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PLANE KINEMATICS IN LORENTZIAN HOMOTHETIC MULTIPLICATIVE CALCULUS

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Abstract. In this study, Lorentzian plane homothetic multiplicative calculus kinematics is discussed. Lorentzian plane homothetic multiplicative calculus movement, the pole points of a point X relative to the moving and fixed plane are discussed. In this motion, the velocities and accelerations of a point X are obtained. In this motion, the relations between the velocities and accelerations of a point X are obtained. In addition, new theorems and results are given.

1. Introduction

Using different arithmetic operations based on classical analysis alternative analysis have also been described. In 1887, the Volterra type of analysis was determined by [1]. Since this new approach is based on multiplication, this analysis is called multiplicative analysis (also called multiplicative analysis). In recent years, studies have been carried out by revealing some areas for the application of this analysis [2, 3, 4].

After the definition of Volterra analysis, some new studies were conducted by Michael Grossman and Robert Katz between 1967 and 1970. As a result of the studies, new analysis called geometric analysis, bigeometric analysis and anageometric analysis were defined. Some basic definitions and concepts regarding this new analysis, also called non-Newtonian analysis, are given [5].There are also studies in which non-Newtonian analysis is applied. Among these analysis, Dick Stanley's geometric analysis was referred to as multiplicative analysis [6]. Later, in 2008, studies were conducted in which the basic concepts of multiplicative analysis were defined and some of its applications were discussed [7]. The aim of this article is to examine one-parameter lorentzian homothetic multiplicative analysis plane kinematics using matrices. Selahattin Aslan, Murat Bekar and Yusuf Yaylı defined multiplicative quaternions and achieved some results using quaternions [8]. Semra Kaya Nurkan, Ibrahim Gürgil and Murat Kemal Karacan are given in geometric calculus, vectors and their properties, matrix, determinant, vector product and Gram–Schmidt

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in geometric space [9]. Hasan ES gave plane kinematics in multiplicative calculus [10]. The aim of this article is to examine one-parameter Lorentzian homothetic multiplicative calculus plane kinematics using matrices.

2. BASIC CONCEPTS

In [28], the set of the multiplicative calculus real numbers $\mathbb{R}(G)$ are determined as

$$\mathbb{R}(G) = \{\exp(m) = e^m : m \in \mathbb{R}\}.$$

Then $(\mathbb{R}(G), \oplus, \otimes)$ is a field with multiplicative calculus(geometric) zero 1 and multiplicative calculus (geometric) identy e.

The relations between the basic multiplicative operations and ordinary arithmetic operations can be given for all $m, n \in \mathbb{R}(G)$ as

$$\begin{aligned}
m \oplus n &= mn, \\
m \ominus n &= \frac{m}{n}, \\
m \otimes n &= m^{\ln n} = n^{\ln m}, \\
m \oslash n &= x^{\frac{1}{\ln n}},\ n \neq 1, \\
\sqrt{m}^{G} &= e^{(\ln m)^{\frac{1}{2}}}, \\
m^{-1_G} &= e^{\frac{1}{\log m}}, \\
\sqrt{m^{2_G}}^{G} &= |m|^{G}, \\
m^{2_G} &= m \otimes m = m^{\ln m}, \\
m \otimes e &= m \\
m \oplus 1 &= m, \\
|m|^{G} &= \begin{cases} m & , \quad m > 1, \\ 1 & , \quad m = 1, \\ m^{-1} & , \quad m < 1, \end{cases}
\end{aligned}$$

Additionally, for each $e^m, e^n \in \mathbb{R}(G)$, the multiplicative addition and multiplicative multiplication operations can be given as follows

$$\begin{aligned}
e^m \oplus e^n &= e^{m+n} \\
e^m \otimes e^n &= e^{mn}
\end{aligned}$$

and thus we can write

$$\begin{gathered}
e^m \otimes e^n = e^{mn}, e^m \oplus e^n = e^{m+n}, \\
e^m \ominus e^n = e^{m-n}, e^m \oslash e^n = e^{\frac{m}{n}}, \\
\sqrt{e^m}^{G} = e^{\sqrt{m}}.
\end{gathered}$$

Positive geometric real numbers and negative geometric real numbers are defined as

$$\mathbb{R}^+(G) = \{m \in \mathbb{R}(G) : m > 1\}$$

and

$$\mathbb{R}^-(G) = \{m \in \mathbb{R}(G) : 0 < m < 1\},$$

respectively [8, 9, 10, 28].

The sentence $\mathbb{R}^2(G)$ is defined as follows

$$\mathbb{R}^2(G) = \{s^\circ = (e^{s_1}, e^{s_2}) : e^{s_1}, e^{s_2} \in \mathbb{R}(G)\} \subset \mathbb{R}^2$$

$$\begin{aligned} s^\circ \oplus z^\circ &= (e^{s_1}, e^{s_2}) \oplus (e^{z_1}, e^{z_2}) \\ &= (e^{s_1} \oplus e^{z_1}, e^{s_2} \oplus e^{z_2}) \\ &= \left(e^{s_1+z_1}, e^{s_2+z_2}\right) \end{aligned}$$

and the multiplicative scalar multiplication as

$$\begin{aligned} e^c \otimes s^\circ &= e^c \otimes (e^{s_1}, e^{s_2}) \\ &= (e^c \otimes e^{s_1}, e^c \otimes e^{s_2}) \\ &= (e^{cs_1}, e^{cs_2}), \end{aligned}$$

where $e^c \in \mathbb{R}(G),\ s^\circ,\ z^\circ \in \mathbb{R}^2(G)$.

Definition 2.1. The relationship between the multiplicative derivative and the classical derivative is determine as

$$h^{*(n)}(x) = e^{(\ln h(x))^{(n)}}.$$

[11, 12, 13, 17, 20, 25].

Definition 2.2. The multiplicative distance defined by [13, 25]. This allows to define the multiplicative distance $d^G(m,n)$ between $m, n \in \mathbb{R}^+(G)$ as

$$d^G(m,n) = \left|\frac{m}{n}\right|^G$$

[11, 12, 13, 25].

Definition 2.3. The relationship between trigonometry and multiplicative trigonometry is determine as $\sin_g \omega = e^{\sin \omega}, \cos_g \omega = e^{\cos \omega}, \tan_g \omega = e^{\tan \omega} = \frac{\sin_g \omega}{\cos_g \omega}$ [5, 6, 11, 12, 13, 14, 15, 30].

Definition 2.4. An $2x2$ multiplicative matrix is defined by

$$K = \begin{bmatrix} e^{k_{11}} & e^{k_{12}} \\ e^{k_{21}} & e^{k_{22}} \end{bmatrix}$$

where $e^{k_{11}}, e^{k_{12}}, e^{k_{21}}, e^{k_{22}} \in \mathbb{R}(G)$. Let K and M be two multiplicative matrices and $K \otimes M = N$ be the multiplication of these matrices, where

$$N = \begin{bmatrix} e^{k_{11}m_{11}+k_{12}m_{21}} & e^{k_{11}m_{12}+k_{12}m_{22}} \\ e^{k_{21}m_{11}+k_{22}m_{21}} & e^{k_{21}m_{12}+k_{22}m_{22}} \end{bmatrix}.$$

Definition 2.5. $2x2$ type identity matrix in multiplicative calculus is

$$I = \begin{bmatrix} e & 1 \\ 1 & e \end{bmatrix}.$$

If matrix D is a $2x2$ type matrix and $D^T \otimes D = D \otimes D^T = I$, then D is called a multiplicative orthogonal matrix.

3. PLANE KINEMATICS IN LORENTZIAN HOMOTHETIC MULTIPLICATIVE CALCULUS

Definition 3.1. The dot product in $\mathbb{R}^2(G)$ is determined as in the equation 3.1

$$\langle m,n\rangle_L^G = e^{m_1 n_1 - m_2 n_2}, \tag{3.1}$$

where $\langle m,n\rangle_L^G$ is the dot product in the multiplicative Lorentz sense and $m = (m_1, m_2)$, $n = (n_1, n_2) \in \mathbb{R}^2(G)$.

Definition 3.2. The norm of a multiplicative vector $m = (m_1, m_2)$ is

$$\|m\|_L^G = \sqrt{\langle m,m\rangle_L^G}^G = e^{\sqrt{m_1^2 - m_2^2}}. \tag{3.2}$$

Definition 3.3. The multiplicative unit circle $S^1(G)$ in $\mathbb{R}^2(G)$ can be defined as

$$\begin{aligned} S^1(G) &= \left\{ m = (m_1, m_2) \in \mathbb{R}^2(G) : \langle m,m\rangle_L^G = e \right\} \\ &= (\cosh_g \omega, \sinh_g \omega) = \left(e^{\cosh \omega}, e^{\sinh \omega} \right). \end{aligned} \tag{3.3}$$

Definition 3.4. Let $m = (e^{m_1}, e^{m_2})$ and $n = (e^{n_1}, e^{n_2})$ be unit vectors in $\mathbb{R}^2(G)$. Then the equation

$$\begin{bmatrix} \cosh_g \omega & \sinh_g \omega \\ \sinh_g \omega & \cosh_g \omega \end{bmatrix} \otimes \begin{bmatrix} e^{m_1} \\ e^{m_2} \end{bmatrix} = \begin{bmatrix} e^{n_1} \\ e^{n_2} \end{bmatrix} \tag{3.4}$$

represents a rotation in $\mathbb{R}^2(G)$ of the multiplicative vector m by a multiplicative angle $\omega \in \mathbb{R}$ in positive direction around the origin $O = (1,1)$ of the Cartesian coordinate system of $\mathbb{R}^2(G)$. We will call this rotation as multiplicative planar rotation. After this rotation multiplicative vector m turns to the multiplicative vector n as given [8]. Where $A(\omega) = \begin{bmatrix} \cosh_g \omega & \sinh_g \omega \\ \sinh_g \omega & \cosh_g \omega \end{bmatrix}$ is a rotation matrix in multiplicative plane.

Definition 3.5. The Lorentzian homothetic multiplicative plane equation of motion in $\mathbb{R}^2(G)$ is determine as,

$$\begin{aligned} y_1 &= x \otimes (h \otimes \cosh_g \omega) \oplus y \otimes (h \otimes \sinh_g \omega) \oplus c_1 \\ y_2 &= x \otimes (h \otimes \sinh_g \omega) \oplus y \otimes (h \otimes \cosh_g \omega) \oplus c_2 \end{aligned} \tag{3.5}$$

If ω, c_1, and c_2 are given by the functions of time parameter t, then this motion is called as a one-parameter Lorentzian homothetic multiplicative motion.

Definition 3.6. The equation of a one-parameter Lorentzian homothetic multiplicative motion in $\mathbb{R}^2(G)$ is defined by

$$Y(t) = B(t) \otimes X(t) \oplus C(t) \tag{3.6}$$

$$Y = \begin{bmatrix} e^{y_1} \\ e^{y_2} \end{bmatrix}, X = \begin{bmatrix} e^{x_1} \\ e^{x_2} \end{bmatrix}, C = \begin{bmatrix} e^{c_1} \\ e^{c_2} \end{bmatrix},$$

where Y and X are the position vectors of the same point R, respectively, for the multiplicative fixed and multiplicative moving systems, and C is the multiplicative translation vector.

If we take the multiplicative derivative of the 3.6 equation with respect to the parameter t. In that case the equation of

$$Y^\star = B^* \otimes X \oplus B \otimes X^* \oplus C^* \tag{3.7}$$

is obtained. Here, $V_a = Y^{\star}$ is called the absolute speed of the motion, $V_r = B \otimes X^*$ is determine the relative speed of the motion, and $V_f = B^* \otimes X \oplus C^*$ is defined the sliding speed of the motion.

We represent movements in the E_G^2 plane as $\frac{L_{MC}}{L'_{MC}}$; One of which is Lorentzian homothetic multiplicative fixed plane L'_{MC} and the other one is a Lorentzian homothetic multiplicative moving plane L_{MC} that moves relative to the fixed plane.

If the matrices B and C are functions of a parameter t, this motion is called a one-parameter Lorentzian homothetic multiplicative motion and is denoted by $B_1 = \frac{L_{MC}}{L'_{MC}}$. By taking the derivatives with respect to t in 3.7, we get

$$Y^{\star\star} = B^{**} \otimes X \oplus e^2 \otimes (B^* \otimes X^*) \oplus B \otimes X^{**} \oplus C^{**}, \tag{3.8}$$

$$b_a = b_r \oplus b_c \oplus b_f \tag{3.9}$$

where the velocities

$$b_a = Y^{\star\star}, b_f = B^{**} \otimes X \oplus C^{**}, b_r = B \otimes X^{**} \text{ and } b_c = e^2 \otimes (B^* \otimes X^*) \tag{3.10}$$

are called multiplicative absolute acceleration, multiplicative sliding acceleration, multiplicative relative acceleration and multiplicative Coriolis accelerations, respectively.

Definition 3.7. Let X be a point in the plane L_{MC}. The speed of this point X while drawing its orbit in the plane L_{MC} is called relative speed. And this speed is defined by V_r.

Definition 3.8. The relationship between the speeds of motion B_1 is defined as

$$V_a = V_f \oplus V_r \tag{3.11}$$

If X is a fixed point in plane L_{MC} of motion B_1, V_r is zero in the multiplicative sense. Therefore $V_a = V_f$.

The expression $V_a = V_f \oplus V_r$ is called the law of velocities in the motion B_1.

Theorem 3.9. *In lorentzian homothetic multiplicative motion, the absolute velocity vector is equal to the sum of the sliding velocity vector and the relative velocity vectors. So it is*

$$V_a = V_f \oplus V_r.$$

4. POLES OF ROTATING AND ORBIT IN LORENTZIAN HOMOTHETİC MULTIPLICATIVE CALCULUS

Definition 4.1. In the sense of multiplicative calculus, the points where $V_f = 1$ are both L_{MC} and L'_{MC} fixed points. These points are called pole points of the movement.

Theorem 4.2. *In a motion B_1 whose angular velocity is not zero (in the sense of multiplicative calculus), there is a single point that remains constant in both L_{MC} and L'_{MC} at each time t.*

Proof. $V_r = 1$ because point X is fixed at L_{MC}. and since the same point X is fixed at L'_{MC}, $V_f = 1$. For such points the equation $V_f = 1$ gives

$$B^* \otimes X \oplus C^* = 1, \tag{4.1}$$

and

$$X = e^{-1} \otimes (B^*)^{m-inv} \otimes C^* \tag{4.2}$$

where $(B^*)^{m-inv}$ is the multiplacative inverse of B^*. Since

$$B = e^h \otimes \begin{bmatrix} e^{\cosh\omega} & e^{\sinh\omega} \\ e^{\sinh\omega} & e^{\cosh\omega} \end{bmatrix} = \begin{bmatrix} e^{h\cosh\omega} & e^{h\sinh\omega} \\ e^{h\sinh\omega} & e^{h\cosh\omega} \end{bmatrix}, \quad C = \begin{bmatrix} e^{c_1} \\ e^{c_2} \end{bmatrix},$$

$$B^* = \begin{bmatrix} e^{h'\cosh\omega + h\omega'\sinh\omega} & e^{h'\sinh\omega + h\omega'\cosh\omega} \\ e^{h'\sinh\omega + h\omega'\cosh\omega} & e^{h'\cosh\omega + h\omega'\sinh\omega} \end{bmatrix}, \quad C^* = \begin{bmatrix} e^{c_1'} \\ e^{c_2'} \end{bmatrix}$$

we get $\det^G(B^*) = e^{(h')^2 - (h\omega')^2}$Thus B^* is regular and

$$(B^*)^{m-inv} = e^{\frac{1}{(h')^2-(h\omega')^2}} \otimes \begin{bmatrix} e^{h'\cosh\omega + h\omega'\sinh\omega} & e^{-h'\sinh\omega - h\omega'\cosh\omega} \\ e^{-h'\sinh\omega - h\omega'\cosh\omega} & e^{h'\cosh\omega + h\omega'\sinh\omega} \end{bmatrix}.$$

Therefore, the equation of $V_f = 1$ has only one X solution. This point X is the pole point of L_{MC}. Accordingly, from 4.2 equation the result

$$X = P = e^{\frac{1}{(h')^2-(h\omega')^2}} \otimes \begin{bmatrix} e^{-c_1'(h'\cosh\omega + h\omega'\sinh\omega) + c_2'(h'\sinh\omega + h\omega'\cosh\omega)} \\ e^{c_1'(h'\sinh\omega + h\omega'\cosh\omega) - c_2'(h'\cosh\omega + h\omega'\sinh\omega)} \end{bmatrix} \tag{4.3}$$

$$= e^{\frac{1}{(h')^2-(h\omega')^2}} \otimes \begin{bmatrix} e^{(-c_1'h' + c_2'\omega'h)\cosh\omega + (-c_1'h\omega' + c_2'h')\sinh\omega} \\ e^{(c_1'h\omega' - c_2'h')\cosh v + (c_1'h' - c_2'h\omega')\sinh\omega} \end{bmatrix} \tag{4.4}$$

is reached.

The pole point in the multiplicative fixed plane is

$$P' = B \otimes P \oplus C \tag{4.5}$$

setting these values in their planes and calculating we have

$$P' = e^{\frac{1}{(h')^2-(h\omega')^2}} \otimes \begin{bmatrix} e^{-c_1'h'h + h^2c_2'\omega'} \\ e^{h^2c_1'\omega' - h'hc_2'} \end{bmatrix} \oplus \begin{bmatrix} e^{c_1} \\ e^{c_2} \end{bmatrix} \tag{4.6}$$

$$= \begin{bmatrix} e^{\frac{-c_1'h'h + h^2c_2'\omega'}{(h')^2-(h\omega')^2} + c_1} \\ e^{\frac{h^2c_1'\omega' - h'hc_2'}{(h')^2-(h\omega')^2} + c_2} \end{bmatrix} \tag{4.7}$$

or as a vector

$$P' = \left(e^{\frac{-c_1'h'h + h^2c_2'\omega'}{(h')^2-(h\omega')^2} + c_1}, e^{\frac{h^2c_1'\omega' - h'hc_2'}{(h')^2-(h\omega')^2} + c_2} \right). \tag{4.8}$$

□

Here we assume that multiplicative $\omega^*(t) \neq 1$ for all t. That is, multiplicative angular velocity is not 1. In this case there exists a unique pole point in each of the moving and fixed planes of each moment t.

Corollary 4.1. If $\omega(t) = t$, then equation 4.3 will be obtained as

$$X = P = e^{\frac{1}{(h')^2-h^2}} \otimes \begin{bmatrix} e^{(-c_1'h' + c_2'h)\cosh\omega + (c_1'h - c_2'h')\sinh\omega} \\ e^{(c_1'h - c_2'h')\cosh\omega + (a'h' - b'h)\sinh\omega} \end{bmatrix}.$$

Corollary 4.2. For $\omega(t) = t$ and $h(t) = 1$, then equation 4.3 will be obtained as

$$X = P = \begin{bmatrix} e^{c_1'\sinh\omega - c_2'\cosh\omega} \\ e^{-c_1'\cosh\omega + c_2'\cosh\omega} \end{bmatrix}.$$

Corollary 4.3. Let $\omega(t) = t$, then equation 4.8 will be obtained as

$$P' = \left(e^{\frac{-c_1' h' h + h^2 c_2'}{(h')^2 - h^2} + c_1}, e^{\frac{h^2 c_1' - h' h c_2'}{(h')^2 - h^2} + c_2} \right).$$

Corollary 4.4. For $\omega(t) = t$ and $h(t) = 1$, then equation 4.8 will be obtained as

$$P' = \left(e^{-c_2' + c_1}, e^{-c_1' + c_2} \right).$$

Definition 4.3. The point $P = (p_1, p_2)$ is called multiplicative instantaneous rotation center or the pole at moment t of the one parameter motion $B_1 = L_{MC} / L'_{MC}$

Theorem 4.4. *The length of vector* V_f *is*

$$\|V_f\|_L^G = \exp\left(\sqrt{\left(\frac{h'}{h}\right)^2 - (\theta')^2} \, \|P'Y\|_L \right).$$

Proof. The pole point in multiplicative moving plane $Y = B \otimes X \oplus C$ implies that

$$X = (B)^{m-inv} \otimes \left(Y \oplus (e^{-1}) \otimes C) \right), \tag{4.9}$$

$$V_f = B^* \otimes X \oplus C^* \text{ and } B^* \otimes X \oplus C^* = 1$$

that leads to$X = P = e^{-1} \otimes (B^*)^{m-inv} \otimes C^*$. Now let us find pole points in multiplicative fixed plane. Then we have from equation

$$Y = B \otimes X \oplus C.$$

$$Y' = P' = B \otimes \left(e^{-1} \otimes (B^*)^{m-imv} \otimes C^*) \oplus C \right), \text{ Hence, we get}$$

$$C^* = B^* \otimes (B)^{m-inv} \otimes \left(C \oplus (e^{-1} \otimes P') \right)$$

we substitute this values in the equation $V_f = B^* \otimes X \oplus C^*$ we have $V_f = B^* \otimes (B)^{m-inv} \otimes P'Y$. Now let us calculate the value of $B^* \otimes (B)^{m-inv} \otimes P'Y$, where $P'Y = (e^{y_1 - p_1}, e^{y_2 - p_2})$, then

$$V_f = \begin{bmatrix} e^{\frac{h'}{h}(y_1 - p_1) - \omega'(y_2 - p_2)} \\ e^{\omega'(y_1 - p_1) + \frac{h'}{h}(y_2 - p_2)} \end{bmatrix}$$

or as a vector

$$V_f = \left(e^{\frac{h'}{h}(y_1 - p_1) + \omega'(y_2 - p_2)}, e^{\omega'(y_1 - p_1) + \frac{h'}{h}(y_2 - p_2)} \right). \tag{4.10}$$

then,

$$\|V_f\|_L^G = \exp\left(\sqrt{\left(\frac{h'}{h}\right)^2 - (\theta')^2} \, \|P'Y\|_L \right).$$

□

Corollary 4.5. If the scalar matrix h is constant, then the length of the sliding velocity vector is

$$\|V_f\|_L^G = \exp\left(|x| \, \|P'Y\|_L \right). \tag{4.11}$$

Corollary 4.6. The speed that occurs when drawing the curve (P) at point L_{MC} at X is called V_r. At the same time, V_a is the speed that occurs when drawing the $(P)'$ curve of this point in the plane L'_{MC}. These velocities are equal to each other at time t.

Proof. Since $V_f = 1$, it is concluded from expression $V_a = V_f \oplus V_r$ that $V_a = V_r$. □

Definition 4.5. The vector V_a is called multiplicative absolute acceleration vector with respect to the plane L'_{MC} of the point X and is denoted by b_a. Since $V_a = Y^*$ then $b_a = V_a^* = Y^{**}$.

Definition 4.6. Let $X \in L_{MC}$ be a fixed point in motion $B_1 = L_{MC}/L'_{MC}$. Multiplicative acceleration vector of X with respect to L'_{MC} is called multiplicative sliding acceleration vector. This multiplicative sliding acceleration vector is denoted by b_f.

Since acceleration of the multiplicative sliding acceleration X is a fixed point of L_{MC}, then $b_f = V_f^* = B^{**} \otimes C^{**}$.

5. Accelerations And Union Of Accelerations In Lorentzian Homothetic Multiplicative Calculus

Definition 5.1. We know that point X is multiplicative relative velocity vector V_r to L_{MC}. The vector b_r obtained by taking the derivative of V_r is called multiplicative relative acceleration vector of X in plane L_{MC}. This multiplicative relative acceleration vector is represented by b_r. Considering point X as a moving point in plane L_{MC}, matrix B is taken as constant

Theorem 5.2. *There is the following relationship between $b_a,\ b_r,\ b_c$ and b_f*

$$b_a = b_r \oplus b_c \oplus b_f.$$

Here $b_c = (e^2 \otimes (B^* \otimes X^*)$ is called multiplicative Corilois acceleration.

Corollary 5.1. Let X be a point in the plane L_{MC} . If point X is fixed at L_{MC}, then $b_a = b_f$.

Proof. Note that

$$V_a = B^* \otimes X \oplus B \otimes X^* \oplus C^*,$$

Differentiating the both sides we have

$$V_a^* = B^{**} \otimes X \oplus e^2 \otimes (B^* \otimes X^*) \oplus B \otimes X^{**} \oplus C^{**},$$

since the point X is constant its derivative is 1. Hence

$$\begin{aligned} b_a &= V_a^* \\ &= B^{**} \otimes X \oplus C^{**} \\ &= b_f. \end{aligned}$$

□

Theorem 5.3. *The result of the multiplicative inner product of vectors b_c and V_r is*

$$\langle b_c, V_r \rangle_L^G = \exp\left(2hh'({x'_1}^2 - {x'_1}^2\right). \tag{5.1}$$

Proof.

$$\begin{aligned} V_r &= B \otimes X^*, \\ b_c &= e^2 \otimes (B^* \otimes X^*), \end{aligned}$$

So it is obvious that

$$\langle b_c, V_r \rangle_L^G = \exp\left(2hh'({x'_1}^2 - {x'_1}^2\right).$$

□

Corollary 5.2. If the h value is taken as constant in 5.1 equation, then the Coriolis acceleration b_c is perpendicular to the relative velocity vector V_r at each instant moment t.

6. The Acceleration Poles Of The Motions

The solution of the equation $V_f^* = B^{**} \otimes X \oplus C^{**}$ gives us multiplicative acceleration pole of multiplicative motion.$V_f^* = B^{**} \otimes X \oplus C^{**}$ implies $X = e^{-1} \otimes (B^{**})^{m-inv} \otimes C^{**}$. Now calculating the matrices $e^{-1} \otimes (B^{**})^{m-inv}$ and C^{**}, and setting these in $X = P_1 = e^{-1} \otimes (B^{**})^{m-inv} \otimes C^{**}$, we obtain

$$X = P_1 = \begin{bmatrix} e^{\frac{1}{T}\left(c_1''(-r\cosh\omega + z\sinh\omega) - c_2''(r\sinh\omega + z\cosh\omega)\right)} \\ e^{\frac{1}{T}\left(c_1''(r\sinh\omega + z\cos\omega) + c_2''(-r\cosh\omega + z\sinh\omega)\right)} \end{bmatrix}, \tag{6.1}$$

where $(B^{**})^{m-inv}$ is the multiplicative inverse of B^{**}. Here P_1 is called multiplicative pole curve in multiplicative moving plane. If multiplicative pole curve in multiplicative fixed plane is denoted by P_1' we get

$$P_1' = B \otimes P_1 \oplus C \tag{6.2}$$

Hence

$$P_1' = \begin{bmatrix} e^{\frac{1}{T}\left(-hrc_1'' - hzc_2''\right) + c_1} \\ e^{\frac{1}{T}\left(hzc_1'' - hrc_2''\right) + c_2} \end{bmatrix} \tag{6.3}$$

where $r = h'' + h(\omega')^2, z = 2h'\omega' + h\omega'', T = r^2 - z^2$

Corollary 6.1. If $\omega(t) = t$, then equation 6.1 will be obtained as

$$X = P_1 = \begin{bmatrix} e^{\frac{1}{(h''+h)^2 - 4(h')^2}\left(c_1''(-(h''+h)\cosh\omega + 2h'\sinh\omega) - c_2''((h''+h)\sinh\omega + 2h'\cos\omega)\right)} \\ e^{\frac{1}{(h''+h)^2 - 4(h')^2}\left(c_1''((h''+h)\sinh\omega + 2h'\cosh\omega) + c_2''(-(h''+h)\cosh\omega + 2h'\sinh\omega)\right)} \end{bmatrix} \tag{6.4}$$

Corollary 6.2. If $\omega(t) = t$ and $h(t) = 1$, then equation 6.1 will be obtained as

$$X = P_1 = \begin{bmatrix} e^{-c_1''\cosh\omega + c_2''\sinh\omega} \\ e^{c_1''\sinh\omega - c_2''\cosh\omega} \end{bmatrix} \tag{6.5}$$

Corollary 6.3. If $\omega(t) = t$, then equation 6.3 will be obtained as

$$P_1' = \left(e^{\frac{1}{(h''+h)^2 - 4(h')^2}\left(h(h''+h)c_1'' - 2hh'c_2''\right) + c_1}, e^{\frac{1}{(h''+h)^2 - 4(h')^2}\left(-2hh'c_1'' - h(h''+h)c_2''\right) + c_2}\right). \tag{6.6}$$

Corollary 6.4. If $\omega(t) = t$ and $h(t) = 1$, then equation 6.3,will be obtained as

$$P_1' = \left(e^{-c_1'' + c_1}, e^{-c_2'' + c_2}\right). \tag{6.7}$$

7. CONCLUSIONS

In multiplicative Lorentz multiplicative homothetic motions, velocities in plane motion, the relationship between velocities, pole points, and pole curves are given. Additionally, multiplicative Lorentz accelerations and multiplicative Lorentz acceleration combinations have been found.

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