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Investigating the validity of the gibrat law at the provincial level in Türkiye: parametric and semiparametric panel data models

Türkiye'de iller düzeyinde gibrat yasasinin geçerliliğinin incelenmesi: parametrik ve semiparametrik panel veri modelleri

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ABSTRACT

The growth of cities is closely linked to the overall economic growth of nations. Especially in urban planning, predicting and modelling the growth trajectory of cities is crucial for ensuring sustainable economic growth. The growth of cities brings with it many social and economic gains, but it also increases many economic and social demands. The growth of cities is analysed based on geographical foundations, the assumption of increasing returns, and the random growth approach. The growth of cities is usually discussed in terms of three different models. The random growth approach is called Gibrat's law, and this approach allows cities to be analysed empirically. Gibrat's law is analysed comparatively with parametric analyses as well as nonparametric and semiparametric models. This study aims to examine the validity of Gibrat's law at the provincial level in Türkiye using parametric and semiparametric panel data models. The study utilizes annual data from 2007 to 2019 at the provincial level. The analysis reveals that parametric models provide weak evidence for the validity of Gibrat's law, while semiparametric models provide stronger evidence.

Keywords: City Growth, Gibrat's Law, Parametric and Semiparametric Models.

Jel Classification: C14, P25, R11.

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ÖZ

Şehirlerin büyümesi, ülkelerin ekonomik olarak büyümesiyle eş tutulmaktadır. Şehirlerin büyümesinin izlediği yörüngenin tahmin edilmesi ve modellenmesi, şehir planlamaları başta olmak üzere sürdürülebilir ekonomik büyümenin sağlanması açısından büyük önem taşımaktadır. Şehirlerin büyümesi, sosyal ve ekonomik birçok kazanımı beraberinde getirmesiyle birlikte birçok sosyal ve ekonomik gereksinimi de artırmaktadır. Şehirlerin büyümesi, genellikle üç farklı model üzerinden tartışılmaktadır. Şehirlerin büyümesi, coğrafi temeller, artan getiri varsayımı ve rastgele büyüme yaklaşımı temelinde incelenmektedir. Rastgele büyüme yaklaşımı Gibrat yasası olarak ifade edilmekte ve ampirik olarak incelenmesine olanak vermektedir. Gibrat yasası parametrik analizlerin yanı sıra nonparametrik ve semiparametrik modellerle karşılaştırmalı olarak incelenmektedir. Bu çalışmada Türkiye'de iller düzeyinde Gibrat yasasının geçerliliğinin parametrik ve semiparametrik panel veri modelleri ile incelenmesi amaçlanmaktadır. Çalışmada iller düzeyinde 2007-2019 dönemine ait yıllık veriler kullanılmıştır. Analizler sonucunda, parametrik modellerin Gibrat yasasının geçerliliğine yönelik zayıf kanıtlar sunarken, semiparametrik modellerin ise daha güçlü kanıtlar sunduğu tespit edilmiştir.

Anahtar Kelimeler: Şehir Büyümesi, Gibrat Yasası, Parametrik ve Semiparametrik Modeller.

Jel Sınıflaması: C14, P25, R11.

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1. Introduction

Cities significantly contribute to regional and national economies in line with their potential and contributions. There have been essential approaches to the growth of cities, and analyses have tested these approaches for a long time. These methodologies are commonly perceived as contingent upon economies of scale, the significance of geographic fundamentals, and the lack thereof (Gonz´alez-Val, 2023). Approaches assuming that increasing returns on scale are effective in the growth of cities draw attention to the endogenous factors of cities. Eaton and Eckstein (1997) argue that there is a highly correlated relationship between the human capital of cities and their growth. This conceptual framework asserts that the overall factor productivity of cities correlates with the average human capital level within these urban centers. A key attribute ascribed to cities is their capacity to furnish a convenient environment for acquiring human capital. Analogous to the collaborative contribution of human capital stocks at the national level to the learning process, the accumulation of human capital stocks within cities similarly plays a contributory role in fostering learning (Eaton and Eckstein, 1997). It is emphasized that local governments' increasing and diversifying local knowledge and human capital resources through various public investments significantly contribute to cities' growth (Black and Henderson 1999). Simultaneously, a proximate association exists between the intensity of economic activities and the population mobility observed within cities. The formation of high employment-generating firms and/or sectors in cities plays an essential role in increasing the population of that city. Likewise, a city's population decreases due to firms shifting their production to other places or giving up production (Eeckhout, 2004).

In addition to the intrinsic characteristics of cities, geographical factors, which are referred to as extrinsic factors, are also influential. It is stated that some cities' being located by a river or sea adds important characteristics to those cities (Davis and Weinstein, 2002). The third approach to the growth of cities is referred to as Gibrat's law. This approach assumes that urban growth is a random variable and that city size is independent of its initial size (Gonz´alez-Val, 2023). Gibrat's law posits that it is not possible to deduce a consistent pattern or principle from the correlation between growth and the initial size (Gonza´lez-Val et al., 2014). It posits that cities exhibit random growth patterns characterized by equivalent expected rates and variances, irrespective of their initial sizes (Giesen and Südekum, 2011). Gibrat's law postulates that the expansion of a city is independent of its initial size or age, and further implies that the growth rate of cities can be characterized as a random walk with a trend (Clark and Stabler, 1991).

Gibrat's law has been applied to various economic and non-economic variables (Córdoba, 2008). Its application to city and firm growth stands out among these socioeconomic variables. Parametric and nonparametric models have been widely used to analyse these variables. However, González-Val (2023) emphasized the significance of applying semiparametric models and tests to avoid the reduction of inferences regarding Gibrat's law to a single test statistic. This can be achieved by considering unit effects and allowing for a detailed examination of city sizes (Giesen and Südekum, 2011). In this paper, the validity of Gibrat's law in Türkiye is analysed with the help of parametric and semiparametric models. The data from 2007 to 2019 is utilised for the study. The analysis yielded results indicating that parametric models offer limited support for the invalidity of Gibrat's law, whereas semiparametric models present compelling evidence affirming the validity of the law.

In studies in the literature, the validity of Gibrat's law has mostly been examined with parametric models. However, the reliability of the parametric approach depends on the satisfaction of the restrictive assumptions that require a priori. Therefore, the nonparametric and semiparametric approach offer a stronger alternative. This new approach, which has rarely been used for Gibrat's law, has not been applied to Türkiye. The originality of this study is the use of the semiparametric approach, which brings innovation to Gibrat's law literature, at the panel data level for the provinces of Türkiye.

After the introduction, the study includes a literature review. Then it introduces the data and the models used; and presents the empirical applications. The fifth section presents policy recommendations, and the conclusion is at the end.

2. Literature Review

The assumptions and propositions developed for the growth of cities have a long history. Since the 1990s, many empirical studies have been conducted to test the validity of Gibrat's law, one of the most important models for city growth and city size distributions (Gonza´lez-Val et al., 2014). Clark and Stabler (1991) scrutinized the validity of Gibrat's law concerning the growth of Canadian cities through the application of unit root tests. The investigation revealed that Gibrat's law holds true for seven prominent Canadian cities.

Glaeser et al. (1995) analysed the growth of 203 US cities from 1960 to 1990. They state that the growth of US cities is related to their initial state. It is emphasized that the initial human capital, schooling rate, and education level are positively effective in the growth of cities.

In the study to determine the growth trajectory in French cities from 1831 to 1990, Guerin-Pace (1995), states that there will be deviations from the Gibrat model of city growth.

Eaton and Eckstein (1997) analysed growth in thirty-nine French cities for 1876-1990 and forty Japanese cities for 1925-1985. They emphasize that total factor productivity growth is reflected in the growth of cities due to human capital accumulation based on endogenous growth in cities. The fact that French and Japanese cities with similar total factor productivity growth have similar growth processes is presented as evidence.

Glaeser and Shapiro (2003) examined the growth of US cities in the 1990s and found no correlation between initial and subsequent population growth. They also emphasize that human capital and automobile use positively affect the growth of some cities, while public transportation has a negative effect.

Ioannides and Overman (2003) assessed the validity of Gibrat's and Zipf's laws by employing parametric and nonparametric tests to analyse the growth of cities in metropolitan areas in the USA from 1900 to 1990. The results of nonparametric tests suggest no justification for rejecting Gibrat's law concerning the growth dynamics of cities.

Black and Henderson (2003) examined the evolutionary developments in the growth process of cities in the USA. The study states that parametric tests do not support Gibrat's law for the growth of cities. In addition to parametric tests, the authors emphasize that growth processes should be checked with nonparametric tests.

Wheeler (2003), as a result of his study on metropolitan cities in the US, states that the population growth of cities moves independently of their initial size and that Gibrat's law is valid.

Eeckhout (2004) argues that the population size and its development over time in US cities are in line with Gibrat's law. Also, it is emphasized that wages and prices affect people's choice of location and, more than that, affect population mobility.

Anderson and Ge (2005) examined the growth trajectory of cities in China and the impact of the Chinese government's one-child policy on the population of cities. Their findings reveal that the expansion of cities in China is unassociated with their initial size, and the progression in city size adheres to the principles articulated by Gibrat's law.

According to Soo (2007), Gibrat's law of proportional growth does not find applicability to Malaysian cities, with observed deviations from the predictions posited by the law.

Henderson and Wang (2007) examined the effects of institutional variables on city growth in 142 countries from 1960 to 2000. It is seen that there is no growth model pursuant to Gibrat's law in the cities considered in the study. Simultaneously, it is asserted that, despite a negative correlation between education and city growth, technological advancements play a positive role in fostering growth.

Bosker et al. (2008) examined the validity of Gibrat’s law for 62 cities in Germany for the period 1925-1999 using parametric and nonparametric models. While parametric models provide weak evidence for Gibrat law, nonparametric tests provide strong evidence for large cities. The most important reasons for the deviations from Gibrat's law are the war and post-war periods. When the growth of cities is divided into pre-war and post-war periods, the pre-war period is consistent with Gibrat's law.

González-Val and Sanso-Navarro (2010) adapted Gibrat's law, which has been empirically analysed, to determine the growth of city populations and the growth of country populations. The study finds that Gibrat's law is also valid for country populations, although weak evidence is obtained for city populations. It is emphasized that it is especially effective for the population growth of small countries.

Giesen and Südekum (2011) examined the validity of the Gibrat law in seventy-one leading cities in Germany. The study emphasizes that the Gibrat law is valid nationally and regionally in Germany. At the same time, it is stated that there were deviations from the Gibrat law in the growth of cities after World War II, but there was a return to the Gibrat law in the following periods.

Zeren and Savrul (2012) scrutinized the cities' growth in Türkiye for the year 2000. The study states that the neighbourhood relations of cities at the provincial level in Türkiye are effective in urbanization and industrialization, and schooling rates have a positive effect.

Gonza´lez-Val et al. (2014) investigated the validity of Gibrat’s law for the growth of cities in the US, Spain, and Italy for 1900-2001 using parametric and nonparametric models. The parametric models confirm the validity of Gibrat’s law in the USA and Italy, but not in Spain. On the other hand, nonparametric models show strong support for the Gibrat law in the US in the 20th century but weak support for the law in Italy and Spain. The weak evidence for Italy and Spain is that these countries experienced wars in the 20th century that affected natural growth (Gonza´lez-Val et al., 2014). Nevertheless, upon extending the time frame across the countries, it is posited that the law holds true for all three nations in the long term.

González-Val and Olmo (2015) analysed the growth trajectories of 1,173 regions defined as cities in the US during the 1990-2000 period. As a result of linear and non-linear models, it is stated that the growth trajectories of cities are complex and that both endogenous and spatial factors are effective. In addition, it is stated that the growth of cities can be explained by more than one approach in a mutually supportive manner.

Seyfettinoğlu and Akın (2020) studied the economic variables determining population growth in cities in Türkiye in 2007 and 2016. The study states that the level of human capital and educational conditions positively affect the economic growth of cities.

Gonz´alez-Val (2023) examined the validity of Gibrat’s law with parametric, semiparametric, and nonparametric models for Spain for the period 1900-2011 and the USA for the period 1900-2000. While the parametric model yields different results for the cities of Spain and the US, the nonparametric models show that Gibrat's law is not valid for the cities of both countries. However, it is stated in the literature that using semiparametric models as an alternative to parametric and nonparametric models gives more robust results. Semiparametric models are emphasized as strong evidence that Gibrat's law is valid.

A review of the literature reveals that, in addition to testing the validity of Gibrat’s law, other approaches are considered complementary to each other. It is understood that local dynamics, technological developments, and human capital levels are effective in realizing the assumptions of Gibrat's law. Moreover, while parametric models are used in the first stages of the analysis of Gibrat's law, nonparametric tests have been applied comparatively over time. Gonz´alez-Val (2023) applied semiparametric models in addition to parametric and nonparametric models and emphasized that they yield more robust results than nonparametric models. This study uses parametric and semiparametric models following Gonz´alez-Val (2023). At the same time, this study seeks to enhance the scholarly discourse by undertaking semiparametric analyses of Gibrat's Law. In addition, the study is unique in terms of the models used.

3. Data and Methodology

In this study, the validity of Gibrat’s law for cities in Türkiye is analysed with parametric and semiparametric panel data models. Gibrat's law, as introduced by Gibrat (1931), pertains to the principle of proportional growth, wherein the growth rate of firms is posited to be independent of their initial size relative to one another. This study, which was put forward for firms, was later used to analyse cities' growth rates and took part in the literature (Eechout, 2004). The study utilizes city population data from the Turkish Statistical Institute (TurkStat). The data were collected starting from 2007, when the address-based population registration system started, and regular annual data is available. The data collection ended in 2019, the year before the pandemic. This protects other variables taken as control variables from the pandemic effect. For cities, GDP per capita (gdp_pc), number of hospital beds per 100,000 people (no_beds), secondary school enrolment rate (sec_sc_rate), and total electricity consumption per capita (elc_cons_pc) are used as control variables. Since externalities resulting from schooling rates are particularly prevalent in cities, a link between growth and initial schooling rates in urban centers supports the view that schooling determines growth. The relationship between schooling and growth for employment and income growth in cities provides evidence supporting the positive role of education in economic development (Glaeser et al., 1995). Simultaneously, the growth of a city is deemed contingent upon factors such as the availability of specialized inputs, the average levels of human capital possessed by resident workers, and the elevation of living standards (Wheeler, 2003). Improving health conditions is important for raising social welfare and living standards in urban life. In this respect, the number of hospital beds is included in the model as an important indicator representing the increase in living standards.

In analysing the validity of Gibrat’s law for city growth rates, we follow the methodology of González-Val (2023). Accordingly, $Population_{it}$ is defined as the population of city *i* in year *t*. The relative size of a city is delineated as the proportion of the city's population to the concurrent population of the entire country:

$$s_{it} = \frac{Population_{it}}{Country\,Population_t}$$

Then, the logarithmic growth rate of the city in the year, over the logarithmic value of its relative size, is calculated as follows:

$$Growth_{it} = \ln s_{it} - \ln s_{it-1}$$

Then, the normalization process, frequently used in literature applications, was applied to this variable. To normalize the variable, a normalized logarithmic relative growth rate variable was created and used as the dependent variable with the help of Z score. A similar normalization process was applied to the control variables to avoid unit of measurement differences. The first approach analysing Gibrat's law is to examine the relationship between growth rate and relative initial size using a linear regression model:

$$zee\_g_{it} = \alpha + \beta \ln s_{it-1} + X_{it}^T \varphi + e_{it} \qquad (1)$$

In this model, the growth rate is modelled depending on the initial size and control variables $\left(X_{it}^T\right)$. The statistical significance of its coefficient β examines the validity of Gibrat's law . Should the coefficient be deemed significant, Gibrat's law stands invalidated; conversely, if the coefficient lacks significance, the law is considered valid. In the event of the law's validity, the growth rate is asserted to be independent of the initial size, with a corresponding average growth rate prevailing. In this model, error terms are assumed to be independent and identically distributed $(e_{it} \sim i.i.d)$, and panel unit effects are not considered. However, in most cases, this assumption is not valid in practice, and the heterogeneous nature of the units leads to heteroskedasticity.

Panel data approaches that consider these effects can be used to improve the first approach and consider possible panel unit (and/or time) effects. The following model is obtained by adding unit effects as a constant in the panel data model:

$$zee\_g_{it} = \alpha + \beta \ln s_{it-1} + X_{it}^T \varphi + \mu_i + e_{it} \qquad (2)$$

Here, the term μ_i denotes unit effects, and with the help of this term, differences arising from the characteristics of cities are taken into account in the model. In this model, the validity of the law is analysed in the same way, but unlike the first one, the term allows the average growth rate to vary from city to city. In the next step to improve the model, the linear relationship structure is relaxed and quadratic (or higher order) terms for the initial size are added to the model in case a non-linear relationship holds:

$$zee\_g_{it} = \alpha + \beta_1 \ln s_{it-1} + \beta_2 \left(\ln s_{it-1}\right)^2 + X_{it}^T \varphi + \mu_i + e_{it} \qquad (3)$$

While this model structure helps to model the non-linear relationship, since it is a parametric approach, it again assumes that the functional relationship structure is known in advance and is a restrictive approach. It is unclear what degree polynomial can capture the correct relationship, and it is determined experimentally. Moreover, as in previous models, the assumption that error terms are independent and identically distributed is also assumed here, and this assumption is difficult to fulfill, especially due to the problem of heteroscedasticity.

All parametric models were used with variables that had a statistically significant effect. Since the presence of insignificant variables in econometric models affects the significance of other variables and the results of assumption tests, continuing with models containing insignificant variables risks the reliability of the results. For this reason, insignificant variables were removed from the parametric models, and significant ones in the models were presented in tables.

At this point, an alternative approach is to use nonparametric and semiparametric models. In these models, the relationship under investigation is treated as nonparametric. Thus, there is no need to make prior assumptions about the functional relationship structure and the distribution of error terms. The structure of the relationship is estimated locally from the data. Therefore, no parameter estimation is performed, and graphical interpretation is made without parameter interpretation. In the semiparametric model, the relevant variables are included nonparametrically, while other variables are added parametrically. Based on the model structure given above, the semiparametric panel model can be shown as follows:

$$zee\_g_{it} = m\left(\ln s_{it-1}\right) + X_{it}^{T}\varphi + \mu_i + e_{it} \qquad (4)$$

Here, the initial magnitude is nonparametrically added to the model. The function $m(.)$ is an unknown nonparametric function that is both smooth and continuous. By locally estimating it, the association between the growth rate and initial size can be established without relying on any specific assumptions. We employ the methodology introduced by Baltagi and Li (2002) to estimate the semiparametric model. Accordingly, the parametric part of the model, including unit effects, is estimated first. The residuals obtained from this estimation are used to estimate the nonparametric part:

$$\hat{e}_{it} = zee\_g_{it} - X_{it}^{T}\hat{\varphi} - \hat{\mu}_i = m\left(\ln s_{it-1}\right) + u_{it} \qquad (5)$$

This model uses the standard local polynomial approach to estimate the $m(.)$ function. Here, the residuals of the parametric model and the initial size are used instead of the growth rate. Hence, the conditional independence of growth from initial size is estimated. In essence, this involves the exclusion of the impact of observed or unobservable characteristics that may exhibit variation at the city level, allowing for an investigation into how growth fluctuates concerning city size (González-Val, 2023).

4. Empirical Analysis and Findings

The validity of Gibrat's law for cities in Türkiye is first examined with a parametric linear panel data model that excludes possible unit and time effects. Table 1 presents the model estimation results.

Table 1: Parametric Linear Panel Data Model

Dependent Variable: zee_g **Number of observations:** 972			**F (4,967)** = 18.36 **Prob > F** = 0.000	
Variables	**Coeff.**	**Std. Error**	**t**	**P > | t |**
lag_ln_s	0.154	0.034	4.50	0.000
gdp_pc	0.088	0.038	2.31	0.021
no_beds	-0.160	0.032	-5.02	0.000
elc_cons_pc	0.104	0.037	2.79	0.005
cons	0.753	0.172	4.38	0.000

The estimation results show that the lag_ln_s variable, which represents the relative initial size of cities, has a significant and positive effect (0.154) on the growth rate of cities. This suggests that Gibrat's law is not valid. Among the control variables, GDP per capita and electricity consumption positively affect the growth rate, while the number of hospital beds has a negative effect. However, one of the basic assumptions made for the error terms in this model is the absence of heteroscedasticity. As a result of the Breusch-Pagan Test conducted to test this assumption, it is found that there is a heteroscedasticity in the error terms of the model [Chi2(1)= 56.62; prob=0.000], and the assumption is not met. Moreover, this model does not consider possible unit and time effects. In the next step, the validity of the law is tested by using panel data models that take these effects into account. The F Test conducted to test for unit effects reveals significant unit effects in the model [F(80,887)=3.96; prob=0.000]. However, in line with the relatively short time compared to the dominant number of units, time effects are generally insignificant. Intending to decide which model approaches treat unit effects as fixed or random is appropriate, the Hausman Test shows that the fixed effect panel data model is appropriate [Chi2(4)= 163.75; prob=0.000]. Table 2 presents the results of the fixed effects model estimation.

Table 2: Fixed Effect Parametric Linear Panel Data Model

Dependent Variable: zee_g **Number of observations:** 972			**F (4,887)** = 34.96 **Prob > F** = 0.000	
Variables	**Coeff.**	**Std. Error**	**t**	**P > | t |**
lag_ln_s	-9.788	0.866	-11.31	0.000
gdp_pc	0.156	0.051	3.05	0.002
no_beds	-0.219	0.104	-2.11	0.035
sec_sc_rate	-0.361	0.081	-4.44	0.000
cons	-48.32	4.274	-11.31	0.000
F Test μ_i **= 0;**	**F (80, 887)** = 3.96		**Prob > F** = 0.000	

The estimation result shows that relative initial size has a significant effect on the growth rate of cities. However, unlike the first model, it is noteworthy that the effect has become negative

(-9.788). In this case, it is understood that taking unit effects into account has a significant effect. However, it still supports the impression that Gibrat's Law is invalid. Among the effects of the control variables, GDP per capita, and the number of hospital beds remained similar. However, in the fixed effect model, unlike the first model, the secondary school enrolment rate variable was found to have a significant and negative effect instead of electricity consumption per capita. This is consistent with the findings of Deliktas et al. (2013). However, as a result of the Modified Wald Test applied to test for heteroscedasticity in this model, it was again found that there was a heteroscedasticity problem in the model [Chi2(81) = 37416.07; prob=0.000].

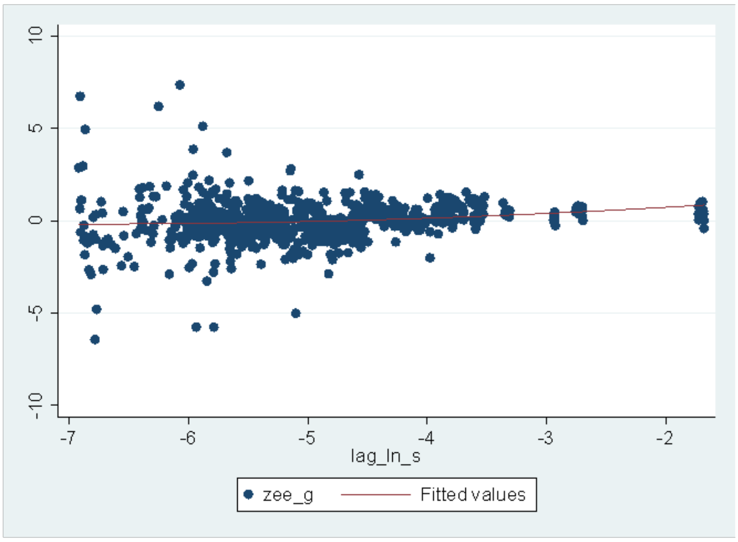
Another parametric approach to examining the validity of Gibrat's law is to relax the linearity assumption by using polynomial terms for the initial size variable whose effect is being tested. As a result of the examination, it was determined that a quadratic model is appropriate for the variable. The results of the parametric polynomial model are presented in Table 3.

Table 3: Fixed Effect Parametric Polynomial Panel Data Model

Dependent Variable: zee_g **Number of observations:** 972			**F (4,887)** = 43.45 **Prob > F** = 0.000	
Variables	**Coeff.**	**Std. Error**	**t**	**P > | t |**
lag_ln_s	18.14	4.893	3.71	0.000
lag_ln_s^2	2.699	0.463	5.82	0.000
gdp_pc	0.108	0.051	2.11	0.035
sec_sc_rate	-0.426	0.075	-5.65	0.000
cons	21.43	12.80	1.67	0.094
F Test μ_i **= 0;**	**F (80, 887)** = 4.77		**Prob > F** = 0.000	

When the estimation results are analysed, it is seen that the square of the initial size variable is significant, as well as the variable itself. In this case, although Gibrat's law is not valid, It is discerned that the impact of initial size on a city's growth rate does not follow a linear pattern. The curve showing the relationship structure is shown in Figure 1.

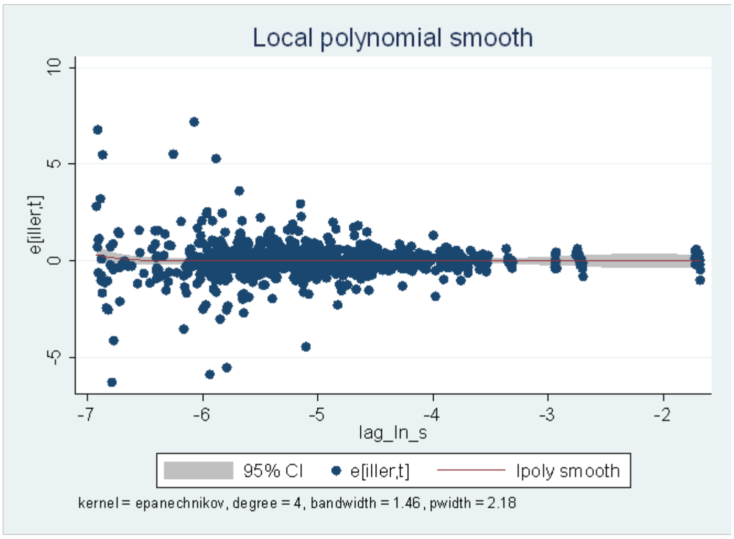
Figure 1: Parametric Polynomial Model Estimation



When the relationship structure is analysed, it is observed that while initial size constantly affects the growth rate at the beginning, it becomes relatively more effective for cities with a higher initial size. Accordingly, it is concluded that cities with a higher initial size grow relatively faster than smaller cities. Among the control variables, the effect of income and schooling rate is similar to the previous model, while the number of hospital beds becomes insignificant in the polynomial model. However, the Modified Wald Test was applied to the model again to test for heteroskedasticity, and it was found that the problem of heteroskedasticity persists in the polynomial model [Chi2(81)= 38163.82; prob=0.000].

Although all of the parametric approaches used so far to determine the validity of Gibrat's law conclude that the law is not valid, it is observed that the assumptions required by parametric models are not met. Moreover, the coefficients representing the validity of the law are highly sensitive to the unit effects in the model and a priori-determined functional relationship structure. This suggests that it is appropriate to use a nonparametric approach to test the validity of the law, which does not require any assumptions about the error terms or the functional form. This approach enables us to assess the validity of the law without concerning for a priori assumptions and by determining the functional relationship structure from the data. Instead of directly using a nonparametric model, a semiparametric model structure is used here to eliminate the unit effects, that are understood to exist in the model and the effects of control variables. Accordingly, the growth rate, the dependent variable, is first modelled parametrically through unit effects and control variables. Then, the residuals of this model are taken, and the relationship between these residuals and the relative initial size variable is modelled nonparametrically. As a result, an investigation into the conditional independence between growth and initial size is pursued by removing the influence stemming from observed or unobserved characteristics that might manifest variability at the city level. Since the parametric part of the semiparametric model is done to remove the effect of city characteristics, the model results are not included here, but only the estimation result of the nonparametric part, which is important to examine the validity of Gibrat's law. The nonparametric part of the model is estimated with a local polynomial approach. For this purpose, the Epanechnikov kernel function and fourth-order polynomial model are used. Similar results were obtained for alternative kernel functions and polynomial degrees. The automatically selected smoothing parameter value was set to 1.46. As it is known, in nonparametric models, the prediction result is interpreted directly graphically, not through any parameter. The prediction result is presented in Figure 2.

Figure 2: Local Polynomial Model Estimation



When the result of the local polynomial model is analysed, it is observed that the curve showing the relationship structure hovers around zero, and the confidence intervals support this result. Thus, after the correction for both observed and unobserved city-specific effects, the growth rate is deemed conditionally independent of the relative initial size of cities. In essence, it is established that Gibrat's law holds true for cities in Türkiye, indicating a stochastic growth pattern that is uncorrelated with their initial sizes. This result, which is different from the findings obtained in parametric models, reveals the importance of determining the appropriate approach to examine the validity of the law. The findings of semiparametric analyses are consistent with Gonz´alez-Val (2023) and support the appropriateness of using the semiparametric model. Similarly, the assumption that the relationship structure is linear or predetermined, and how city-specific differences are addressed or not taken into account, can lead to the false conclusion that the law is invalid. In this respect, the superiority of the semiparametric approach comes to the fore.

5. Discussion and Policy Implications

Parametric and semiparametric models were used in the study. Following the parametric analysis, although findings indicating that Gibrat's law is not valid are obtained, it is seen that the basic assumptions are not met and the results remain weak. The findings obtained as a result of semiparametric analyses provide strong evidence for the validity of Gibrat's law. We can confirm that Gibrat's law is valid for the growth of cities in Türkiye for 2007-2019. Furthermore, we have found that cities exhibit a rather random growth regardless of their initial size. These findings are compatible with the studies of Ioannides and Overman (2003), Black and Henderson (2003), Bosker et al. (2008), González-Val and Olmo, (2015), and Gonz'alez-Val (2023). Predicting and modeling the trajectory of the growth of cities in Türkiye is of great importance in ensuring a sustainable economy, especially in city planning.

The growth of cities brings with it many social and economic gains and increases many needs. Especially due to the growth of cities, the infrastructure requirement per capita increases. For this reason, infrastructure investments need to be replanned to help increase the efficiency of transportation technology or urban management techniques.

At the same time, as a result of cities growing at higher rates than expected, environmental degradation and pollution there occurs especially congestion in urban transportation. Advanced city planning is required to eliminate the negative externalities that may arise as a result of growth.

Henderson and Wang (2007) assert that technological advancements promote the expansion of individual cities, potentially resulting in larger and more efficient urban sizes. Simultaneously, they posit that innovative solutions may hold greater significance for larger cities compared to smaller ones, given the elevated transportation costs, traffic congestion, and heightened management challenges inherent in larger urban settings (Henderson and Wang, 2007). For this reason, highly technology-oriented solutions need to be prioritised in solving cities' problems, such as traffic and administrative difficulties. Moreover, positive externalities generated through knowledge spillover can be expected to be reflected in the planned growth of other small cities.

It is understood that GDP growth in Türkiye positively affects the growth of cities. Therefore, the effects of economic growth on the population of cities should be accurately measured, and the required investments should be scheduled.

Health and education services are seen as the most important elements in the socioeconomic development of cities. Improvements in educational opportunities and human capital accumulation positively affect the economic growth performance of cities (Seyfettinoğlu and Akın, 2020). Simultaneously, it contributes substantially to the development of human capital, a factor of considerable significance in the internal progress of cities and the shaping of their social environments. For this reason, education and health services, the most important elements of city plans prepared from a holistic perspective, should be evaluated and planned.

At the same time, training a qualified workforce and attracting talent to small cities are important to reduce population growth and migration pressures in big cities in Türkiye, especially Ankara and Istanbul. The establishment of comprehensive educational institutions, vocational training programs, and initiatives promoting lifelong learning should be prioritised, increasing the attractiveness of small cities for businesses and individuals.

6. Conclusion

Cities' growth significantly contributes to national economic growth, especially regional development. However, the faster-than-expected growth of cities brings significant negative and positive externalities. For this reason, it is essential to predict accurately and model cities' growth trajectories. Gibrat's law, which primarily by stating that the growth rate of companies is independent of their relative initial size, was later applied to the growth of cities, and it was revealed that the law could also be valid for the growth of cities. Various modeling approaches are used to examine the validity of this law. The correct results from the approaches used are important to offering correct policy recommendations. At this point, selecting the appropriate modeling approach is key to obtaining reliable results. This study examined the validity of Gibrat's law for cities in Türkiye with panel data models. Parametric and semiparametric approaches were employed to ascertain the appropriate methodology, and the outcomes were comparatively interpreted. According to the results obtained from parametric models, Gibrat's law is not valid for cities in Türkiye, and their relative initial size affects their growth rate. Although this result is valid for linear and non-linear models, it has been determined that the necessary assumptions are not met in these models. At this point, it was decided that it would be appropriate to examine the validity of the law with local polynomial models that do not require these assumptions. Since city-specific effects and characteristics were purified before estimating the local polynomial model, the model used turned into a semiparametric approach. According to the findings obtained from this approach, Gibrat's law is valid for cities in Türkiye, and the growth rate of cities is not affected by their relative initial size. This result, different from parametric models, reveals the importance of the appropriate approach. The fact that the law, which appears to be invalid, is actually valid requires the development of different policies. The independence of urban growth from initial size implies that, through the implementation of suitable policies tailored for smaller cities, they have the potential to attain growth rates comparable to those observed in larger cities. At this point, accurate population and infrastructure planning can achieve a balanced population distribution and a fair distribution of the national development burden among cities.

The results obtained here reveal that nonparametric or semiparametric approaches are a direct alternative to parametric models for similar studies to be conducted in the future or an indirect alternative that can be used to check the reliability of the results.

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