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Sedenionic matrices and their properties

Sedeniyonik matrisler ve özellikleri

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Abstract

In this article, the matrix algebra is well-known concept in mathematics, has been extended to sedenionic-coefficient matrices using sedenions, which have many applications in recent years. Subsequently, sedenionic-coefficient matrices have been obtained in their real, complex, quaternionic and octonionic forms. Based on these definitions, the arithmetic operations of addition, multiplication, conjugation, transpose and conjugate transpose for sedenionic matrices and their complex, quaternionic and octonionic variations have been established and their algebraic properties scrutinized. Lastly, vector space over real and complex numbers and module structure over quaternions of sedenionic matrices has been searched.

Keywords: Octonion, Sedenion, Sedenionic matrices, Quaternion

Öz

Bu makalede matematikte iyi bilinen bir konu olan matris cebiri, son yıllarda pek çok uygulamalaya sahip sedeniyonlar kullanılarak sedeniyonik katsayılı matrislere genişletilmiştir. Daha sonra sedeniyonik katsayılı matrislerin, reel, kompleks, kuaterniyonik ve oktoniyonik katsayılı versiyonları elde edilmiştir. Bu tanımlar doğrultusunda, sedeniyonik matrislerin ve diğer varyasyonlarının toplama, çarpma, eşlenik, tranpoz ve eşlenik transpoz işlemleri tanımlanmış ve cebirsel özellikleri incelenmiştir. Son olarak sedeniyonik matrislerin reel ve kompleks sayılar üzerindeki vektör uzayları ve kuaterniyonlar üzerindeki modül yapısı araştırılmıştır.

Anahtar kelimeler: *Oktoniyon, Sedeniyon, Sedeniyonik matrisler, Kuaterniyon*

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1. Introduction

In abstract algebra the term 'sedenion' refers to any 16-dimensional algebra over the real numbers. The Cayley–Dickson construction (Dickson, 1919) shows how octonions can be constructed as two dimensional algebra over quaternions. If this doubling process applied to octonions, Cayley–Dickson sedenions are obtained. Sedenions are non-commutative, non-associative, non-alternative, but power-associative algebra over the real numbers. Unlike octonions, sedenions are not composition algebras or normed division algebras, because they have zero divisors. The most fundamental and comprehensive source on sedenions is the study entitled "Sedenionic algebra and analysis" (Imeada & Imeada, 2000). Carmody dealt with the investigation of circular, hyperbolic quaternions, octonions and sedenions (Carmody, 1988). Moreno studied zero divisors of the Cayley-Dickson algebras over the real numbers (Moreno, 1997) and also the most comprehensive study on zero divisors of Cayley-Dickson sedenions has been conducted by Cawagas (Cawagas, 2004). The conjugacy class of subalgebras in the domain of real sedenions is investgated by Chan and Dokovic (Chan & Dokovic, 2018). The application scope of sedenions can be demonstrated through various recent examples. For instance, solving plasma physics' Maxwell equations using sedenions can be cited as the most current application (Sumer, 2022). The complex, quaternionic, and octonionic forms of sedenions, along with their matrix representations, were obtained by Bektas (Bektas, 2021). Additionally, Bektas conducted research on fuzzy sedenions and fuzzy sedenion-valued series (Bektas, 2023). Sedenions have recently emerged in number theories in pure mathematics. Research in this field has been applied in computer science and combinatorics. Some examples of these studies can be provided. Firstly, a new generalization of Fibonacci and Lucas sedenions can be given (Kirlak & Kizilates, 2022). Akpinar and Tasyurdu investigated Perrin octonions and sedenions (Tasyurdu & Akpinar, 2020). Catarino investigated modified pell and modified k-pell quaternions and octonions (Catarino, 2016) and also k-pell, k-pell-Lucas and modified k-pell sedenions (Catarino, 2019). The expansion of Mersenne and Mersenne-Lucas numbers through sedenions resulted in obtaining Mersenne and Mersenne-Lucas sedenions (Devi & Devibala, 2021). Similar to Mersenne sedenions, Fibonacci and Lucas sedenions and their generating functions and Binet's formulas obtained by (Bilgici et al, 2017). These studies showcase the diverse applications of sedenions in different fields. They highlight the role of sedenions not only in mathematical theories but also in applied sciences, thereby indicating their relevance in interdisciplinary research. In thispaper, we aimed to expand the scope of standard matrix algebra and its properties within the context of sedenions. The article initially provided information about sedenions and their general characteristics. Subsequently, based on this information, sedenionic coefficient matrices were defined. Algebraic properties were examined through operations including addition, multiplication, conjugate, transpose, conjugate transpose and trace. Finally, within this study, the vector space structures of sedenionic matrix sets over real numbers and complex numbers and also the module structure over quaternions were investigated.

Definition 1.1 (Imaeda & Imaeda, 2000) The sedenions are constructed over real numbers. Let $\mathrm{E}_{16} = \{e_{\mathrm{i}} \in \mathbb{S} \mid \mathrm{i} = 0,1,2,\dots,15\}$ is canonical basis of sedenions, where $e_0 = 1$ is multiplicative scalar element and e_{i}'s $(\mathrm{i} = 1,2,\dots,15)$ are imaginary units. Sedenions are written as a linear combination of E_{16}. The set of sedenions can be written in the form

$$\mathbb{S} = \left\{ \mathcal{X} = \sum_{\mathrm{i}=0}^{15} x_{\mathrm{i}} e_{\mathrm{i}} \mid x_{\mathrm{i}} \in \mathbb{R}, 0 \leq \mathrm{i} \leq 15 \right\}.$$

Sedenionic units are satisfy the following properties:

1. $e_0 = 1$ and $e_0 e_{\mathrm{i}} = e_{\mathrm{i}}\, e_0 = e_{\mathrm{i}}$, $(\mathrm{i} \neq 0)$
2. $e_{\mathrm{i}}\, e_{\mathrm{i}} = (e_{\mathrm{i}})^2 = -1,$ $(\mathrm{i} \neq 0)$
3. $e_{\mathrm{i}}\, e_{\mathrm{j}} = -e_{\mathrm{j}}\, e_{\mathrm{i}},$ $(\mathrm{i} \neq \mathrm{j}), (\mathrm{i}, \mathrm{j} \neq 0).$

Let us first give some fundamental notions of sedenions. $\mathcal{X}$ is a Cayley-Dickson Sedenion, $\mathcal{X} = \sum_{\mathrm{i}=0}^{15} x_{\mathrm{i}} e_{\mathrm{i}}$, $\mathrm{S}_{\mathcal{X}} = x_0 e_0 = \mathrm{Re}(\mathcal{X})$ is the real part of sedenion, $\vec{\mathrm{V}}_{\mathcal{X}} = \sum_{\mathrm{i}=1}^{15} x_{\mathrm{i}} e_{\mathrm{i}} = \mathrm{Im}(\mathcal{X})$ is the vectorial part of sedenion. Thus, sedenions can be expression as

$$\mathcal{X} = x_0 e_0 + \sum_{\mathrm{i}=1}^{15} x_{\mathrm{i}} e_{\mathrm{i}} = \mathrm{S}_{\mathcal{X}} + \vec{\mathrm{V}}_{\mathcal{X}}.$$

The sum of two sedenions is defined by

$$\mathcal{X} + \mathcal{Y} = \sum_{\mathrm{i}=0}^{15}(x_{\mathrm{i}} + y_{\mathrm{i}})e_{\mathrm{i}} = \left(\mathrm{S}_{\mathcal{X}} + \mathrm{S}_{y}\right) + \left(\vec{\mathrm{V}}_{\mathcal{X}} + \vec{\mathrm{V}}_{y}\right).$$

The multiplication of two sedenions is defined by

$$\mathcal{X} \cdot \mathcal{Y} = \left(\sum_{\mathrm{i}=0}^{15} x_{\mathrm{i}} e_{\mathrm{i}}\right) \cdot \left(\sum_{\mathrm{i}=0}^{15} y_{\mathrm{i}} e_{\mathrm{i}}\right).$$

The multiplication of sedenionic unit basic elements are given in **Table 1.**

Table 1. Multiplication table of unit sedenion basic elements

$\boldsymbol{e_0}$	$\boldsymbol{e_1}$	$\boldsymbol{e_2}$	$\boldsymbol{e_3}$	$\boldsymbol{e_4}$	$\boldsymbol{e_5}$	$\boldsymbol{e_6}$	$\boldsymbol{e_7}$	$\boldsymbol{e_8}$	$\boldsymbol{e_9}$	$\boldsymbol{e_{10}}$	$\boldsymbol{e_{11}}$	$\boldsymbol{e_{12}}$	$\boldsymbol{e_{13}}$	$\boldsymbol{e_{14}}$	$\boldsymbol{e_{15}}$
$\boldsymbol{e_0}$	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}
$\boldsymbol{e_1}$	$-e_0$	e_3	$-e_2$	e_5	$-e_4$	$-e_7$	e_6	e_9	$-e_8$	$-e_{11}$	e_{10}	$-e_{13}$	e_{12}	e_{15}	$-e_{14}$
$\boldsymbol{e_2}$	$-e_3$	$-e_0$	e_1	e_6	e_7	$-e_4$	$-e_5$	e_{10}	e_{11}	$-e_8$	$-e_9$	$-e_{14}$	$-e_{15}$	e_{12}	e_{13}
$\boldsymbol{e_3}$	e_2	$-e_1$	$-e_0$	e_7	$-e_6$	e_5	$-e_4$	e_{11}	$-e_{10}$	e_9	$-e_8$	$-e_{15}$	e_{14}	$-e_{13}$	e_{12}
$\boldsymbol{e_4}$	$-e_5$	$-e_6$	$-e_7$	$-e_0$	e_1	e_2	e_3	e_{12}	e_{13}	e_{14}	e_{15}	$-e_8$	$-e_9$	$-e_{10}$	$-e_{11}$
$\boldsymbol{e_5}$	e_4	$-e_7$	e_6	$-e_1$	$-e_0$	$-e_3$	e_2	e_{13}	$-e_{12}$	e_{15}	$-e_{14}$	e_9	$-e_8$	e_{11}	$-e_{10}$
$\boldsymbol{e_6}$	e_7	e_4	$-e_5$	$-e_2$	e_3	$-e_0$	$-e_1$	e_{14}	$-e_{15}$	$-e_{12}$	e_{13}	e_{10}	$-e_{11}$	$-e_8$	e_9
$\boldsymbol{e_7}$	$-e_6$	e_5	e_4	$-e_3$	$-e_2$	e_1	$-e_0$	e_{15}	e_{14}	$-e_{13}$	$-e_{12}$	e_{11}	e_{10}	$-e_9$	$-e_8$
$\boldsymbol{e_8}$	$-e_9$	$-e_{10}$	$-e_{11}$	$-e_{12}$	$-e_{13}$	$-e_{14}$	$-e_{15}$	$-e_0$	e_1	e_2	e_3	e_4	e_5	e_6	e_7
$\boldsymbol{e_9}$	e_8	$-e_{11}$	e_{10}	$-e_{13}$	e_{12}	e_{15}	$-e_{14}$	$-e_1$	$-e_0$	$-e_3$	e_2	$-e_5$	e_4	e_7	$-e_6$
$\boldsymbol{e_{10}}$	e_{11}	e_8	$-e_9$	$-e_{14}$	$-e_{15}$	e_{12}	e_{13}	$-e_2$	e_3	$-e_0$	$-e_1$	$-e_6$	$-e_7$	e_4	e_5
$\boldsymbol{e_{11}}$	$-e_{10}$	e_9	e_8	$-e_{15}$	e_{14}	$-e_{13}$	e_{12}	$-e_3$	$-e_2$	e_1	$-e_0$	$-e_7$	e_6	$-e_5$	e_4
$\boldsymbol{e_{12}}$	e_{13}	e_{14}	e_{15}	e_8	$-e_9$	$-e_{10}$	$-e_{11}$	$-e_4$	e_5	e_6	e_7	$-e_0$	$-e_1$	$-e_2$	$-e_3$
$\boldsymbol{e_{13}}$	$-e_{12}$	e_{15}	$-e_{14}$	e_9	e_8	e_{11}	$-e_{10}$	$-e_5$	$-e_4$	e_7	$-e_6$	e_1	$-e_0$	e_3	$-e_2$
$\boldsymbol{e_{14}}$	$-e_{15}$	$-e_{12}$	e_{13}	e_{10}	$-e_{11}$	e_8	e_9	$-e_6$	$-e_7$	$-e_4$	e_5	e_2	$-e_3$	$-e_0$	$-e_1$
$\boldsymbol{e_{15}}$	e_{14}	$-e_{13}$	$-e_{12}$	e_{11}	e_{10}	$-e_9$	e_8	$-e_7$	e_6	$-e_5$	$-e_4$	e_3	e_2	$-e_1$	$-e_0$

Additionaly, the multiplication operation can be defined as $\mathcal{X} \cdot \mathcal{Y} = \sum_{\mathrm{i,j,k}=1}^{15} \mathrm{f_{ij}} \gamma_{\mathrm{ij}}^{\mathrm{k}} e_{\mathrm{k}}$, where $e_{\mathrm{i}}, e_{\mathrm{j}}, e_{\mathrm{k}} \in \mathrm{E}_{16}$, $\mathrm{f_{ij}} = x_{\mathrm{i}} y_{\mathrm{j}}$. $\gamma_{\mathrm{ij}}^{\mathrm{k}} \in \{-1, 0, +1\}$. $\gamma_{\mathrm{ij}}^{\mathrm{k}}$ are called field parameter. The list of all triplets indices of the ordered triplets $(\mathrm{i}, \mathrm{j}, \mathrm{k})$ which provide the loops here, can be found in **Table 2.**

Table 2. Sedenionic triplets

			$(\mathbf{i}, \mathbf{j}, \mathbf{k})$			
(1,2,3)	(1,4,5)	(1,7,6)	(1,8,9)	(1,11,10)	(1,13,12)	(1,14,15)
(2,4,6)	(2,5,7)	(2,8,10)	(2,9,11)	(2,14,12)	(2,15,13)	(3,4,7)
(3,6,5)	(3,8,11)	(3,10,9)	(3,13,14)	(3,15,12)	(4,8,12)	(4,9,13)
(4,10,14)	(4,11,15)	(5,8,13)	(5,10,15)	(5,12,9)	(5,14,11)	(6,8,14)
(6,11,13)	(6,12,10)	(6,15,9)	(7,8,15)	(7,9,14)	(7,12,11)	(7,13,10)

Let $\mathcal{X} = \left(\sum_{i=0}^{15} x_i e_i\right)$ and $\mathcal{Y} = \left(\sum_{i=0}^{15} y_i e_i\right)$ be two sedenions. $<,>_s$ operation is called sedenionic inner product and it is defined as

$$<\mathcal{X},\mathcal{Y}>_s = \frac{1}{2}(\mathcal{X}\cdot\bar{\mathcal{Y}} + \mathcal{Y}\cdot\bar{\mathcal{X}}) = x_0y_0 + x_1y_1 + \cdots + x_{15}y_{15}.$$

Hence, the norm of the sedenion $\mathcal{X}$ denoted by $\|\mathcal{X}\| = \sqrt{\mathcal{X}\cdot\bar{\mathcal{X}}} = \sqrt{\sum_{i=0}^{15}(x_i)^2}$. If $\|\mathcal{X}\| = 1$, then $\mathcal{X}$ is called the unit sedenion. The inner product and norm operations mention above provide the following properties:

1. $<\mathcal{X},\mathcal{Y}>_s = <\mathcal{Y},\mathcal{X}>_s$,
2. $<\mathcal{X},\mathcal{X}>_s = \|\mathcal{X}\|^2 \geq 0$,
3. $<\mathcal{X}\cdot\mathcal{Y},\mathcal{Z}>_s = <\mathcal{Y},\bar{\mathcal{X}}\cdot\mathcal{Z}>_s = <\mathcal{X},\mathcal{Z}\cdot\bar{\mathcal{Y}}>_s$,
4. $\|\mathcal{X}+\mathcal{Y}\| \leq \|\mathcal{X}\| + \|\mathcal{Y}\|$,
5. $\|\mathcal{X}\| = \|-\mathcal{X}\| = \|\bar{\mathcal{X}}\| = \|-\bar{\mathcal{X}}\|$,
6. $\|\mathcal{X}\|^2 + \|\mathcal{Y}\|^2 = \frac{1}{2}(\|\mathcal{X}+\mathcal{Y}\|^2 + \|\mathcal{X}-\mathcal{Y}\|^2)$,
7. $\|\mathcal{X}\cdot\mathcal{Y}\| = \|\mathcal{Y}\cdot\mathcal{X}\| = \|\bar{\mathcal{X}}\cdot\mathcal{Y}\| = \|\mathcal{X}\cdot\bar{\mathcal{Y}}\|$.

$\bar{\mathcal{X}}$ is called conjugate of sedenion and it is defined by

$$\bar{\mathcal{X}} = x_0e_0 - \sum_{i=1}^{15} x_i e_i = S_{\mathcal{X}} - \vec{V}_{\mathcal{X}}.$$

$\mathcal{X}$ non-zero sedenion, inverse of sedenion is defined by

$$\mathcal{X}^{-1} = \frac{\bar{\mathcal{X}}}{\|\mathcal{X}\|^2}, \quad (\|\mathcal{X}\| \neq 0).$$

2. Material and method

In this section, we will investigate properties of sedenionic matrices. Firstly, we define real, complex, quaternionic and octonionic coefficient combination of sedenionic matrices. According to definition of basic matrix algebra operations are defined for each combination. These operations can be listed as follows: Addition, multiplication, conjugate, transpose, conjugate transpose and trace operation of sedenionic matrices. Finally, algebraic structures on sedenionic matrices set are investigated.

Definition 2.1 Sedenionic matrix is defined by $\breve{A} = [\breve{a}_{st}]_{mxn}$, where $\breve{a}_{st} = \sum_{i=0}^{15} a_{st}^i e_i \in \mathbb{S}$, $a_{st}^i \in \mathbb{R}$, $(1 \leq s \leq m, 1 \leq t \leq n)$. The set of sedenionic matrices denoted by $M_{mxn}(\mathbb{S})$. If m=n the set of sedenionic square matrices denoted by $M_n(\mathbb{S})$.

Lemma 2.1 (Bektas, 2021) The algebras obtained through the construction known as the Cayley-Dickson doubling process always contain a sub-algebra. Therefore, there exists a relationship between these sets such $\mathbb{R} \subset \mathbb{C} \subset \mathbb{H} \subset \mathbb{O} \subset \mathbb{S} \subset \cdots$ that every sedenion can be expressed as a linear combination of octonions, quaternions, and complex numbers.

i. Let $z_1 = x_0 + x_1e_1$, $z_2 = x_2 + x_3e_1$, $z_3 = x_4 + x_5e_1$, $z_4 = x_6 + x_7e_1$, $z_5 = x_8 + x_9e_1$, $z_6 = x_{10} + x_{11}e_1$, $z_7 = x_{12} + x_{13}e_1$, $z_8 = x_{14} + x_{15}e_1 \in \mathbb{C}$ and $\mathcal{X} = \sum_{i=0}^{15} x_i e_i \in \mathbb{S}$ be given. Sedenion $\mathcal{X}$ can be written linear combination of 8 complex numbers as follows:

$$\mathcal{X} = z_1 + z_2e_2 + z_3e_4 + z_4e_6 + z_5e_8 + z_6e_{10} + z_7e_{12} + z_8e_{14}.$$

ii. Let $q_1 = x_0 + \sum_{i=1}^{3} x_i e_i$, $q_2 = x_4 + \sum_{i=5}^{7} x_i e_{i-4}$, $q_3 = x_8 + \sum_{i=8}^{10} x_i e_{i-7}$, $q_4 = x_{12} + \sum_{i=11}^{13} x_i e_{i-10} \in \mathbb{H}$ and $\mathcal{X} = \sum_{i=0}^{15} x_i e_i \in \mathbb{S}$ be given. Sedenion $\mathcal{X}$ can be written linear combination of 4 quaternionic numbers as follows:

$$\mathcal{X} = q_1 + q_2e_4 + q_3e_8 + q_4e_{12}.$$

iii. Let $o_1 = x_0 + \sum_{i=1}^{7} x_i e_i$, $o_2 = x_8 + \sum_{i=9}^{15} x_i e_{i-8} \in \mathbb{O}$ be given and $\mathcal{X} = \sum_{i=0}^{15} x_i e_i \in \mathbb{S}$ be given.

Sedenion $\mathcal{X}$ can be written linear combination of 2 octonionic numbers as follows:

$\mathcal{X} = o_1 + o_2 e_8$.

Theorem 2.1 Let $\breve{A} = [\breve{a}_{st}] = \left[\sum_{i=0}^{15} a_{st}^i e_i\right]$ be given. The sedenionic matrices can be written as real, complex, quaternionic and octonionic coeffecient matrix as follows:

1. $\breve{A} = [\breve{a}_{st}] = \left[\sum_{i=0}^{15} a_{st}^i e_i\right] = [a_{st}^0] + [a_{st}^1]e_1 + [a_{st}^2]e_2 + \cdots + [a_{st}^{15}]e_{15}$ a sedenionic matrix is written as a linear combination of 16 real matrices as follows

$\breve{A} = A_0 + A_1 e_1 + A_2 e_2 + \cdots + A_{15} e_{15}$,

where $A_0 = [a_{st}^0]$,…, $A_{15} = [a_{st}^{15}]e_{15} \in M_{mxn}(\mathbb{R})$.

2. $\breve{A} = \sum_{i=0}^{15} A_i e_i$, a sedenionic matrix is written as a linear combination of 8 complex matrices as follows

$\breve{A} = \sum_{i=1}^{8} \widehat{A}_i e_{2i-2} = \widehat{A}_1 + \widehat{A}_2 e_2 + \widehat{A}_3 e_4 + \widehat{A}_4 e_6 + \widehat{A}_5 e_8 + \widehat{A}_6 e_{10} + \widehat{A}_7 e_{12} + \widehat{A}_8 e_{14}$,

where $A_1 = A_0 + A_1 e_1$, $A_2 = A_2 + A_3 e_1$, $A_3 = A_4 + A_5 e_1$, $A_4 = A_6 + A_7 e_1$,
$A_5 = A_8 + A_9 e_1$, $A_6 = A_{10} + A_{11} e_1$, $A_7 = A_{12} + A_{13} e_1$, $A_8 = A_{14} + A_{15} e_1 \in M_{mxn}(\mathbb{C})$.

3. $\breve{A} = \sum_{i=0}^{15} A_i e_i$, a sedenionic is written as a linear combination of 4 quaternionic matrices as follows

$\breve{A} = \sum_{i=1}^{4} \widetilde{A}_i e_{4i-4} = \widetilde{A}_1 + \widetilde{A}_2 e_4 + \widetilde{A}_3 e_8 + \widetilde{A}_4 e_{12}$,

where $\widetilde{A}_1 = A_0 + \sum_{i=1}^{3} A_i e_i$, $\widetilde{A}_2 = A_4 + \sum_{i=5}^{7} A_i e_{i-4}$, $\widetilde{A}_3 = A_8 + \sum_{i=8}^{10} A_i e_{i-7}$,
$q_4 = A_{12} + \sum_{i=11}^{13} A_i e_{i-10} \in M_{mxn}(\mathbb{H})$.

4. $\breve{A} = \sum_{i=0}^{15} A_i e_i$, a sedenionic matrix is written as a linear combination of 2 octonionic matrices as follows

$\breve{A} = \widetilde{\widetilde{A}}_1 + \widetilde{\widetilde{A}}_2 e_8$,

where $\widetilde{\widetilde{A}}_1 = A_0 + \sum_{i=1}^{7} A_i e_i$, $\widetilde{\widetilde{A}}_2 = A_8 + \sum_{i=9}^{15} A_i e_{i-8} \in M_{mxn}(\mathbb{O})$.

Proof: It can be obtained easily from lemma 2.1.

Definition 2.2 Let $\breve{A} = [\breve{a}_{st}]$, $\breve{B} = [\breve{b}_{st}] \in M_{mxn}(\mathbb{S})$ be sedenionic matrices. If $\breve{a}_{st} = \breve{b}_{st}$, then it is called $\breve{A}$ is equal to $\breve{B}$ and written $\breve{A} = \breve{B}$.

Remark 2.1 According to sedenionic matrix definition, if two sedenionic matrices are equal then their real, complex, quaternionic and octonionic coeffiecient matrices are equal.

Definition 2.3 Let $\breve{A} = [\breve{a}_{st}]$ be given. $\forall\, \breve{a}_{st} \in \mathbb{S}$, if $\breve{a}_{st} = 0$, then it is called sedenionic zero matrix and written as $\breve{A} = \breve{0}_{mxn}$. In case of $m = n$, it is called sedenionic zero square matrix and it is written as $\breve{0}_{mxn} = \breve{0}_n$.

Definition 2.4 Let $\breve{A} = [\breve{a}_{st}]$, $\breve{B} = [\breve{b}_{st}] \in M_{mxn}(\mathbb{S})$, $(1 \le s \le m, 1 \le t \le n)$ be sedenionic matrices. The addition operation of sedenionic matrices can be written as follow:

$$\breve{A} \oplus \breve{B} = [\breve{a}_{st} + \breve{b}_{st}] = \begin{pmatrix} \breve{a}_{11} + \breve{b}_{11} & \breve{a}_{12} + \breve{b}_{12} & \cdots & \breve{a}_{1n} + \breve{b}_{1n} \\ \breve{a}_{21} + \breve{b}_{21} & \breve{a}_{22} + \breve{b}_{22} & \cdots & \breve{a}_{2n} + \breve{b}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \breve{a}_{m1} + \breve{b}_{m1} & \breve{a}_{m2} + \breve{b}_{m2} & \cdots & \breve{a}_{mn} + \breve{b}_{mn} \end{pmatrix}_{mxn}.$$

Remark 2.2 The symbol $\oplus$ is defined as sedenionic matrix addition operation. Addition of sedenionic matrices can be written in the term of matrices with real, complex, quaternionic and octonionic coefficient respectively

1. $\breve{A}\oplus\breve{B} = \sum_{i=1}^{15}(A_i+B_i)e_i$,
2. $\breve{A}\oplus\breve{B} = \sum_{i=1}^{8}\left(\widehat{A}_i + \widehat{B}_i\right)e_{2i-2}$,
3. $\breve{A}\oplus\breve{B} = \sum_{i=1}^{4}\left(\widetilde{A}_i + \widetilde{B}_i\right)e_{4i-4}$,
4. $\breve{A}\oplus\breve{B} = \widetilde{\widetilde{A}}_1 + \widetilde{\widetilde{B}}_1 + \left(\widetilde{\widetilde{A}}_2 + \widetilde{\widetilde{B}}_2\right)e_8$.

Corollary 2.1 $(M_{mxn}(\mathbb{S}),\oplus)$ is an Abelian group. $\breve{0}_{mxn}$ is unit element of $M_{mxn}(\mathbb{S})$. The inverse of $\breve{A} = \sum_{i=0}^{15} A_i e_i$ according to the addition operation is $-\breve{A} = -\sum_{i=0}^{15} A_i e_i$.

Definition 2.5 Let $\breve{A} = [\breve{a}_{st}] \in M_{mxn}(\mathbb{S})$, $\breve{B} = \left[\breve{b}_{tr}\right] \in M_{nxp}(\mathbb{S})$ $(1 \le s \le m, 1 \le t \le n,$ $1 \le r \le p)$ be sedenionic matrices. The multiplication of two sedenionic matrices can be define as follows:

$$\breve{A} \otimes \breve{B} = \left[\sum_{t=1}^{n} \breve{a}_{st} \cdot \breve{b}_{tr}\right]_{mxp}.$$

The symbol $\otimes$ is defined as sedenionic matrix multiplication operation. If $\sum_{t=1}^{n} \breve{a}_{st}\breve{b}_{tr} = \breve{c}_{sr}$ $(1 \le s \le m,$ $1 \le r \le p)$, then we can write

$$\breve{c}_{11} = \sum_{t=1}^{n} \breve{a}_{1t} \cdot \breve{b}_{t1} = \breve{a}_{11} \cdot \breve{b}_{11} + \breve{a}_{12} \cdot \breve{b}_{21} + \cdots + \breve{a}_{1n} \cdot \breve{b}_{n1},$$
$$\breve{c}_{12} = \sum_{t=1}^{n} \breve{a}_{1t} \cdot \breve{b}_{t2} = \breve{a}_{11} \cdot \breve{b}_{12} + \breve{a}_{12} \cdot \breve{b}_{22} + \cdots + \breve{a}_{1n} \cdot \breve{b}_{n2},$$
$$\vdots$$
$$\breve{c}_{1p} = \sum_{t=1}^{n} \breve{a}_{1t} \cdot \breve{b}_{tp} = \breve{a}_{11} \cdot \breve{b}_{1p} + \breve{a}_{12} \cdot \breve{b}_{2p} + \cdots + \breve{a}_{1n} \cdot \breve{b}_{np}.$$

On the other hand, the multiplication of sedenionic matrices can be written as

$$\breve{A} \otimes \breve{B} = \begin{pmatrix} \breve{a}_{1t} & \breve{a}_{1t} & \cdots & \breve{a}_{1t} \\ \breve{a}_{1t} & \breve{a}_{1t} & \cdots & \breve{a}_{1t} \\ \vdots & \vdots & \ddots & \vdots \\ \breve{a}_{1t} & \breve{a}_{1t} & \cdots & \breve{a}_{1t} \end{pmatrix}_{mxn} \begin{pmatrix} \breve{a}_{1t} & \breve{a}_{1t} & \cdots & \breve{a}_{1t} \\ \breve{a}_{1t} & \breve{a}_{1t} & \cdots & \breve{a}_{1t} \\ \vdots & \vdots & \ddots & \vdots \\ \breve{a}_{1t} & \breve{a}_{1t} & \cdots & \breve{a}_{1t} \end{pmatrix}_{nxp} = \left(\breve{c}_{mp}\right) = \breve{C}.$$

Remark 2.3 Let $\breve{A} = \sum_{i=0}^{15} A_i e_i \in M_{mxn}(\mathbb{S})$, $\breve{B} = \sum_{i=0}^{15} B_i e_i \in M_{nxp}(\mathbb{S})$, $(1 \le s \le m, 1 \le t \le n,$ $1 \le r \le p)$ be two sedenionic matrices. For each $i = 0,1,2 \ldots,15$, let $A_i \in M_{mxn}(\mathbb{R})$ and $B_i \in M_{nxp}(\mathbb{R})$ real coefficient matrices. Then, the multiplication operation in terms of real combination of two sedenionic matrices can be define as follows:

$$\begin{aligned}
\breve{A} \otimes \breve{B} &= \left(\sum_{i=0}^{15} A_i e_i\right) \otimes \left(\sum_{i=0}^{15} B_i e_i\right) \\
&= (A_0B_0 - A_1B_1 - A_2B_2 - A_3B_3 - A_4B_4 - A_5B_5 - A_6B_6 - A_7B_7 - A_8B_8 \\
&-A_9B_9 - A_{10}B_{10} - A_{11}B_{11} - A_{12}B_{12} - A_{13}B_{13} - A_{14}B_{14} - A_{15}B_{15})e_0 \\
&+(A_0B_1 + A_1B_0 + A_2B_3 - A_3B_2 + A_4B_5 - A_5B_4 - A_6B_7 + A_7B_6 + A_8B_9 \\
&-A_9B_1 - A_{10}B_{11} + A_{11}B_{10} - A_{12}B_{13} - A_{13}B_9 - A_{14}B_{15} - A_{15}B_{14})e_1 \\
&+(A_0B_2 - A_1B_3 + A_2B_0 + A_3B_1 + A_4B_6 + A_5B_7 - A_6B_4 - A_7B_5 + A_8B_{10} \\
&+A_9B_{11} - A_{10}B_8 - A_{11}B_9 - A_{12}B_{14} - A_{13}B_{15} + A_{14}B_{12} + A_{15}B_{13})e_2 \\
&+(A_0B_3 + A_1B_2 - A_2B_1 + A_3B_0 + A_4B_7 - A_5B_6 + A_6B_5 - A_7B_4 + A_8B_{11} \\
&-A_9B_{10} + A_{10}B_9 - A_{11}B_8 - A_{12}B_{15} + A_{13}B_{14} - A_{14}B_{13} + A_{15}B_{12})e_3 \\
&+(A_0B_4 - A_1B_5 - A_2B_6 - A_3B_7 + A_4B_0 + A_5B_1 + A_6B_2 + A_7B_3 + A_8B_{12} \\
&-A_9B_{13} + A_{10}B_{14} + A_{11}B_{15} - A_{12}B_8 - A_{13}B_9 - A_{14}B_{10} - A_{15}B_{11})e_4 \\
&+(A_0B_5 + A_1B_4 - A_2B_7 + A_3B_6 - A_4B_1 + A_5B_0 - A_6B_3 + A_7B_2 + A_8B_{13} \\
&-A_9B_{12} + A_{10}B_{15} - A_{11}B_{14} + A_{12}B_9 - A_{13}B_8 + A_{14}B_{11} - A_{15}B_{10})e_5 \\
&+(A_0B_6 + A_1B_3 + A_2B_4 - A_3B_5 - A_4B_2 + A_5B_3 + A_6B_0 - A_7B_1 - A_8B_{14} \\
&-A_9B_{15} - A_{10}B_{12} + A_{11}B_{13} + A_{12}B_{10} - A_{13}B_{11} - A_{14}B_8 + A_{15}B_9)e_6 \\
&+(A_0B_7 - A_1B_6 + A_2B_5 + A_3B_4 - A_4B_3 - A_5B_2 + A_6B_1 + A_7B_0 + A_8B_{15}
\end{aligned}$$

$$+A_9B_{14} - A_{10}B_{13} + A_{11}B_{12} + A_{12}B_{11} + A_{13}B_{10} - A_{14}B_9 - A_{15}B_8)e_7$$
$$+(A_0B_8 - A_1B_9 - A_2B_{10} - A_3B_9 - A_4B_{12} - A_5B_{13} - A_6B_{14} - A_7B_{15} + A_8B_0$$
$$+A_9B_1 + A_{10}B_2 + A_{11}B_3 + A_{12}B_4 + A_{13}B_5 + A_{14}B_6 + A_{15}B_7)e_8$$
$$+(A_0B_9 + A_1B_8 - A_2B_{11} + A_3B_{10} - A_4B_{13} + A_5B_{12} + A_6B_{15} - A_7B_{14} - A_8B_1$$
$$+A_9B_0 - A_{10}B_3 + A_{11}B_2 - A_{12}B_5 + A_{13}B_4 + A_{14}B_7 - A_{15}B_6)e_9$$
$$+(A_0B_{10} + A_1B_{11} + A_2B_8 - A_3B_9 - A_4B_{14} - A_5B_{15} + A_6B_{12} + A_7B_{13}$$
$$-A_8B_2 + A_9B_3 + A_{10}B_0 - A_{11}B_1 - A_{12}B_6 - A_{13}B_7 + A_{14}B_4 + A_{15}B_5)e_{10}$$
$$+(A_0B_{11} - A_1B_{10} + A_2B_9 + A_3B_8 - A_4B_{15} + A_5B_{14} - A_6B_{13} + A_7B_{12} - A_8B_3 - A_9B_2$$
$$+A_{10}B_1 + A_{11}B_0 - A_{12}B_7 + A_{13}B_6 - A_{14}B_5 + A_{15}B_4)e_{11}$$
$$+(A_0B_{12} + A_1B_{13} + A_2B_{14} + A_3B_{15} + A_4B_8 - A_5B_9 - A_6B_{10} - A_7B_{11} - A_8B_4$$
$$+A_9B_5 + A_{10}B_6 + A_{11}B_7 + A_{12}B_0 - A_{13}B_1 - A_{14}B_2 - A_{15}B_3)e_{12}$$
$$+(A_0B_{13} - A_1B_{12} + A_2B_{15} - A_3B_{14} + A_4B_9 + A_5B_8 + A_6B_{11} - A_7B_{10} - A_8B_5$$
$$-A_9B_4 + A_{10}B_7 - A_{11}B_6 + A_{12}B_1 + A_{13}B_0 + A_{14}B_3 - A_{15}B_2)e_{13}$$
$$+(A_0B_{14} - A_1B_{15} - A_2B_{12} + A_3B_{13} + A_4B_{10} - A_5B_{11} + A_6B_8 + A_7B_9 - A_8B_6$$
$$-A_9B_7 - A_{10}B_4 + A_{11}B_5 + A_{12}B_2 - A_{13}B_3 + A_{14}B_0 + A_{15}B_1)e_{14}$$
$$+(A_0B_{15} + A_1B_{14} - A_2B_{13} - A_3B_{12} + A_4B_{11} + A_5B_{10} - A_6B_9 + A_7B_8 - A_8B_7$$
$$+A_9B_6 - A_{10}B_5 - A_{11}B_4 + A_{12}B_3 + A_{13}B_2 - A_{14}B_1 + A_{15}B_0)e_{15}.$$

Remark 2.4 Sedenionic matrix set, unlike other matrix algebras, none of the properties of multiplication operation is not safisfied in general.

Definition 2.6 Let $\breve{A} = [\breve{a}_{st}] \in M_{mxn}(\mathbb{S})$, $(1 \leq s \leq m, 1 \leq t \leq n)$ be given. $\overline{\breve{A}}$ is define as conjugate of sedenionic matrix of $\breve{A}$ and written as follow:

$$\overline{\breve{A}} = \overline{[\breve{a}_{st}]} = \overline{\left[\sum_{i=0}^{15} a_{st}^i e_i\right]} = \left[a_{st}^0 - \sum_{i=1}^{15} a_{st}^i e_i\right].$$

Remark 2.5 The conjugate of sedenionic matrices can be written in terms of matrices with real, complex, quaternionic and octonionic coefficient respectively

1. $\overline{\breve{A}} = A_0 - \sum_{i=1}^{15} A_i e_i$,
2. $\overline{\breve{A}} = \widehat{A}_1 - \sum_{i=2}^{8} \widehat{A}_i e_{2i-2}$,
3. $\overline{\breve{A}} = \widetilde{A}_i - \sum_{i=2}^{4} \widetilde{A}_i e_{4i-4}$,
4. $\overline{\breve{A}} = \widetilde{\widetilde{A}}_1 - \widetilde{\widetilde{A}}_2 e_8$.

Definition 2.7 Let $\lambda \in \mathbb{S}$ be given. The multiplication of a sedenion and a sedenionic matrix is defined as follow:

$$\odot_{\mathbb{S}}: \ \mathbb{S} \times M_{mxn}(\mathbb{S}) \longrightarrow M_{mxn}(\mathbb{S})$$

$$(\lambda, \breve{A}) \longrightarrow \lambda \odot_{\mathbb{S}} \breve{A} = \lambda \odot_{\mathbb{S}} [\breve{a}_{st}] = [\lambda \cdot \breve{a}_{st}].$$

Due to $\lambda \in \mathbb{S}$ and $\breve{a}_{st} \in \mathbb{S}$ it can be written as $\lambda \cdot \breve{a}_{st} \in \mathbb{S}$ and $\lambda \odot_{\mathbb{S}} \breve{A} \in M_{mxn}(\mathbb{S})$. The scalar multiplication operation is scalar operation.

Theorem 2.2 Let $\breve{A}$, $\breve{B} \in M_{mxn}(\mathbb{S})$, $\check{C} \in M_{nxr}(\mathbb{S})$ and $\lambda \in \mathbb{S}$ be given. Then, the following properties are safisfied.

1. $\overline{\left(\overline{\breve{A}}\right)} = \breve{A}$,
2. $\bar{\lambda} \odot_{\mathbb{S}} \overline{\breve{A}} = \overline{\breve{A} \odot_{\mathbb{S}} \lambda}$,
3. $\overline{(\breve{A} \oplus \breve{B})} = \overline{\breve{A}} \oplus \overline{\breve{B}}$,
4. $\overline{(\breve{A} \otimes \check{C})} \neq \overline{\breve{A}} \otimes \overline{\check{C}}$.

Proof: (1), (2), (3) can be easily shown. Then, we will prove the condition of (4).

(4) $\overline{(\breve{A}\otimes\breve{C})} = A_0C_0 - \sum_{i=1}^{15} A_iC_i$
$-(A_0C_1 + A_1C_0 + A_2C_3 - A_3C_2 + A_4C_5 - A_5C_4 - A_6C_7 + A_7C_6 + A_8C_9$
$-A_9C_1 - A_{10}C_{11} + A_{11}C_{10} - A_{12}C_{13} - A_{13}C_9 - A_{14}C_{15} - A_{15}C_{14})e_1$
$\vdots$
$-(A_0C_{15} + A_1C_{14} - A_2C_{13} - A_3C_{12} + A_4C_{11} + A_5C_{10} - A_6C_9 + A_7C_8 - A_8C_7$
$+A_9C_6 - A_{10}C_5 - A_{11}C_4 + A_{12}C_3 + A_{13}C_2 - A_{14}C_1 + A_{15}c_0)e_{15}$,

and

$\bar{\bar{A}} \otimes \bar{\bar{C}} = A_0C_0 - \sum_{i=1}^{15} A_iC_i$
$-(A_0C_1 + A_1C_0 + A_2C_3 - A_3C_2 + A_4C_5 - A_5C_4 - A_6C_7 + A_7C_6 + A_8C_9$
$-A_9C_1 - A_{10}C_{11} + A_{11}C_{10} - A_{12}C_{13} - A_{13}C_9 - A_{14}C_{15} + A_{15}C_{14})e_1$
$\vdots$
$-(A_0C_{15} - A_1C_{14} - A_2C_{13} - A_3C_{12} + A_4C_{11} + A_5C_{10} - A_6C_9 + A_7C_8 - A_8C_7$
$+A_9C_6 - A_{10}C_5 - A_{11}C_4 + A_{12}C_3 + A_{13}C_2 - A_{14}C_1 + A_{15}c_0)e_{15}$.

Then, it is obtained that $\overline{(\breve{A}\otimes\breve{C})} \neq \bar{\bar{A}} \otimes \bar{\bar{C}}$.

Corollary 2.2 Let $\bar{\bar{A}} = A_0 - \sum_{i=1}^{15} A_ie_i \in M_n(\mathbb{S})$ be given. As a result of theorem 2.2, it can be written the following expression: $\left(\bar{\bar{A}}\right)^2 \neq \left(\bar{\bar{A}}^2\right)$.

Definition 2.8 Let $\breve{A} = [\breve{a}_{st}] \in M_{mxn}(\mathbb{S})$, $(1 \leq s \leq m, 1 \leq t \leq n)$ be sedenionic matrix. The transpose of sedenionic matrix of $\breve{A}$ is defined as $\breve{A}^t = [\breve{a}_{ts}] \in M_{nxm}(\mathbb{S})$.

Remark 2.6 The tranpose of sedenionic matrices can be written in terms of matrices with real, complex, quaternionic and octonionic coefficients, respectivly as follows

1. $\breve{A}^t = \sum_{i=0}^{15} {A_i}^t e_i$,
2. $\breve{A}^t = \sum_{i=1}^{8} {\hat{A}_i}^t e_{2i-2}$,
3. $\breve{A}^t = \sum_{i=1}^{4} {\tilde{A}_i}^t e_{4i-4}$,
4. $\breve{A}^t = {\tilde{\tilde{A}}_1}^t + {\tilde{\tilde{A}}_2}^t e_8$.

Theorem 2.3 Let $\breve{A}, \breve{B} \in M_{mxn}(\mathbb{S})$, $\breve{C} \in M_{nxr}(\mathbb{S})$ and $\lambda \in \mathbb{S}$ be given. The following properties are safisfied.

1. $\left(\breve{A}\oplus\breve{B}\right)^t = \breve{A}^t\oplus\breve{B}^t$,
2. $\left(\breve{A}^t\right)^t = \breve{A}$,
3. $\left(\lambda \odot_{\mathbb{S}} \breve{A}\right)^t = \lambda \odot_{\mathbb{S}} \breve{A}^t$,
4. $\left(\breve{A} \otimes \breve{C}\right)^t \neq \breve{C}^t \otimes \breve{A}^t$.

Proof: (1), (2) can be easily shown. Then, we will prove the last two conditions.

(3) $\left(\lambda \odot_{\mathbb{S}} \breve{A}\right)^t = (\lambda \odot_{\mathbb{S}} [\breve{a}_{st}])^t = [\lambda \cdot \breve{a}_{ts}] = \lambda \odot_{\mathbb{S}} [\breve{a}_{ts}] = \lambda \odot_{\mathbb{S}} \breve{A}^t$.

(4) $\left(\breve{A} \otimes \breve{C}\right)^t = \left(\left(\sum_{i=0}^{15} A_ie_i\right) \otimes \left(\sum_{i=0}^{15} C_ie_i\right)\right)^t$
$= {C_0}^t{A_0}^t - \sum_{i=0}^{15} \breve{C}^t\breve{A}^t$
$+\left({C_0}^t{A_1}^t + {C_1}^t{A_0}^t + {C_2}^t{A_3}^t - \cdots - {C_{15}}^t{A_{14}}^t\right)e_1$
$\vdots$
$+\left({C_0}^t{A_{15}}^t - {C_1}^t{A_{14}}^t - \ldots + {C_{15}}^t{A_0}^t\right)e_{15}$

and

$$\breve{C}^t \otimes \breve{A}^t = \left(\sum_{i=0}^{15} C_i{}^t e_i\right)\left(\sum_{i=0}^{15} A_i{}^t e_i\right)$$
$$= C_0{}^t A_0{}^t - \sum_{i=0}^{15} \breve{C}^t \breve{A}^t$$
$$+\left(C_0{}^t A_1{}^t + C_1{}^t A_0{}^t + C_2{}^t A_3{}^t - \cdots - C_{15}{}^t A_{14}{}^t\right) e_1$$
$$\vdots$$
$$+\left(C_0{}^t A_{15}{}^t + C_1{}^t A_{14}{}^t + \ldots + C_{15}{}^t A_0{}^t\right) e_{15}.$$

Based on obtained results, it can be written $\left(\breve{A} \otimes \breve{C}\right)^t \neq \breve{C}^t \otimes \breve{A}^t$.

Definition 2.9 Let $\breve{A} = [\breve{a}_{st}] \in M_{mxn}(\mathbb{S})$, $(1 \leq s \leq m, 1 \leq t \leq n)$ be given. Then, the conjugate traspose of $\breve{A}$ is define as $\overline{\left(\breve{A}\right)}^t = \overline{[\breve{a}_{st}]}^t \in M_{nxm}(\mathbb{S})$.

Remark 2.7 The conjugate tranpose of sedenionic matrices can be written in terms of matrices with real, complex, quaternionic and octonionic coefficients, respectively as follows

1. $\overline{\left(\breve{A}\right)}^t = A_0{}^t - \sum_{i=1}^{15} A_i{}^t e_i$,
2. $\overline{\left(\breve{A}\right)}^t = \widehat{A}_1{}^t - \sum_{i=2}^{8} \widehat{A}_i{}^t e_{2i-2}$,
3. $\overline{\left(\breve{A}\right)}^t = \widetilde{A}_1{}^t - \sum_{i=2}^{4} \widetilde{A}_i{}^t e_{4i-4}$,
4. $\overline{\left(\breve{A}\right)}^t = \widetilde{\widetilde{A}}_1{}^t - \widetilde{\widetilde{A}}_2{}^t e_8$.

Theorem 2.4 Let $\breve{A}$, $\breve{B} \in M_{mxn}(\mathbb{S})$, $\breve{C} \in M_{nxr}(\mathbb{S})$ and $\lambda \in \mathbb{S}$ be given. The following properties are safisfied.

1. $\left(\overline{\lambda \odot_{\mathbb{S}} \breve{A}}\right)^t = \left(\overline{\breve{A}}\right)^t \odot_{\mathbb{S}} \bar{\lambda}$,
2. $\left(\overline{\breve{A}}\right)^t = \overline{\left(\breve{A}^t\right)}$,
3. $\left(\overline{\breve{A}} \oplus \overline{\breve{B}}\right)^t = \left(\overline{\breve{A}}\right)^t \oplus \left(\overline{\breve{B}}\right)^t$,
4. $\left(\overline{\breve{A} \otimes \breve{C}}\right)^t \neq \left(\overline{\breve{C}}\right)^t \otimes \left(\overline{\breve{A}}\right)^t$.

Proof: (2), (3), (4) can be easily shown. Then, we will prove condition of (1).

(1) $\left(\overline{\lambda \odot_{\mathbb{S}} \breve{A}}\right)^t = \left(\overline{\breve{A}} \odot_{\mathbb{S}} \bar{\lambda}\right)^t = \left(\overline{\breve{A}}\right)^t \odot_{\mathbb{S}} \bar{\lambda}$.

Definition 2.10 Let $\breve{A} = [\breve{a}_{st}] = \sum_{i=0}^{15} A_i e_i \in M_n(\mathbb{S})$ be given. The sum of the elements of sedenionic square matrix $\breve{A}$ on principal diagonal is called the trace of matrix $\breve{A}$ and denoted by $tr\left(\breve{A}\right)$.

$$tr\left(\breve{A}\right) = \sum_{r=1}^{n} \breve{a}_{rr} = \sum_{r=1}^{n}\left(\sum_{i=1}^{15} \breve{a}_{rr}{}^i\right)$$
$$= \sum_{r=1}^{n}\left(\breve{a}_{rr}{}^0 + \breve{a}_{rr}{}^1 + \cdots + \breve{a}_{rr}{}^{15}\right)$$
$$= \sum_{r=1}^{n}\left(\breve{a}_{rr}{}^0\right) + \sum_{r=1}^{n}\left(\breve{a}_{rr}{}^1\right) + \cdots + \sum_{r=1}^{n}\left(\breve{a}_{rr}{}^{15}\right).$$

On the other hand $tr\left(\breve{A}\right)$ can be written in terms of matrices with real coefficient as:

$$tr\left(\breve{A}\right) = \sum_{i=0}^{15} tr(A_i) e_i = tr(A_0) + tr(A_1) e_1 + tr(A_2) e_2 + \cdots + tr(A_{15}) e_{15}.$$

Theorem 2.5 Let $\breve{A} = \sum_{i=0}^{15} A_i e_i$, $\breve{B} = \sum_{i=0}^{15} B_i e_i \in M_n(\mathbb{S})$ and $\lambda \in \mathbb{S}$ be given. The following properties of trace operation are safisfied.

1. $tr\left(\breve{A} \oplus \breve{B}\right) = tr\left(\breve{A}\right) + tr\left(\breve{B}\right)$,
2. $tr\left(\breve{A} \otimes \breve{B}\right) \neq tr\left(\breve{A}\right) \cdot tr\left(\breve{B}\right)$,
3. $tr\left(\breve{A} \otimes \breve{B}\right) \neq tr\left(\breve{B} \otimes \breve{A}\right)$,

4. $\mathrm{tr}(\breve{A} \odot_{\mathbb{S}} \lambda) \neq \mathrm{tr}(\breve{A}) \cdot \lambda$ or $\mathrm{tr}(\lambda \odot_{\mathbb{S}} \breve{A}) \neq \lambda \cdot \mathrm{tr}(\breve{A})$,
5. $\mathrm{tr}(\breve{A}^t) = \mathrm{tr}(\breve{A})$.

Proof: (1), (3), (4), (5) can be easily shown. Then, we will prove condition of (2).

(2) $$\begin{aligned}\mathrm{tr}(\breve{A} \otimes \breve{B}) = {} & \mathrm{tr}(A_0B_0) - \mathrm{tr}\left(\textstyle\sum_{i=1}^{15} A_iB_i\right) \\ & -\mathrm{tr}(A_0B_1 + A_1B_0 + A_2B_3 - A_3B_2 + A_4B_5 - A_5B_4 - A_6B_7 + A_7B_6 + A_8B_9 \\ & -A_9B_1 - A_{10}B_{11} + A_{11}B_{10} - A_{12}B_{13} - A_{13}B_9 - A_{14}B_{15} - A_{15}B_{14})e_1 \\ & \vdots \\ & -\mathrm{tr}(A_0B_{15} + A_1B_{14} - A_2B_{13} - A_3B_{12} + A_4B_{11} + A_5B_{10} - A_6B_9 + A_7B_8 - A_8B_7 \\ & +A_9B_6 - A_{10}B_5 - A_{11}B_4 + A_{12}B_3 + A_{13}B_2 - A_{14}B_1 + A_{15}B_0)e_{15}\end{aligned}$$

and

$$\begin{aligned}\mathrm{tr}(\breve{A}) \cdot \mathrm{tr}(\breve{B}) = {} & \mathrm{tr}(A_0B_0) - \mathrm{tr}\left(\textstyle\sum_{i=1}^{15} A_iB_i\right) \\ & -\mathrm{tr}(A_0B_1 + A_1B_0 + A_2B_3 - A_3B_2 + A_4B_5 - A_5B_4 - A_6B_7 + A_7B_6 + A_8B_9 \\ & -A_9B_1 - A_{10}B_{11} + A_{11}B_{10} - A_{12}B_{13} - A_{13}B_9 - A_{14}B_{15} - A_{15}B_{14})e_1 \\ & \vdots \\ & -\mathrm{tr}(A_0B_{15} + A_1B_{14} - A_2B_{13} - A_3B_{12} + A_4B_{11} + A_5B_{10} - A_6B_9 + A_7B_8 - A_8B_7 \\ & +A_9B_6 - A_{10}B_5 - A_{11}B_4 + A_{12}B_3 + A_{13}B_2 - A_{14}B_1 + A_{15}B_0)e_{15}.\end{aligned}$$

The properties of trace of real matrix is $\mathrm{tr}(A \cdot B) \neq \mathrm{tr}(A) \cdot \mathrm{tr}(B)$, so the results are not the same and it is obtained $\mathrm{tr}(\breve{A} \otimes \breve{B}) \neq \mathrm{tr}(\breve{A}) \cdot \mathrm{tr}(\breve{B})$.

Definition 2.11 Let $\breve{A} = [\breve{a}_{st}] \in M_{mxn}(\mathbb{S})$ and $\lambda \in \mathbb{R}$ be given. The multiplication of a real number and a sedenionic matrix is defined as follows:

$$\odot\colon \mathbb{R} \times M_{mxn}(\mathbb{S}) \longrightarrow M_{mxn}(\mathbb{S})$$

$$(\lambda, \breve{A}) \longrightarrow \lambda \odot \breve{A} = \lambda \odot [\breve{a}_{st}] = [\lambda \cdot \breve{a}_{st}].$$

Due to $\lambda \in \mathbb{R}$ and $\breve{a}_{st} \in \mathbb{S}$ it can be written as $\lambda \cdot \breve{a}_{st} \in \mathbb{S}$ and $\lambda \odot \breve{A} \in M_{mxn}(\mathbb{S})$. The scalar multiplication operation is scalar operation.

Theorem 2.6 Let $\breve{A}, \breve{B} \in M_{mxn}(\mathbb{S})$ and $\lambda_1, \lambda_2 \in \mathbb{R}$ be given. Following properties of the scalar multiplication are safisfied.

1. $\lambda_1 \odot (\breve{A} \oplus \breve{B}) = \lambda_1 \odot \breve{A} \oplus \lambda_1 \odot \breve{B}$,
2. $(\lambda_1 + \lambda_2) \odot \breve{A} = \lambda_1 \odot \breve{A} \oplus \lambda_2 \odot \breve{A}$,
3. $(\lambda_1\lambda_2) \odot \breve{A} = \lambda_1 \odot (\lambda_2 \odot \breve{A})$,
4. 1_λ is the unit element of $\mathbb{R}$ field, $1_\lambda \odot \breve{A} = \breve{A}$.

Proof: (1), (2), (4) can be easily shown. Then, we will prove condition of (3).

(3) $$\begin{aligned}(\lambda_1\lambda_2) \odot \breve{A} &= [(\lambda_1\lambda_2) \cdot \breve{a}_{st}] \\ &= [\lambda_1(\lambda_2 \cdot \breve{a}_{st})] \\ &= \lambda_1 \odot [(\lambda_2 \cdot \breve{a}_{st})] \\ &= \lambda_1 \odot (\lambda_2 \odot [(\breve{a}_{st})]) \\ &= \lambda_1 \odot (\lambda_2 \odot \breve{A}).\end{aligned}$$

Corollary 2.3 Due to $(M_{mxn}(\mathbb{S}), \oplus)$ being an Abelian group and from theorem 2.6, it is concluded that $\{M_{mxn}(\mathbb{S}), \oplus, \mathbb{R}+, \cdot, \odot\}$ is vector space.

Definition 2.12 Let $\breve{A} = [\breve{a}_{st}] \in M_{mxn}(\mathbb{S})$, where $\breve{a}_{st} = \sum_{i=0}^{15} a_{st}^{i} e_i \in \mathbb{S}$ be given.

$$\breve{A} = \begin{bmatrix} \breve{a}_{11} & \breve{a}_{12} & \dots & \breve{a}_{1n} \\ \breve{a}_{21} & \breve{a}_{22} & \dots & \breve{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \breve{a}_{m1} & \breve{a}_{m2} & \dots & \breve{a}_{mn} \end{bmatrix}_{mxn}$$ is written more explicitly as follows:

$$\begin{aligned} \breve{A} = {} & a_{11}^{0}\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} + a_{11}^{1}\begin{bmatrix} e_1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} + \dots + a_{11}^{15}\begin{bmatrix} e_{15} & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} \\ & + \dots + a_{1n}^{0}\begin{bmatrix} 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} + a_{1n}^{1}\begin{bmatrix} 0 & 0 & \dots & e_1 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} + \dots + a_{1n}^{15}\begin{bmatrix} 0 & 0 & \dots & e_{15} \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} \\ & + \dots + a_{m1}^{0}\begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \dots & 0 \end{bmatrix} + a_{m1}^{1}\begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ e_1 & 0 & \dots & 0 \end{bmatrix} + \dots + a_{m1}^{15}\begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ e_{15} & 0 & \dots & 0 \end{bmatrix} \\ & + \dots + a_{mn}^{0}\begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix} + a_{mn}^{1}\begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e_1 \end{bmatrix} + \dots + a_{mn}^{15}\begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e_{15} \end{bmatrix}. \end{aligned}$$

Considering this expansion,

$$\begin{aligned} M_{mxn}(\mathbb{S}) = Sp\{S_1\} = Sp\Bigg\{ & \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix}, \begin{bmatrix} e_1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix}, \dots, \begin{bmatrix} e_{15} & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} \\ & , \dots, \begin{bmatrix} 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & \dots & e_1 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix}, \dots, \begin{bmatrix} 0 & 0 & \dots & e_{15} \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} \\ & , \dots, \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \dots & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ e_1 & 0 & \dots & 0 \end{bmatrix}, \dots, \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ e_{15} & 0 & \dots & 0 \end{bmatrix} \\ & , \dots, \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e_1 \end{bmatrix}, \dots, \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e_{15} \end{bmatrix} \Bigg\} \end{aligned}$$

thus, S_1 is obtained. If S_1 system linearly independent, it is obtained that S_1 is the base of $M_{mxn}(\mathbb{S})$ vector space over field $\mathbb{R}$. Let q_{11}, q_{12},…, q_{1n}, q_{21}, q_{22},…, q_{2n}, q_{m1}, q_{m2},…, q_{mn}, r_{11}, r_{12},…, r_{1n}, r_{21}, r_{22},…, r_{2n}, r_{m1}, r_{m2},…, r_{mn}, t_{11}, t_{12},…, t_{1n}, t_{21}, t_{22},…, t_{2n}, t_{m1}, t_{m2},…, $t_{mn} \in \mathbb{R}$ and

$$\begin{aligned} & q_{11}\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} + r_{11}\begin{bmatrix} e_1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} + \dots + t_{11}\begin{bmatrix} e_{15} & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} \\ & + \dots + q_{1n}\begin{bmatrix} 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} + r_{1n}\begin{bmatrix} 0 & 0 & \dots & e_1 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} + \dots + t_{1n}\begin{bmatrix} 0 & 0 & \dots & e_{15} \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} \\ & + \dots + q_{m1}\begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \dots & 0 \end{bmatrix} + r_{m1}\begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ e_1 & 0 & \dots & 0 \end{bmatrix} + \dots + t_{m1}\begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ e_{15} & 0 & \dots & 0 \end{bmatrix} \end{aligned}$$

$$+\cdots+q_{mn}\begin{bmatrix}0&0&\dots&0\\0&0&\dots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\dots&1\end{bmatrix}+r_{mn}\begin{bmatrix}0&0&\dots&0\\0&0&\dots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\dots&e_1\end{bmatrix}+\cdots+t_{mn}\begin{bmatrix}0&0&\dots&0\\0&0&\dots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\dots&e_{15}\end{bmatrix}=\breve{0}_{mxn}\text{ be given.}$$

In that case, $q_{11}, q_{12},\dots, q_{1n}, q_{21}, q_{22},\dots, q_{2n}, q_{m1}, q_{m2},\dots, q_{mn}, r_{11}, r_{12},\dots, r_{1n}, r_{21}, r_{22},\dots, r_{2n}, r_{m1}, r_{m2},\dots, r_{mn}, t_{11}, t_{12},\dots, t_{1n}, t_{21}, t_{22},\dots, t_{2n}, t_{m1}, t_{m2},\dots, t_{mn} = 0$ is obtained. This result tells us S_1 system is linearly independent. Due to $M_{mxn}(\mathbb{S}) = Sp\{S_1\}$, S_1 matrix system is $M_{mxn}(\mathbb{S})$ vector space's standart base and it is obtained $\dim(M_{mxn}(\mathbb{S})) = 16mn$.

Remark 2.8 The relationship between the number sets such $\mathbb{R} \subset \mathbb{C} \subset \mathbb{H} \subset \mathbb{O} \subset \mathbb{S} \subset \cdots$ is known from lemma 2.1. Similar to definition 2.11, it can be define the scalar multiplication operation involving complex numbers or quaternionic numbers with sedenionic matrices as follows:

Let $\breve{A} = [\breve{a}_{st}] \in M_{mxn}(\mathbb{S})$ and $\lambda_1, \lambda_2 \in \mathbb{C}, \lambda_3, \lambda_4 \in \mathbb{H}$ be given.

i. The multiplication of a complex number and a sedenionic matrix is defined as follow:

$\odot_{\mathbb{C}}: \mathbb{C} \times M_{mxn}(\mathbb{S}) \longrightarrow M_{mxn}(\mathbb{S})$

$$(\lambda_1, \breve{A}) \longrightarrow \lambda_1 \odot_{\mathbb{C}} \breve{A} = \lambda_1 \odot_{\mathbb{C}} [\breve{a}_{st}] = [\lambda_1 \cdot \breve{a}_{st}].$$

ii. The multiplication of a quaternion and a sedenionic matrix is defined as follow:

$\odot_{\mathbb{H}}: \mathbb{H} \times M_{mxn}(\mathbb{S}) \longrightarrow M_{mxn}(\mathbb{S})$

$$(\lambda_3, \breve{A}) \longrightarrow \lambda_3 \odot_{\mathbb{H}} \breve{A} = \lambda_3 \odot_{\mathbb{H}} [\breve{a}_{st}] = [\lambda_3 \cdot \breve{a}_{st}].$$

Considering these two scalar operations and given that sedenions are non-commutative, property number (4) in theorem 2.6 can be expressed as follows for complex numbers and quaternions, respectively

1. $(\lambda_1\lambda_2) \odot_{\mathbb{C}} \breve{A} \neq \lambda_1(\lambda_2 \odot_{\mathbb{C}} \breve{A})$,
2. $(\lambda_3\lambda_4) \odot_{\mathbb{H}} \breve{A} \neq \lambda_3(\lambda_4 \odot_{\mathbb{H}} \breve{A})$.

Corollary 2.4 $\{M_{mxn}(\mathbb{S}), \oplus, \mathbb{C}, +, \cdot, \odot_{\mathbb{C}}\}$ is not left (right) vector space.

Corollary 2.5 $\{M_{mxn}(\mathbb{S}), \oplus, \mathbb{H}, +, \cdot, \odot_{\mathbb{H}}\}$ is not left (right) module.

4. Discussion

Taking into account the evolving technology and current conditions, it has become inevitable to explore new areas mathematically and create various alternatives. In the 21st century, the application areas of sedenions are increasingly expanding. Its usage is widespread in fields such as robotic movements, quantum mechanics and prediction of time series. In this study unlike the known matrix algebra, matrices with sedenionic coefficients have been defined and their algebraic properties have been investigated and contributed to the literature. Considering the applications of standard matrix algebra, it can be stated that this study opens a new area in this field.

5. Conclusions

The exploration of sedenions and their algebraic operations in the first section are given. Sedenionic algebra properties are used for on defining sedenionic coefficient matrices. This definition extends to real, complex, quaternionic and octonionic coefficient sedenionic matrices, followed by the comprehensive definition of various essential operations tailored for each type of sedenionic matrix. Addition, multiplication, conjugate, transpose, conjugate transpose, trace operation of sedenionic matrices are defined for each combinations. The vector space of $M_{mxn}(\mathbb{S})$ over $\mathbb{R}$ field and $\dim(M_{mxn}(\mathbb{S})) = 16mn$ is obtained.

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Author contribution

All authors contributed to all parts of the article.

Declaration of ethical code

The authors of this manuscript declare that the materials and methods used in study to do not require ethical committee approval and/or legal-specific permission.

Conflicts of interest

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this manuscript.

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