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Research Article

Logistic and CSG Growth Models for Predicting Life Expectancy

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Abstract: The tables that allow calculating the probability of death at a certain age by recording the number of births/deaths in a population are called life tables. The concept of life expectancy, which is a measure that determines how long a creature will live, is also determined by mortality rates obtained from life tables. It is also possible to model the expected lifetime with some nonlinear mathematical functions. One of the functions that is often used in modeling mortality rates is the logistic growth function. This study aims to propose a model that can be used as an alternative to the logistic growth model and to interpret the mortality rates of countries. In this study, the life expectancy of males and females in Türkiye, Singapore, Norway, and China was modeled using the logistic and the CSG growth model, which was newly introduced to the literature. When modeling the life expectancy of countries, the adjusted graph was drawn following the data of each growth model. Then, the performances of the logistic growth model and the CSG growth model were compared with R^2, *RMSE*, and *MAPE* statistical criteria. As a result of the comparison, it was revealed that the CSG growth model is more suitable than the logistic model for estimating life expectancy for overall data and for each gender. The originality of this study is the CSG model which is a new nonlinear model that predicts life expectancy effectively for related datasets.

Yaşam Beklentisi Tahmininde Lojistik ve CSG Modelleri

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Anahtar Kelimeler
Büyüme modelleri,
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Lojistik model,
Yaşam beklentisi,
Yaşam tabloları

Öz: Bir popülasyondaki doğum/ölüm sayıları kullanılarak belirli bir yaştaki ölüm olasılığının hesaplanmasını sağlayan tablolara yaşam tabloları denir. Bir canlının ne kadar yaşayıp yaşamayacağının ölçüsü olan yaşam beklentisi, yaşam tablolarından elde edilen ölüm oranıyla hesaplanmaktadır. Beklenen yaşam süresini, lineer olmayan fonksiyonlarla modellemek mümkündür. Ölüm oranlarının modellenmesinde sıklıkla kullanılan fonksiyonlardan biri lojistik büyüme modelidir. Bu çalışma ile lojistik büyüme modeline alternatif olarak kullanılabilecek literatüre yeni kazandırılan bir model kullanarak Türkiye, Singapur, Norveç ve Çin'e ait ölüm oranlarını yorumlamak amaçlanmıştır. Ülkelerin total ve cinsiyete göre yaşam beklentileri lojistik ve CSG büyüme modelleri kullanılarak tahmin edilmiştir. Ülkelerin yaşam beklentileri tahmin edilirken her büyüme modelinin tahmini grafiklerle desteklenmiştir. Daha sonra lojistik ve CSG büyüme modellerinin performansları R^2, *RMSE* ve *MAPE* istatistikleri kullanılarak karşılaştırılmıştır Karşılaştırma sonucunda CSG büyüme modelinin yaşam beklentisi tahmininde hem total veri hem de cinsiyetler açısından lojistik büyüme modeline göre daha iyi tahminde bulunduğu tespit edilmiştir. Bu çalışmanın özgünlüğü, çalışmada kullanılan veri kümeleri için yaşam beklentisini etkili bir şekilde tahmin edebilen yeni bir doğrusal olmayan büyüme modelinin sunulmasıdır.

1. Introduction

To establish a detailed population structure favorable to both economic and social decision-making, life tables are created (Bowers et al., 1997; Namboodiri & Suchindran, 2013). In their simplest form, life tables are tables containing data on the survival of living and non-living things (telephones, computers) as a function of time and age. Data from these tables are used in the estimation of life expectancy at any age or mortality rates using mathematical growth models (Dinçer, 1998; Şençelikel & Öner, 2017). As these models are mathematical functions, it should be noted that they enable forward-looking estimation of life expectancies (Taylan & Yapar, 2013). There are many different approaches to studies on life expectancy. For example, Aje et al. (2024) used the ARIMA model in their study on life expectancy at birth for men in developing countries. Levantesi et al. (2023), proposed a new clustering method to forecast the healthy life expectancy of countries which allows for the clustering of different countries according to similarities in their healthy life expectancy.

Given their importance in establishing mortality tables and predicting life expectancy, mathematical growth models are the subject of constant study (Burton et al., 2021). Classical models such as the Gompertz model and the logistic model are the most frequently used, as illustrated by the studies of İskender (2021) in the prediction of population and gender, Gavrilov & Gavrilova (2019) in the prediction of mortality in elderly US citizens, and Lee et al. (2021) in the estimation of life expectancy in a chemical plant. Hifzan et al. (2024) used Gompertz and logistic functions to estimate mature size in Katjang X Boer goats. In this study, the coefficient of determination was used to compare the models. Both models have been found to be ideal for estimating body length. Thus, the Gompertz model predicts body weight and height at withers better than the logistic model, which is fitted for chest circumference. Although these functions give successful results, growth models that can always give better results are tried to be suggested in the literature. There are many studies in the literature on comparisons of models. So that, Makgopa et al. (2024) made a summary by reviewing dozens of articles about growth models for animal production industry. As a result, they stated that Gompertz, logistic and Brody are the most used models, respectively. In another study, Santos et al. (2024) proposed a model based on combining models frequently used in the literature. In this study, comparisons between models were made using body weight data of Norfork rabbits. They showed that the proposed model and the Von Bertalanfy model gave better results than the Gompertz and Richards models according to the goodness of fit criterion. However, they also emphasized that further studies are needed on this subject. Other researchers, however, have developed their models. For example, Weon & Je (2009) and Carla & Sumathi (2021) have established mathematical growth models based on the Weibull distribution.

However, these mathematical models generally present problems related to prediction accuracy (Panik, 2014). It's therefore important to address the issue of choosing the model with optimal goodness of fit thus favoring prediction results closer to reality. In this study, the logistic growth model used in the literature and the Sloboda Gompertz Combined Growth Model (CSG), which is a new model introduced in the literature, are used to estimate life expectancy (Ünal & Çığşar, 2021). For that purpose, general and gender-specific life expectancies, from the first month to over 100 years, obtained in four different countries, namely Türkiye, Singapore, Norway, and China, have been used. The most important factors taken into account in the selection of the countries used in the study are preference of the first and last quartile of the world quality of life rankings and the availability of reliable data. Particular attention was paid to the fact that the data were produced by the official statistical institutions of the countries. The performance of these models is compared using the statistical criteria of R^2, *RMSE*, and *MAPE*, and the model-data fit is illustrated graphically.

2. Material and Methods

The logistic growth and CSG functions are those used in this study. Remember that these are sigmoidal (S) functions, and in this case were applied to analyze life table data from four countries. Let the life expectancy at aged x be represented by T_x, then the expected value of this value (i.e., $E(T_x)$) gives us the life expectancy for age x and denoted by e_x^0.

Logistic Growth Model: The logistic model is a growth model frequently used to describe population growth (Schacht, 1980; Arosio, 2015; İskender, 2021; Prasad, 2022). For logistic growth, population size at time *t* can be given as (Tsoularis & Wallace, 2002; Longhi et al., 2017; Windarto et al., 2018):

$$y(t) = \frac{y_\infty}{1 + a\, e^{-bt}}. \tag{1}$$

The value $y_\infty = y(t)$ in the equation represents the asymptotic magnitude, in other words, the maximum magnitude. *a* is any constant and *b* represents the growth in the positive or negative direction and *b≠0*.

CSG Growth Model: The CSG model procure the population growth at time *t* from the instantaneous change in the expression:

$$\frac{\frac{dy(t)}{dt}}{y(t)} = \beta . ln\left(\frac{y_\infty}{y_t}\right) + \ (1-k).b_1.t^{-b_2}\ ln\ ln\ \left(\frac{y_\infty}{y_t}\right). \tag{2}$$

Here $k \in R$ is in the range *0≤k≤1*. If the instantaneous variation is resolved, we achieve the expression

$$y(t) = y_\infty .exp\ exp\left(-c_1.e^{-\beta\, t}\right)\ exp\ exp\left(-c_2.e^{-\,(1-k).b_1.\frac{t^{1-b_2}}{1-b_2}}\right). \tag{3}$$

If we write e^c=$(-c_1)$.$(-c_2)$ where c_1 and c_2 are random constants and $\gamma = 1 - b_2,\ m = \frac{-b_1}{b_{2-1}}$, the CSG growth model is defined by the following function (Ünal & Çığşar, 2021).

$$y(t) = \ y_\infty.e^{-c_1.e^{-\beta\, t}}\ e^{-c_2.e^{-\,m.(1-k).t^\gamma}} \tag{4}$$

2.1. Datasets

The life tables for each country were analyzed using the latest datasets published by the countries' official statistical agencies as of October 2022 (Singapore Department of Statistics (DOS), Statistics Norway (SSB), National Bureau of Statistics of China (NBS), Turkish Statistical Institute (TÜİK)).

Analyses were performed considering the life expectancy e_x^0 column for *1+* ages due to the dramatic difference in mortality rates between *0* and *1* year of age.

The graphs were produced using the MATLAB program, and the CSG and logistic models were compared using the R^2, *RMSE*, and *MAPE* criteria.

2.2. Statistical criteria

Metrics are tools which are used to measure the performance of models in general, focusing on the difference between real and fitted values. In this study, *MAPE*, R^2 and *RMSE* criteria were used in model comparisons. Their mathematical formulas are listed below, considering $\hat{y_i}$ as the predicted value, y_i as the observed value and $\underline{y_i}$ as the mean:

$$MAPE = \frac{100\%}{N} \sum_{i=1}^{n} \left| \frac{y_i - \widehat{y_i}}{y_i} \right| \tag{5}$$

It is a form of measurement that focuses on the difference between the real values and the fitted values and expresses it proportionally. Since this value calculates an error, minimal value of *MAPE* is preferred.

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \widehat{y_i})}{\sum_{i=1}^{n}(y_i - \underline{y_i})} \tag{6}$$

It calculates the performance of the fitted model using real values and averages. It is assumed that the closer the R^2 value is to *1*, the better the model fits.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \widehat{y_i})^2}{n}} \tag{7}$$

RMSE takes into account the square root of the mean difference between the real values and the predicted values. For a model with good performance, it is preferable for *RMSE* to take values approaching *0* (Barnston, 1992; Chai & Draxler, 2014).

3. Results

In this section, logistic and CSG models were applied to life expectancy data from four countries, both overall and separately for each gender. Model-data fit was compared for each country using the R^2, *RMSE* and *MAPE* criteria, and graphs were plotted for each country.

Table 1. Statistical results for models

	Logistic				**CGS**		
	Overall	**Female**	**Male**		**Overall**	**Female**	**Male**
				NORWAY			
R^2	0.9890	0.9881	0.9899	R^2	0.9959	0.9954	0.9963
RMSE	0.1127	0.1164	0.1117	**RMSE**	0.0668	0.0707	0.0676
MAPE	4.9111	5.0009	4.9061	**MAPE**	2.7430	2.8708	2.9151
				CHINA			
R^2	0.9967	0.9967	0.9965	R^2	0.9983	0.9983	0.9983
RMSE	0.0390	0.0372	0.0414	**RMSE**	0.0277	0.0265	0.0292
MAPE	1.0182	0.9483	1.1277	**MAPE**	0.7309	0.6825	0.7929
				SINGAPORE			
R^2	0.9923	0.9912	0.9922	R^2	0.9971	0.9965	0.9972
RMSE	0.0870	0.0905	0.0913	**RMSE**	0.0523	0.0561	0.0531
MAPE	3.1098	3.1236	3.5397	**MAPE**	1.8123	1.8783	2.0026
				TÜRKİYE			
R^2	0.9969	0.9967	0.9958	R^2	0.9996	0.9988	0.9987
RMSE	0.0491	0.0499	0.0600	**RMSE**	0.0189	0.0302	0.0365
MAPE	1.5824	1.5211	2.1210	**MAPE**	0.6022	0.9463	0.9569

In both logistic and CSG life expectancy modeling, the R^2 values were 99% in all countries, indicating that the data is well explained by these two models. However, the R^2 values with the CSG model were consistently higher than those with the logistic model. Likewise, Table 1 also shows that all *RMSE* and *MAPE* values for the CSG model for the entire population are lower than those of the logistic model. Therefore, it can be said that the CSG model outperforms the logistic model in predicting life expectancy for the general population of these countries. The same observations can also be made for predicting life expectancy for men and women. Table 1 indicates that the CSG model performs better for all three criteria.

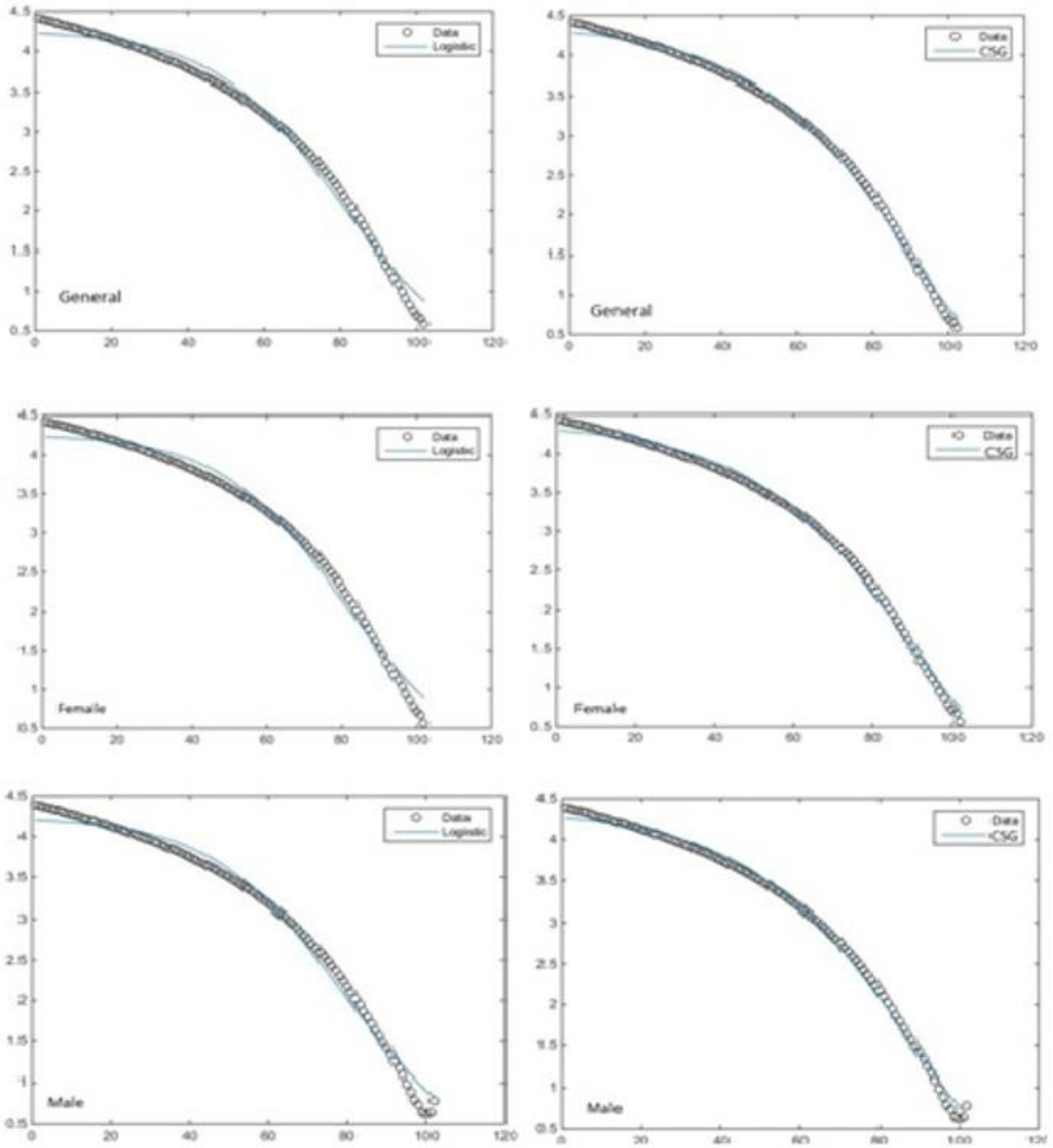

Figure 1. Norway model-data fit plot.

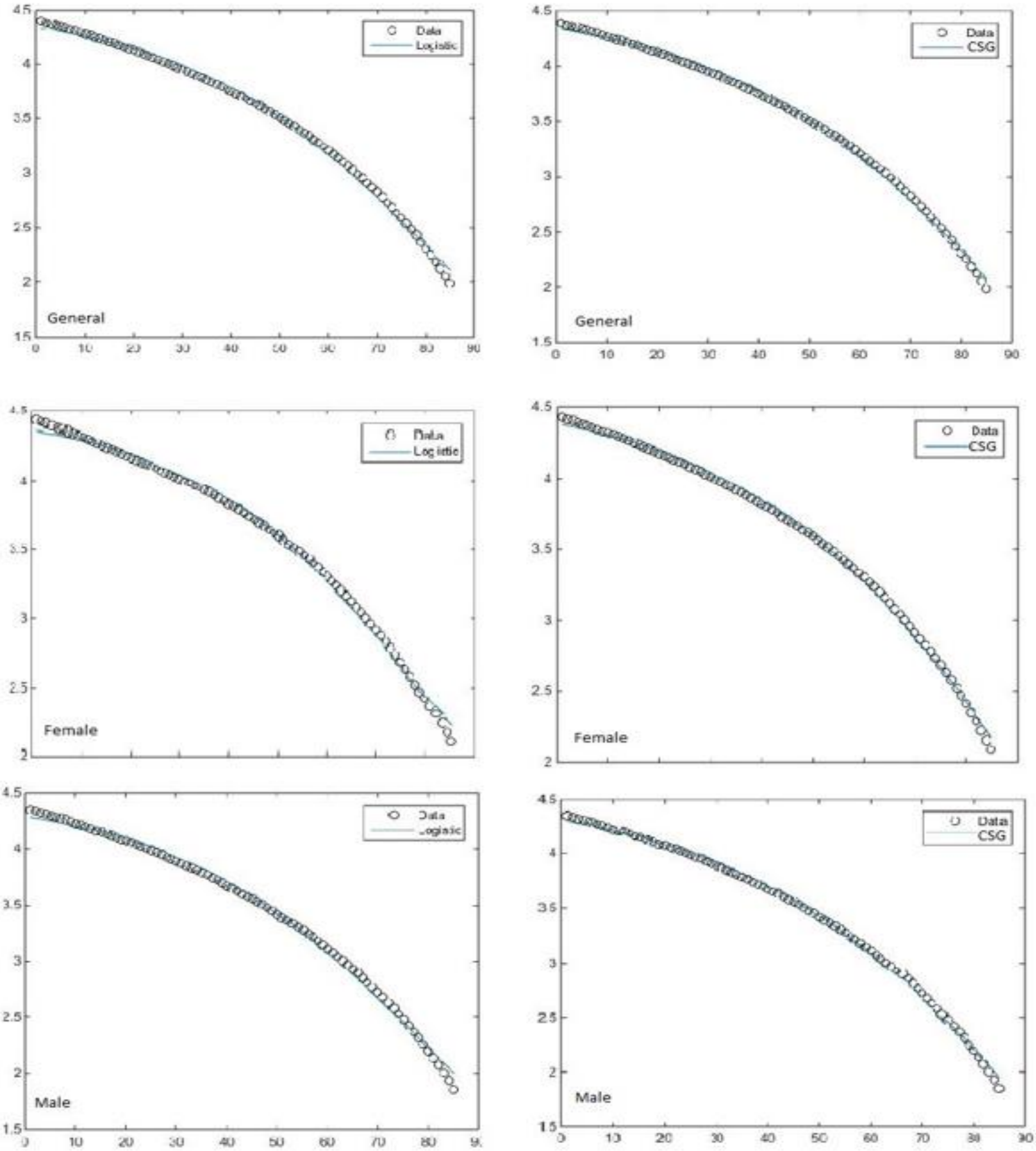

Figure 2. China model-data fit plot.

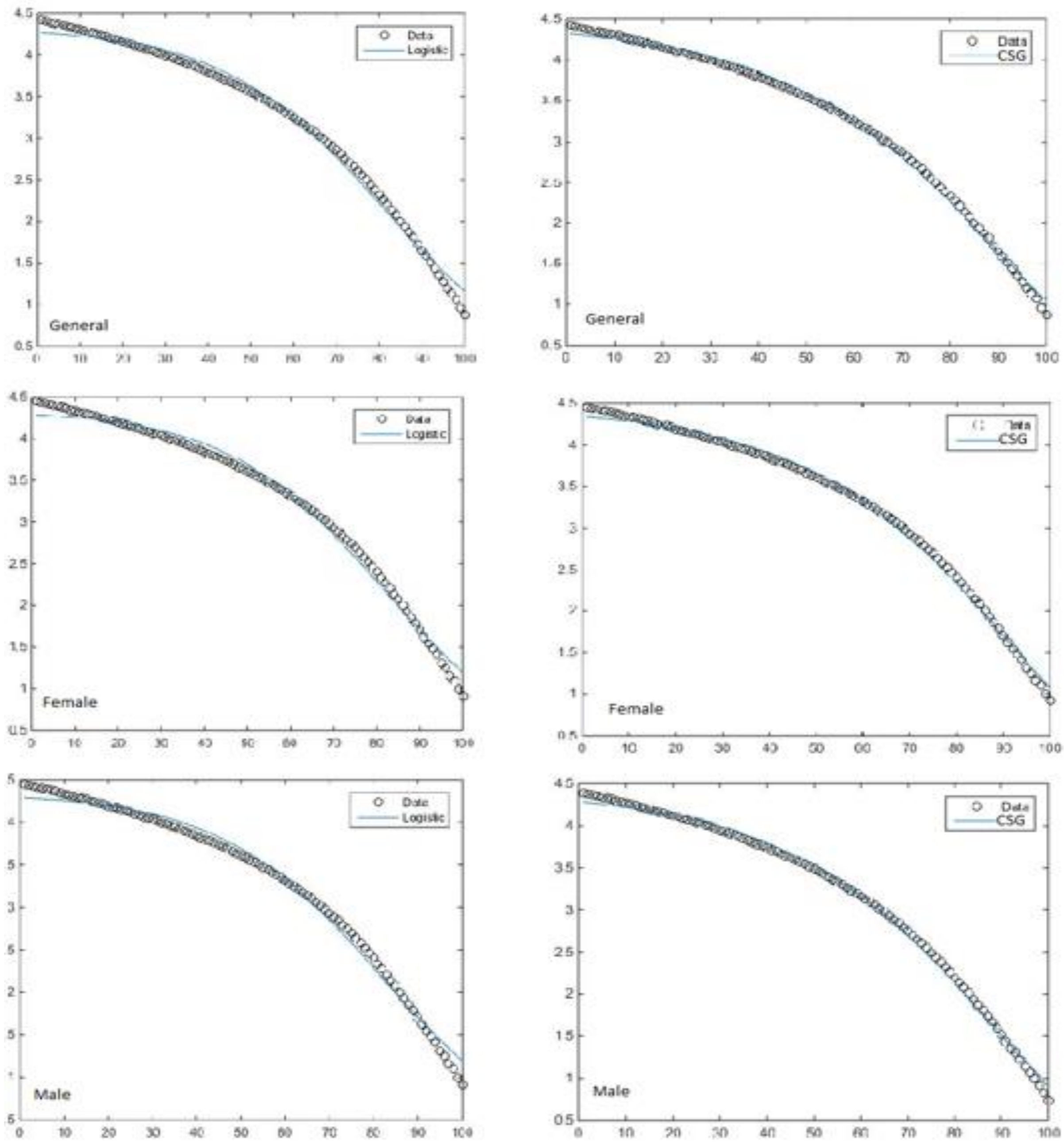

Figure 3. Singapore model-data fit plot.

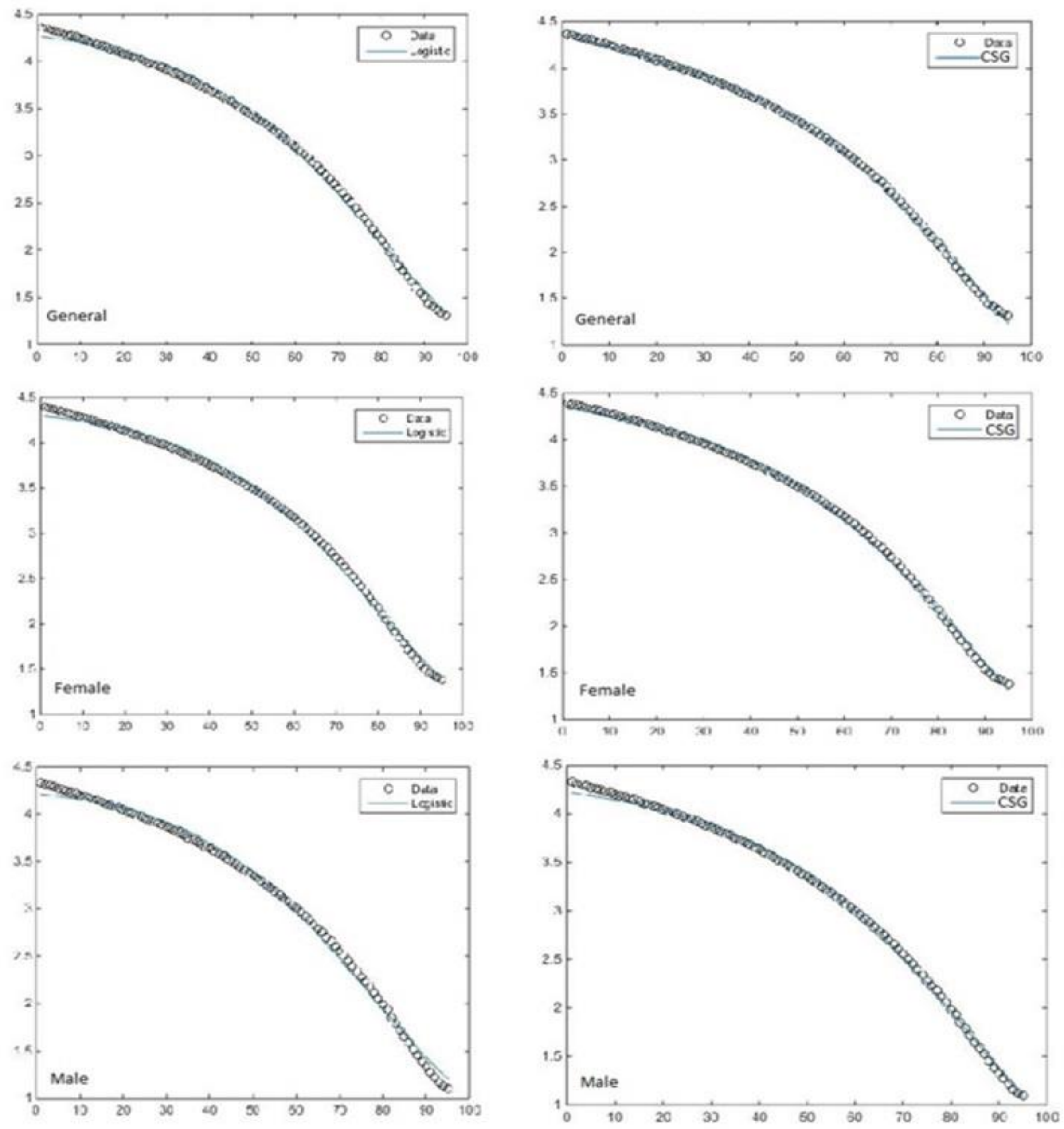

Figure 4. Türkiye model-data fit plot.

Graphs 1, 2, 3, and 4 present the results of logistic growth (all figures on the left) and CSG growth (all figures on the right) for the general population of each country, as well as their distribution by gender. The analysis of these graphs generally shows discrepancies between actual values and predicted values in the logistic model, both for general data and gender-specific data. At the same time, the two types of values are in perfect harmony in the CSG model, except for a few cases where a slight discrepancy is observed, which is much smaller than the discrepancies in the logistic model (as in the case of the *1-10* age group in Turkish men). This demonstrates that the CSG model fits the data of the four countries better than the logistic model, supporting the results obtained with the R^2, *RMSE*, and *MAPE* criteria mentioned earlier.

4. Discussion and Conclusion

Mortality tables provide information about the life expectancy of individuals in a country calculated from precise mathematical models. The present study conducts an assessment and comparison of two mathematical models used to estimate and predict the lifespan of individuals in a population. These models are the logistic model, developed over a century ago, and the recently introduced CSG model (Ünal & Çığşar, 2021). The objective is to determine whether older and newer models yield similar results when applied to current realities. These two models were chosen because of their non-linear structure, making them suitable for life expectancy data (which is also non-linear). The data is derived from mortality tables of four countries with different conditions and income levels (Singapore, China, Norway, and Türkiye).

The logistic model is recognized as one of the most suitable models that better describe and predict the data from mortality tables. This model has undergone numerous parameterizations and re-parameterizations over time, making it much more robust. Pham (2011) conducted a study on the mortality rate in the United States using data collected over six decades (1946-2005). The Gompertz, Gompertz-Makeham, logistic, log logistic, loglog, and Weibull models were used in this study. The different results show, with the lowest mean square error for the general population and for both genders (0.00996, 0.054, 0.01 respectively), that the logistic model is the most suitable among the six models for the mortality table data and in predicting life expectancy in the United States during this period. Vanfleteren et al. (1998) also drew the same conclusions by applying the three-parameter logistic model, the Gompertz model, and the Weibull model to suvival data obtained from 77 cohorts of Caenorhabditis elegans in axenic culture. The logistic model showed better results and was more suitable for the mortality data. Trappey & Wu (2008) also conducted studies on models capable of best predicting the short life cycles of products. Among the simple logistic model, the Gompertz model, and the time-varying extended logistic models, the time-varying extended logistic models came out as the best in predicting the data. This model fits the data 70% better than the Gompertz and simple logistic models. The performance of the models was evaluated using the mean absolute deviation and the root mean square error. Many other studies in the literature also demonstrate that the logistic model provides better results compared to classical models such as the Gompertz model and the Weibull model (Chen, 2007; Huishuo et al., 2020). Despite the literature emphasizing the efficiency of the logistic model as one of the best models to fit living mortality data and life expectancy estimation, in our study it was determined that a new model introduced in the literature outperforms the logistic model using all criteria.

In brief, the results of this study demonstrate a good fit between both model types and the data. However, graphically, the data fits almost perfectly with the CSG model, regardless of gender, the general population, and is valid for all four countries. The misfit of the logistic model with the data is more pronounced. This is evidenced by a higher margin of error in this model compared to the CSG model. This is confirmed by calculating indicators such as R^2, MAPE, and RMSE. All R^2values for the CSG model are higher than all R^2values for the logistic model for the four countries studied and for all genders. MAPE and RMSE are much higher with the logistic model than with the CSG model.

In conclusion, it can be said that the CSG model provides a better estimation of life expectancy at all ages and less biased predictions of demographic growth for selected countries. However, will these results be the same when these models are applied to countries in Africa, Asia, or South America with very low life expectancy or high mortality rates, which may be due to more complex factors than Western and North American countries? This will be the subject of future studies.

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