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Influence of Leg Geometry on the Performance of Bi_2Te_3 Thermoelectric Generators

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Highlights

- The best performance has been obtained in the square-leg structure in TEM.
- Insulation and conductors have played a very important role in TEM performance.
- The use of rectangular prism-shaped thermoelectric pairs provides ease of manufacturing.

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Abstract

This study analyzed the significant performance using COMSOL Multiphysics software of thermoelectric modules (TEMs) fabricated from aluminium oxide (Al_2O_3), copper (Cu), and bismuth telluride (Bi_2Te_3) materials, with a particular focus on investigating various leg geometries. The TEM design had Al_2O_3 for insulation, Cu for conducting, and Bi_2Te_3 for TE legs among the Cu. Investigated the influence of square and rectangular TE legs with heights of 2.0, 2.75, and 3.5 mm on critical parameters such as the normalized current density, electric potential, temperature gradient, and total internal energy within the TEM. Furthermore, the impact of varying thicknesses in the insulator and conductor layers of the TEM was explored. The results consistently demonstrated that the square leg geometry, particularly when configured with a height of 2.75 mm, outperformed other leg geometries. Consequently, it is suggested to adopt a square-shaped Bi_2Te_3 TEM measuring 1 mm × 1 mm × 2.75 mm with a 0.50 mm Al_2O_3 thickness and 0.125 mm Cu thickness during the manufacturing process. Investigate how temperature differences in TE device leg design are influenced by parameters such as the Seebeck coefficient (S), thermal conductivity (k), and electrical conductivity (σ). At lower temperatures, modeling reveals lower electrical conductivity and enhanced thermal conductivity, highlighting the significance of $S = \pm 2.37\times10^{-4}$ V/K. This illustrates the high potential of TEM for applications in thermoelectric generator (TEG) manufacturing.

1. INTRODUCTION

In recent years, there have been a growing number of research endeavors aimed at addressing the pressing challenge of reducing our dependence on fossil fuels and identifying environmentally friendly alternative energy sources. Among these initiatives, the field of direct thermal-to-electrical energy conversion using thermoelectricity stands out as one of the most promising areas of investigation with respect to both waste heat management and energy efficiency. The remarkable potential of this technique to contribute to air conditioning and electrical power generation while causing minimal environmental harm has captured the attention of scientists worldwide. TE materials have emerged as a highly desirable choice for electricity generation due to their unique ability to convert heat energy into electrical power. Currently, researchers are actively exploring various materials characterized by intricate crystal structures and exciting thermal properties [1]. Bi_2Te_3 stands out as a unique material in TE applications due to its low thermal conductivity combined with its remarkable electrical conductivity, which makes it a perfect option for TEGs [2]. In fact, Bi_2Te_3 ranks as the most commonly used TE material in the field of electrical power generation [3]. Our study has focused on developing and evaluating Bi_2Te_3 and materials made from it by nanostructuring for

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different TE uses, as described in prior publications [4–8]. The effectiveness of using TEGs to generate power is significantly dependent on both the characteristics of the TE materials and the particular operating circumstances [9]. TEGs featuring Bi_2Te_3 semiconductors find exceptional utility as stable and reliable power sources in different applications, such as aerospace probes, remote sensors, and automobile waste heat recovery systems [10]. In a TEM, the creation of a temperature differential between n-type and p-type semiconductor surfaces in TE materials. gives rise to a voltage across the two semiconductor ends. This transformation of a temperature gradient into an electrical voltage, facilitated by these TE materials, is known as the Seebeck effect. Bi_2Te_3 semiconductor materials exhibit significantly higher voltage production compared to other TE semiconductor materials [11]. Furthermore, Bi_2Te_3 is readily available and cost-effective, rendering it an exceptionally appealing choice for large-scale energy harvesting applications [12]. The distinctive blend of electrical and thermal properties inherent to Bi_2Te_3 positions it as a promising material for integration into TEGs.

Efficient conversion of electrical energy can be achieved in TEGs through the strategic connection of multiple TEMs in series and/or parallel. In recent years, extensive research efforts have been dedicated to developing precise models for TEGs at the device level [13]. The Seebeck effect is inherent in TEGs, which, as solid-state devices, enables them to harness temperature gradients for the generation of electric power. This effect allows the conversion of temperature variations into electrical energy, making TEGs an efficient solution for applications involving temperature differentials [14]. The TE effect, on the other hand, signifies the direct conversion of a temperature gradient into an electric voltage and vice versa, facilitated by a thermoelement [15]. Generally, the properties of materials employed in TEG fabrication are contingent upon the operating temperature. The utilization of numerical simulations and investigations within COMSOL Multiphysics reveals significant potential for advancing research and applying TEG technology across diverse industries. We used a computer program called COMSOL Multiphysics to do 3D simulations, which helped us understand things better and has been instrumental in determining the optimal dimensions of TEMs [16]. Some studies have looked into the physics and form of TE materials, examining how the height and shape of legs affect the performance of TEMs [17–19]. For nearly seven decades, materials based on Bi_2Te_3 have consistently led the way in achieving high figures of merit (ZT) at room temperature [20]. Notably, a critical consideration in TEG design revolves around the choice of semiconductor, as it determines the maximum operating temperature of the TEG [21]. Furthermore, the length and shape of the TE elements employed impact the electrical resistance of the TEG [22]. To enhance the performance of TEGs, it is crucial to minimize their internal resistance, thereby increasing the electrical power output while reducing power consumption [23]. Another factor contributing to this resistance is contact resistance, which should ideally be minimized as well [24]. The thermal conductivity within TEGs varies with temperature, which is desirable. It is important that this thermal conductivity remain lower than that of the enclosure material [25]. Notably, the shape of the TE legs can significantly influence both thermal and electrical conductivity, and it is essential to optimize the leg shape to achieve high performance for the TEG.

We conducted this study using COMSOL Multiphysics software to carefully analyze how the size of TE legs affects heat transfer and the formation of temperature differences in TEMs. Furthermore, an optimal solution has been derived by taking into account the influence of insulator and conductor thicknesses on Bi_2Te_3-based TEMs. The TE characteristics of Bi_2Te_3-based TEMs have been computed employing the heat transfer module within COMSOL Multiphysics software. Subsequently, significant findings have been established concerning the ideal leg shape, height, conductor thickness, and insulator thickness, which can be adopted by manufacturers when incorporating Bi_2Te_3 TE materials into their TEG/TEC structures [26]. The study begins by elucidating the importance of leg shapes and sizes for TEG manufacturers and provides an overview of previous research on these topics. This study examines the influence of leg geometry on heat transfer and temperature differences in Bi_2Te_3. Previous studies have only looked at leg height or shape individually, but our research considers both factors simultaneously using this software. By finding optimal solutions that consider insulator and conductor thicknesses, our study offers valuable insights for TEG manufacturers looking to improve device performance. This comprehensive approach sets our research apart and provides a deeper understanding of the factors that affect TEG performance. Following this, the fundamental criteria underlying the proposed leg shapes and sizes are detailed in the materials and methods section (Section 2). Subsequently, the results obtained in the other section (Section 3) are presented and analyzed, leading to conclusive insights based on the determined values. The objective of this study is to investigate the performance of Bi_2Te_3 in TEM and optimize the design by investigating leg geometries and material configurations. The software is used to assess the impact of different factors. The paper also proposes optimized parameters to improve performance in TEG manufacturing. Through detailed simulations and analysis, this research offers valuable insights for TEG manufacturers looking to enhance their energy harvesting systems and address the challenge of reducing reliance on fossil fuels.

2. MATERIAL METHOD

The process involves using alumina as insulators, copper as conductors, and Bi_2Te_3 as TE legs between them. The characteristics of the materials used in the simulation are shown in Table 1.

Table 1. *The properties of the materials were utilized for simulation*

Materials	Seebeck coeff., S (V/K)	Thermal cond., k (W/(m×K))	Electrical cond., σ (S/m)	Density, ρ (kg/m^3)	Spec. heat cap., Cp (J/(kg×K))
Al_2O_3	0.0	27	0.0	3900	900
Cu	0.0	400	5.998×10^7	8700	385
p-type Bi_2Te_3	S (T)	k (T)	Sigma (T)	7700	154
n-type Bi_2Te_3	–S (T)	k (T)	Sigma (T)	7700	154
T→Temperature dependent; cond.→ conductivity; coeff.→ coefficient; Spec. heat cap.→ Specific heat capacity					

This study examined how factors like the height and shape of Bi_2Te_3 TEM legs, thickness of alumina and copper layers, impact electric potential, temperature gradient, the current density normalized, and whole internal energy. 3D numerical models were conducted by means of COMSOL Multiphysics to accomplish the experiment, with Figure 1 illustrating the typical geometry for the Bi_2Te_3 TEM.

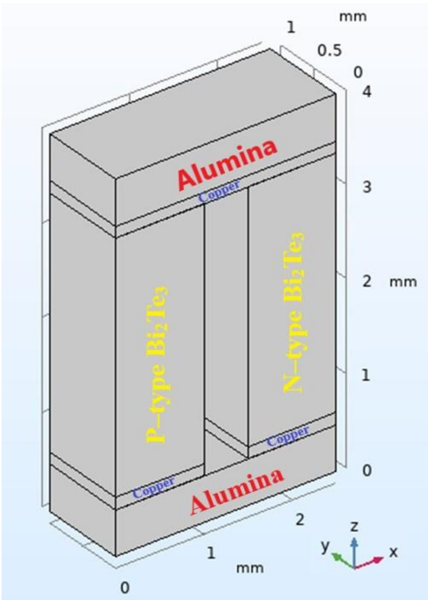

Figure 1. *The typical geometry for the Bi_2Te_3 TEM is 1 mm × 1 mm × 2.75 mm, incorporating a copper layer thickness of 0.125 mm and an alumina thickness of 0.50 mm*

The equations (for heat transfer in solids) utilized in the COMSOL multiphysics simulation according to the approach using finite elements are:

$$\rho C_P u \nabla T + \nabla q = Q + Q_t \, , \tag{1}$$

where ρ indicates material density, C_P indicates specific heat, u indicates velocity, Q indicates heat source, Q_t indicates thermoelastic effects, q indicates heat flux in conduction, and T indicates temperature. The heat flux in conduction is provided below:

$$q = -k\nabla T + JST\ , \tag{2}$$

where k denotes thermal conductivity, J the induced electric current density, and S the Seebeck coefficient. The equations for electric currents are described below:

$$\nabla J = Q_j\ , \tag{3}$$

$$J = \sigma E + J_e\ , \tag{4}$$

$$E = \nabla V\ , \tag{5}$$

where $Q_j$represents the current source; σ is the electrical conductivity; J_e represents the external current density; E is the electric field; and V represents the electric potential. The following equations describe thermoelectric effects:

$$q_p = PJ\ , \tag{6}$$

$$P = ST\ , \tag{7}$$

$$J_e = -\sigma S\nabla T\ , \tag{8}$$

where q_p represents the Peltier heat or power; P represents the Peltier coefficient. When there is a temperature difference between the TEG's hot and cold surfaces, it generates an open circuit voltage, as seen below:

$$V_{oc} = S\Delta T, \tag{9}$$

where V_{oc} stands for the TEG's open circuit voltage. When the load resistor is connected to the TEG, the power generated by the TEG is expressed as follows:

$$P_{out} = \left(\frac{V_{oc}}{R_L+R_{in}}\right)^2 R_L, \tag{10}$$

where the internal resistance of the TEG is represented by R_{in}, in, while R_L indicates its external load resistance. The TEG generates its maximum power when the values of R_L = R_{in}. The term used for the heat flow on the hot side of the TEG is:

$$q_{in} = K\Delta T + SIT_h - \frac{1}{2}I^2R, \tag{11}$$

where T_h indicates the temperature of the hot side of the TEG, whereas I indicates the electric current.

3. RESULT AND DISCUSSION

3.1. Influence of Leg Height

A TEM's height withinthe legs is able to have a significant influence on how well it works in a lot of different techniques. If the legs are overly short from the perspective of electrical conductivity, it leads to an increase in resistance between the two sides of the TEG [27]. This rise in resistance diminishes the module's capability to convert a temperature gradient into electrical energy, resulting in reduced efficiency. Conversely, if the legs are too tall, it can limit heat flow between the two sides, reducing the module's thermal conductivity. This can lead to suboptimal power generation. Additionally, the height of a TEM's legs affects its mechanical stability. Short legs may not support mechanical stress well, while tall legs can make the module too rigid to adapt to pressure or temperature changes. The perfect height for a TEM's legs depends on different situations, like the materials used, its purpose, and the performance standards required. This study specifically explores how various leg heights (2 mm, 2.75 mm, 3.5 mm) affect a Bi_2Te_3 TEM measuring 1×1 mm². The copper conductor size is 0.125 mm, while the insulator consists of 0.50 mm alumina. The way a Bi_2Te_3 TEM reacts to a 2 mm leg height depends on various factors, like the module's material properties, temperature, and the state of electrical and thermal loads. When a consistent current of 4.725 A as well as a load resistance of 0.42 Ω are applied across leg heights, the difference in potential between the hot and cold sides is approximately 0.29 V each. This potential greatly affects the temperature difference inside the TEM, which drives the thermoelectric effect. As the potential difference increases, so does the disparity in temperature between the heated and chilled areas, resulting in increased production of electricity. Potentially, electrically, this can influence the thermoelectric material's Seebeck coefficient and electrical conductivity in the module. When the electrical potential increases, the Seebeck coefficient may decrease while the electrical conductivity rises, which could affect the whole performance of the TEG [28]. Furthermore, the heat transfer of the module can be affected by the electric potential. If the electric potential increases, more heat may pass through the module, affecting the distribution of temperature and, consequently, the overall performance of the TEG. It is important to note that, depending on the electric potential, the temperature difference between the hot and cold surfaces could range from 20 to 246°C. Analyzing the application needs and operating conditions thoroughly is crucial for optimizing TEG performance. A temperature gradient of 20–246°C is considered moderate and achievable with Bi_2Te_3. This level of temperature gradient may be well-suited for specific applications of TEGs, which demand moderate cooling capacity or power output. Such applications include thermal management in electronic devices or waste heat recovery systems in automobiles. It is crucial to understand that a TEG's performance is greatly influenced by how the variations in temperature throughout the module relate to the potential it produces. Additionally, changes in temperature and material characteristics within the module affect the current density in a TEM. The current density ranges from 143 to 4.08×10^7 A/m^2, depending on the electric potential. To achieve higher current densities, reducing electrical resistance in the module and using materials with better electrical conductivity are essential. The achievable current density of a TEG mostly depends on its design and material properties. It is important to mention that the total internal energy of a TEM is indirectly affected by the electric potential it receives, as this impacts both the electrical current and the disparity in temperature. The disparity in temperature throughout the entire module, along with the materials' electrical resistance and Seebeck coefficient, are the primary factors affecting the total internal energy. The highest total internal energy, considering electric potential, was nearly 7000 J/kg. Even though leg height may not have an immediate impact on the module's internal energy, leg height should be taken into account when evaluating other performance factors to maximize the TEG's overall efficacy. Figure 2 illustrates a typical performance assessment of different Bi_2Te_3 TEM properties. When maintaining the leg height at 2.75 mm and keeping the constant load resistance and applied current, the potential ends of the TEMs hot as well as cold vary from -2.61×10^{-3} to 0.39 V. Consequently, there's a temperature difference of 20 to 300°C between the hot and cold surfaces, while the normalized current density varies from 75.4 to 4.08×10^7 A/m^2. Remarkably, the highest total internal energy recorded for the TEM is approximately 46000 J/kg, depending on the electric potential. Bi_2Te_3 might offer a reasonably adequate temperature gradient within the range of 20 to 300°C. However, achieving and maintaining such a temperature gradient might necessitate more advanced TEG designs, such as cascaded TEGs or segmented TEGs [29]. Managing temperature fluctuations across such a wide range poses significant challenges in terms of ensuring optimal performance, reliability, and efficiency. This temperature gradient could find application in TEG scenarios demanding elevated cooling capacity or power output, such as solid-state refrigeration or power generation from waste heat in industrial processes. Considering the same load resistance, current, and leg height of 3.5 mm, it was observed that the potential throughout the hot and cold sides of the TEM varied between -8.05×10^{-3} and 0.47 V. This generated a temperature difference spanning from 20 to 481°C between the two sides, while the normalized current density varied between 18.2 and 4.07×10^7 A/m^2. Also, the highest total internal energy of the module reached approximately 83000 J/kg, which was linked to the electric potential. The leg height stands out as a pivotal factor in TEG design. Longer legs, while potentially increasing thermal resistance, can conversely reduce the overall performance of the TEG [30]. Hence, when constructing a TEG for a specific purpose, it is essential to consider the trade-offs between overall efficiency, thermal resistance, output voltage and current, and leg height. Additionally, the temperature gradient of 20 to 481 °C represents a considerably high range, which may necessitate the employment of more advanced TE materials like silicon, germanium, and skutterudites. These materials serve as potential alternatives to Bi_2Te_3. The ZT value of silicon germanium is higher at higher temperatures, making it suitable for electricity production in vehicle exhaust systems [31]. On the other hand, ZT of skutterudites is higher at lower temperatures. For this reason, these are more suitable for medium-temperature waste heat. But it is essential to acknowledge that both of these alternative materials are less readily available and come at a higher cost compared to Bi_2Te_3.

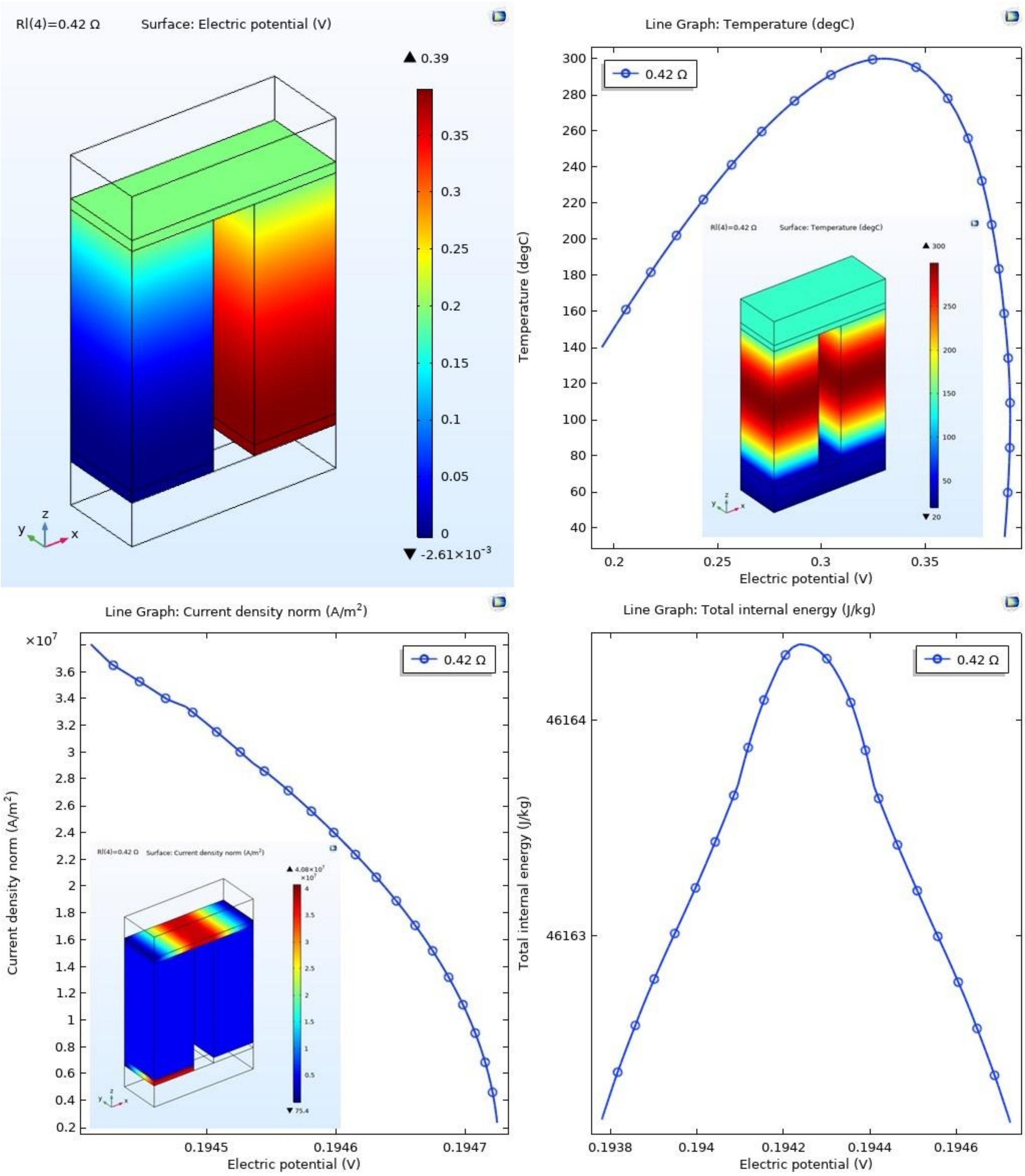

Figure 2. *Assessing the suitability of various parameters for the* Bi_2Te_3 *TEM*

Bi_2Te_3 remains popular in TEG applications due to its reliability, affordability, and availability, despite having a lower *ZT* value compared to other materials. Table 2 represents a detailed assessment of the performance of different factors for the TEM with various leg dimensions.

Table 2. *Performance evaluation of Bi_2Te_3 TEM at different leg heights*

Height of the legs (mm)	Gradient of temperature (ºC)	Electric potential (Volt)	Current density normalized (A/m^2)	Total internal energy (J/kg)
2.00	20 ~ 246	~0.29	143 ~ 4.08×10^7	~ 7000
2.75	20 ~ 300	~0.39	75.4 ~ 4.08×10^7	~ 46000
3.50	20 ~ 481	~0.47	18.2 ~ 4.07×10^7	~ 83000

The best TEG leg height depends on the intended functionality goals, the TE legs' material properties, and the feature of the utilization. Typically, the leg heights of the TEs are kept between 1 and 3 mm to ensure that the TEG output voltage and output current are balanced. In light of the foregoing discussion, it is advisable that, for the purpose of TEG manufacturing, the optimum leg height for a square (1×1 mm^2) Bi_2Te_3 TEM stands at 2.75 mm. This particular leg height has been determined to generate a medium temperature difference, rendering it well-suited for TEG applications that necessitate medium cooling capacity or power output [32]. The standard geometry of a Bi_2Te_3 TEM, featuring a 2.75 mm leg height, is selected for several significant reasons. This specific leg height strikes a balance crucial for TEG performance. Firstly, it provides ample material volume for efficient TE conversion, facilitating the generation of essential temperature gradients within the TE material. Secondly, it contributes to the TEG's overall size and dimensions, rendering it suitable for space-constrained applications and emphasizing miniaturization. Furthermore, this leg height aligns optimally with the material properties of Bi_2Te_3, enhancing its thermoelectric performance. Lastly, it serves as a practical height for manufacturing, ensuring both cost-effectiveness and reliability during mass production.

3.2. Influence of Different Sizes or Shapes

The performance of a Bi_2Te_3-based TEM with alumina insulator layers and copper conductor layers is significantly impacted by the TE layer's dimensions and structure. The size of this layer is pivotal; a larger size can capture more heat, potentially resulting in a higher temperature differential and increased power generation. Nevertheless, a larger layer may also introduce higher electrical resistance, affecting overall efficiency. The shape of the TE layer is equally critical. An optimized shape enhances heat transfer, ensuring efficient conversion of heat into electricity. Irregular or inefficient shapes can lead to temperature variations and decreased overall performance. Balancing the thickness of the TE layer is vital since it affects both heat flow and electrical resistance. Finding the right thickness is essential for maximizing performance. Furthermore, ensuring effective contact between the TE layer and the copper conductor layers is necessary for proficient electrical conductivity and heat transfer. Maintaining high material purity in the Bi_2Te_3 layer is crucial to minimizing TE losses, and the alumina insulator layers should possess excellent thermal insulation properties to prevent heat loss and maintain a high temperature gradient within the TE layer. In summary, optimizing the size, shape, thickness, material purity, and thermal insulation properties of the TE layer is imperative for maximizing the performance of this TEG configuration.

3.2.1. Influence of different leg shape

The form of the legs is vital for TEG manufacturing and significantly affects TEG performance. The geometry of the TEM legs determines both the area of cross-section and the region of contact between the metal electrodes and the TE materials [33]. Hence, the Seebeck coefficient, total efficiency, and electrical and thermal conductivities of the TEG could be influenced. Therefore, it is crucial to optimize the square leg form with a changing cross-sectional area to enhance the TEG's conversion of energy efficiency and meet the intended performance. The research thoroughly examines the effect of square leg forms, including 0.75×0.75 mm^2, 1.00×1.00 mm^2 (as mentioned before), and 1.25×1.25 mm^2, on TEM performance metrics. The choice of leg shape is influenced by the temperature difference, material characteristics of the TE semiconductors, and application areas. So, optimizing leg forms is important in TEM design to increase overall performance and efficiency. This study evaluates the performance of a Bi_2Te_3 TEG with a cross-sectional size of 0.75×0.75 mm² and a leg height of 2.75 mm. During the TEG evaluations, both the load resistance and the applied current were constant, keeping the potential difference between the heated and cooled surfaces at approximately 0.65 V. The range of the normalized current density was between 172 and 5.56×10^{7} A/m^2, while the disparity in temperature between these two sides varied from 20 to 914°C. The maximum total internal energy observed reached approximately 10^5 J/kg concerning the electric potential. For a TEG operating at such elevated temperatures, larger leg dimensions are preferable as they enhance heat transfer and electrical conductivity, contributing to improved TE performance. Smaller leg dimensions can result in inefficient heat transfer and restricted electrical conductivity, adversely affecting the generator's capacity to convert temperature disparities into electricity. It is worth mentioning that smaller dimensions also deviate from the practical application of Bi_2Te_3 due to its high melting point, which is approximately 585°C (where the temperature of 914°C has lost its physical meaning). Decreasing the size of the legs may reduce their mechanical stability and add complexity to the construction process [34]. An electric potential of roughly 0.23 V was observed across the hot and cold surfaces throughout the TEM experiments due to the maintenance of a constant applied current, load resistance, leg height of 2.75 mm, and cross-section area of 1.25×1.25 mm². The temperature gradient between these surfaces ranged from –6.33 to 81.8°C, while the normalized current density varied from 25.9 to 3.23×10^{7} A/m^2. Remarkably, the total internal energy exhibited a negative value concerning the electric potential. However, it is essential to recognize that increasing the dimensions of the TEM legs can also elevate thermal resistance, potentially diminishing the overall device efficiency. Thus, a trade-off exists between the TEM leg dimensions and their performance parameters. Careful consideration of the application requirements and material properties of the TEG legs is imperative when designing a TEG with larger dimensions.

Choosing the area of a TEM's cross-section is crucial and should align with the features of the application. It will be important to get help from researchers working on this subject to determine the leg dimensions of TEGs with Bi_2Te_3 semiconductors. Because the working areas of TEGs, their operating temperatures, and the maximum temperature values to which their surfaces will be exposed are seen as important criteria in TEG formation [35]. The temperature gradient range provided by each size option for a single TE element impacts the TE performance of the TEG module. Increasing the cross-sectional area typically leads to higher power output, although this may necessitate greater input power. The performances of TEMs depending on different leg shapes are given in Table 3. Based on the previous statements, the best performance values for Bi_2Te_3 TEMs were obtained in the leg geometry with dimensions of 1.00 × 1.00 mm². This choice is recommended because it streamlines the manufacturing process, guarantees uniform material properties, and ensures even heat distribution. The symmetry inherent in this square configuration contributes to the preservation of consistent thermoelectric performance and simplifies integration into diverse applications.

Table 3. *Performance of Bi_2Te_3 TEMs with different forms*

Square shape of the legs (mm)	Gradient of temperature (ºC)	Electric potential (Volt)	Current density normalized (A/m^2)	Total internal energy (J/kg)
0.75×0.75	20 ~ 914	~0.65	172 ~ 5.56×10^{7}	~ 100000
1.00×1.00	20 ~ 300	~0.39	75.4 ~ 4.08×10^{7}	~ 46000
1.25×1.25	–6.33 ~ 81.8	~0.23	25.9 ~ 3.23×10^{7}	–10000

3.2.2. Influence of rectangular leg shape

The rectangular shape of the leg significantly affects the material's thermal and electrical conductivity. When the rectangular shape of the leg's length is considerably higher than its width, heat conductivity along the longer dimension may surpass that along the lower dimension because of material anisotropy [36]. These anisotropic features also impact the device's overall performance, influencing the passage of electrical current and heat transfer. Additionally, the mechanical properties of the leg may be compromised; excessively long and thin rectangular legs are more susceptible to mechanical breakdown, such as buckling or bending under stress, potentially reducing the device's longevity and reliability. Moreover, the shape of the legs can affect manufacturing costs and feasibility. In contrast to square legs, rectangular legs may require more complex machining or processing techniques, which would increase the time and expense of production. When selecting leg shapes, factors such as sample size, the material being examined, and specific application requirements should be considered. For instance, the Bi_2Te_3 TEM's rectangular shape (1 mm × 1.5 mm) should be chosen based on the application's needs and production capabilities. Square-leg TEM production provides manufacturers with better symmetry and alignment. Rectangular leg structures provide a better viewing opportunity. However, square-leg forms are isotropic. Additionally, they have better thermal and electrical characteristics. During the TEM investigations, a constant applied current and load resistance were maintained, resulting in an electric potential of approximately 0.25 V across the surfaces that are hot and cold. The normalized current density ranged between 40.1 and 3.90×10^7 A/m^2, and the temperature gradient between these sides varied from 4.32 to 98.2°C. Remarkably, the total internal energy exhibited a negative value relative to the electrical strength. Table 4 illustrates an assessment of how various leg designs impact specific characteristics of the Bi_2Te_3 TEM. Typically, a TEM with square-shaped legs of the same height is more efficient compared to a module with a rectangular design. This efficiency stems from the square shape's capability to uphold a consistent temperature across the module, enhancing its thermal performance throughout the operation. Additionally, square legs offer ease of manufacturing and assembly, potentially leading to reduced production costs, thereby increasing the accessibility of TEGs for a wide range of applications. Furthermore, square legs contribute to better mechanical stability for the TEG module, mitigating the risk of deformation or bending under specific conditions.

Table 4. *Comparison of the impact of various leg shapes*

Shape of the legs (mm)	Gradient of temperature (ºC)	Electric potential (Volt)	Current density normalized (A/m^2)	Total internal energy (J/kg)
Square	20 ~ 300	~0.39	75.4 ~ 4.08×10^7	~ 46000
Rectangular	4.32 ~ 98.2	~0.25	40.1 ~ 3.90×10^7	–6000

3.3. Influence of Pitch

Pitch is the distance, in TEM units, between adjacent legs that is crucial for its operation. The relative pitch had been systematically altered at distances of 0.25, 0.50, and 0.75 mm, and its influence on the temperature difference was thoroughly examined. Remarkably, the temperature gradient consistently remained within the range of 20 to 300ºC, with no significant alterations attributed to pitch variation. As a result, a pitch of 0.50 mm is recommended as the optimal choice for TEG manufacturing purposes. Figure 3 presents the optimized pitch of the TEM and evaluates various parameters for the Bi_2Te_3 TEM.

3.4. Influence of Conductor Thickness

The conductor layer thickness within a TEG is a critical factor that significantly impacts the TE module's performance [37]. Typically composed of highly electrically conductive materials like copper, the primary role of the conductor layer is to facilitate a low-resistance pathway for electrical current throughout the module. A thick conductive layer used in TEM fabrication has low electrical resistance. This increases conductivity and performance. Increasing the thickness of the conductor causes the thermal resistance to increase. This reduces the TEM's ability to conduct heat. The optimum value of the thickness of this conductor must be determined. The materials, temperature range of operation, and efficiency level parameters are important in determining the optimum thickness value. Throughout the study, the electric potential remained approximately 0.39 V, and the utilized current and load resistance stayed unchanged between the hot and cold sides of a Bi_2Te_3 TEM with a square-shaped leg height of 2.75 mm and a conductor thickness of 0.05 mm. The normalized current density ranged from 10.4 to 9.77×10^7 A/m^2, while the temperature gradient between the hot and cold surfaces changed from 20 to 301°C. At the highest electric potential, approximately 46,000 J/kg of total internal energy was achieved. However, there wasn't much change in either the temperature difference or the electric potential when the conductor thickness in the Bi_2Te_3 TEM was increased to 0.200 mm with a 2.75 mm leg height (square shape). Nonetheless, alterations in conductor thickness did influence the total internal energy and the normalized current density. Table 5 provides a summary of the effect of conductor thickness on various TEM parameters. When manufacturing TEMs, the conductor's 0.125 mm thickness is recommended. This recommendation is based on the careful consideration of balancing effective heat conduction with cost-efficient production. A 0.125 mm conductor thickness provides the necessary thermal conductivity for efficient heat transfer within the module, all while maintaining a thin profile that allows for precise manufacturing processes. This balanced approach ensures both optimal performance and manufacturability.

Table 5. *The influence of conductor thickness on various TEM factors*

Thickness of conductor (mm)	Gradient of temperature (ºC)	Electric potential (Volt)	Current density normalized (A/m^2)	Total internal energy (J/kg)
0.050	20 ~ 301	~0.39	10.4 ~ 9.77×10^7	~ 46000
0.125	20 ~ 300	~0.39	75.4 ~ 4.08×10^7	~ 46000
0.200	20 ~ 300	~0.39	29.6 ~ 2.67×10^7	~ 6000

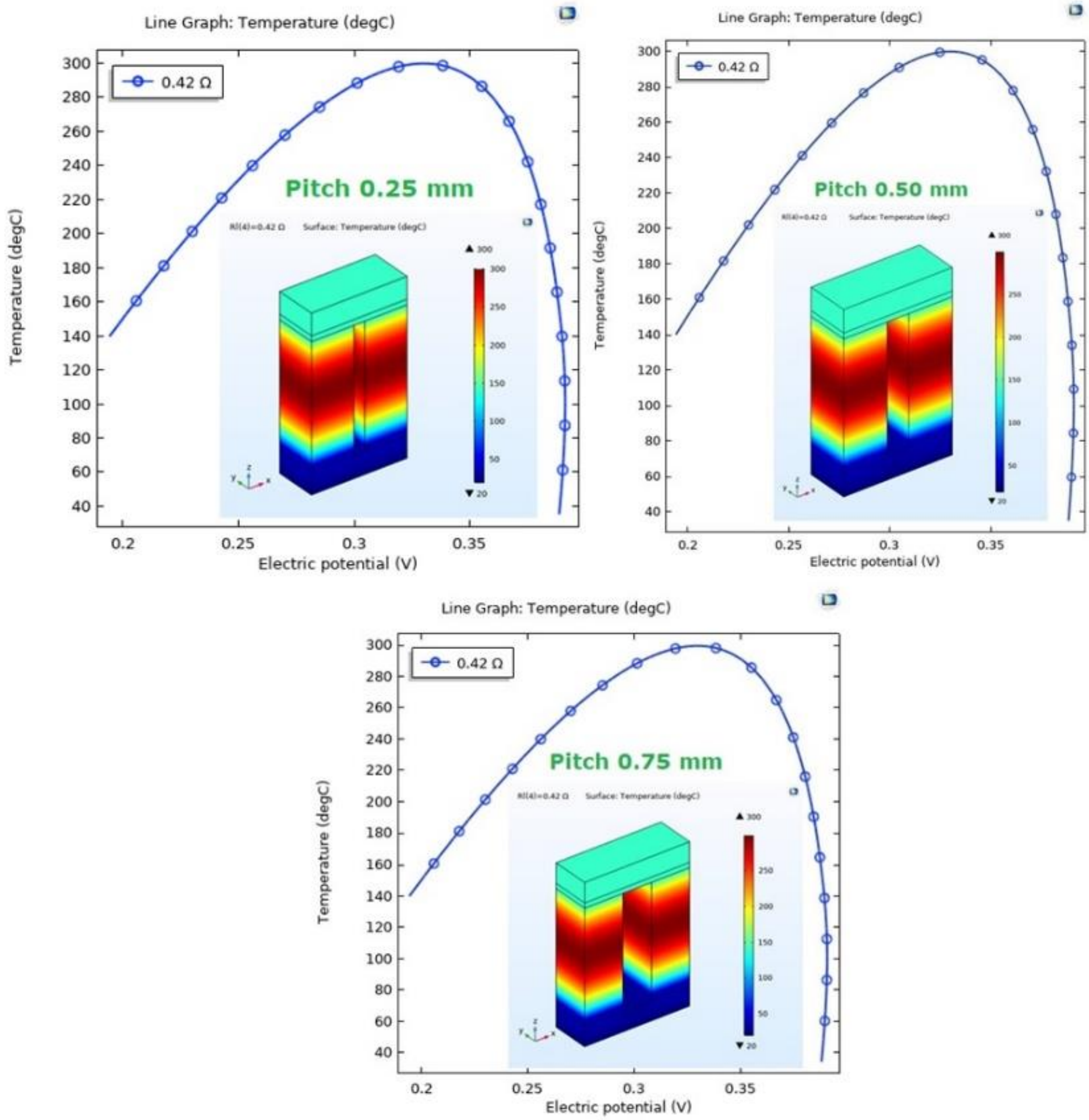

Figure 3. *Influence of pitch on temperature gradient for the* Bi_2Te_3 *TEM*

3.5. Influence of Alumina Thickness

The TEM's insulator layer thickness is a critical factor that significantly influences the functionality of the module. These insulator layers typically consist of alumina, a substance that conducts heat poorly, and their primary purpose is to inhibit the direct flow of thermal energy from the module's hot side to its cold side

[38]. When insulator layers are thicker, they exhibit higher thermal resistance, effectively impeding heat transfer. It is important to note that raising the TEM's electrical resistance also increased the insulator layer's thickness, potentially reducing its overall efficiency. For a Bi_2Te_3 TEM with a square-shaped leg height of 2.75 mm, the conductor's 0.125 mm thickness, and an insulating layer's thickness of 0.25 mm, the potential voltage difference between the module's hot and cold sides amounted to 0.39 V. The resistance of the load and the applied current remained constant throughout the study. The temperature variance between hot and cold surfaces varied between 20 and 296°C, with current density normalized ranging between 30.1 and 4.10×10^7 A/m^2. At the highest electric potential, approximately 45,000 J/kg of total internal energy was achieved. However, there was no noticeable change in the temperature difference, electric potential, or whole internal energy when the insulator thickness was increased to 0.75 mm. However, changes in the insulator thickness did affect the normalized current density. Table 6 offers an overview of how the insulator thickness affects various TEM parameters. It is recommended to use an insulator thickness of 0.50 mm for TEM production. This recommended insulator thickness achieves a harmonious equilibrium between efficient electrical insulation and mechanical stability. At 0.50 mm, it delivers sufficient electrical isolation between the TE legs while also upholding the structural integrity of the device. This practical choice promotes both dependable electrical performance and streamlined manufacturing processes.

Table 6. *The thickness of an insulator affects various TEM factors*

Thickness of insulator (mm)	Gradient of temperature (ºC)	Electric potential (Volt)	Current density normalized (A/m^2)	Total internal energy (J/kg)
0.25	20 ~ 296	~0.39	30.1 ~ 4.10×10^7	~ 45000
0.50	20 ~ 300	~0.39	75.4 ~ 4.08×10^7	~ 46000
0.75	20 ~ 304	~0.39	27 ~ 4.05×10^7	~ 47000

Based on the preceding discussion, we can elucidate the TE properties of the Bi_2Te_3 TEM with the following specifications: sizes of 1.00 mm × 1.00 mm × 2.75 mm, with a pitch of 0.50 mm, insulation that is 0.50 mm thick, and conductors that are 0.125 mm thick.

3.6. Seebeck Coefficient

The Seebeck coefficient, also referred to as TE power, quantifies a material's capacity to produce an electric potential in reaction with a temperature gradient. A heightened Seebeck coefficient augments the performance of the TEG, resulting in the generation of a greater voltage when subjected to a temperature gradient spanning from 20 to 300°C. This voltage differential plays a vital role in the TEG's electricity generation process, underscoring the pivotal importance of optimizing the Seebeck coefficient to enhance TEG performance within this specific configuration. As shown in Figure 4, the Seebeck coefficient of a Bi_2Te_3 TEM reaches a maximum value of 2.37×10^{-4} V/K for p-type legs and a minimum value of -2.37×10^{-4} V/K for n-type legs, with a load resistance of 0.42 ohms.

In Figure 5, in the case of p-type materials, the dominant charge carriers are positive holes (p-holes). When a temperature gradient is applied across a p-type TE material, p-holes migrate from the hot surface to the cold surface, resulting in the generation of a positive electric potential. This means that a positive voltage is established when comparing the hot surface to the cold surface. Conversely, for n-type materials, the primary charge carriers are negative electrons (n-electrons). When a temperature gradient is applied across an n-type TE material, n-electrons travel from the hot surface to the cold surface, producing a negative voltage. The Seebeck coefficient for n-type materials is negative, leading to the creation of a negative voltage when comparing the hot and cold sides. The obtained Seebeck coefficient of ±237 µV/K closely aligns with the findings of a previous report [39,40].

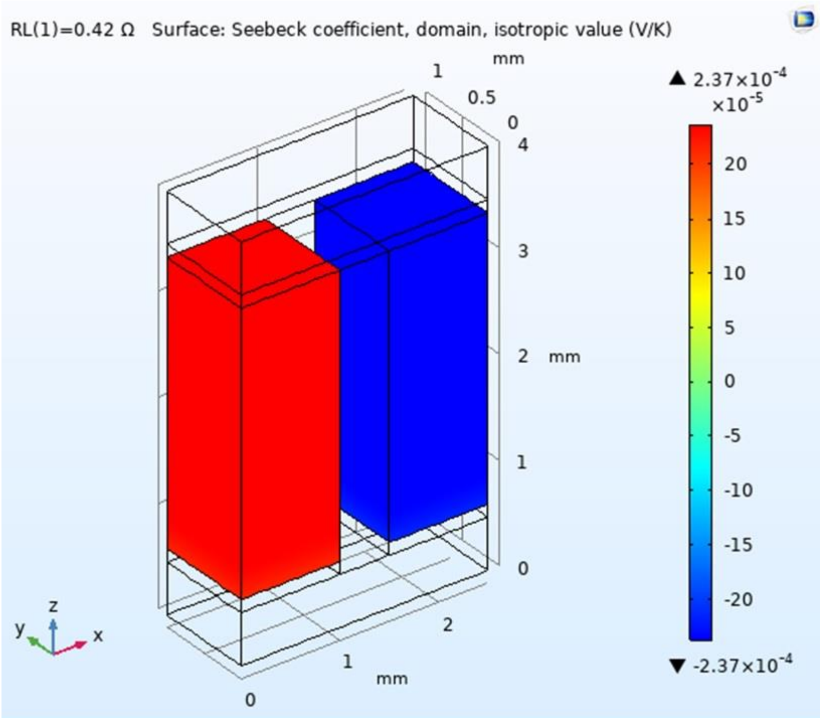

Figure 4. *Seebeck coefficient for Bi_2Te_3 TEM*

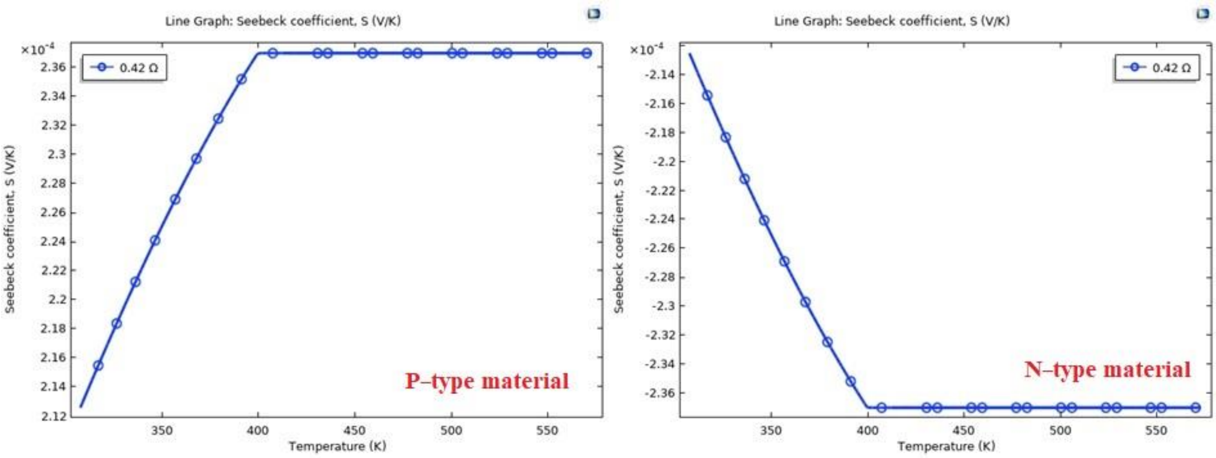

Figure 5. *Seebeck coefficient for both the p- and n-type legs of Bi_2Te_3 TEM*

3.7. Thermal Conductivity

The thermal conductivity of Bi_2Te_3, which is widely used in TEGs, varies with temperature and material purity. At room temperature (around 300K), the thermal conductivity of Bi_2Te_3 typically falls within the range of 1–2 W/(m*K). However, the exact value can depend on factors such as the specific composition of the material and any doping or alloying that may have been applied to enhance its TE properties.

As depicted in Figure 6, the thermal conductivity of Bi_2Te_3 in a TEG exhibits a notable trend concerning temperature. Within the temperature range of 305 to 325K, there is a slight decrease in thermal conductivity, shifting from its initial value of 1.58 W/(m*K) to a minimum of 1.57 W/(m*K). However, as the temperature surpasses 325K and rises up to approximately 400K, the thermal conductivity undergoes an increase, reaching a peak value of 1.75 W/(m*K). Beyond 400 K, the thermal conductivity stabilizes at this level. This behavior highlights the temperature-dependent nature of thermal conductivity in Bi_2Te_3 TEGs. A similar trend was identified in the thermal conductivity of another researcher [41].

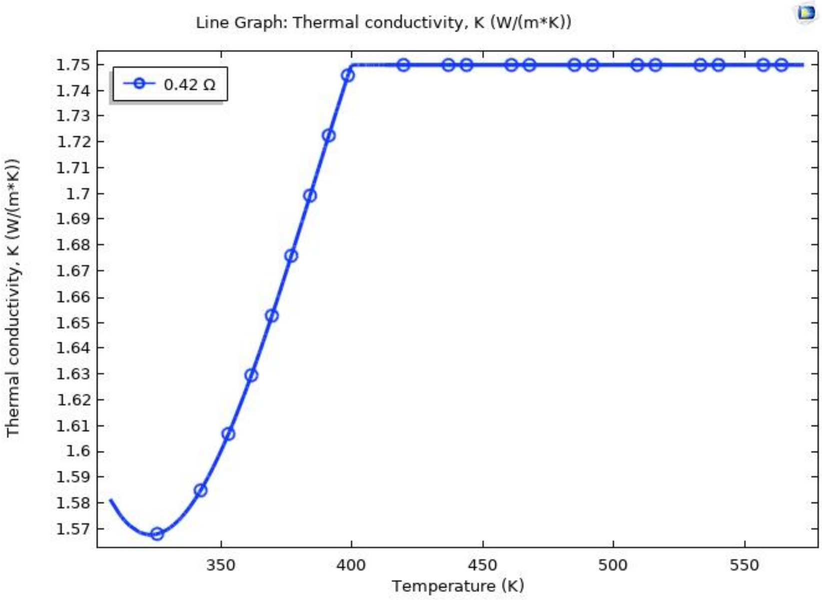

Figure 6. *Illustrates the thermal conductivity for* Bi_2Te_3 *TEM*

3.8. Electrical Conductivity

The electrical conductivity of Bi_2Te_3 is a fundamental property that allows a way to effectively convert temperature differential into electrical current, and that is the primary concept of TE power generation. Doping and controlling the temperature are crucial variables in enhancing the electrical conductivity of such materials in practical applications.

As illustrated in Figure 7, the electrical conductivity of Bi_2Te_3 within the TEM exhibits a notable trend in response to temperature variations. Over the temperature range of 300 to 400K, the electrical conductivity decreases, transitioning from its initial value of 84,000 S/m to a minimum of 58,000 S/m. Beyond 400K, the electrical conductivity remains constant, sustaining this level up to 550K. This behavior underscores the temperature-dependent nature of electrical conductivity in Bi_2Te_3 TEMs. Another researcher [42] observed a similar behavior in electrical conductivity.

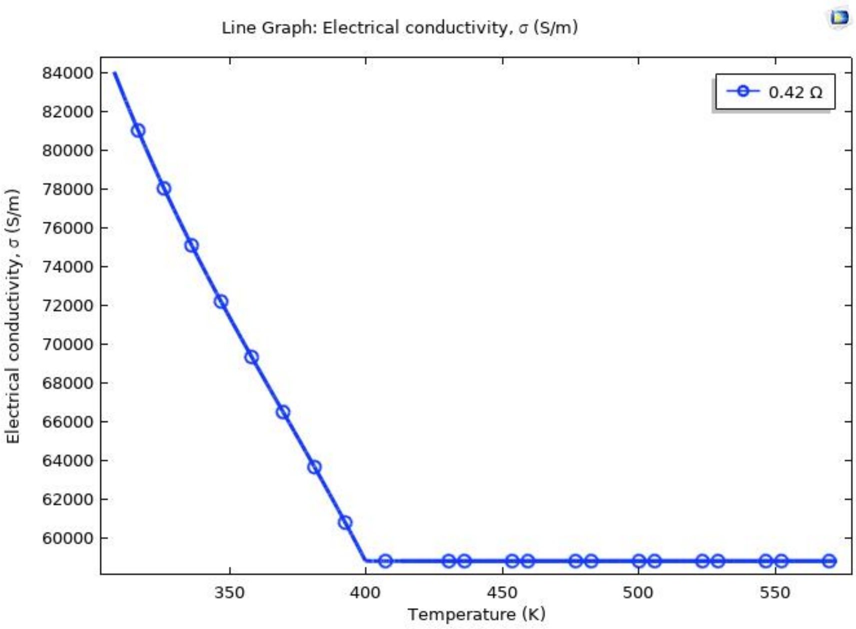

Figure 7. *Electrical conductivity for* Bi_2Te_3 *TEM*

4. CONCLUSIONS

In conclusion, the results strongly suggested that square-leg configurations consistently deliver superior performance, which makes them an attractive choice for manufacturers. It is important to underscore that each of the insulating and conducting layers plays a pivotal role in maximizing TEM performance. The thin copper layer in this design optimizes heat conduction, while the thicker alumina layer ensures effective electrical insulation. To conclude, it is recommended to adopt a Bi_2Te_3 TEM with optimized sizes of 1.00 mm × 1.00 mm × 2.75 mm, with an insulator thickness of 0.50 mm and a conductor thickness of 0.125 mm. The specific configuration exhibits superior functionality for TEGs and opens up exciting prospects for further research in this domain. This geometry is well-suited for applications characterized by spatial constraints, offering a practical solution for TE energy conversion while remaining cost-effective in terms of production. Furthermore, within the scope of the simulations that ascertained a Seebeck coefficient of ±237 μV/K. This measurement showcases an increase in thermal conductivity and a decrease in electrical conductivity at lower temperatures. This study suggests areas for future research in TE energy generation. Exploring different material compositions beyond Bi_2Te_3 could expand the performance range of TEGs. Investigating methods to manipulate the temperature-dependent properties of Bi_2Te_3, such as through doping or alloying, shows promise for optimizing TEG performance. Exploring alternative compositions or layer configurations may help balance electrical insulation and thermal conductivity. Further investigation into the underlying mechanisms of the Seebeck coefficient in Bi_2Te_3 materials could provide strategies for enhancing TEG performance. A multidisciplinary approach that combines materials science, thermodynamics, and engineering could drive advancements in TEG efficiency and sustainability. Addressing these recommendations could lead to the development of more robust and efficient TEGs for numerous uses involving waste heat recycling and portable power generation.

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CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

REFERENCES

[1] Shastri, S. S., and Pandey, S. K., Thermoelectricity and Advanced Thermoelectric Materials, Woodhead Publishing, (2021).

[2] Cao, T., Shi, X. L., Li, M., Hu, B., Chen, W., Liu, W. D., Lyu, W., MacLeod, J., and Chen, Z. G., "Advances in bismuth-telluride-based thermoelectric devices: progress and challenges", eScience, 3(3): 100122, (2023).

[3] Mamur, H., Bhuiyan, M. R. A., Korkmaz, F., and Nil, M., "A review on bismuth telluride (Bi_2Te_3) nanostructure for thermoelectric applications", Renewable and Sustainable Energy Reviews, 82: 4159-4169, (2018).

[4] Mamur, H., and Bhuiyan, M. R. A., "Characterization of Bi_2Te_3 nanostructure by using a cost-effective chemical solution route", Iranian Journal of Chemistry and Chemical Engineering (IJCCE), 39(3): 23-33, (2020).

[5] Bhuiyan, M. R. A., Mamur, H., and Dilmaç, Ö. F., "Review on performance evaluation of Bi_2Te_3-based and some other thermoelectric nanostructured materials", Current Nanoscience, 17: 423-446, (2020).

[6] Mamur, H., Üstüner, M. A., Dilmaç, Ö. F., and Bhuiyan, M. R. A., "Performance evaluation of $Bi_2Te_{3-x}Se_x$ (0.10≤ X ≤ 1.80) thermoelectric nanostructured materials", Cleaner Chemical Engineering, 4: 100063, (2022).

[7] Mamur, H., Üstüner, M. A., Korucu, H., and Bhuiyan, M. R. A., "A review of the performance evaluation of thermoelectric nanostructure materials $Bi_{2-x}Sb_xTe_3$ (0.20≤ X ≤ 1.80)", Cleaner Chemical Engineering, 6: 100101, (2023).

[8] Bhuiyan, M. R. A., Korucu, H., Mamur, H., and Haque, M. M., "Growth and characterization of $Bi_2Te_{2.70}Se_{0.30}$ nanostructured materials by using a cost-effective chemical solution route", Journal of Alloys and Metallurgical Systems, 4: 100032, (2023).

[9] Sanin-Villa, D., Montoya, O. D., and Grisales-Noreña, L. F., "Material property characterization and parameter estimation of thermoelectric generator by using a master–slave strategy based on metaheuristics techniques", Mathematics, 11(6): 1326, (2023).

[10] Demir, M. E., and Dincer, I., "Performance assessment of a thermoelectric generator applied to exhaust waste heat recovery", Applied Thermal Engineering, 120: 694-707, (2017).

[11] Chiba, T., Yabuki, H., and Takashiri, M., "High thermoelectric performance of flexible nanocomposite films based on Bi2Te3 nanoplates and carbon nanotubes selected using ultracentrifugation", Scientific Reports, 13(1): 3010, (2023).

[12] Jaziri, N., Boughamoura, A., Müller, J., Mezghani, B., Tounsi, F., and Ismail, M., "A comprehensive review of thermoelectric generators: technologies and common applications", Energy Reports, 6: 264-287, (2020).

[13] Soleimani, Z., Zoras, S., Ceranic, B., Cui, Y., and Shahzad, S., "A comprehensive review on the output voltage/power of wearable thermoelectric generators concerning their geometry and thermoelectric materials", Nano Energy, 89: 106325, (2021).

[14] Aridi, R., Faraj, J., Ali, S., Lemenand, T., and Khaled, M., "Thermoelectric power generators: state-of-the-art, heat recovery method, and challenges", Electricity, 2(3): 359-386 (2021).

[15] Li, W., Paul, M. C., Montecucco, A., Knox, A. R., Siviter, J., Sellami, N., Meng, X. L., Fernandez, E. F., Mallick, T. K., Mullen, P., Ashraf, A., Samarelli, A., Llin, L. F., Paul, D. J., Gregory, D. H., Gao, M., Sweet, T., Azough, F., Lowndes, R., and Freer, R., "Multiphysics simulations of a thermoelectric generator", Energy Procedia, 75: 633-638, (2015).

[16] Li, W., Paul, M. C., Montecucco, A., Siviter, J., Knox, A. R., Sweet, T., Gao, M., Baig, H., Mallick, T. K., Han, G., Gregory, D. H., Azough, F., and Freer, R., "Multiphysics simulations of thermoelectric generator modules with cold and hot blocks and effects of some factors", Case Studies in Thermal Engineering, 10: 63-72, (2017).

[17] Hasan, M. K., Üstüner, M. A., Mamur, H., and Bhuiyan, M. R. A., "Enhancing $Bi_2Te_{2.70}Se_{0.30}$ thermoelectric module performance through COMSOL simulations", Thermo, 4: 185-201, (2024).

[18] Şişik, B., and LeBlanc, S., "The influence of leg shape on thermoelectric performance under constant temperature and heat flux boundary conditions", Frontiers in Materials, 7: 595955, (2020).

[19] Xu, H., Zhang, Q., Yi, L., Huang, S., Yang, H., Li, Y., Guo, Z., Hu, H., Sun, P., Tan, X., Liu, G. G., Song K., and Jiang, J., "High performance of Bi_2Te_3-based thermoelectric generator owing to pressure in fabrication process", Applied Energy, 326: 119959, (2022).

[20] Wu, X., Wang, Z., Liu, Y., Sun, X., Xu, Y., Tian, Y., Wang, B., Sang, X., Shi, J., and Xiong, R., "Enhanced performance of Bi2Te3-based thermoelectric materials by incorporating $Bi_2Fe_4O_9$ magnetic nanoparticles", Journal of Alloys and Compounds, 904: 163933, (2022).

[21] Rogl, G., and Rogl, P., "Skutterudites, a most promising group of thermoelectric materials", Current Opinion in Green and Sustainable Chemistry, 4: 50-57, (2017).

[22] Nan, B., Xu, G., Liu, W. M., Yang, Q., Zhang, B., Dong, Y., Tie, J., Guo, T., and Zhou, X., "High thermoelectric performance of PNP abrupt heterostructures by independent regulation of the electrical conductivity and Seebeck coefficient", Materials Today Communications, 31: 103343, (2022).

[23] Jouhara, H., Żabnieńska-Góra, A., Khordehgah, N., Doraghi, Q., Ahmad, L., Norman, L., Axcell, B., Wrobel, L., and Dai, S., "Thermoelectric generator (TEG) technologies and applications", International Journal of Thermofluids, 9: 100063, (2021).

[24] Luo, Y., Li, L., Chen, Y., and Kim, C. N., "Influence of geometric parameter and contact resistances on the thermal-electric behavior of a segmented TEG", Energy, 254: 124487, (2022).

[25] Wilhelmy, S., Zimare, A., Lippmann, S., and Rettenmayr, M., "A temperature gradient evaluation method for determining temperature dependent thermal conductivities", Measurement Science and Technology, 32(10): 105601, (2021).

[26] Hasan, M. K., Haque, M. M., Üstüner, M. A., Mamur, H., and Bhuiyan, M. R. A., "Optimizing the performance of Bi_2Te_3 TECs through numerical simulations using COMSOL multiphysics", Journal of Alloys and Metallurgical Systems, 5: 100056, (2024).

[27] Liang, Z., Deng, L., Shi, X., Song, S., Xu, C., Chu, C. W., and Ren, Z., "Composition-dependent contact resistivity in an n-type $Mg_3Sb_xBi_{2-x}$ thermoelectric single leg", Materials Today Energy, 29: 101099, (2022).

[28] Khan, S., Cheema, T. A., Hassan, M., Malik, M. S., and Park, C. W., "Thermoelectric investigation of low-cost modular night-time electricity generation", Heat and Mass Transfer, 58(8): 1381-1391, (2022).

[29] Aljaghtham, M., and Celik, E. "Design of cascade thermoelectric generation systems with improved thermal reliability", Energy, 243: 123032, (2022).

[30] Shittu, S., Li, G., Zhao, X. and Ma, X., "Review of thermoelectric geometry and structure optimization for performance enhancement", Applied Energy, 268: 115075, (2020).

[31] Basu, R., and Singh, A., "High temperature Si–Ge alloy towards thermoelectric applications: a comprehensive review", Materials Today Physics, 21: 100468, (2021).

[32] Maduabuchi, C., Njoku, H., Eke, M., Mgbemene, C., Lamba, R., and Ibrahim, J. S., "Overall performance optimisation of tapered leg geometry based solar thermoelectric generators under isoflux conditions", Journal of Power Sources, 500: 229989, (2021).

[33] Kondaguli, R. S., and Malaji, P. V., "Geometry design and performance evaluation of thermoelectric generator", The European Physical Journal Special Topics, 231(8): 1587-1597, (2022).

[34] Ibeagwu, O. I., "Modelling and comprehensive analysis of TEGs with diverse variable leg geometry", Energy, 180: 90-106, (2019).

[35] Luo, Y., and Kim, C. N., "Effects of the cross-sectional area ratios and contact resistance on the performance of a cascaded thermoelectric generator", International Journal of Energy Research, 43(6): 2172-2187, (2019).

[36] Siddique, A. R. M., Mahmud, S., and Heyst, B. V, "Performance comparison between rectangular and trapezoidal-shaped thermoelectric legs manufactured by a dispenser printing technique", Energy, 196: 117089, (2020).

[37] Ma, Q., Fang, H., and Zhang, M., Theoretical analysis and design optimization of thermoelectric generator", Applied Thermal Engineering, 127: 758-764, (2017).

[38] Liu, J., Li, Y., Li, S., and Chen, P., "Novel nanometer alumina-silica insulation board with ultra-low thermal conductivity", Ceramics International, 48(8): 10480-10485, (2022).

[39] Zhang, Y., Wang, X. L., Yeoh, W. K., Zheng, R. K., and Zhang, C., "Electrical and thermoelectric properties of single-wall carbon nanotube doped Bi_2Te_3", Applied Physics Letters, 101(3): 031909, (2012).

[40] Zhang, W., Li, M., Jia, M., Fan, Y., Zhang, Y., Tian, Z., Li, X., Liu, Y., Yang, D., Song, H, and Cabot, A., "Realizing high mechanical and thermoelectric properties of N-type $Bi_2Te_{2.7}Se_{0.3}$ ingots through powder sintering and carrier concentration regulation", Journal of the European Ceramic Society, 44: 5088-5095, (2024).

[41] Alrebdi, T. A., Wudil, Y. S., Ahmad, U. F., Yakasai, F. A., Mohammed, J., and Kallas, F. H. "Predicting the thermal conductivity of Bi_2Te_3-based thermoelectric energy materials: a machine learning approach", International Journal of Thermal Sciences, 181: 107784, (2022).

[42] Malik, I., Srivastava, T., Surthi, K. K., Gayner, C., and Kar, K. K., "Enhanced thermoelectric performance of n-type Bi_2Te_3 alloyed with low cost and highly abundant sulfur", Materials Chemistry and Physics, 255: 123598, (2020).