BITLIS EREN UNIVERSITY
JOURNAL OF SCIENCE AND TECHNOLOGY

E-ISSN: 2146-7706

THE PRACTICAL INTEGRATION OF LINEAR ALGEBRA IN GENETICS, CUBIC SPLINE INTERPOLATION, ELECTRIC CIRCUITS AND TRAFFIC FLOW

Khadeejah James Audu [1,*], Yak Chiben Elisha [1], Yusuph Amuda Yahaya [2], Sikirulai Abolaji Akande[1]

[1] *Department of Mathematics, Federal University of Technology, Minna, Nigeria,* *k.james@futminna.edu.ng, chiben.m1804731@st.futminna.edu.ng, siqlam@yahoo.com*
[2] *Department of Mathematics, Pen Resource University, Gombe, Nigeria,yusuph.yahaya@pru.edu.ng*
* *Corressponding author*

KEYWORDS

Linear algebra
Genetics
Cubic spline
Traffic flow
Electric circuits

ARTICLE INFO

Research Article
DOI: 10.17678/beuscitech.1418696

Received	12 January 2024
Accepted	13 May 2024
Year	2024
Volume	14
Issue	1
Pages	23-42

ABSTRACT

A fundamental mathematical field with many applications in science and engineering is linear algebra. This paper investigates the various applications of linear algebra in the fields of traffic flow analysis, electric circuits, cubic spline interpolation, and genetics. This research delves into individual applications while emphasizing cross-disciplinary insights, fostering innovative solutions through the convergence of genetics, cubic spline interpolation, circuits, and traffic flow analysis.

The research employs specific methodologies in each application area to demonstrate the practical integration of linear algebra in genetics, cubic spline interpolation, electric circuits, and traffic flow analysis.

In genetics, linear algebra techniques are utilized to represent genetic data using matrices, analyze genotype distributions across generations, and identify genotype-phenotype associations. For cubic spline interpolation, linear algebra is employed to construct smooth interpolating curves, involving the derivation of equations for spline functions and the determination of coefficients using boundary conditions and continuity requirements. In electric circuit analysis, linear algebra is crucial for modeling circuit elements, formulating systems of linear equations based on Kirchhoff's laws, and solving for voltage and current distributions in circuits. In traffic flow analysis, linear algebra techniques are used to represent traffic movement in networks, formulate systems of linear equations representing traffic flow dynamics, and solve for traffic flow solutions to optimize transportation networks.

By addressing contemporary challenges, emerging research frontiers, and future trajectories at the intersection of linear algebra and diverse domains, this study underscores the profound impact of mathematical tools in advancing understanding and resolving complex real-world problems across multiple fields.

1 INTRODUCTION

This research work delves into the versatile applications of linear algebra in genetics, cubic spline interpolation, electric circuits, and traffic flow analysis. As a foundational mathematical discipline, linear algebra serves as a unifying framework transcending disciplinary boundaries [9]. It plays a pivotal role in genetic data analysis, employing techniques like eigenvalues eigenvectors and diagonalization of matrix [8]. In cubic spline interpolation, linear algebra forms the bedrock, influencing applications in image processing, computer graphics, and motion analysis as demonstrated [10].

This study extends its focus to electric circuits, where linear algebra simplifies simple circuit analysis, fostering efficient design and optimization [2]. Additionally, linear algebra contributes significantly to modelling and analysing traffic flow systems, offering insights into network behaviour, optimization, and control. This research unravels the interconnected contributions of linear algebra across these diverse domains, emphasizing its cross-disciplinary significance and potential to revolutionize problem-solving in genetics, cubic spline interpolation, circuits, and traffic flow analysis [5].

The research identifies a critical gap in existing studies that have individually explored the application of linear algebra in genetics, cubic spline interpolation, electric circuits, and traffic flow. The lack of comprehensive research addressing the synergies and cross-disciplinary implications of employing linear algebra across these diverse domains constitutes a significant research gap. The study aims to bridge this gap by systematically applying linear algebraic principles, offering a unified perspective on the interconnectedness of mathematical principles across these scientific realms. The motivation behind the study lies in the potential to discover novel connections, optimize mathematical models, and improve solutions within each domain, contributing to advancements and fostering interdisciplinary collaboration. The novelty of the research lies in its integrative approach, providing a fresh perspective on problem-solving methodologies and offering new avenues for research and practical applications. Overall, the study is expected to contribute significantly to knowledge by uncovering hidden patterns and relationships, fostering a nuanced understanding of underlying structures.

The literature presents a multifaceted exploration of various research studies. In one instance, [5] underscores the significance of a strategic approach to modeling electrical power systems within communication networks, highlighting diverse methods such as experimental, software-based, and analytical modeling. [6] employs the cubic spline interpolation method to effectively interpolate original noise data, ensuring the seamless connection of noise data for a detailed description and recognition of characteristics within the output layer. In a different context, [1] introduces a cubic spline interpolation algorithm to categorize emotions in speech using curve-fitting techniques, utilizing datasets like Ryerson Audio-Visual Berlin (Emo-DB), Surrey Audio-Visual Expressed Emotion (SAVEE), and Database of Emotional Speech and Song (RAVDESS). Furthermore, [7] delves into the significance of linear algebra methods in solving systems of linear equations, exploring techniques such as scalar augmentation, determinants, and vector space. Each study contributes unique insights and methodologies to their respective fields of research.

The paper presents a comprehensive exploration of the practical integration of linear algebra across genetics, cubic spline interpolation, electric circuits, and traffic flow analysis. In genetics, the research provides a systematic approach for analyzing genotype distributions and identifying genetic markers associated with diseases. For cubic spline interpolation, it offers a methodical process for constructing accurate interpolating curves, enhancing applications in computer graphics and data analysis. In electric circuits, the study advances understanding by demonstrating how linear algebra facilitates the analysis and design of circuits, crucial for electronics and power systems. Moreover, in traffic flow analysis, the research elucidates how linear algebra techniques optimize transportation networks, leading to insights into traffic dynamics and urban planning. Overall, the paper's findings underscore the profound impact of linear algebra in advancing understanding and resolving complex real-world problems across diverse scientific and engineering disciplines, offering valuable contributions to each field.

2 METHODOLOGY

2.1 Genetic Data Representation Using Matrices

The genotype distribution of a particular trait in a population in the *nth* generation can be represented by a genotype vector;

$$P^{n} = \begin{bmatrix} f_n \\ g_n \\ h_n \end{bmatrix} \tag{1}$$

where f_n, g_n and h_n are the portion with population AA, Aa and aa respectively in the *nth* generation. Since the genotype distribution changes over time, the succession of genotype distributions from one generation to the next will be represented in the form;

$$P^{n} = TP^{n-1} \tag{2}$$

for a suitable matrix *T* as *n* = 1,2,3,… an explicit description of P^n whose formula for each P^n does not depend on T or on the preceding terms in the sequence other than the initial term P^0 (the initial genotype distribution in the population),will given as;

$$P^{n} = TP^{n-1} = T^2P^{n-2} = \ldots = T^nP^0 \tag{3}$$

Therefore, the explicit expression for T^n will be obtained by diagonalizing *T*. Hence,

$$T^n = VD^nV^{-1} \tag{4}$$

where *V*= *Eigenvector* and *D*=*Diagonal Matrix of T*

Solving for V, D^n and V^{-1}, the obtained outcome becomes

$$D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}, \quad V = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } V^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

Substituting (5) in (4), it gives as shown in (Kizilaslan & Acer, 2023);

$$T^n = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \tag{6}$$

Putting (6) into (3), the result is

$$P^n = \begin{bmatrix} f_n \\ g_n \\ h_n \end{bmatrix} = \begin{bmatrix} 1 & 1-\left(\frac{1}{2}\right)^n g_0 & 1-\left(\frac{1}{2}\right)^{n-1} h_0 \\ 0 & \left(\frac{1}{2}\right)^n g_0 & \left(\frac{1}{2}\right)^{n-1} h_0 \\ 0 & 0 & 0 \end{bmatrix} \quad for\, n = 1, 2, 3, \ldots \tag{7}$$

(7) is the expression for the distribution of the three possible genotypes $\underline{AA}, \underline{Aa}$ and $\underline{aa}$ in the population after any number of generations.

2.1.1 Genetics problem algorithm

Step 1: Define the genetics problem and identify the aspect it relates to.
Step 2: Create a matrix or set of linear equations considering the relevant genetic variables and principles.
Step 3: Collect the necessary genetic information, including genotype/phenotype frequencies and known inheritance patterns.
Step 4: Solve the matrix equation using linear algebraic techniques.
Step 5: Evaluate the outcomes in relation to the genetics puzzle and verify the solution by comparing it with empirical or experimental data.
It is noteworthy that in the Genetics Problem Algorithm (Step 2), the selection of genetic variables and principles is guided by fundamental genetic principles and the specific traits under investigation. This includes considering variables relevant to the traits being studied and applying principles such as Mendelian inheritance and allele frequencies. The rationale for these choices ensures that the resulting matrices accurately represent the genetic dynamics of the problem at hand. By aligning with established genetic principles, the algorithm facilitates the creation of matrices that effectively capture the genetic variation within populations across generations.

2.2 Cubic Spline Representation Using Linear Algebra

The general form of a cubic spline is given as;

$$\underline{S}(x) = \begin{cases} \underline{S}_0(x), x_0 \le x \le x_1 \\ \underline{S}_1(x), x_1 \le x \le x_2 \\ \vdots \\ \underline{S}_{n-1}(x), x_{n-1} \le x \le x_n \end{cases} \tag{8}$$

where $\underline{S}_1(x),\ \underline{S}_2(x),\ldots,\ \underline{S}_{n-1}(x)$ are cubic polynomials. For convenience, (6) is written in the form;

$$\begin{aligned}
&\underline{S}_0(x) = e_0 + f_0(x-x_0) + g_0(x-x_0)^2 + h_0(x-x_0)^3,\ x_0 \le x \le x_1 \\
&\underline{S}_1(x) = e_1 + f_1(x-x_1) + g_1(x-x_1)^2 + h_1(x-x_1)^3,\ x_1 \le x \le x_2 \\
&\vdots \\
&\underline{S}_{n-1}(x) = e_{n-1} + f_{n-1}(x-x_{n-1}) + g_{n-1}(x-x_{n-1})^2 + h_{n-1}(x-x_{n-1})^3,\ x_{n-1} \le x \le x_n
\end{aligned} \tag{9}$$

The e_k's, f_k's, g_k's and h_k's constitute a total of $4n-4$ coefficients that must be determined to specify $\underline{S}(x)$ completely as demonstrated in [1]. If these coefficients are chosen so that $\underline{S}(x)$ interpolates the n specified points in the plane $\underline{S}(x),\ \underline{S}'(x)$ and $\underline{S}''(x)$ are continuous, then the resulting interpolating curve is called a Cubic Spline [3-4]. A cubic spline has four condition and two boundary condition which are used to get the equations required. These conditions are stated below;

1. $\underline{S}(x)$ interpolates the points $(x_k, y_k),\ \mathrm{k} = 0,1,\ldots,n$.
2. $\underline{S}(x)$ is continuous on $[x_0, x_n]$.
3. $\underline{S}'(x)$ is continuous on $[x_0, x_n]$
4. $\underline{S}''(x)$ is continuous on $[x_0, x_n]$.

The boundary conditions are;

1. Free/Natural Cubic Spline; $\underline{S}_0''(x_0) = 0$, $\underline{S}_{n-1}''(x_n) = 0$
2. Clamed/Complete Cubic Spline; $\underline{S}_0'(x_0) = f'(x_0)$, $\underline{S}_{n-1}'(x_n) = f'(x_n)$

Applying the four cubic spline conditions on (9) and the boundary conditions, the following equations are obtained;

$$\begin{aligned} &e_0 = y_0 \\ &e_1 = y_1 \\ &\vdots \\ &e_{n-1} = y_{n-1} \end{aligned} \tag{10}$$

$$e_{n-1} + f_{n-1}(x - x_{n-1}) + g_{n-1}(x - x_{n-1})^2 + h_{n-1}(x - x_{n-1})^3 = y_n \tag{11}$$

$$\begin{aligned} &e_0 + f_0(x - x_{n-1}) + g_0(x - x_{n-1})^2 + h_0(x - x_{n-1})^3 = y_1 \\ &e_1 + f_1(x - x_{n-1}) + g_1(x - x_{n-1})^2 + h_1(x - x_{n-1})^3 = y_2 \\ &\vdots \\ &e_{n-1} + f_{n-1}(x - x_{n-1}) + g_{n-1}(x - x_{n-1})^2 + h_{n-1}(x - x_{n-1})^3 = y_n \end{aligned} \tag{12}$$

$$\begin{aligned} &f_0 + 2g_0(x - x_{n-1}) + 3h_0(x - x_{n-1})^2 = f_1 \\ &f_1 + 2g_1(x - x_{n-1}) + 3h_1(x - x_{n-1})^2 = f_2 \\ &\vdots \\ &f_{n-1} + 2g_{n-1}(x - x_{n-1}) + 3h_{n-1}(x - x_{n-1})^2 = f_n \end{aligned} \tag{13}$$

$$\begin{aligned} &2g_0 + 6h_0(x - x_{n-1}) = 2g_1 \\ &2g_0 + 6h_0(x - x_{n-1}) = 2g_2 \\ &\vdots \\ &2g_{n-1} + 6h_{n-1}(x - x_{n-1}) = 2g_n \end{aligned} \tag{14}$$

The equations (10), (11), (12), (13), (14) and either the natural cubic spline boundary condition or the clamped cubic spline boundary condition are utilized to constitute a linear system which are then solved by Gauss-Jordan elimination method is used to get the values of the constants in (9) [6].

2.2.1 Cubic spline interpolation algorithm

Step 1: Identify data points for cubic spline interpolation.
Step 2: Calculate the number of intervals between data points.
Step 3: Formulate a system of linear equations.
Step 4: Express the equations as ($Ty = b$).
Step 5: Solve the matrix equation to obtain coefficients for the cubic spline function.

In the Cubic Spline Interpolation Algorithm (Step 1), identifying data points for interpolation involves a thoughtful selection process aimed at capturing the behavior of the function or dataset accurately. Firstly, data points should ideally be uniformly distributed across the range to ensure comprehensive coverage of the function's behavior. This entails including critical points such as extrema and inflection points, which offer valuable insights into the curve's behavior. However, it's crucial to strike a balance between data sparsity and richness; too few points may lead to inaccuracies, while an excessive number could result in overfitting. Additionally, the quality of data points must be ensured, avoiding inaccuracies or outliers that could compromise the reliability of the interpolation. Context is also vital; data points should align with the specific application's requirements and provide meaningful insights into the problem at hand. By adhering to these guidelines, the selection of suitable data points lays the foundation for an accurate and meaningful interpolation process, facilitating a robust analysis of the function or dataset.

2.3 Matrix Representation of Circuits

Linear algebra stands as a crucial tool in the arsenal of electricians working with electrical circuits. Among the fundamental principles applied in this domain is the use of matrices, particularly in constructing systems of linear equations based on Kirchhoff's voltage law, Kirchhoff's current law, and Ohm's law for simple electrical circuits. While a single permutation suffices to solve equations in straightforward circuits, more intricate circuits demand node analysis. To elucidate, consider the electrical circuit depicted in Figure 1 as an example.

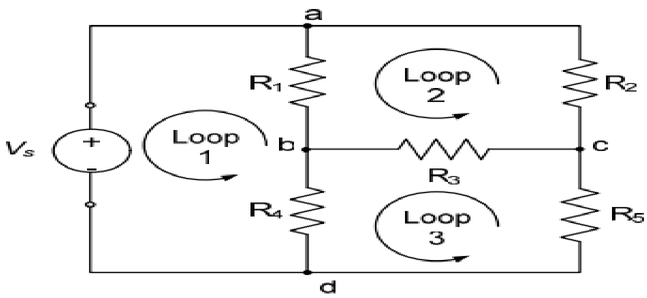

Figure 1. *Simple Electric Circuit*

Central Concept: Kirchhoff's Voltage Law dictates the flow of current, asserting that the algebraic sum of voltages or voltage drops along any closed path within a network, oriented in a single direction, equals zero. This principle serves as the foundational concept for establishing the system of equations. The resulting system of linear equations follows a specific form due to this fundamental law.

$$RI = V \tag{15}$$

where R= Resistance (Ohms), I= Current (Ampere) and V= Voltage (Volts). In this scenario, equation (15) signifies the fundamental principle known as Ohm's Law, which relates the current (I) flowing through a resistor to its resistance (R) and the resulting voltage drop (V) across it. Ohm's Law serves as a cornerstone in electrical engineering, establishing a direct relationship between a circuit component's resistance, the current passing through it, and the voltage across it. By applying Ohm's Law, we can predict the behavior of current and voltage in response to variations in resistance or applied voltage, crucial for comprehending and engineering electric circuits. Employing Kirchhoff's voltage law on each loop, the following equations will be constituted;

Loop 1: $I_1R_4 + I_1R_1 - I_2R_1 - I_3R_4 = -V_s$

$$\left(R_1 + R_4\right)I_1 - I_2R_1 - I_3R_4 = -V_s \tag{16}$$

Loop 2: $-I_1R_1 + I_2R_1 + I_2R_2 + I_2R_3 + I_3R_3 = 0$

$$-I_1R_1 + \left(R_1 + R_2 + R_3\right)I_2 + I_3R_3 = 0 \tag{17}$$

Loop 3: $-I_1R_4 + I_2R_3 + I_3R_3 + I_3R_4 + I_3R_5 = 0$

$$-I_1R_4 + I_2R_3 + \left(R_3 + R_4 + R_5\right)I_3 = 0 \tag{18}$$

Equations (16), (17), and (18) undergo transformation into a linear system, yielding:

$$\begin{bmatrix} \left(R_1 + R_4\right) & -R_1 & -R_4 \\ -R_1 & \left(R_1 + R_2 + R_3\right) & R_3 \\ -R_1 & R_3 & \left(R_3 + R_4 + R_5\right) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -V_s \\ 0 \\ 0 \end{bmatrix}$$

The augmented matrix of the linear system is obtained as;

$$\left[\begin{array}{ccc|c} \left(R_1 + R_4\right) & -R_1 & -R_4 & -V_s \\ -R_1 & \left(R_1 + R_2 + R_3\right) & R_3 & 0 \\ -R_1 & R_3 & \left(R_3 + R_4 + R_5\right) & 0 \end{array}\right]$$

The Gauss-Jordan elimination method is utilized to achieve the reduced-row echelon form of the matrix, which is subsequently employed to obtain the solution for the circuit, as exemplified in [7].

2.3.1 Electric circuit algorithm

Step 1: Describe the electric circuit components and connections.
Step 2: Assign variables to unknown quantities in the circuit.
Step 3: Apply Kirchhoff's laws to formulate equations.
Step 4: Create a system of linear equations representing relationships.
Step 5: Solve the matrix equation using linear algebra techniques to analyze voltages and currents.

In the Electric Circuit Algorithm (Step 1), understanding Kirchhoff's laws is pivotal for analyzing electrical circuits. Kirchhoff's Voltage Law (KVL) dictates that the sum of voltage changes around any closed loop in a circuit equals zero, providing insights into voltage distribution. Similarly, Kirchhoff's Current Law (KCL) states that the total current entering a junction equals the total current leaving, facilitating the analysis of current flow. These laws serve as foundational principles, guiding engineers in formulating equations that represent circuit behavior. By adhering to Kirchhoff's laws, analysts can systematically analyze complex circuits, predict their behavior, and solve for circuit variables using linear algebra techniques. This understanding is crucial for designing, troubleshooting, and optimizing electrical circuits effectively.

2.4 Network Representation of Traffic System

The movement of traffic along the branches is examined to derive systems of linear equations. To establish these equations, an assumption is made that the traffic entering an intersection must exit that intersection, ensuring a balance where the flow into and out of each node is equal. Additionally, it is assumed that the total traffic entering the network equals the total traffic leaving the network, emphasizing the importance of a well-balanced network. To gain a deeper understanding of traffic flow dynamics, Figure 2 is analyzed.

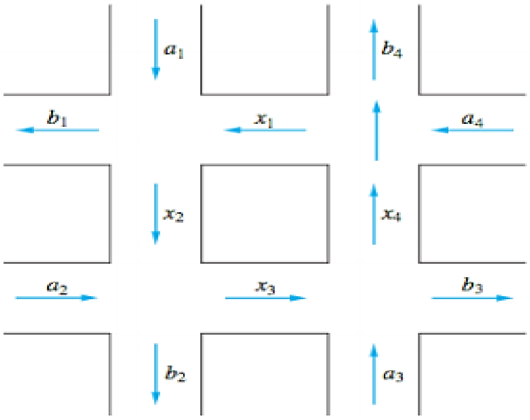

Figure 2. *A Traffic Flow Pattern*

The system of linear equations will have the form; $Flow\ In = Flow\ Out$

At the 1^{st} $Node$; $Flow\ In = Flow\ Out$

$$a_1 + x_1 = b_1 + x_2$$

$$x_1 - x_2 = b_1 - a_1 \tag{19}$$

At the 2^{nd} $Node$; $Flow\ In = Flow\ Out$

$$x_2 + a_2 = x_3 + b_2$$

$$x_2 - x_3 = b_2 - a_2 \tag{20}$$

At the 3^{rd} $Node$; $Flow\ In = Flow\ Out$

$$x_3 + a_3 = x_4 + b_3$$

$$x_3 - x_4 = b_3 - a_3 \tag{21}$$

At the 4^{th} $Node$; $Flow\ In = Flow\ Out$

$$x_4 + a_4 = x_1 + b_4$$

$$x_4 - x_1 = b_4 - a_4 \tag{22}$$

Using (3.40), (3.41), (3.42) and (3.43) to generate a linear system in the form *Ax*=*b*;

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 - a_1 \\ b_2 - a_2 \\ b_3 - a_3 \\ b_4 - a_4 \end{bmatrix}$$

The augmented matrix of the linear system is represented as;

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & b_1 - a_1 \\ 0 & 1 & -1 & 0 & b_2 - a_2 \\ 0 & 0 & 1 & -1 & b_3 - a_3 \\ -1 & 0 & 0 & 1 & b_3 - a_3 \end{array}\right]$$

Utilizing the Gauss-Jordan elimination technique, the reduced-row echelon form of the matrix will be obtained, allowing for the solution of the traffic flow problem.

2.4.1 Traffic flow algorithm

Step 1: Define the traffic flow problem.
Step 2: Gather relevant data on the road network.
Step 3: Formulate the traffic flow problem mathematically.
Step 4: Solve the system of equations using linear algebra techniques.
Step 5: Interpret the mathematical solution to gain insights into traffic flow dynamics.

In the Traffic Flow Algorithm (Step 1), understanding the importance of a balanced network is crucial for effectively modeling traffic dynamics. A balanced network ensures that the flow of traffic into and out of each intersection or node is equal, maintaining equilibrium and efficient traffic movement throughout the network. For example, in a well-balanced network, the total number of vehicles entering an intersection matches the total number exiting, preventing congestion or bottlenecks. This balance is essential for optimizing traffic flow, minimizing delays, and ensuring smooth transportation operations. By considering the significance of network balance, analysts can accurately model traffic flow dynamics, formulate systems of linear equations, and devise effective strategies for traffic management and urban planning. Therefore, ensuring a balanced network is a fundamental aspect of the Traffic Flow Algorithm, facilitating the accurate analysis and optimization of transportation systems.

2.5 Numerical Methods for Linear Systems

Linear algebraic methods, notably Gaussian Elimination and Gauss-Jordan Elimination, play a crucial role in efficiently and accurately finding solutions to problems in various scientific, engineering, and mathematical fields. Gaussian

Elimination systematically transforms linear systems into upper triangular form, facilitating efficient determination of variable values through back substitution. Gauss-Jordan Elimination refines systems into reduced row-echelon form, simplifying solution extraction and providing comprehensive insights into system properties, even in complex scenarios. These methods rely on basic row operations and are fundamental in solving linear systems across diverse domains. In the Traffic Flow Algorithm (Step 1), employing techniques like Gaussian elimination or Gauss-Jordan elimination enhances comprehension and analysis of traffic flow dynamics, streamlining solution processes and optimizing transportation networks effectively. Understanding these numerical methods enhances the Traffic Flow Algorithm's efficacy in traffic flow dynamics analysis and optimization.

3 NUMERICAL ANALYSIS

Experiment 1: In a controlled experimental farm, there exists a substantial population of flowers encompassing all potential genotypes (AA, Aa, and aa) with initial frequencies denoted as f0 = 0.05, g0 = 0.90, and h0 = 0.05. It is assumed that these genotypes determine flower color, and each flower is fertilized by a flower sharing a similar genotype. The task is to derive an expression for the genotype distribution in the population after four generations and to make predictions regarding the long-term genotype distribution after an infinite number of generations.

A concise explanation of the significance of the experiment involves investigating the dynamics of genotype distribution in a flower population across generations. Initially, the population encompasses three genotypes (AA, Aa, and aa) with frequencies f0 = 0.05, g0 = 0.90, and h0 = 0.05, respectively, presumed to determine flower color. Flowers with similar genotypes fertilize each other, and through genetic inheritance principles, the experiment aims to deduce the genotype distribution after four generations. This analysis offers insights into short-term genotype frequency changes and predicts long-term distributions. Understanding such dynamics has practical applications in agriculture and conservation, guiding breeding strategies, preserving genetic diversity, and effectively managing populations, thereby enhancing crop yields, biodiversity preservation, and ecosystem sustainability.

Experiment 2: Employ a linear algebra technique to construct a natural cubic spline that passes through the points (1, 2), (2, 3) and (3, 5).
To accurately interpolate data points and capture the underlying behavior of the dataset in experiment 2, it is imperative to design a natural cubic spline. When piecewise cubic polynomial functions are connected smoothly across data points and function and derivative values remain continuous, the result is a natural cubic spline. Natural cubic splines offer a trustworthy representation of the data by minimizing interpolation mistakes and guaranteeing smoothness and continuity. Due to the ability to precisely approximate data and fit curves, this is advantageous for applications including image processing, computer graphics, and motion analysis. For instance, using the linear algebra approach, in the scenario where the points (1, 2), (2, 3), and (3, 5) are supplied, matrices are created and equations are solved to find the coefficients of the cubic spline that passes through these points.

Experiment 3: Use linear algebra method to determine the three loop currents I_1, I_2 and I_3 in the circuit.

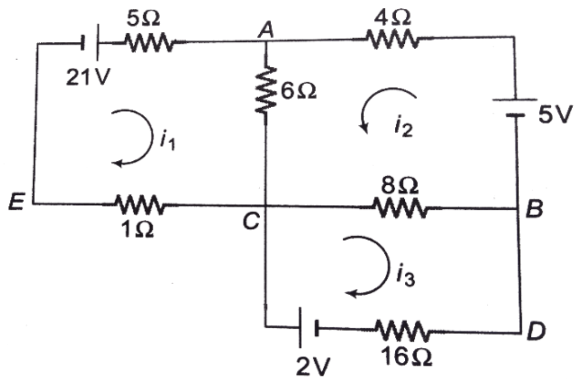

Figure 3. *Designed Circuit Problem*

Experiment 4: The network in the Figure 4.3 shows the traffic flow (in vehicles per hour) over several one-way street in the downtown area of a certain city during a typical lunch time. Determine the general flow pattern for the network. What is the maximum possible value of x_4?

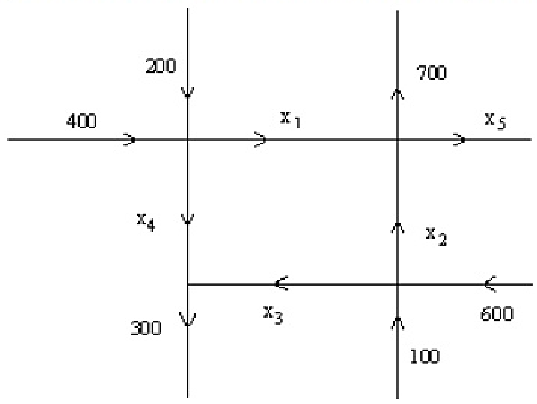

Figure 4. *Traffic Flow Diagram*

The numerical solutions of experiment 1, 2, 3 and 4 are obtained using the algorithms described in (2.1.1), (2.2.1), (2.3.1) and (2.4.1), the results are shown in the tables below.

Table 1. *Computational Result for Experiment 1*

No. of Generation (n)	f_n	g_n	h_n
0	0.05	0.90	0.05
1	0.28	0.45	0.28
2	0.39	0.23	0.39
3	0.44	0.11	0.44
4	0.47	0.06	0.47
∞	0.5	0	0.5

Table 2. *Computational Result of Experiment 2*

Constants	Values
e_0	2
f_0	$\frac{3}{4}$
g_0	0
h_0	$\frac{1}{4}$
e_1	3
f_1	$\frac{3}{2}$
g_1	$\frac{3}{4}$
h_1	$-\frac{1}{4}$

Table 3. *Comparative Result of Experiment 3*

Linear Algebra Method	Iteration	Time (millisecond)
Gaussian Elimination	-	0.00
Gauss-Jordan Elimination	-	0.00
Jacobi	11	1.27
Gauss-Seidel	6	1.17

Table 4. *Computational Result of Experiment 4*

Unknowns	Flow Pattern
x_1	$600 - x_4$
x_2	$400 + x_4$
x_3	$300 - x_4$
x_4	Free
x_5	300

Table 1 shows the expression for the genotype distribution of the population after 4 generation and after an infinite number of generations. In Table 1, the genetic distributions represent the frequencies of different genotypes (AA, Aa, and aa) within a population across multiple generations. As the generations progress from 0 to 4, there are observable changes in the genotype frequencies. Initially, at generation 0, the frequencies are f0 = 0.05, g0 = 0.90, and h0 = 0.05 for genotypes AA, Aa, and aa, respectively. Subsequently, with each generation, there are shifts in these frequencies due to genetic recombination, mutation, selection, and other evolutionary processes. For instance, by generation 4, the frequencies have changed to f4 = 0.47, g4 = 0.06, and h4 = 0.47. These changes in genotype frequencies over generations provide insights into the genetic dynamics of the population, including patterns of inheritance, allele frequencies, and the impact of evolutionary forces. Analyzing these trends provides valuable insights into genetic dynamics, inheritance patterns, and population evolution.

Table 2 presents the computed values of constants necessary for constructing a natural cubic spline in Experiment 2. These constants determine the coefficients of cubic polynomials, ensuring smooth interpolation between specified data points. The table's data facilitates accurate representation of data and curve fitting, benefiting applications like image processing and motion analysis. The values of the constants have been calculated as depicted in Table 2. Substituting the values of the constants in (9), the natural cubic spline function will now be stated as;

$$\underline{S}_0(x) = 2 + \tfrac{3}{4}(x-1) + \tfrac{1}{4}(x-1)^3,\ 1 \le x \le 2$$

$$\underline{S}_1(x) = 3 + \tfrac{3}{2}(x-2) + \tfrac{3}{4}(x-2)^2 - \tfrac{1}{4}(x-2)^3,\ 2 \le x \le 3$$

Table 3 showcases the application of different linear algebra techniques to determine the loop currents in Experiment 3's electric circuit problem. Direct methods like Gaussian Elimination and Gauss-Jordan Elimination, alongside iterative approaches such as Jacobi and Gauss-Seidel, were employed. These methods yield crucial insights into electric circuit analysis, elucidating how electric charge flows through the circuit's branches. Interpretations of the obtained loop currents, denoted as $I_1 = 2.06A$, $I_2 = -0.41A$, and $I_3 = 0.22A$, provide clarity on current distribution, voltage drops, and power dissipation within the circuit. Such insights facilitate advancements in electrical engineering, aiding in circuit analysis, design, and troubleshooting by discerning component behavior and assessing circuit performance. Direct methods like Gaussian elimination and Gauss-Jordan elimination provide precise solutions efficiently, suitable for complex circuit configurations. These methods guarantee accurate results but have a computational complexity of O(n^3). In contrast, iterative methods such as Jacobi and Gauss-Seidel offer advantages for large-scale circuit problems, with a computational complexity of O(n^2). Although iterative methods may converge slowly, they are more suitable for systems with numerous equations.

Table 4 depicts the flow pattern or general solution that describes the flow of the traffic system in experiment 4. Examining the flow pattern, it is observed from a practical point of view that all flow must be non-negative and this force $x_4 \le 300$

The unknowns in experiment 4 stand in for different sections or segments of the downtown road network. The numbers assigned to these unknowns in Table 4 indicate the traffic flow (in vehicles per hour) along each road segment. Each unknown $(x_1, x_2, x_3, x_4 \text{ and } x_5)$ corresponds to a particular road segment. For example, the variables x_4 indicates freely fluctuating variables, and $x_1, x_2, x_3 \text{ and } x_5$ reflect flow values on specific streets, respectively. The traffic patterns and directional flow along these route segments are depicted in Table 4. Comprehending the road network's flow pattern is essential to enhancing traffic control tactics, enhancing transportation effectiveness, and mitigating urban congestion. Therefore, in order to improve the efficiency and security of urban transport systems, policymakers and urban planners can make better decisions by examining the flow patterns of road networks. Experiment 4 provides insights into the flow pattern of vehicles in a traffic network, offering practical implications for traffic system management. By analyzing the flow pattern, transportation authorities can identify congestion points, optimize signal timings, and improve overall traffic efficiency. This information informs decision-making in traffic management, leading to reduced travel times and enhanced transportation functionality. Therefore, the findings from Experiment 4 contribute to the development of strategies for mitigating congestion and improving urban transportation networks.

Recognizing the inherent constraints and assumptions within each application field is crucial when practically integrating linear algebra into areas such as traffic flow analysis, cubic spline interpolation, genetics, and electric circuits. The accuracy of genetic predictions depends on factors such as the complexity of genetic models, data availability, and assumptions regarding genetic inheritance patterns. In cubic spline interpolation, the assumptions of continuity and smoothness may be challenged by noisy or sparse data. In electric circuit analysis, simplifying circuit models is common, leading to discrepancies between theoretical predictions and real-world behavior. Similarly, traffic flow analysis often simplifies real-world traffic systems, assuming uniform traffic conditions and homogeneous behavior. Acknowledging these constraints enhances study transparency, aids in interpreting findings, and directs future research towards resolving these issues, thereby increasing the adaptability of linear algebraic approaches across various contexts.

4 CONCLUSION

In conclusion, this study delved into the practical integration of linear algebra across various domains, including genetics, cubic spline interpolation, electric circuits, and traffic flow analysis. Through detailed methodologies and analyses, key insights were gained. The investigation of genetic distributions over generations provided understanding of evolutionary processes and inheritance patterns. The application of cubic spline interpolation facilitated precise data representation and curve fitting in diverse fields. Electric circuit analysis showcased the importance of linear algebra techniques in understanding current flow, voltage distribution, and circuit performance. Additionally, traffic flow analysis highlighted the role of linear systems in modeling and optimizing transportation networks. Overall, this study underscores the profound impact of linear algebra in solving complex real-world problems and advancing understanding across multiple disciplines, paving the way for future research and innovation in interdisciplinary studies. The implications of this research emphasize the indispensability of linear algebra in addressing complex real-world problems across diverse domains. By harnessing the computational power and analytical rigor of linear algebra, researchers and practitioners can gain deeper insights into the underlying mechanisms of biological systems, develop more accurate predictive models, design efficient electrical systems, and optimize transportation infrastructures. Ultimately, the integration of linear algebra contributes to advancements in science, engineering, and technology, driving innovation and progress in the modern world.

Conflict of Interest

There is no conflict of interest between the authors.

Authors Contributions

In the context of this research paper, Khadeejah James Audu formulated the conceptual framework and played a key role in shaping the methodology. Yak Chiben Elisha was instrumental in preparing the data and drafting the content, while Yusuph Amuda Yahaya provided valuable supervision throughout the project. Sikirulai Abolaji Akande contributed by thoroughly reviewing and editing the manuscript.

Acknowledgment and Support

The authors express their sincere gratitude to Dr. A. T. Tiamiyu from Department of Mathematics at the Chinese University of Hong Kong, Hong Kong. His unwavering support, care, guidance, contributions, motivation, and encouragement were invaluable and greatly contributed to the success of this research work.

Statement of Research and Publication Ethics

The study is complied with research and publication ethics.

REFERENCES

[1] H. A. Abdulmohsin, H. B. Abdul Wahab, and A. M. Jaber Abdul Hossen, "A Novel Classification Method with Cubic Spline Interpolation", *Intelligent Automation & Soft Computing*, vol. 31, no. 1, pp. 780-863, 2022.

[2] K. M. M. Aburaas, "Matrices and their Applications in Electrical Circuits", *African Journal of Advanced Pure and Applied Sciences (AJAPAS)*, pp. 364- 370, 2023.

[3] S. Andrilli and D. Hecker, *Elementary linear algebra*. Academic Press, Elsevier, Netherlands, 2022.

[4] M. Ashraf, V. De Filippis, and M. A. Siddeeque, *Advanced Linear Algebra with Applications*. Springer Nature Singapore, Gateway East, Singapore, 2022.

[5] M. L. De Klerk and A. K. Saha, "A review of the methods used to model traffic flow in a substation communication network", *IEEE Access*, vol. 8, pp. 204545-204562, 2020.

[6] W. Hao, D. Rui, L. Song, Y. Ruixiang, Z. Jinhai, and C. Juan, “Data processing method of noise logging based on cubic spline interpolation”, *Applied Mathematics and Nonlinear Sciences*, vol. 6, no. 1, pp. 93-102, 2021.

[7] A. D. Madhuri, P. V. N. Sagar, P. G. Raju, and A. A. Tanuja, "A Study on Applications of Linear Algebra", *Mathematical Statistician and Engineering Applications*, vol. 71, no. 4, pp. 2463-2473, 2022.

[8] C. D. Meyer and I. Stewart, “Matrix analysis and Applied linear algebra”, Society for Industrial and Applied Mathematics (SIAM), 2023.

[9] M. M. Moghaddama, "11th Seminar on Linear Algebra and its Applications", arXiv preprint arXiv:2206.05494, 2022.

[10] T. T. Moh, "Linear Algebra and its Applications", *Monatshefte für Mathematik*, vol. 199, pp. 929-931, 2022.