{Min } \sum_{i=1}^{n} p_i \ln \frac{p_i}{q_i} \qquad …(22)$$

optimization has been done. In the minimum entropy model, portfolio weights were selected randomly so that the weight total is “1” as a priori distribution.

The summarizing statistics of the investment instruments are given on Table 1.

	AKBNK	ARCLK	ASELS	ASYAB	BIMAS	DOHOL	EKGYO	ENKAI	EREGL	GARAN	HALKB	IHLAS	IPEKE	ISCTR	KCHOL
Mean	-0,000649	-0,001091	0,0031998	-0,000289	-0,000373	-0,000148	-0,001473	0,0003561	-0,001137	-0,000874	-0,000313	-0,005659	0,0001881	-0,00059	0
Standard Error	0,0012716	0,0014253	0,0014741	0,0013944	0,0011484	0,0015188	0,0019916	0,0009887	0,0010495	0,0013765	0,0014653	0,0036083	0,0015765	0,0015581	0,0012692
Median	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Mode	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Standard Deviation	0,0066074	0,0074062	0,0076594	0,0072457	0,0059674	0,0078917	0,0103485	0,0051373	0,0054535	0,0071525	0,0076139	0,0187493	0,008192	0,0080962	0,006595
Sample	4,366E-05	5,485E-05	5,867E-05	5,25E-05	3,561E-05	6,228E-05	0,0001071	2,639E-05	2,974E-05	5,116E-05	5,797E-05	0,0003515	6,711E-05	6,555E-05	4,349E-05
Kurtosis	1,1676283	-0,059599	-0,142984	0,6934873	0,2646003	1,1643943	5,3856953	0,098848	0,0625839	0,6447022	0,0885777	5,2071891	3,1204397	0,8350431	1,851963
Skewness	-1,15716	0,3370841	0,6559101	-0,616185	-0,086263	0,5645576	-2,093337	-0,074489	0,1978734	-0,704726	-0,172424	-1,848766	1,3293851	-0,843732	0,5334856
Range	0,0265306	0,0289776	0,0301913	0,0312946	0,025676	0,0352726	0,0482364	0,0210668	0,0223528	0,0288315	0,0309323	0,1009419	0,0370279	0,0304178	0,0327504
Minimum	-0,018525	-0,012965	-0,011021	-0,018885	-0,01399	-0,015794	-0,037398	-0,011451	-0,010508	-0,016626	-0,015466	-0,068355	-0,013364	-0,019063	-0,013906
Maximum	0,0080057	0,0160127	0,0191706	0,0124093	0,0116858	0,0194784	0,0108386	0,0096158	0,0118451	0,0122058	0,0154661	0,0325874	0,0236639	0,0113548	0,0188444
Sum	-0,01753	-0,029455	0,0863939	-0,00779	-0,010071	-0,004003	-0,039767	0,0096158	-0,030711	-0,023587	-0,008457	-0,152804	0,0050795	-0,015939	0
Count	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27
	KOZAA	KOZAL	KRDMD	MGROS	PETKM	SAHOL	SISE	TCELL	THYAO	TOASO	TTKOM	TTRAK	TUPRS	VAKBN	YKBNK
Mean	-0,001067	-0,000629	-0,00041	0,0006813	-5,18E-05	-0,000485	-0,001463	0,0016276	0,0021171	0,0011009	-4,33E-05	0,0027591	0,0005649	0,0004132	-0,000621
Standard Error	0,0012502	0,0013433	0,0017757	0,0011042	0,0017008	0,000937	0,0013348	0,0009496	0,0018118	0,0017078	0,0009811	0,0015996	0,0010808	0,0015484	0,0015243
Median	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Mode	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Standard Deviation	0,006496	0,0069798	0,0092266	0,0057377	0,0088374	0,0048686	0,006936	0,004934	0,0094141	0,0088741	0,0050977	0,0083119	0,0056161	0,0080455	0,0079203
Sample	4,22E-05	4,872E-05	8,513E-05	3,292E-05	7,81E-05	2,37E-05	4,811E-05	2,434E-05	8,863E-05	7,875E-05	2,599E-05	6,909E-05	3,154E-05	6,473E-05	6,273E-05
Kurtosis	2,7104125	0,8055835	-0,747098	1,6282431	-0,21991	0,027994	-0,252793	0,1803719	1,6920376	0,8664537	-0,563916	-0,098522	0,3437404	0,235304	1,1302886
Skewness	-0,712395	0,0501744	0,0967938	-0,38914	0,0945417	0,1165781	-0,035067	0,5618765	0,0749624	-0,402164	-0,242269	0,1849021	0,5940716	-0,549823	-1,197852
Range	0,0333837	0,0326964	0,0332471	0,0269742	0,0331943	0,0197899	0,0261772	0,0198415	0,0475062	0,0380558	0,0189693	0,0317685	0,0238424	0,0316353	0,03087
Minimum	-0,0211	-0,016786	-0,015988	-0,01542	-0,01524	-0,009171	-0,013986	-0,00599	-0,02234	-0,021969	-0,010873	-0,012965	-0,010777	-0,018271	-0,021189
Maximum	0,0122839	0,01591	0,017259	0,0115539	0,0179543	0,010619	0,0121915	0,0138511	0,0251662	0,0160868	0,0080963	0,0188036	0,0130653	0,013364	0,0096807
Sum	-0,028818	-0,016994	-0,011065	0,0183964	-0,001399	-0,013095	-0,039509	0,0439441	0,0571611	0,0297232	-0,001169	0,0744956	0,0152532	0,0111568	-0,01677
Count	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27

Table 1: Summarizing Statistics of the Stocks

The study first compared portfolio optimization with Mean- Variance (MV), Maximum Entropy (MaxEnt) and Minimum Entropy (MinxEnt) models and standard deviation values obtained at a certain return level. The performance of each portfolio was calculated with the data obtained and performances were evaluated.

Equally Weighted Portfolio	
Portfolio Variance	1,71286E-05
Portfolio STDV	0,004138669
Portfolio Return	-0,00014387

Table 2: Equal Weight Portfolio

For the equal weight portfolio created with stocks from BIST-30 Index, the return is calculated as -0.000144 and risk as 0.0041387. The fact that the return of the portfolio is negative is thought to be the result of the banks that is in the BIST-30 Index, since there is an expectation that the Competition Institution will punish bank in the month of February and this had a negative impact on the stocks.

STANDART DEVIATION OF MV, MAXENT, MINXENT ON DIFFERENT PORTFOLIO RETURN LEVELS			
	STANDART DEVIATION		
RETURN	MV	MAXENT	MINXENT
0,003062278	0,006158	0,006188	0,006199311
0,002562278	0,004075	0,004583	0,004868353
0,002062278	0,002937	0,004117	0,004192571
0,001562278	0,002325	0,003819	0,003925343
0,001262278	0,002114	0,003674	0,003679633
0,001162278	0,002071	0,003676	0,003694070
0,001062278	0,002035	0,003590	0,003688293
0,000962278	0,002013	0,003587	0,003678028
0,000862278	0,002003	0,003595	0,003673007
0,000762278	0,002000	0,003610	0,003697124
0,000562300	0,002026	0,003671	0,003759523
0,000062300	0,002184	0,003996	0,004046582

Table 3: MV, MaxEnt and MinxEnt Standard Deviations at Different Portfolio Return Levels

When standard deviations of the portfolios created at the same return level, it is found that the most risky portfolios are created by the MinxEnt model while the least risky portfolios are created by the MV model. The reason for the portfolio returns being higher than the equal weighted portfolio is the fact that the banking sector is not included in the portfolios created with the MV model, while banking sector stocks have very low weight in MaxEnt and MinxEnt models.

PORTFOLIOS CREATED ON DIFFERENT RETURN LEVELS							
MV MODEL		MAXIMUM ENTROPY MODEL			MINIMUM ENTROPY MODEL		
STANDART DEVIATION	RETURN	MAXENT	STANDART DEVIATION	RETURN	MINXENT	STANDART DEVIATION	RETURN
0,006158340	0,003062	0,429794	0,006187549	0,003062	24,670273	0,006199311	0,003062
0,004074836	0,002562	0,743487	0,004582716	0,002562	7,880613	0,004868353	0,002562
0,002937063	0,002062	0,844188	0,004117085	0,002062	4,687897	0,004192571	0,002062
0,002324689	0,001562	0,901634	0,003818750	0,001562	2,926106	0,003925343	0,001562
0,002114306	0,001262	0,924922	0,003674133	0,001262	2,110007	0,003679633	0,001262
0,002070851	0,001162	0,931640	0,003675834	0,001162	1,905903	0,003694070	0,001162
0,002035073	0,001062	0,938130	0,003589810	0,001062	1,717096	0,003688293	0,001062
0,002013156	0,000962	0,943131	0,003587151	0,000962	1,578012	0,003678028	0,000962
0,002003343	0,000862	0,947492	0,003594828	0,000862	1,452720	0,003673007	0,000862
0,001999572	***0,000762***	0,951201	0,003609823	0,000762	1,344355	0,003697124	0,000762
0,002026066	0,000562	0,956671	0,003670588	0,000562	1,175134	0,003759523	0,000562
0,002184260	0,000062	**0,961527**	**0,003995779**	**0,000062**	**1,000254**	**0,004046582**	**0,000062**

Table 4: Portfolios Created at Different Return Levels

The return of the optimal portfolio created with the MV model is 0.00076228 while the risk is 0.00199957. The return of the optimal portfolio obtained by maximizing the entropy measure is 0.00006230 while its risk is calculated as 0.00397578. As it can be seen the MaxEnt mode is both more risky and bringing lower returns than the MV model. It is also found that the portfolio return obtained by minimizing entropy measure is the same as the MinxEnt model, but the portfolio is more risky. Looking at the created portfolios; the stocks that are at the 0.00076228 return level with the MV model are; ASELS, EKGYO, IPEKE, KOZAL, KRDMD, PETKM, SAHOL, TCELL, THY, TTRAK and TUPRS. In the MaxEnt and MinxEnt models on the other hand, 29 stocks that are in the BIST-30 Index, except IHLAS, have the same return level.

PORTFOLIO WEIGHTS ON 0,0076228 LEVEL											
	MV	MAXENT	MINXENT		MV	MAXENT	MINXENT		MV	MAXENT	MINXENT
AKBNK	0,000%	0,002%	1,219%	HALKB	0,000%	3,065%	2,343%	SAHOL	20,922%	2,769%	2,818%
ARCLK	0,000%	1,755%	0,843%	IHLAS	0,000%	0,000%	0,000%	SISE	0,000%	0,012%	1,061%
ASELS	4,427%	9,005%	5,373%	IPEKE	1,692%	3,916%	3,281%	TCELL	31,640%	15,868%	7,280%
ASYAB	0,000%	3,106%	1,805%	ISCTR	0,000%	2,588%	2,147%	THYAO	7,115%	7,173%	8,482%
BIMAS	0,000%	2,964%	1,792%	KCHOL	0,000%	3,588%	3,184%	TOASO	0,000%	5,454%	6,507%
DOHOL	0,000%	3,339%	2,141%	KOZAA	0,000%	1,778%	1,472%	TTKOM	0,000%	3,515%	4,150%
EKGYO	1,687%	0,011%	0,256%	KOZAL	13,744%	2,522%	2,278%	TTRAK	6,073%	8,259%	10,527%
ENKAI	0,000%	4,194%	2,970%	KRDMD	2,684%	2,900%	2,737%	TUPRS	2,156%	4,547%	5,733%
EREGL	0,000%	0,006%	1,075%	MGROS	0,000%	4,744%	4,825%	VAKBN	0,000%	4,286%	5,502%
GARAN	0,000%	2,121%	1,483%	PETKM	7,861%	3,500%	3,568%	YKBNK	0,000%	2,536%	3,149%

Table 5: Portfolio Weights at the 0.0076228 Return Level

PORTFOLIOS PERFORMANCE ON DIFFERENT RETURN LEVELS									
	MV MODEL			MAXIMUM ENTROPY MODEL			MINIMUM ENTROPY MODEL		
RETURN	SHARP	TREYNOR	JENSEN	SHARP	TREYNOR	JENSEN	SHARP	TREYNOR	JENSEN
0,003062	1,085748	0,063838	0,005986	0,494910	0,031678	0,002766	0,496373	0,030542	0,002755
0,002562	1,518197	0,052774	0,002262	0,559118	0,016681	0,002169	0,526313	0,016154	0,002156
0,002062	1,936085	0,047018	0,001813	0,500907	0,010234	0,001647	0,491889	0,010099	0,001641
0,001562	2,231010	0,039785	0,001359	0,419107	0,006449	0,001184	0,409365	0,006332	0,001177
0,001262	2,311115	0,037217	0,001097	0,343558	0,004787	0,000929	0,343792	0,004623	0,000918
0,001162	2,311322	0,036504	0,001010	0,316194	0,004178	0,000839	0,314633	0,004106	0,000833
0,001062	2,302818	0,035612	0,000922	0,295915	0,003774	0,000763	0,288013	0,003680	0,000756
0,000962	2,278216	0,034736	0,000835	0,268257	0,003323	0,000684	0,261629	0,003238	0,000676
0,000862	2,239458	0,033446	0,000747	0,239866	0,002901	0,000606	0,234761	0,002838	0,000600
0,000762	2,193672	0,032239	0,000659	0,211168	0,002500	0,000530	0,206181	0,002448	0,000525
0,000562	2,066283	0,030981	0,000486	0,153191	0,001753	0,000382	0,149567	0,001724	0,000379
0,000062	1,687723	0,022837	0,000052	0,015591	0,000173	0,000040	0,015396	0,000171	0,000040

Table 6: Portfolio Performances at Different Return Levels

The portfolio performances at the same return level are calculated according to the Sharpe Ratio, Treynor Index and Jensen Measure. When the performance measures are compared, the performances of the portfolios that are created in the MV model can said to be higher than portfolios created in the MaxEnt and MinxEnt models.

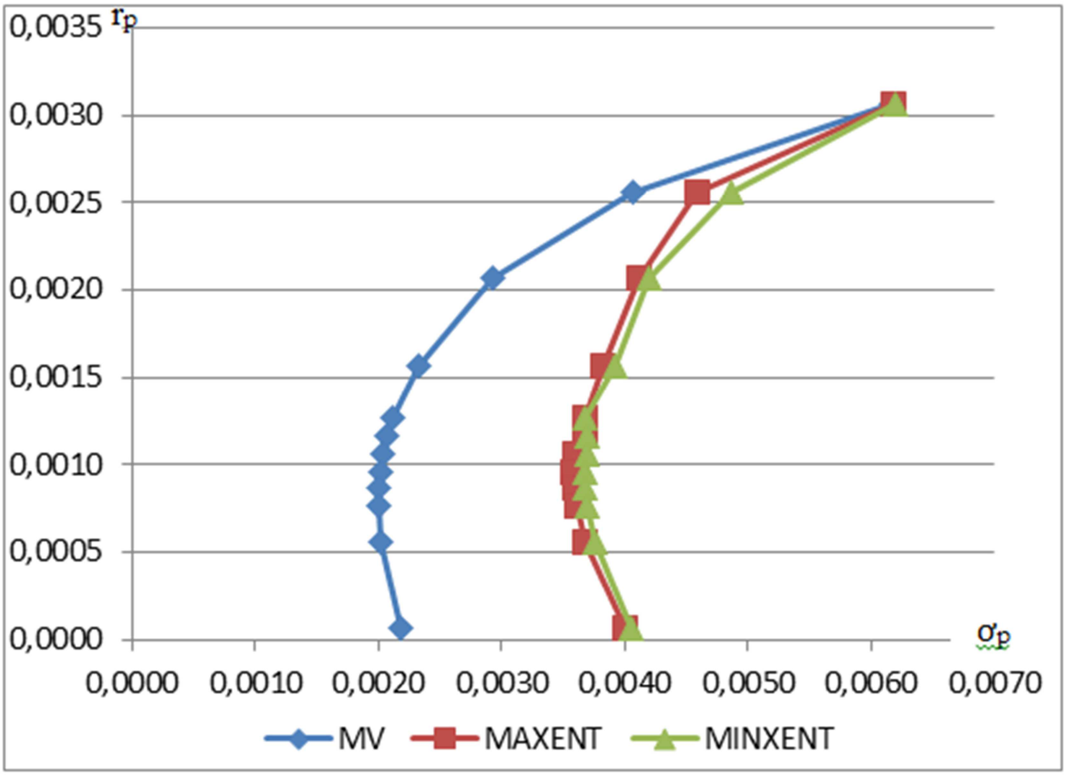

Figure 1: Efficient Frontier of MV, MaxEnt, MinxEnt

In addition to the limits used in analysis, Pearson Skewness measure was added in the second part of the study and optimization was repeated for MV, MaxEnt and MinxEnt models. The purpose here is to show how the skewness measure affects our portfolio. The reason for trying to find the weights of the portfolio return series that will create negative skewness is the thought that positive return investments can be included in the portfolio (Altayligil, 2008).

Pearson's Skewness Measure: $3*(\bar{R}_p - \tilde{R}_p \text{ Stdv}) < 0$ …(23)

$\bar{R}_p$ =Portfolio Return

$\tilde{R}_p$ = Portfolio Median

STANDART DEVIATION OF MV, MAXENT,MINXENT ON DIFFERENT PORTFOLIO RETURN LEVELS			
	STANDART DEVIATION		
RETURN	MV	MAXENT	MINXENT
0,00306228	0,00615834	0,00618894	0,00619772
0,00256228	0,00407755	0,00458281	0,00486851
0,00206228	0,00294299	0,00413839	0,00438557
0,00156228	0,00232755	0,00376546	0,00394986
0,00126228	0,00211601	0,00368555	0,00374875
0,00116228	0,00207491	0,00369539	0,00371362
0,00106228	0,00203835	0,00359548	0,00368903
0,00096228	0,00201511	0,00359385	0,00367803
0,00086228	0,00200783	0,00360810	0,00368384
0,00076228	0,00200399	0,00363546	0,00370162
0,00056230	0,00202622	0,00370543	0,00375925
0,00006230	0,00222480	0,00397959	0,00404659

Table 7: MV, MaxEnt and MinxEnt Standard Deviations at Different Return Levels

Portfolios optimized by added Pearson's skewness limit are compared at the same return level and the lowest risk level at each return level is found in the MV model.

PORTFOLIOS CREATED ON DIFFERENT RETURN LEVELS							
MV MODEL		MAXIMUM ENTROPY MODEL			MINIMUM ENTROPY MODEL		
STANDART DEVIATION	RETURN	MAXENT	STANDART DEVIATION	RETURN	MINXENT	STANDART DEVIATION	RETURN
0,00615834	0,00306228	0,42932008	0,00618894	0,00306228	24,70283661	0,00619772	0,00306228
0,00407755	0,00256228	0,74339212	0,00458281	0,00256228	7,88353906	0,00486851	0,00256228
0,00294299	0,00206228	0,84420170	0,00413839	0,00206228	4,48547819	0,00438557	0,00206228
0,00232755	0,00156228	0,90253925	0,00376546	0,00156228	2,73336840	0,00394986	0,00156228
0,00211601	0,00126228	0,92621492	0,00368555	0,00126228	2,05755134	0,00374875	0,00126228
0,00207491	0,00116228	0,93263377	0,00369539	0,00116228	1,87727510	0,00371362	0,00116228
0,00203835	0,00106228	0,93837090	0,00359548	0,00106228	1,71748472	0,00368903	0,00106228
0,00201511	0,00096228	0,94350349	0,00359385	0,00096228	1,57614498	0,00367803	0,00096228
0,00200783	0,00086228	0,94804710	0,00360810	0,00086228	1,45226879	0,00368384	0,00086228
0,00200399	***0,00076228***	0,95206001	0,00363546	0,00076228	1,34419722	0,00370162	0,00076228
0,00202622	0,00056230	0,95851637	0,00370543	0,00056230	1,17533891	0,00375925	0,00056230
0,00222480	0,00006230	**0,96605000**	**0,00399959**	**0,00006230**	**1,00025411**	**0,00404659**	**0,00006230**

Table 8: Portfolios Created at Different Return Levels

The return of the optimal portfolio created with the MV model is 0.00076228 while its risk is found as 0.00200399. Although slightly, the skewness limit increased the risk. The return of the optimal portfolio created by maximizing entropy is 0.00006230 while its risk is calculated as 0.00399978. It is seen that the MaxEnt model is both more risky and bringing less return that the MV model. When the entropy measure is minimized the portfolio return is same as the MaxEnt model but the portfolio risk increases. In the MV model, the stocks that are included in the portfolio at the 0.00076228 return level are; ASELS, EKGYO, EREGL, IPEKE, KOZAL, KRDMD, PETKM, SAHOL, TCELL, THY, TTRAK and TUPRS. In the MaxEnt and MinxEnt models on the other hand, 29 stocks that are in the BIST-30 Index, except IHLAS, have the same return level.

PORTFOLIO WEIGHTS ON 0,0076228 LEVEL											
	MV	MAXENT	MINXENT		MV	MAXENT	MINXENT		MV	MAXENT	MINXENT
AKBNK	0,000%	2,147%	1,211%	HALKB	0,000%	2,777%	2,336%	SAHOL	22,522%	2,452%	2,807%
ARCLK	0,000%	1,317%	0,829%	IHLAS	0,000%	0,000%	0,000%	SISE	0,000%	0,619%	0,915%
ASELS	4,126%	9,366%	5,388%	IPEKE	0,182%	3,714%	3,275%	TCELL	30,839%	15,868%	7,290%
ASYAB	0,000%	2,823%	1,798%	ISCTR	0,000%	2,256%	2,131%	THYAO	6,839%	7,335%	8,495%
BIMAS	0,000%	2,664%	1,782%	KCHOL	0,000%	3,365%	3,176%	TOASO	0,000%	5,429%	6,508%
DOHOL	0,000%	3,088%	2,132%	KOZAA	0,000%	1,363%	1,455%	TTKOM	0,000%	3,284%	4,141%
EKGYO	0,219%	0,603%	0,562%	KOZAL	15,016%	2,184%	2,263%	TTRAK	7,146%	8,539%	10,549%
ENKAI	0,000%	4,031%	2,966%	KRDMD	3,430%	2,594%	2,727%	TUPRS	2,361%	4,428%	5,730%
EREGL	0,227%	1,226%	1,059%	MGROS	0,000%	4,640%	4,824%	VAKBN	0,000%	4,139%	5,497%
GARAN	0,000%	1,732%	1,470%	PETKM	7,093%	3,268%	3,558%	YKBNK	0,000%	2,199%	3,125%

Table 9: Portfolio Weights at the 0.0076228 Return Level

PORTFOLIOS PERFORMANCE ON DIFFERENT RETURN LEVELS									
	MV MODEL			MAXIMUM ENTROPY MODEL			MINIMUM ENTROPY MODEL		
RETURN	SHARP	TREYNOR	JENSEN	SHARP	TREYNOR	JENSEN	SHARP	TREYNOR	JENSEN
0,00306228	1,08574787	0,06383773	0,00598607	0,49790969	0,03168266	0,00276629	0,49637279	0,03054720	0,00275429
0,00256228	1,51718671	0,05286446	0,00226243	0,55911777	0,01668279	0,00216874	0,52631314	0,01615890	0,00215598
0,00206228	1,93218545	0,04704936	0,00181303	0,50090732	0,01022632	0,00164639	0,49188860	0,00993819	0,00163433
0,00156228	2,22826781	0,03978960	0,00135864	0,41910720	0,00642675	0,00118660	0,40936483	0,00630100	0,00117492
0,00126228	2,30925151	0,03723648	0,00109663	0,34385808	0,00475191	0,00092697	0,34379199	0,00464773	0,00091646
0,00116228	2,30680378	0,03639661	0,00100943	0,31619433	0,00422983	0,00084291	0,31463349	0,00414000	0,00083598
0,00106228	2,29911219	0,03553421	0,00092218	0,29591480	0,00375486	0,00076175	0,28801346	0,00368116	0,00075574
0,00096228	2,27600464	0,03461067	0,00083476	0,26825692	0,00331110	0,00068262	0,26162876	0,00324841	0,00067722
0,00086228	2,23445808	0,03344433	0,00074661	0,23986626	0,00289114	0,00060510	0,23476079	0,00283888	0,00060037
0,00076228	2,18883787	0,03257179	0,00065962	0,21116780	0,00249224	0,00052913	0,20618146	0,00244953	0,00052506
0,00056230	2,06612713	0,02998644	0,00048380	0,15319071	0,00175225	0,00038186	0,14956685	0,00172394	0,00037889
0,00006230	1,65696644	0,02305009	0,00005234	0,01559145	0,00017370	0,00003995	0,01539571	0,00017128	0,00003964

Table 10: Portfolio Performances at Different Return Levels

When portfolio performances are compared at the same return levels with Pearson's skewness limit is added to optimization, the performances of the portfolios created in the MV model can said to be higher than portfolios created in MaxEnt and MinxEnt models.

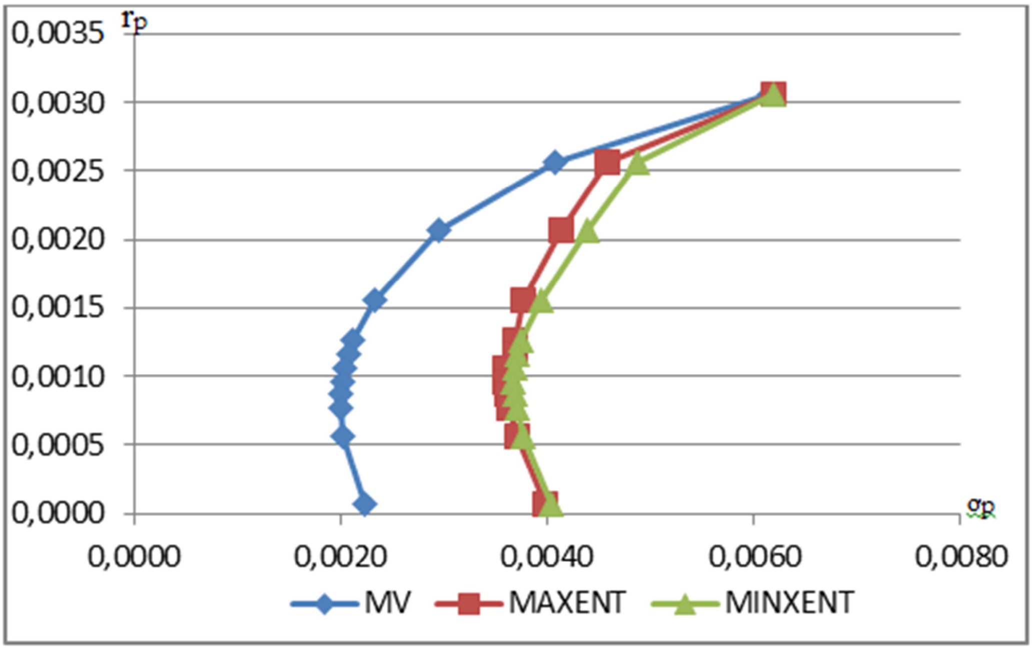

Figure 2: Efficient Frontier of MV, MaxEnt, MinxEnt

4. Discussion And Conclusion

In this study, portfolios are created with MV, MaxEnt and MinxEnt models by using stocks from the BIST-30 index. First of all return and risk has been calculated after equal weighted portfolio is created. Later, limits for each model were added in order to create portfolios at certain return levels. The optimal portfolio was found and its return and risk was calculated. Finally performances of the three models created at the return level of the optimal portfolio. As a result of the application, we are able to say that the MV model is more optimal than portfolios created with MaxEnt and MinxEnt models at the same return level. Portfolios created with the MV model are both less risky and higher performing. MaxEnt and MinxEnt models give almost equal results. However, if we need to compare, MaxEnt model can said to be more optimal than the MinxEnt model. MV model is also superior to the other two when Pearson’s skewness limit is added to all three models as the optimization limit. However, the effect of Pearson’s skewness limit is very low on all three models. It raises the risk of the portfolios but this increase is ignorably low. In other words, we can say that Pearson’s skewness limit has no effect on the models.

Another important finding is that diversification is higher in MaxEnt and MinxEnt models than in the MV model, meaning the number of stocks included in the than in the MV model. In the light of the findings, assuming that negative expectations about the banking sector for the month of February will not be reflected in the following months, monthly returns of banks will increase and their stocks will be included more in portfolios and/or their weights will increase in portfolio optimizations that will be conducted T period of time later.

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