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RESEARCH ARTICLE

EXAMINING INVESTMENT STRATEGIES WITH EVOLUTIONARY GAME THEORY

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Abstract

This study provides a comprehensive analysis of the investment strategies used in stock markets by utilizing evolutionary game theory. The main objective is to investigate the conditions necessary for achieving an evolutionary stable equilibrium, which is crucial for a successful investment strategy and a rational market process. To achieve a stable investment strategy, investors must focus on returns and be wary of yield differences. Yet, empirical observation of this situation can be challenging. Therefore, evolutionary theory is selected as the ideal tool to model emotional states and non-rational behaviors, such as reciprocity, altruism, and selfishness. The study is divided into three parts. The first part presents a literature review on the modeling of investment strategies. In the second part, investment strategies are modeled using evolutionary game theory. Finally, in the last part, a behavioral dimension is added to the model, revealing the difficulty of rational preferences and evolutionary stable balances in the presence of human behavioral preferences. We emphasize the importance of a stable investment strategy dominating the market to achieve an equilibrium state. The study highlights the challenge of achieving rational preferences and evolutionary stable balances, given the behavioral dimension of human preferences.

Keywords: Evolutionary Game Theory, Evolutionary Stable Equilibrium, Replicator Dynamics, Conformism, Investment Straregies
JEL Classification: B52, C62, C73, D10

1. Introduction

By utilizing evolutionary game theory, this study delves into investment strategies in the stock market and explores the conditions necessary to achieve evolutionary stable equilibrium. While it is natural for some stock market participants to come out on top while others fall behind, it's crucial to comprehend the underlying factors. The ultimate goal is to identify an investment approach that benefits all parties. Limited rationality and adaptive expectations of economic

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agents based on past results can lead to investment setbacks in stock markets. To prevent such setbacks, investors engage with each other and strive to adopt portfolio rules that yield higher returns than their current strategies. However, this doesn't always result in an evolutionary equilibrium where all investors adopt a uniform approach.

Social interactions play a vital role in preventing investment failures, and their impact depends on various factors such as group size, reciprocity, selfishness, feelings of fairness, and altruism. For example, if players know they will be punished for a failed investment, they may be more careful. Similarly, a player living in a community with a sense of reciprocity may try to adapt to the community's investment practices. A sense of trust is also crucial for successful interactions. However, since the sensitivity to payoffs differs from player to player, a player with a lower payoff strategy may continue to invest in the same stocks. They may believe that the payoff difference needs to be more significant to change their strategy. Emotional elements with a behavioral dimension should also be considered when developing models. Models should consider the effect of conformism and show how unsuccessful strategies with lower returns can continue to exist. In other words, the most successful investment strategy cannot eliminate low-return strategies.

Evolutionary game theory is preferred to model conformism and non-yield-dominant motives as it uses replicator dynamic equations. It also allows emotional relationships to be established between periods. Our study has shown that a stable investment strategy is theoretically possible. However, it has yet to be observed in practice. This is an important finding because the behavioral dimension of individuals' preferences leads to irrational preferences.

This study is exceptional, as it employs innovative evolutionary game theory tools to investigate the decision-making processes of risk-neutral investigators within a utility function. By revealing that low-return investment strategies can conquer the market, this groundbreaking approach can significantly advance our understanding of decision-making processes in specific contexts. Furthermore, the study provides crucial insights into the social and emotional dimensions that impact our preferences and can lead to systematic errors in our choices. This knowledge is crucial in developing effective strategies and policies across various fields.

1.1. Literature Review

Investors use different approaches to determine their strategies. These approaches have developed and diversified since the 1950s. Kelly (1956) studied a player with different criteria than a typical gambler. This player aims to maximize the expected value of the logarithm of their capital on each bet. The reason for this is not the value of money but the law of large numbers (Kelly, 1956). A follow-up study examined where players can make one-dollar bets every week but cannot reuse the earnings. The player must maximize their expectation (expected capital value) on each bet this time. In the second case, the strategy is to select the bet with the highest expected return. This way, the player can earn more than others who divide their money between bets. This idea shows that rational investors maximize the expected logarithmic value of the portfolio growth rate (Amir et al., 2005).

Another investment model, the discounted cash flow approach, accepts the stock price as the present value of future cash flows. This approach is also known as net present value. It compares the present value of the stock's future return with the return of a different investment instrument. (Williams, 1997). Another model, the Gordon (1959) growth model, proved that an investor's potential return from a stock is the stock's cash dividend yield. Evstigneev et al. (2002) showed that investors consider the rational and future value of stocks when determining their strategies. They also consider the discounted value of future dividends. In the long term, stock prices coincide with the dividend trend. In other words, two determinants of share prices in the long term are dividend payment success and quality.

Stock prices can deviate significantly from their fundamental values in the short-medium term. Shiller (1981) first brought attention to this problem of extreme volatility. It makes it difficult to determine the investment strategy. Models that assume rational behavior may not be enough to explain excessive volatilities. Markowitz (1952) divided the portfolio selection process into two stages. The first stage starts with observation and experience. It ends with belief in the future performance of existing securities. The second stage starts with belief in the future performance of securities. It ends with portfolio selection (Markowitz, 1952). This study focused on the second stage. It examined the situation where the investor maximizes the discounted expected return. Moreover, it looked at the rule where the investor sees the expected return variance as an undesirable variable. Based on this, the geometric relationships between beliefs and portfolio selection were demonstrated using the expected returns and returns variance rule. So, the optimal strategy for investors is to maximize the discounted value of future returns (Markowitz, 1952). Hicks (1939) used a similar approach to Markowitz's expected value for investment preferences of companies instead of individual portfolio selection. In this study, the risks involved in the concept of expected value were also examined (Hicks, 1939).

Friedman (1953) and Fama (1965) also showed that the market dynamics naturally select rational strategies. These strategies lead to market efficiency in the long run and eliminate other strategies. Cowen and High (1988) examined investment strategies in an evolutionary game theoretical framework. They demonstrated stationary equilibria for different situations, such as the St. Petersburg Paradox, competitive investment, the posterior probability model, and the multi-stage investment process. Another critical study by Mehra and Prescott (1985) emphasized that investors likely only base their strategies on something other than well-defined preferences and objectives.

De Long et al. (1990) stated that if there are rational and many other irrational strategies in the market, any set of irrational strategies will be easily eliminated. Accordingly, the market may lose its rational valuation with irrational strategy pressure but eventually return to rational strategies with market selection pressure. With this stability feature, the market, which has irrational values in the short and medium term, becomes rational in the long term.

Shleifer (2000) stated that random choice strategies can make more money than rational ones. So, it is unclear if a randomly chosen investment strategy can be stable over time. Hens and Hoppe (2005a) conducted a study that showed financial and insurance markets are stable when a strategy that dominates market selection mechanisms can increase its market share against another strategy. Brock et al. (2005) presented a general strategy selection framework for a market with many different types of investors. This framework is called Large Type Limit. The study concluded that in a market with the characteristics of evolutionary systems, the system may become unstable if investors are too sensitive to a tiny return difference in another strategy (Brock et al., 2005).

Critical studies (Chiarella et al., 2006) argued that a market with only a few investors (ones who act based on momentum, short-sighted investors, etc.) could not have a stable strategy. Kane (1984) did one of the first studies on how financial regulations affect the market and investors' strategies. Financial innovations can either help or hurt the market and elections. Regulations can make information more transparent and help investors succeed, but if the regulations fail, the opposite can happen.

Cheng et al. (2011) emphasized how rules and policies to ensure financial stability can harm investment strategies by making the financial system riskier. Rubio and Gallego (2016) demonstrated that similar policies can promote stability and lower the risk of investment strategies. Boz and Mendoza (2014) believe that specific financial regulations can make the market overly confident, causing stock prices to rise. They argued that this can mislead investors into thinking assets are risk-free, leading to failed strategies.

A study by Alos-Ferrer and Ania (2005) treated asset markets like a game where investors get paid based on a return matrix. Investors divide their wealth among different assets. The prices of assets are set at market-clearing levels. The study assumed that players copied the best strategy from the previous period. The study found that when investors allocate their wealth based on expected returns, they reach Nash equilibrium. In this equilibrium, asset prices match the relative expected returns. The uniqueness of Nash equilibrium means there must be arbitrage prices whenever asset prices are far from equilibrium prices. So, arbitrage transactions bring prices closer to equilibrium values. This equilibrium strategy is also evolutionarily stable. In the dynamic evolutionary model, wealth tends to flow towards strategies with higher returns in the previous period.

Amir et al. (2005) examined the success of investment strategies and market choices with evolutionary tools. In the model, asset returns were assumed to depend on external and random factors. Market players use dynamic investment strategies, taking into account available information. Current information is based on both current and past experiences. This means the investment strategy is affected by intertemporal dynamics. It has been shown that an investor who distributes his wealth based on the conditional expected returns on assets can eventually accumulate to the extent of total market wealth.

Hens and Hoppe (2005b) studied how wealth shares change in imperfect markets with short-lived assets. They examined how asset prices are determined and how portfolio selection rules perform in repeated reinvestment processes. The study used stochastic dynamical systems theory to find the necessary and sufficient conditions for the evolutionary stability of portfolio selection rules. It was found that when returns are consistent with Markov returns, local stationarity conditions can lead to an evolutionarily stable portfolio selection rule. They also proved that the CAPM rule always copies the most successful portfolio selection rule and survives. Nevertheless, the mean-variance optimization rule is not evolutionarily stable (Hens & Hoppe, 2005).

Evstigneev et al. (2006) found that a stock market is stable if stocks are valued based on their expected relative dividend performance. If this condition is not met, a new portfolio selection rule with minimal initial wealth can conquer the market. Even if rational strategies face risk in a market with irrational strategies, irrational strategies can easily be replaced by a new set of strategies. This means that rational strategies can eventually lead to rational valuation in a market where assets are undervalued due to irrational strategies. This stability may explain why stock markets seem more rational in the long term than in the short and medium term. These findings support Friedman's (1953) belief that the market naturally chooses strategies that promote efficiency.

These studies show that investors use different criteria when determining their strategies. If the success of the chosen investment strategy is directly based on yield, the strategy that provides more return is more successful. Thus, in such a situation, other agents in the market will try to copy the most successful strategy.

2. Model

In this section, we'll analyze an investor utility function with evolutionary game theory tools. Again, the primary motivation is to copy the most successful strategy. Suppose there is a stock market in which

- There are $N \geq 1$ long-term assets (stocks).
- Each asset pays dividends at the start of each period. Dividends are the primary source of income.
- Time index will be showed as t.
- Usually, dividends are distributed every year.
- Businesses that make profits at the end of the period distribute dividends at certain rates.

- The distribution of dividends depends on the company's economic performance, cyclical fluctuations, and external factors.
- This stock market has only a few investment strategies. A group of investors implements each strategy, and a limited number of portfolio construction rules are applied in the market.
- $D_t^n \geq 0$ shows the total dividend distributed by stock n in period t.
- At the outset, assets were made available for acquisition. Each asset was allocated an initial supply, which is standardized and unchanging in each period.

2.1. Investment Strategy

The portfolio held at the beginning of period t by a player with wealth w_t^x who has chosen investment investment strategy *x* is

$$\Gamma_{t,n}^x = {}^{x w_t^x}\!\big/_{p_t^n} \qquad n = 0,1, \ldots., N \quad (1)$$

The denominator expression represents the price of stock *n*. Given that long-term asset supply is normalized, $\Gamma_{t,n}^x$ also indicates the proportional size that investment rule x purchased within the total quantity of asset n supplied.

The classical equilibrium condition is valid, and the equilibrium price is the price at the point where demand equals supply. The budget constraint limits players' portfolio size according to their investment strategies. Let us define the budget constraint as

$$\sum_{n=0}^{N} p_t^n \Gamma_{t,n}^x = w_t^x \quad (1.1)$$

The objective of players is to increase their yields by selecting stocks that offer higher dividends. At the beginning of period *t*+1, a player's wealth applying investment strategy *x* is

$$w_{t+1}^x = \sum_{n=0}^{N} (D_{t+1}^n + p_{t+1}^n) \Gamma_{t,n}^x \ (2)$$

Let us define a new equation which shows the divitends as below

$$TD_t^x = \sum_{n=1}^{N} D_{t+1}^n \, \Gamma_{t,n}^x \quad (3)$$

The time index serves as a measure of the d dependence on wealth, whereby investment strategy x, dividend distributions (TD_t^x), and the portfolio acquired by strategy x $(\Gamma_{t,n}^x)$ are all correlated with the initial wealth of the investor. So, now we can define evolutionary wealth equation

$$w_{t+1}^x = TD_t^x + \sum_{n=1}^{N} \Gamma_{t,n}^x \mathrm{X'}_{t+1,n}\, w_{t+1} \quad (4)$$

Here w_{t+1} shows the wealth of the other investors. With the trust and investment of fellow investors in the same stock, player x can look forward to a rewarding return on her investment in the market. $\mathrm{X'}_{t+1,n}$ statement describes the investment portfolios of those players who choose to invest in partly similar stocks as that of player x. This information can be useful for gaining insights into the investment strategies of other players. So expression $\mathrm{X'}_{t+1,n} w_{t+1}$ shows the portfolio preferred by other investors according to their wealth size and investment strategies. Other investors' wealth and investment strategies are important factors to consider when making investment decisions. Their portfolio sizes reflect their preferences, so analyzing these variables can help you make informed choices and understand the market better.

To summarize, the wealth of player x, who favors investment strategy x, is determined by multiple crucial factors. These include the dividends received at the end of period t (beginning of period t+1), the investment strategies and wealth of other players, as well as the portfolio sizes determined by other investors based on their own preferred investment strategies and wealth.

This section describes the stock market that is primarily suitable for risk-neutral investors. For risk-averse investors, the rule to create a portfolio is to ensure that the risk premium satisfies their satisfaction level. This analysis excludes external shocks and sudden fluctuations caused by them. The focus of this examination is on the situation where investors invest their wealth in the stock market. In summary, this market assumes that long-term volatility is less than short-term, risk-neutral investors dominate the market, information is reachable and returns from dividends are reinvested.

Utility functions will be defined and evolutionary game theory tools will be utilized under the given limitations. The model can be generalized to include multiple categories of investors, unexpected market instabilities, asymmetric information and the presence of consumer goods in the analysis.

2.2. Utilitiy Functions

As humans are inherently social creatures, it is imperative that investors broaden their utility functions beyond solely focusing on returns. It is crucial to consider non-return effects in order to make informed investment decisions.

The utility function of player x (the player who determines strategy x) will be

$$U_x = (1-\vartheta)w_t^x + \left(\vartheta\left[(\rho-\varphi)+\sum\nolimits_b \nu_{xb}U_b\right]\right) \quad (5)$$

The coefficients for ρ and φ in the equation above play a crucial role in our analysis. The former coefficient represents the fraction of individuals who follow the same investment strategy in all conditions, even if it may lead to potential loss. This fraction is a powerful indicator of the level of conformism among individuals. Conformism is a complex topic debated by scholars and philosophers for many years. At its core, it refers to respecting and adapting to the opinions and values of society or one's close circle without opposing them.

Conformism is not just about adapting to a specific environment or situation; it's about accepting and obeying without questioning or researching. However, conformity can be both beneficial and harmful. While conforming to societal norms can foster a sense of unity and belonging that benefits individuals and communities, blindly following the beliefs and values of others can stifle critical thinking and independent thought, hindering personal growth and societal progress. Even if agents experience financial losses, the individual derives satisfaction from sticking to the same strategy.

In this particular context, φ denotes the percentage of individuals who would not choose a particular strategy, despite its potential financial gains. This particular group of people may be influenced by various factors, such as their cultural, political, or ideological beliefs, or they may have a narrow perspective. As a result, the conformist effect is also present in this situation.

Another important factor which determines investor x's intangible benefit is a certain proportion (ν_{xb}) of the sum of the utilities of other investors that is shown as $\sum_b \nu_{xb} U_b$. Here $b = 1,2,\ldots\ldots.., Y$ refers to all other investors $(x \neq b)$ and ν_{xb} refers to the sensitivity of player x to the return/benefit level of other players.

This model surpasses previous ones by incorporating non-return effects and analyzing the impact of key human emotions such as altruism, selfishness, and reciprocity. Player x's sensitivity to the benefit of other investors can be written similar as Bowles (2006) study:

$$\nu_{xb} = (a_x + D_x a_b)/1 + D_x \quad (6)$$

where $a_x \in [-1,1], a_b \in [-1,1]$ and $D_x \geq 0$. a_x represents the unconditional good or bad will (altruism or feeling of grudge/anger/hate) of player x. a_b indicates player x's belief about other players' opinions of her and lastly D_x shows the level of importance that player x attaches to other players' opinions about her.

- If $a_x = 0$ and $D_x > 0$, player x is not altruistic and attaches importance to reciprocity in behavior. This player does not have an unconditionally good or bad norm of behavior and shapes his thoughts based on the behavior of others.

If $a_x \neq 0$ and $D_x = 0$, player x acts as an unconditional altruist according to the sign of a_x (if $a_x > 0$). Therefore, in these conditions, unconditional hatred may also arise.

The denominator in the equation v_{xb} increases according to D_x. Since the maximum value that a_x can take is 1, it can be written $v_{xb} \leq 1$. Hence, player x's valuation of other players' payoffs (utilities) cannot be greater than her own payoff (utility).

The derivative of the expression with respect to D_x varies depending on the sign of $(a_b - a_x)$. Due to reciprocity, if the player x interacts with is kinder, more gentle or better than herself, the weight of the other player's payoff in x's utility function increases. Therefore, moral and subjective values come to the fore here.

If $a_x = a_b$ then $v_{xb} = a_x$. In this situation sensitivity to other investors' returns (utilities) is determined directly based on the player x's unconditional opinion.

It's important to note that as ϑ decreases, yield-dependent factors become increasingly critical. This also means that the utility function becomes more return-oriented. As a result, a return-oriented individual focuses solely on returns and neglects the behavioral and emotional aspects of their decisions.

Briefly, each player is believed to have a benefit function that aligns with the strategy they choose. This utiility function takes into account various factors, including both material and behavioral aspects. Essentially, the benefit that a player receives from a particular strategy is determined by how well that strategy aligns with their goals and preferences. As the game progresses, players may choose to change their strategy based on how their benefit function evolves over time. This can be influenced by a variety of factors, including changes in the game environment, the actions of other players, and the player's own internal motivations and preferences. Ultimately, successful game play requires a deep understanding of the benefit functions of all players involved, as well as an ability to adapt to changing circumstances and anticipate the strategic moves of one's opponents.

2.2.1. Evidence of Irrational Investors

This study provides compelling evidence that investors are capable of making irrational decisions while strictly adhering to them, as a result of the behavioral dimension or conformist effect. Our novel model, which has not been studied before, enables us to analyze this phenomenon in greater depth and with greater certainty. Notwithstanding the divergence in mechanics, several crucial empirical studies have arrived at comparable conclusions to ours.

In 2015, Mamun et al. conducted a study on 200 individual investors in the Dhaka Stock Exchange (DSE) to gain insights into their behavior. The researchers analyzed the participants' responses to various questions related to the concept of rationality and irrationality. The aim of the study was to determine the actual investing behaviors of the participants and to understand whether they acted rationally or irrationally. Through empirical research, it has been determined that a hybrid approach, combining aspects of efficient market hypotheses (the idea of rational investment markets) and behavioral modeling, yields a deeper understanding of investor behavior in financial brands. One of the fundamental principles considered in our study is the assumption presented here in. This premise provides the basis for our research, and its importance is paramount in our analysis.

A study conducted in China proves that investors in the real world often exhibit irrational behavior, such as herd behavior, due to market frictions and psychological factors, which contradicts the efficient market hypothesis. The research proves that irrational investors are influenced by herding behavior and behavioral dimensions; herding behavior is caused by loss aversion and predictive expectation (Jianhua et al., 2020).

Mushinada analyzed detailed survey data from 384 Indian investors to investigate whether self-attribution bias and overconfidence bias exist in the Indian stock market. He also examined if individual investors are adaptable to market dynamics. The study found evidence for the existence of cognitive biases alongside rationality. It was observed that investors tend to adapt to the changing environment once they experience losses or uncertain events. However, during this process, investors sometimes exhibit apparently irrational behavior (Mushinada, 2020).

Hirsleifer et al. (2006) proposed a model in which investors trade irrationally based on factors that are not necessarily connected to the actual fundamentals of the market. However, since trading activity has an impact on market prices, irrational trades can also influence the underlying cash flows. In certain situations, it is proved that irrational investors can earn profits that are higher than the profits made by informed, rational investors.

The results of the replicator that we theoretically modeled in this study unequivocally align with the empirical studies presented in this section. The present study's theoretical framework is ambitious, and it is essential to emphasize that this work is open to improvements, particularly empirically. In this respect, we also emphasize that our study aims to prove in the theoretical framework with evolutionary game theory that the return-dominant strategy cannot capture the stock market in the presence of non-return effects and irrational investors.

2.2.2. Utility Level Differentials and Strategic Behaviour

Economics serves as the cornerstone of utility theory, providing a profound collection of theories that researchers from all utility-related disciplines have employed, expanded, and adapted. The results are rigorous framework that enables researchers to scrutinize and develop a deeper understanding of the rational behavior of consumers (Fishburn, 1968).

While there is general agreement regarding the extent of utility theory, its practical application is subject to varying perspectives. These perspectives arise from differing interpretations of the fundamental tenets of preference and decision-making, which have been shaped by the fields in which the theory is employed, such as economics, psychology, statistics, and management science.

Utility function, which serves as a mathematical tool for evaluating an individual's subjective preferences for varying levels of total wealth. This function enables investors to quantify their satisfaction levels based on the outcomes of their financial decisions. Essentially, it measures the individual's relative preference for various amounts of wealth, and is a critical component in the determination of optimal investment strategies. By modeling investors' utility functions, financial analysts and planners can better understand their clients' investment objectives and design portfolios that align with their risk tolerance and goals.

The foundation of utility theory is based on the acknowledgement of humans' insatiable nature and their tendency to act rationally. The non-satiation property is an economic principle that posits that an individual's utility increases with a higher level of wealth. This implies that an investor always favors a higher level of wealth and is never satiated with their current wealth level (Norstad, 2011). It is imperative to note that investors may possess varying utility functions; however, it is essential to assume that all functions adhere to the non-satiationary property.

So, the utility maximization theory posits that investors, when presented with a range of feasible investment alternatives, will opt for the investment that maximizes their expected utility of wealth. This principle is founded on the tenet that investors are motivated by utility.

As we have seen in the previous chapters, this study is also founded on the notion that human strategic behavior is driven by seeking the maximum benefit. Consequently, strategic behavior is observed, but we have opted for a modern approach known as evolutionary game theory rather than a traditional game theoretical approach. To analyze the process, we leverage the replicator dynamics technique.

Nevertheless, there are instances where individuals may prioritize moral pleasure over material gain, leading them to relinquish material possessions. Our study's most important contribution to the literature will be this finding, which will undoubtedly add immense value to the existing knowledge.

2.3. Replicator Dynamic

The replicator dynamic models the likelihood that the more beneficial strategy will be copied by other players. It can be written as

$$x' = x - \upsilon x(1-x)\varsigma_{y>x}\tau\left(U_y - U_x\right) + \upsilon x(1-x)\left(1-\varsigma_{y>x}\right)\tau\left(U_x - U_y\right) \quad (7)$$

In any period there are x investors (with utility function U_x) who adopt strategy x. Within this fraction υ is eligible to change strategy. With probability *x*(1-*x*) a x person will match with y person and with probability $\varsigma_{y>x}\tau\left(U_y - U_x\right)$ a strategy change will occur as a result of this interaction. The factor that enables this strategy change is the yield difference or information about independent-return factors. If $U_y > U_x$ then $\varsigma_{y>x} = 1$ accepted. Otherwise, the expression will be equal to 0. Replicator dynamics can be arranged as follows:

$$\Delta x = \upsilon x(1-x)\tau\left(U_x - U_y\right) \quad (7.1)$$

In replicator dynamic 7.1, the parameter υ represents the fraction of players who interact to change their strategy. This parameter determines the rate at which the strategies in the population evolve over time. The larger the value of υ, the faster the strategies will change.

The coefficient $\tau \in [-1,1]$ is a measure of the human character and the feasibility of strategic behavior. To better understand this concept, consider a scenario where the coefficient equals 1. In such a case, it can be said that the economic actor is homo-economicus, which means that she is rational, self-interested, and fully responsive to any changes in the environment. Specifically, such an investor is highly sensitive to the differences in her utility level and immediately adjusts strategy to maximize her utility. This type of investor is unlikely to make systematic errors since they are well-versed in all the relevant information and have a clear understanding of the risks and benefits associated with decisions. In other words, this actor is capable of making the best possible decisions at any given time, without being influenced by emotions or external factors. This makes her a valuable asset in the economic landscape, where efficiency, accuracy, and profitability are highly valued.

In the event that the coefficient is zero, strategic behavior will be rendered obsolete. Consequently, the effectiveness of investment strategies will lose significance, and the validity of the utility theory will be called into question. This is due to the failure of the replicator dynamics to function when the coefficient reaches zero. In short, if τ is high, players will be more sensitive to changes in payoff and may switch strategies more frequently. If τ is low, players will be less likely to switch strategies and the population will evolve more slowly.

It is important to note that these parameters are interdependent and influence each other. Changes in the value of one parameter can affect the rate of evolution and stability of strategies in the population.

The velocity of the evolutionary process is determined by the mathematical expression $x(1-x)$. A uniform population tends to decelerate the process, while a diverse population accelerates it. It is apparent that this expression attains its maximum value at x=1/2 As a result, an equitably divided population will maximize the rate of alteration in x, while holding other variables constant.

The residual component of the replicator dynamic can be represented mathematically as $\upsilon\tau\left(U_x(x) - U_y(x)\right)$. The rate of update and utility functions are contingent on the level of x within the population. Should a minute percentage of the population interact, the divergence in the level of benefit and its corresponding sensitivity will diminish, thereby decreasing the exertion of replication pressure.

The condition required to guarantee stationarity can be expressed as:

$$\partial \Delta x / \partial x < 0 \quad (8)$$

$$\partial \Delta x / \partial x = (U_x - U_y)(\upsilon\tau - 2x\upsilon\tau) \quad (8.1)$$

As can be seen from equation 8.1 there are four equilibrium point as

1. $U_x = U_y$
2. $\upsilon = 0$
3. $\tau = 0$
4. $x = \frac{1}{2}, 1, 0$

In the first equilibrium, there is an equal benefit to the player from both strategies. This stability is maintained since there are no conditions for switching between them. Equity can be determined based on returns or behavior. If players are solely interested in returns, then the benefits will be equal only when the material returns are identical. In the case of conformist and altruistic players, some individuals will continue to use the same strategy, regardless of whether they make a profit or a loss, due to the conformist effect. The equality of benefits rests on the players' sensitivity to them. In another equilibrium, there is no interaction among players, which implies that there will not be any alteration in strategy.

When people ignore the difference in their utility levels, it inevitably leads to ineffective interactions. Furthermore, when society is divided and people opt for both strategies with equal frequency, it results in a lack of progress. It is important to note that the model can only be considered in equilibrium when the entire society chooses a single strategy.

The sign of the equation 8.1 can be examined for stationarity point. Accordingly, one of two conditions must be met for stationarity;

$$U_x > U_y \text{ and } x > 1/2 \quad (9)$$

$$U_x < U_y \text{ and } x < 1/2 \quad (10)$$

Equilibria $x = 1$ and $x = 0$, where the entire society adopts a single strategy, are evolutionarily stable. The x=1/2 equilibrium is not stationary and a small deviation may create basins of attraction.

When conformism and the behavioral dimension completely dominate the utility function for $U_x > U_y$ it is required

$$(\rho - \varphi) + \sum_b \nu_{xb} U_b > (\varphi - \rho) + \sum_c \nu_{yc} U_c \quad (11)$$

For a strategy to be truly effective, the difference between the adoption rates of strategies x and y must be substantial in all circumstances. This difference must be significantly greater than the difference between the sensitivity levels of players who have implemented opposing strategies that benefit others.

$$(2\rho - 2\varphi) > \sum_{c} \nu_{yc} U_c - \sum_{b} \nu_{xb} U_b \quad (12)$$

In other words, when conformism is effective $U_x > U_y$ can be achieved by the significant dominance of players who prefer strategy x under all circumstances. Otherwise, even if $\rho > \varphi$ if the altruism of the players adopting strategy y is more than x's, than $U_x > U_y$ can not achieved. This result shows that in cases where comfort and behavioral dimensions are dominant, the most important issue for the level of benefit is the comfort zone.

In the case where all benefit depends on the return, the necessary condition for $U_x > U_y$ is basicly

$$w_t^x > w_t^y \quad (13)$$

This condition achieved only if strategy x is the most profitable strategy in terms of returns. When ϑ is ½, the relationship between utilities becomes more complex. This time, in order to meet the condition $U_x > U_y$ it is required

- The return of the player who has adopted strategy x is higher than the opposite strategy
- Players who will adopt strategy x under all circumstances are dominant in the population
- This dominance should be greater than the altruism of the players who adopt strategy y.

Game theory modeling demonstrates the existence of a non-evolutionary equilibrium among evolutionary equilibria. This relationship is illustrated in Figure 1. The arrows in the figure indicate the basins of attraction, which visualize that if the population is evenly divided, a slight shift in favor of one of the strategies will cause the population to move towards one of the two stable equilibria.

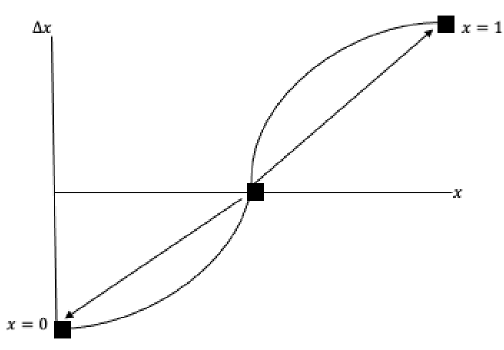

Figure 2 – Equilibrium and Attraction Basins

Source: Prepared by the authors.

Any other stable equilibrium would not meet these conditions. This, in turn, makes it challenging to have a stable investment strategy that is solely dependent on returns. If individuals are unable to make rational decisions, players who choose low-return strategies can prevent the highest returning strategy from dominating the market.

Players in this case tend to prioritize their own return rather than altruism. They also tend to stick to the same strategy even when better options exist. This behavior can prevent a strategy that is solely focused on return from dominating society. The advantage of return obtained by those who adopt this strategy is offset by the fraction that chooses the opposite strategy. This is because of the effect of conformism or inertia, or the altruism of this fraction.

2.3.1. Evolutionary Stability

The equations for investment strategy (Equation 1) and wealth evolution (Equation 4), as discussed in Section 2.1, are contingent upon the financial returns yielded by the players. Additionally, these equations are influenced by the strategies and wealth of other players, as well as the overall economic conditions.

Typically, investors seek out investment strategies that provide the greatest benefits and advantages, ultimately leading to a surge in demand for those particular portfolios. Consequently, the return on investment also increases In this scenario, the equilibrium is achieved when the entire population chooses the same strategy, resulting in the emergence of the investment portfolio that offers the highest return, except in a utopian situation where the conformist and behavioral dimensions dominate completely.

This is the idea that if a particular strategy becomes widespread in the market, it will become a stable strategy with a strong return advantage that makes it difficult for other strategies to compete. Therefore, even a small deviation from this equilibrium (such as introducing a small fraction of people who have adopted a different strategy) will lead to the elimination of the opposing strategy by replicator dynamics.

This section endeavors to provide a mathematical definition of the evolutionary steady state referred to earlier. First, we will define the vector that shows the wealth shares of all investors in period t as below

$$\kappa(t, d_t, W) \quad (14)$$

Within this system, the variable d^t is utilized to denote the current prevailing situation or conjuncture that governs the system. Meanwhile, the variable W^t signifies the distribution of wealth during the initial period.

The dynamic evolution of wealth (equation 4) for each strategy (investment portfolio) can be rewritten as follows:

$$w_{t+1}^{x} = f\left(TD_t^x, w_t, X'_{t+1,n} W_{t+1}\right) \quad (14.1)$$

All variables in this equation are assumed as defined in section 2.1. With a little arrangement of equation 1 we get investment rule x

$$x = {p_t^n \Gamma_{t,n}^x}/{w_t^x} \quad (15)$$

A mathematical representation for evolutionary stasis will be defined through a mutant strategy, say y

$$x = {p_t^n \Gamma_{t,n}^y}/{w_t^y} \quad (16)$$

In the context of evolutionary game theory, an investment strategy $x \neq y$ is said to be evolutionarily stable if, for every mutant strategy, there exists a random variable $a(d) > 0$ that satisfy a given condition $\lim_{t\to\infty} \kappa^x(t, d, w^x) = 1$ for every $w^x \geq 1 - a(d)$. This condition stipulates that the utility level of strategy x must be greater than or equal to the utility level of the mutant strategy under consideration. In other words, the presence of strategy x in a given population makes it invulnerable to invasion by any mutant strategy.

The presence of variable $a(d)$ acts as an entry barrier that hinders the success of any new investment approach that seeks to outperform and displace the dominant, reliable strategy. Mutant strategies are unable to thrive due to the impact of this barrier. The barrier may arise from either a financial return advantage or a moral advantage stemming from behavioral or conformist factors.

Assume that there is distortion in the distribution of wealth shares. Thanks to the barrier, long-term behavior will not be affected and the market selection process will move towards eliminating the mutant strategy. Ultimately, this will lead to the destruction of the mutant strategy. Here, we also need to mention local evolutionary stasis. This new stationarity can also be demonstrated mathematically.

Let us define a new random variable $\mathrm{r(d)} > 0$. If dominant strategy x meets the condition $\|x(d) - \mathrm{y}(d)\| < \mathrm{r(d)}$ it is considered a local evolutionary stable strategy. The concept of local evolutionary stable strategy refers to the strategy that is resistant to local mutations. This means that if a population adopts this strategy, it is unlikely to be replaced by any other strategy that is slightly different and arises due to new information. This stability against mutations is crucial for the long-term survival of a stabile strategies and helps to maintain the diversity of a population. In short, if points near an equilibrium point tend to move towards the equilibrium point over time, the strategy is said to be locally stable.

Note that, locally stable is slightly different from globally evolutionary stability (first case). In a state of global stationarity, a strategy is considered effective if it can successfully resist any mutant strategy and cannot be eliminated by any mutant strategy. When there is no influence of conformism and behavioral considerations, and decisions are solely based on the advantage of returns, the preferred strategy for the entire population is the global evolutionary stable equilibrium.

In a local evolutionary stable equilibrium, small deviations are adjusted to maintain balance. However, if the deviations are too large, it's uncertain whether the system can return to its original equilibrium point. When conformism and behavioral dimension are taken into consideration, the point achieved is called the local evolutionary stable balance. An example of this equilibrium can be found in section 3.4. Although this is sustainable, it's weaker than global stasis.

2.3.2. Replicator Dynamic Under Non-Random Matching

Individuals tend to engage in conversations and discussions with those who share similar viewpoints or are in close proximity. Thus, it becomes crucial to conduct an in-depth analysis of non-random associations. If the pairings are not random, the replicator dynamics can be written as follows

$$\Delta x = \upsilon x(1-\nabla)(1-x)\tau\left(U_x - U_y\right) \quad (17)$$

When a player picks strategy x, the likelihood of them matching with another player who also chose that same strategy is not simply x, but rather $\nabla + (1-\nabla)x$. In the event of complete intolerance, where $\nabla = 1$, there is a new equilibrium that guarantees there will be no change in Δx. In other words, when opposite strategies are completely intolerant, there is no chance of a match. If the derivative is taken to examine the partial effect of intolerance we get

$$\partial\Delta x /_{\partial\nabla} = -\,\upsilon x(1-x)\tau\left(U_x - U_y\right) \quad (17.1)$$

Whether this equation is positive or negative directly depends on the sign of the expression $\left(U_x - U_y\right)$. For $U_x > U_y$, $\partial\Delta x /_{\partial\nabla} < 0$ and for $U_x < U_y$, $\partial\Delta x /_{\partial\nabla} > 0$. Accordingly, the following determinations can be made:

If players find strategy x to be more advantageous, then increasing intolerance will result in a decrease in the potential population of strategy x. During matches, if a player adopts strategy y, they will likely copy strategy x in the next period. However, as intolerance increases, the population of strategy y will avoid mating with the population of strategy x, and this will prevent any strategy changes from occurring.

If strategy y is the optimal approach, then an increase in intolerance would lead to a corresponding increase in the potential population x. Conversely, if intolerance rises, the number of individuals who could be included in the population x also rises when the best strategy is y.

If there is complete intolerance, opposing players cannot be matched. This means that the probability of player x matching with another x becomes 1 according to the formula $\nabla + (1 - \nabla)x$.

2.3.3 Non-Return Effect and Equilibrium's

If the derivative is taken with respect to ϑ for equation 7.1 we get

$$\partial(\Delta x)/_{\partial\vartheta} = (w_t^y - w_t^x) - \left[(2\varphi - 2\rho) + \left(\sum_c \nu_{yc} U_c - \sum_b \nu_{xb} U_b\right)\right] \quad (18)$$

The following assessments can be made:

If strategy *y* provides the highest payoff, but the x fraction is driven by altruism and the dominant fraction always chooses strategy *x*, then the partial derivative of the change in the x fraction with respect to ϑ will be greater than zero $(\partial(\Delta x) \,/\, \partial\vartheta > 0)$. This will increase the weight of non-return effects in the utility function, leading to more players choosing strategy *x*. However, non-return effects like altruism and conformism might prevent some x players from switching to the best strategy, *y*.

Conversely, $\partial(\Delta x)/_{\partial\vartheta} < 0$. Increasing the weight of non-return effects in the utility function will have a reducing effect on the x fraction.

Hence the stationarity of the equilibria can be summarized as Table 1.

Table 1: Equilibriums'

Equilibrium	Is it stable?	Explanation
$x = 1/2$	No.	A slight deviation from equilibrium in any period will provide a return advantage and progress throughout the basin of attraction.
$x = 1$ $\vartheta = 0$	Yes.	The yield dominant strategy has completely dominated the stock market.
$x = 1$ $\vartheta = 1$	Not Certain.	Non-return effects have full weight on the utility function. That's why if $\sum_c \nu_{yc} U_c > \sum_b \nu_{xb} U_b$ than a fraction y introduced into play in the next period can ensure $U_y > U_x$
$x = 1$ $\vartheta = 0.5$	Not Certain.	If $\sum_c \nu_{yc} U_c > \sum_b \nu_{xb} U_b$ the equilibrium is not stable. The return advantage of the x's is balanced by the altruism of the y's. In the utopian situation where altruism is very dominant, $U_y > U_x$, despite the return advantage.
$x = 0$ $\vartheta = 0.5$	Not Certain.	Similar to the previous case, if the altruism of the x fraction is very dominant, the equilibrium is not stationary.
$x = 0$ $\vartheta = 0$	Yes.	When the effect of altruism and conformism disappears, all benefit is return-oriented and this balance is static.

$x = 0$ $\vartheta = 1$	Not Certain.	If $\sum_c \nu_{yc} U_c > \sum_b \nu_{xb} U_b$ the equilibrium is stable. On the other hand, if the altruism of the fraction adopting strategy *x* is dominant, this equilibrium is not stationary.

Source: Prepared by the authors.

When individuals make investment decisions in the stock market, their behavior can be influenced by different factors such as conformism, behavioral dimensions, and altruism. In the absence of these factors, the utility function focuses solely on the returns. In this case, the stable equilibria are x=0 and x=1, where the best investment strategies dominate the stock market. However, when conformism and behavioral dimensions come into play, the return advantage must be greater than the effect of altruism and conformism for a stable equilibrium to be reached. When conformism and altruism dominate, it becomes challenging to maintain a stable balance, which explains why low-return strategies can still survive in evolution.

3. The Effect of Altruism and Reciprocity Feelings on the Utility Function

When the utility function U_x is analyzed, it can be concluded that

- If the utility level of other players increases and if $\nu_{xb} > 0$, $\vartheta > 0$, the utility level of player x also increases (provided that π_x remains constant). Player x is aware of the utility level of other players. Her utility function does not only depend on her own financial payoff. As ϑ or ν_{xb} increases, the benefit level also increases.
- If $\nu_{xb} = 0$, x is only sensitive to his own payoff. In this case, the benefit level changes depending on the value of ϑ. If $\vartheta = 0$, player x is still return-oriented. As ϑ increases, player x begins to care about the existence of people who think like her, as well as her financial return.
- This equation also works in reverse. For instance, if $\nu_{xb} > 0$, $\vartheta > 0$and if $\sum_b U_b$ decreasing than the benefit level of player x will also decrease, provided that π_x remains constant.
- If $\nu_{xb} < 0$ and $\vartheta > 0$, a reverse relationship begins between $\sum_b U_b$ and U_x.
- Unconditional altruism or unconditional anger/selfishness is denoted by a_x. $D_x a_b$ characterizes conditional altruism. This condition can be summarized as player x's character D_x and belief about other players' opinions of her a_b.
- If $a_x = 0$ and D_x has a positive value, player x has an understanding of the other person's feelings, thoughts and behaviors towards him. This leads to conditional altruism.
- If D_x =0, ν_{xb} depends solely on a_x. For negative values of a_x, player x can be said to be unconditionally selfish or spiteful.

- If, $a_x = 1$ player x is unconditionally altruistic. In this case, the expression ν_{xb} will take the maximum value of 1. Therefore, player x may care about others' gains as much as her own.
- If $\nu_{xb} = a_x$ player x's sensitivity to other players' utility levels is directly determined by unconditional altruism/selfishness/anger/hatred.

If player x is an unconditional altruist, $a_x = 1$ and $\nu_{xb} = a_x$. Hence, the utility function becomes

$$U_{x(a)} = (1-\vartheta)w_t^x + \left(\vartheta\left[(\rho-\varphi) + \sum\nolimits_b U_b\right]\right) \quad (19)$$

and the benefit of the unconditionally selfish player is

$$U_{x(b)} = (1-\vartheta)w_t^x + \left(\vartheta\left[(\rho-\varphi) - \sum\nolimits_b U_b\right]\right) \quad (20)$$

For $U_{x(a)} > U_{x(b)}$ it is required $\sum_b U_b > 0$. The unconditionally altruistic player places significant weight on the utility level of other players. If $D_x = 0$, that is, if player x doesn't care about what others think, we can still talk about pure selflessness or selfishness and utility functions will be alike. As D_x increases the importance of player x's belief about what other players think about her becomes crucial.

Let us to compare two different player,

$D_x = 1, a_b = -1, \nu_{xb} = (a_x - 1)/2$ and $D_x = 0, \nu_{xb} = a_x$:

- If $a_x = -1$ (unconditional selfishness), then both ν_{xb} are equal.
- If $a_x = 1$ (unconditional altruism) than the second players' ν_{xb} is bigger. Ceteris paribus her utility will be higher also (if $U_b > 0$).
- If $a_x = 0.5$, the player who does not care about the opinions of others will benefit more. If player cares about other players' opinions of her, sensitivity to the benefits of others will decrease (just because the fact that for admitting that other players have a bad opinion of her).

$D_x = 1, a_b = 1$, and $D_x = 0$:

- If $a_x = 1$ (unconditional altruism), then both ν_{xb} are equal.
- If $a_x = -1$ (unconditional selfishness), then the first players' utility level will be higher (if $U_b > 0$).
- If $a_x = 0.5$, the benefit in the first case will be greater. If the player who cares about the opinions of others and believes that they have a good opinion of her has unconditional

altruism. Her benefit level will be higher than the player who does not value the opinions of others.

- If $a_x > 0$, the player who cares about the opinions of other players and believes that there is a good opinion about her will benefit more
- If $a_x < 0$, again, the benefit of the player who gives full importance to the opinions of other players and thinks that these players' think excellent about her (provided that the sum of the benefits of the other players is not zero) is greater.

If the same analysis is repeated for $D_x = 0.5$, $a_b = 1$ and $D_x = 0.5$, $a_b = -1$ (compare with $D_x = 0$)

- For $D_x = 0.5$ and $a_b = 1$, if $a_x = 1$, then in both cases ν_{xb} will be the same.
- For $D_x = 0.5$ and $a_b = -1$, if $a_x = 1$, then provided that the sum of the other players' utility levels is positive, the unconditional altruist's utility level will be higher.

Same analysis can be repeated under the following conditions

- When comparing, $D_x = 1, a_x = 1$ and $a_b = 1$ with $D_x = 1, a_x = 0$ and $a_b = 1$, the benefit of the player who attaches importance to other players' opinions about herself and believes that these opinions are positive, and who is also unconditionally altruistic, will be higher.
- When comparing $D_x = 1, a_x = 1$ and $a_b = -1$ with $D_x = 1, a_x = -1$ and $a_b = 1$, in both cases ν_{xb} obtained. Unconditional altruism and conditional emotions balance each other.
- When comparing $D_x = 1, a_x = 1$ and $a_b = -1$ with $D_x = 0.5, a_x = 1$ and $a_b = 1$, it can be said that the benefit level of the player with the second condition is higher (if the sum of the benefits of the other players is positive).
- When comparing $D_x = 1, a_x = 1$ and $a_b = 0$ with $D_x = 1, a_x = 0$ and $a_b = 1$, ν_{xb} becomes higher for the unconditionally altruistic player.

3.1. Partial Effects of ν_{xb}

If the derivative is taken to see the partial effect of player x's sensitivity we get

$$\partial U_x / \partial \nu_{xb} = \vartheta \sum_b U_b \quad (21)$$

As can be easily seen, if $\vartheta = 0$, the effect of conditional or unconditional altruism and selfishness disappears. For positive values of U_b, $\partial U_x / \partial v_{xb} > 0$. As the utility level of other players or the weight of the non-return effect increases, U_x increases also.

3.2. Partial Effects of a_x and a_b

If the partial derivative of v_{xb} with respect to a_x and a_b is taken, we get

$$\partial v_{xb} / a_x = \frac{1 + D_x}{(1 + D_x)^2} \quad (22)$$

$$\partial v_{xb} / a_b = \frac{D_x(1 + D_x)}{(1 + D_x)^2} \quad (23)$$

The following two statements hold positive implications. Firstly, an increase in the level of unconditional altruism would result in an elevated level of player x's sensitivity towards the benefits of other players. Secondly, if player x perceives that other players hold a more favorable opinion of her, her sensitivity towards the benefits of other players will also increase. But if player x does not care about other players' opinions about her, then $\partial v_{xb} / a_b = 0$. In this case, the effect of other players' opinions on the utility equation disappear.

The results demonstrate how altruism and reciprocity impact the utility function. If $D_x = 0$, all impact comes from unconditional altruism. If $v_{xb} > 0$ player x is happy with this situation as long as the utility totals of the other players are positive.

3.3. Equilibriums With Conformism and Behavioral Dimension

The results can be summarized:

Table 2: Behavioral Dimension and Equilibriums'

Status	**Effect to the Stability**
Player x is unconditionally altruistic and does not care about other players' opinions.	If $x > 1/2$ for stability it is required $w_t^x > w_t^y$, $\rho > \varphi$ and $\sum_b U_b > \sum_c U_c$ (according to ϑ)
Players x and y are unconditionally altruistic and do not care about other players' opinions while conformism is full-dominant	If $x > 1/2$ for stability it is required $(2\rho - 2\varphi) > [\sum_c U_c - \sum_b U_b]$. Player y gains a benefit from her unconditional altruism. The conformist effect must exceed this benefit in favor of strategy x.

Player x is unconditionally altruistic, cares about other players' opinions, player y is unconditionally selfish, cares about other players' opinions	If $x > 1/2$ for stability it is required $(2\rho - 2\varphi) > [\sum_b U_b - \sum_c U_c]$. The conformist effect in favor of strategy x must exceed the benefit that player y gains from unconditional selfishness.
Utility functions are only return-oriented	If $x > 1/2$ for stability it is required $w_t^x > w_t^y$.
Player x and y are unconditionally altruistic, utility functions are depend on both return and conformism	If $x > 1/2$ for stability it is required $\left(w_t^x - w_t^y\right) + ([(2\rho - 2\varphi)]) > ([\sum_c U_c - \sum_b U_b])$. In this case, the payoff and conformist advantage of strategy x should not be balanced by the unconditional altruism of player y.
Player x is unconditionally altruistic, player y has a sense of reciprocity, utility functions are depend on both return and conformism	If $x > 1/2$ this time, the important issue for stability is the player's belief in what other players think about her, with a sense of reciprocity. Stasis is achieved if player y thinks that other players have a negative opinion of her. Detailed analysis can be made with $\left(w_t^x - w_t^y\right) + ([(2\rho - 2\varphi)]) > \left([\sum_c((D_y a_c)/1 + D_y)U_c - \sum_b U_b]\right)$. Stationarity cannot be achieved if player x's payoff and conformism advantage is balanced by y's sense of reciprocity.
Player x and y have a sense of reciprocity, their utility functions depend on both payoff and conformism	If $x > 1/2$ similar to previous case for stability it is required $\left(w_t^x - w_t^y\right) + ([(2\rho - 2\varphi)]) > \left([\sum_c((D_y a_c)/1 + D_y)U_c - \sum_b((D_x a_b)/1 + D_x)U_b]\right)$. The payoff and conformist advantage should be greater than the advantage that the sense of reciprocity provides to the y.
Players x and y have a sense of full reciprocity, their utility functions depend on both payoff and conformism, player x thinks the other players' negative opinions about her, and y thinks the opposite	If $x > 1/2$ for stability it is required $\left(w_t^x - w_t^y\right) + (2\rho - 2\varphi) > ([\sum_b U_b + \sum_c 0.5U_c\ ])$. Player x must have a higher payoff and conformist advantage than the combined benefits of the other players. If we agree that this situation is hard to achieve, then we have to say it's almost impossible to reach a stable equilibrium under the conditions we studied.

Players are completely conformist influence oriented, x is unconditionally selfish, y has a sense of reciprocity, both players fully attach importance to the opinions of other players.	If $x > 1/2$ for stability it is required $([(2\rho - 2\varphi)]) > \left(\left[\sum_c \frac{a_b}{2} v_{yc} U_c\right] + \sum_b 0.5 U_b\right)$ If most people conform and support strategy x in all matter, the balance is stable.

Source: Prepared by the authors.

3.4. A New Replicator Dynamic with Conformism and Behavioral Dimension

We've already examined the effect of conformism and behavioral dimension, ϑ, in the previous section. But now we'll concentrate on it's importance in the survival of unsuccessful strategies.

Players update their preferences and thoughts based on two pieces of information:

- Payoff of the player with the relevant strategy compared to the player with the opposite preference (strategy *x* and *y*)
- Frequency of strategies in the population and behavioral dimension (independent of returns)

Now, let us define a new fraction that prefers strategy *x* due to non-return (conformism, ρ, and behavioral aspect, including $\sum_b v_{xb} U_b$ effects. It will be referred as η. Now, we'll derive a new replicator dynamic. But first, non-return effects need to be rewrited.

$$r_x = {}^1/_2 \left[\vartheta(\eta - \zeta) + (1 - \vartheta)(w_t^x(\eta) \; - w_t^y(\eta))\right] \quad (24)$$

$$r_y = {}^1/_2 \left[\vartheta(\zeta - \eta) + (1 - \vartheta)(w_t^y(\eta) - w_t^y(\eta))\right] \quad (25)$$

Here r_x and r_y typify the amount of agents who select strategies *x* and *y* respectively. The model only looks at how two strategy multiply. Thus 1/2 is an arbitrary amount, making the model easier to understand. The compliance coefficient determines whether it is more beneficial to change strategies or stick with the it according to interactions. This decision is based on the return-oriented nature $(1 - \vartheta)$ or the conformist and non-return effect (ϑ). ζ is the fraction that can not be affected by non-return dimensions.

While η represent the fraction that prefer strategy *x* with non-return motives, $w_t^x(\eta)$ and $w_t^y(\eta)$ show the yields of the strategies *x* and *y* respectively. Players can update their thoughts and preferences at any time. We will use the replicator dynamics from the base model to analyze how many people use strategy *x* with non-return motives over time.

$$\Delta\eta = \eta^{t+1} - \eta = \beta\eta(1 - \eta)\xi\left(r_x - r_y\right) \quad (26)$$

The probability of a player with strategy x matching a player with the opposite strategy is shown by $\beta\eta(1 - \eta)$. When η has extreme values, this match is unlikely. $\xi\left(r_x - r_y\right)$ shows the speed of spread. Here ξ is the sensitivity to the difference between return and non-return effects.

The model involving the non-return effect can be summarized. The player who prefers strategy x through some non-return aspect, changes the strategy she adheres to with probability $\xi(r_x - r_y)$. According to equation 26, there are four equilibria as $\eta = 0,\ \beta = 0,\ \xi = 0$ and $r_x = r_y$. Necessary condition for the last equilibrim can be written as

$$\vartheta(\eta - \zeta)/_{1-\vartheta} = w_t^y(\eta) - w_t^x(\eta) \quad (26.1)$$

The equation is satisfied by η, which represents the evolutionary steady state. The evolutionary stationary η, balances the effects of the non-return process and the effect of the yield difference. Copying strategy *x* is favored by non-return (fraction η) pressures, but it is offset by the advantage of return strategy *y*. To keep η stable, the derivative of the replicator dynamics with respect to it must be negative. To achieve this, the following inequality has to be met.

$$\vartheta < (1-\vartheta)\left(\frac{\partial w_t^y(\eta)}{\partial \eta} - \frac{\partial w_t^x(\eta)}{\partial \eta}\right) \quad (26.2)$$

Strategies that generate low returns may persist and be resilient in the presence of non-return pressure. Hence, population structure can be shaped by the desire to adapt or behavioral dimension.

Statements 26.1 and 26.2 show an important result. If a dominant part of players who adopted strategy *x* made their choice motivated by non-return effects, as long as the payoff advantage of the opposite strategy does not exceed $\vartheta(1-\zeta)/_{(1-\vartheta)}$, strategy *x* will continue to be an evolutionary stable equilibrium. Even if strategy *y* earns more than strategy *x*, steady state is reached when non-return effects balance the difference in returns. This important result is visualized in Figure 2.

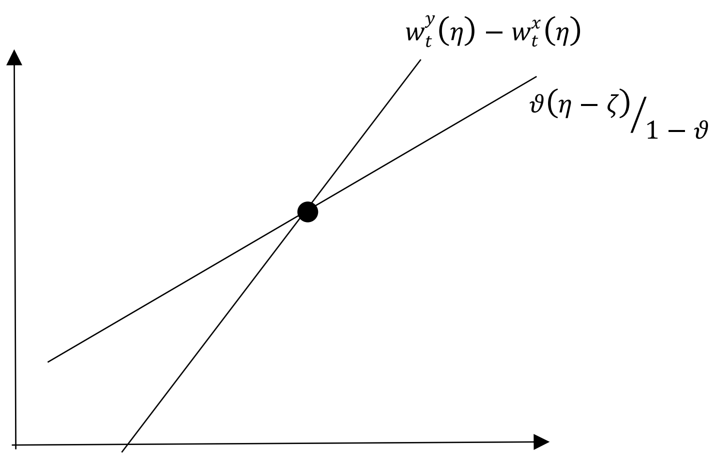

Figure 2 : Equilibrium η fraction
Source: Prepared by the authors, adopted from Bowles (2006).

This figure apparently and clearly emphasizes that the evolutionary stable point is achieved with one strict rule, yield effect should be equal to non-return effect between opposite strategies. So, it is clear that, yield-low strategies can survive in investment market.

4. Conclusion

The present research findings unequivocally demonstrate that individuals who exhibit unconditional altruism are more likely to adopt low-return strategies. These individuals hold the belief that their own utility level is positively correlated with the benefit levels of those with whom they interact. Consequently, they persist in their preference for the same strategy in the subsequent period.

In reciprocal situations, players' behavior is directly influenced by their assumptions about the thoughts of other players. Specifically, players' behavior is shaped by their assumptions about what others are thinking about themselves.

A player with a sense of reciprocity, who believes that other players have a negative opinion of her, subtracts the sum of these players' utilities from her utility function. Conversely, the total benefit of the other players is included in the utility function as positive. In contrast, unconditionally selfish players only interact and determine strategies that are focused on their own benefits.

If the players in this context, the investors, behave in a return-oriented manner or are selfish, the only condition for an evolutionarily stable investment strategy is that the entire group favors the same investment strategy. Therefore, an evolutionarily stable investment strategy that is solely return-oriented seems unlikely. Another reason for this outcome is the players who prefer to continue with a low-return investment strategy, not for a high return.

Striking a balance between the advantage of return and the feeling of unconditional altruism or reciprocity ensures the survival of low-return strategies. This also accounts for individuals who earn less than their loved ones but are content with their success. These findings underscore the importance of the behavioral (non-rational) preferences of humans, who are social beings.

Achieving a return-oriented evolutionary balance requires specific factors, including sensitivity to yield differences, the absence of systematic error, and rational expectations for the future. However, humans are prone to making mistakes, have strong emotions, and are driven by impulses. This dimension is the primary reason why the investment strategy that provides the highest return, the strategy that will capture the whole market, remains purely theoretical. In a market where some individuals make mistakes and non-return processes affect them, an investment strategy that captures the entire market and always generates a profit is unrealistic.

Let us summarize the main contributions of this paper to literature;

- The research reveals a significant insight: while some approaches may not result in substantial gains, they may still be favored by the public. This implies that the triumph of the most advantageous strategy is not a certainty and is influenced by various factors.
- Additionally, the research underscores the reality that humans do not make purely logical decisions but are influenced by emotions and social factors. By applying evolutionary game theory to this context, we can achieve a more precise comprehension of human behavior and decision-making.
- Moreover, the study's findings have significant implications for the analysis of financial markets, revealing why stock prices and investment markets may not reach equilibrium values over time. Nevertheless, it's essential to note that this specific topic falls outside the scope of the research, and additional studies are required to investigate these intricate dynamics fully.
- Our findings, through the use of evolutionary game theory tools, suggest that return-oriented equilibria can only be achieved in stock markets that are dominated by rational investors. Although our study does not disprove the existence of rational investors or the stock market itself, it does raise questions about the likelihood of such an equilibrium. The evaluation of an ideal stock market's efficiency can only be conducted in a theoretical manner due to inherent limitations.
- The efficiency and perfection of a stock market can only be evaluated theoretically, for the same reasons.
- The present study endeavors to explore the implications of non-return effects on strategic changes within a theoretical framework.
- The crucial issue at hand is the decision that people have to make between acquiring material possessions or pursuing spiritual fulfillment.
- It is important to note that our study stands out from others in its emphasis on the achievement of global evolutionary stasis through pure financial return, as opposed to local evolutionary stasis, which is achieved through the weight of non-return effects.

Our study aspires to introduce evolutionary game theory tools which have been relatively underutilized in Turkey, with the aim of highlighting their potential for future research endeavors. Our overarching objective is to demonstrate the efficacy of these tools in the domain of game theory and pave the way for future research in Turkey. We have every confidence in our ability to accomplish this goal and hope that our work will prove to be a valuable contribution to the field.

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