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Research Article

Disjoint maximal arcs in projective planes of order 16

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ABSTRACT

This paper provides the results of some computer searches for disjoint maximal (52, 4)-arcs in the known planes of order 16. *Thirty-seven* new such sets are discovered: *four* in Johnson plane and *thirty-three* in Mathon plane, *eighteen* of which give examples of 104-sets of type (4,8) coming from non-isomorphic pairs of maximal (52, 4)-arcs, providing first examples for such sets. A new lower bound on the number of 104-sets of type (4,8) coming from disjoint maximal (52, 4)-arcs in the known planes of order 16 is obtained. The 104-length binary and ternary linear codes generated by the blocks of 1-designs associated with the known 104-sets of type (4,8) are classified.

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INTRODUCTION

A t-(v, k, λ) design (or shortly, a t-design) is a pair D={X, B} of a set X of cardinality v, called *points*, and a collection B of k-subsets of X, called *blocks*, such that every t distinct points appear together in exactly λ blocks. The incidence matrix of D is a matrix I = (m_{ij}) with rows and columns labeled by the blocks and points of D, respectively, where $m_{i,j}$ equals to 1 if the i^{th} block contains the j^{th} point and 0 otherwise. The dual design of a design D, denoted by $D^{\perp}$, has point set as the block set of D, and the block set as the point set of D. Two designs are isomorphic if there is a one-to-one and onto map between their point sets that maps every block of one design to a block of the other design. A 2-$(q^2 + q + 1, q + 1, 1)$ design with $q \geq 2$ is called a projective plane of order q.

Let P be a projective plane of order ab. A v-set of type (x, y) in P is defined to be a set S of v points of P such that any line of P intersects with S in either x or y points. A 1-design associated with S is the design having the point set as S and block set as the set of intersections of lines of P with S in x points.

A maximal (m, a)-arc in P is a set M of m = (ab – b + 1)a points of P such that M meets with the lines of the plane in either 0 or a points. The set of lines disjoint from M determines a maximal {(ab – a + 1)b, b}-arc in the dual of P. For a maximal (m, a)-arc, a is called the degree of the arc. If a > 1, the nonempty intersection of lines of P with M form blocks of a 2-design.

A *linear code* with parameters $[n, k, d]_q$ is defined to be a k-dimensional subspace of $\mathbb{F}_q^n$ (here, d refers to the smallest distance between distinct code words). The q-ary code of a design D, denoted by C_q(D), is the linear space

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generated by the rows of the incidence matrix of D over $\mathbb{F}_q$. The parameters of $C_q(D)$ depend on the parameters and structure of D and q. If there exists a set of code words in a code whose supports (nonzero positions) corresponds to the points in blocks of a design, then the code is said to support a design. Codes over $\mathbb{F}_2$ and $\mathbb{F}_3$ are called binary and ternary codes, respectively.

The paper is organized as follows. In Section 2, a summary of the known maximal arcs in the known planes of order 16 is given. In Section 3, we report the results of computer searches for disjoint maximal (52, 4)-arcs in the known planes of order 16. It is shown that new disjoint maximal 52-arcs exist in Mathon and Johnson planes. In Section 4, the 104-length binary and ternary linear codes spanned by the blocks of 1-designs associated with the known 104-sets of type (4,8) are classified.

Maxımal ARCS in Planes of Order 16

Currently, up to isomorphism, twenty-two planes of order 16 are known to exist. We denote these planes as in [1]: PG(2,16) for the Desarguesian plane, DEMP for the Dempwolff plane, SEMI4 and SEMI2 for the semifield planes with kernel GF(4) and GF(2), respectively, LMRH for the Lorimer-Rahilly plane, MATH for the Mathon plane, HALL for the Hall plane, BBH1 and BBH2 for the planes obtained from the Hall plane (by Bose-Barlotti derivation), JOWK for the Johnson-Walker plane, JOHN for the Johnson plane, DSFP for the derived semifield plane, and BBS4 for the plane obtained from SEMI4 plane (by Bose-Barlotti derivation). The planes of order 16 given in [2] are used for our computations.

In [3], Dempwolff and Reifart classified the translation planes of order 16. In [4], Johnson showed that by derivation of SEMI4 plane, we can obtain DSFP, LMRH, and JOWK planes. In [3], Dempwolff and Reifart showed that the SEMI2 plane can be obtained by derivation from DEMP plane.

Maximal arcs with $1 < a < q$ do not exist in any Desarguesian planes of odd order [5], and exist in any Desarguesian plane of even order with $a = 2^i < q$, and some non-Desarguesian planes of even order. In [6], the degree 2 maximal arcs in the translation planes of order 16 are classified. In [7], Penttila et al. did a computer search and classified all degree 2 maximal arcs in the known planes of order 16. All known planes of order 16 except two, namely BBH2 and BBS4 planes, are shown to have maximal (52, 4)-arcs [1, 2, 8, 9]. PG(2,16) is the only plane where all inequivalent maximal (52, 4)-arcs are completely classified. In the remaining of the known planes of order 16, maximal (52, 4)-arcs have not been completely classified, yet.

PG(2,16) contains exactly two maximal arcs of degree 4 (denoted by PG(2,16).1 and PG(2,16).2). The numbers of known maximal (52, 4)-arcs in the remaining of the projective planes of order 16 are: DEMP plane contains *five* (denoted by demp.1, demp.2, etc.), SEMI4 plane contains *one* (denoted by semi4.1), SEMI2 plane contains *seven* (denoted by semi2.1, semi2.2, etc.), LMRH plane contains *two* (denoted by lmrh.1 and lmrh.2), MATH plane contains *seven* (denoted by math.1, math.2, etc.), BBH1 plane contains *three* (denoted by bbh1.1, bbh1.2, and bbh1.3), JOWK plane contains *two* (denoted by jowk.1 and jowk.2), JOHN plane contains *four* (denoted by john.1, john.2, etc.), and DSFP plane contains *one* (denoted by dsfp.1) [1]. All known maximal (52, 4)-arcs in the known planes of order 16 can be found in [2].

Disjoint Maxımal ARCS of Degree 4

For a prime power q, let π be a projective plane of order q^2, and M be a maximal (m, a)-arc. If N is a maximal arc of degree a disjoint from M, then there are m lines of π meeting M in a points but external to N, and vice versa. This implies that every line in π meets M ∪ N in either a or 2a points. This observation can be generalized as

Theorem 3.1 [10] *The union of t pairwise disjoint maximal (m, a)-arcs in π is a tm-set of type ((t − 1)a, ta).*

It was reported that some v-sets of type (x, y) might be given by disjoint pairs of maximal arcs and it was shown that pairs of disjoint maximal arcs exist in DESG, SEMI2, SEMI4, MATH, JOWK and BBH1 planes [9, 10].

A disjoint maximal arc M in π is denoted by the abbreviation of the plane name (similar to maximal arcs) followed by three numbers i, j and k, where i stands for the ith maximal arc in π M is disjoint from, j indicates which maximal arc M is isomorphic to, and k runs from 1 to the number of disjoint maximal arcs related to the ith and jth maximal arcs. For example, a disjoint maximal arc denoted by *math.(2, 6).3* implies that this set is the third inequivalent set which is isomorphic to math.6 and disjoint from math.2.

In [10], twenty-one disjoint Denniston maximal arcs of degree 4 with stabilizers of orders 2 (*sixteen* of them), 4 (*three* of them), and 8 (*two* of them) are reported. Previously, it was shown that there are at least *four* disjoint maximal arcs of degree 4 in SEMI4 plane, *four* in SEMI2 plane, *three* in MATH plane, *three* in JOWK plane, and *two* in BBH1 plane [9]. All pairs of disjoint maximal arcs mentioned above are coming from isomorphic copies of the related maximal arcs, that is, in the name of all known disjoint maximal arcs i and j are equal (in the above notation). Some typos in [9] are corrected and the point sets of the isomorphic copies of the previously known disjoint maximal (52,4)-arcs found by our algorithm are listed online at

https://euniversite.nku.edu.tr/kullanicidosyalari/3705/files/KnownDisjoinrMax52Arcs.txt

We developed an algorithm to find disjoint maximal (52, 4)-arcs in all known planes of order 16. The algorithm contains the following steps:

Find the automorphism group of the plane π
for each maximal (52, 4)-arc M of π **do**
for each maximal (52, 4)-arc M′ of π **do**
for each isomorphic copy N of M′ **do**
check the intersections with M
if the intersection is empty, save N in a set S

end for
end for
check equivalencies of the sets in S and print inequivalent 52-length sets
end for

The algorithm described in the previous paragraph found the disjoint maximal (52, 4)-arcs in planes of order 16 presented below. The notation of [2] is used for the point sets and line sets of the projective planes of order 16, and the known maximal arcs of degree 4.

Previously, only three disjoint maximal (52, 4)-arcs were known to exist in MATH plane [9]. However, our computations show that there are more such sets. Details of some statistics for the known disjoint maximal (52, 4)-arcs in MATH plane can be found in Table 1, where rows (and columns) are labeled by the known maximal (52, 4)-arcs in MATH plane, an entry $n^b(c)$ in the cell (i, j) implies that there are b isomorphic copies of math.j disjoint from math.i such that the collineation stabilizers of the union of each of these sets with math.i all have order n, of which c of them are inequivalent. An empty (i, j) cell in Table 1 implies that none of the isomorphic copies of *math.j* is disjoint from *math.i*.

math.(2,2).1=[1 3 6 10 23 24 26 30 54 57 60 62 67 71 77 78 83 90 92 95 97 100 103 111 132 134 135 140 164 168 169 170 184 188 189 191 209 217 222 223 225 230 232 237 243 244 249 253 264 269 271 272],
math.(2,2).2=[2 7 14 16 34 42 43 45 56 57 58 62 85 87 91 94 99 100 103 109 114 115 120 123 132 135 136 138 147 149 152 160 178 180 185 192 197 202 205 208 211 217 221 222 228 229 233 235 263 265 270 273],
math.(2,2).3=[3 4 16 19 28 30 38 46 48 58 59 63 65 71 74 85 89 90 98 99 102 117 119 127 129 133 136 146 148 156 164 166 173 178 189 190 193 201 203 220 221 224 231 232 235 248 249 255 257 258 259 266],
math.(2,3).1=[9 10 12 24 25 32 38 42 48 49 53 62 68 69 77 81 83 84 103 108 112 115 117 127 132 139 143 146 152 156 162 167 170 182 183 184 193 203 205 210 214 217 237 238 239 243 251 254 257 258 259 266],
math.(2,3).2=[2 4 5 9 23 24 25 29 53 58 59 61 68 72 77 78 84 89 91 96 98 99 104 112 131 133 136 139 163 167 169 170 183 187 190 192 210 218 221 224 226 229 231 238 243 244 250 254 264 269 271 272],
math.(2,6).1=[7 10 11 16 19 23 25 26 49 51 56 59 82 85 87 93 97 106 109 112 113 117 127 128 133 138 141 143 167 168 169 173 178 181 184 185 193 194 203 207 210 211 216 223 227 233 235 240 264 269 271 272],
math.(2,6).2=[2 5 8 9 23 24 25 29 53 58 61 63 83 89 91 96 98 99 104 111 113 114 123 127 129 131 136 139 163 167 169 170 183 186 187 192 193 197 207 208 209 218 221 224 226 229 231 237 264 269 271 272],
math.(3,3).1=[6 8 12 13 33 36 39 44 51 52 56 62 81 88 93 95 103 105 106 109 113 121 124 126 132 138 141 142 150 153 158 159 179 182 186 188 196 198 199 207 211 215 216 217 225 227 234 239 263 265 270 273],
math.(3,3).2=[6 7 9 15 24 26 28 31 52 57 58 61 65 67

Table 1. Known disjoint maximal (52,4)-arcs in Mathon plane

i\j	1	2	3	4	5	6	7
1	$4^{12}(1)$, $8^{12}(2)$			$4^{6}(1)$	$4^{6}(1)$		
2		$16^{12}(3)$	$8^{8}(2)$			$8^{8}(2)$	
3			$16^{4}(1)$, $32^{8}(4)$				$8^{8}(2)$
4				$8^{16}(2)$, $16^{8}(2)$, $32^{4}(2)$	$8^{16}(4)$		
5					$4^{8}(1)$, $8^{8}(2)$, $16^{4}(2)$		
6							$4^{8}(2)$
7							

Representatives of the new disjoint maximal (52, 4)-arcs in MATH plane are

math.(1,4).1=[1 6 13 24 25 29 34 39 40 49 51 57 66 69 78 87 90 94 97 103 104 114 116 122 131 132 134 149 151 157 162 165 169 180 185 189 195 196 197 214 216 222 225 230 234 243 250 254 257 259 261 267],

math.(1,5).1=[21 24 25 31 33 39 44 47 49 51 52 56 65 71 73 78 82 85 86 87 114 115 121 127 149 152 156 158 164 166 169 174 177 178 180 181 196 198 204 207 211 214 215 216 242 243 252 254 263 264 272 273],

143 167 168 169 173 178 181 184 185 193 194 203 207 210 211 216 223 227 233 235 240 264 269 271 272],

70 77 84 86 94 95 99 100 104 105 131 135 136 142 161 166 168 170 177 183 185 188 215 218 221 222 225 228 236 238 243 252 253 255 260 261 262 267],
math.(3,3).3=[6 11 15 16 34 41 46 48 49 52 57 63 82 85 86 95 100 102 105 108 116 117 119 123 129 135 142 143 146 148 151 160 177 178 181 188 197 201 203 206 214 215 220 222 225 235 236 240 260 261 262 267],
math.(3,3).4=[3 4 9 13 17 22 24 29 33 41 46 47 72 76 77 79 84 88 89 90 116 118 119 124 145 148 151 159 163 170 172 175 179 183 189 190 198 201 204 206 231 232 234 238 241 243 246 250 264 269 271 272],
math.(3,3).5=[20 28 30 31 36 37 39 48 49 50 60 64 65 70 71 73 97 101 107 108 117 121 126 128 130 134 143 144 146 153 155 158 161 164 166 174 194 196 199 203 213 214 219 223 247 249 252 255 264 269 271 272],
math.(3,7).1=[2 12 15 16 17 24 29 32 39 41 42 45 49 56 57 60 66 70 72 77 97 99 103 108 134 135 138 143 147 151 152 153 162 163 170 175 177 178 182 192 214 217 221 223 243 250 252 256 263 265 270 273],
math.(3,7).2=[4 9 11 15 37 42 46 48 65 66 69 79 81 84 89 96 100 103 104 106 117 120 121 128 146 148 152 155 177 181 183 190 194 199 202 203 216 217 218 222 226 231 238 239 241 251 255 256 263 265 270 273],
math.(4,4).1=[1 3 7 11 17 25 27 29 70 71 73 78 83 88 93 96 97 103 104 106 116 123 125 126 131 132 134 141 151 153 154 160 161 164 169 174 179 184 186 187 228 230 232 240 246 250 254 256 263 269 271 273],
math.(4,4).2=[3 5 8 11 18 20 23 27 51 55 61 62 66 73 75 78 82 90 93 96 103 104 106 110 131 132 137 141 165 169 174 176 181 186 187 189 212 216 217 218 226 227 232 240 244 245 247 256 263 265 270 273],
math.(4,4).3=[1 3 12 13 20 26 29 30 55 60 62 63 67 71 72 73 81 88 90 92 97 102 103 110 129 132 134 137 163 164 168 174 179 182 189 191 212 217 220 223 230 232 234 239 247 249 250 253 260 261 262 267],
math.(4,4).4=[2 3 11 13 18 21 22 31 33 42 45 47 70 75 79 80 82 88 90 91 115 118 120 124 145 147 152 159 161 171 172 176 179 181 189 192 198 202 204 205 229 232 234 240 241 242 245 252 264 269 271 272],
math.(4,4).5=[2 5 8 10 17 19 22 26 50 54 63 64 67 74 76 79 83 91 93 96 101 102 107 111 129 130 140 144 168 172 173 175 184 186 187 192 209 213 219 220 226 227 229 237 241 246 248 253 263 265 270 273],
math.(4,4).6=[19 23 25 26 33 36 39 47 51 54 56 63 67 68 74 78 102 106 109 111 118 121 124 126 129 138 140 141 145 153 158 159 167 168 169 173 196 198 199 204 209 211 216 220 244 248 253 254 263 265 270 273],
math.(4,5).1=[1 6 8 11 22 23 25 31 36 38 41 44 56 57 59 62 83 91 92 95 100 103 104 112 131 132 135 139 184 188 191 192 193 199 206 207 211 217 222 224 225 227 230 240 241 244 252 254 264 269 271 272],
math.(4,5).2=[18 21 22 30 33 41 42 45 70 73 75 80 82 88 90 91 101 104 107 109 115 118 120 121 130 131 138 144 145 147 152 158 161 171 174 176 179 181 189 192 198 202 205 206 241 242 245 249 264 269 271 272],
math.(4,5).3=[17 19 25 26 54 56 63 64 67 74 78 79 83 91 93 96 102 106 107 111 118 121 124 126 129 140 141 144 145 153 158 159 168 169 172 173 184 186 187 192 209 211 219 220 246 248 253 254 263 265 270 273],
math.(4,5).4=[17 19 23 26 33 36 39 47 51 54 63 64 67 68 74 79 83 91 93 96 102 107 109 111 129 138 140 144 167 168 172 173 184 186 187 192 196 198 199 204 209 216 219 220 244 246 248 253 263 265 270 273],
math.(5,5).1=[5 12 14 17 19 27 34 39 43 51 52 55 67 69 76 86 94 96 102 106 111 122 125 126 129 140 141 153 154 157 162 168 175 177 185 187 196 197 208 212 215 216 226 233 239 246 248 256 257 258 259 266],
math.(5,5).2=[2 3 5 13 17 24 29 30 49 51 53 60 72 73 77 79 97 98 108 109 113 121 126 127 133 134 138 143 150 153 156 158 163 170 172 174 210 214 216 223 226 229 232 234 243 246 249 250 263 265 270 273],
math.(5,5).3=[19 24 26 29 33 35 45 47 50 54 60 64 66 69 75 80 101 102 107 108 115 118 124 125 129 130 143 144 145 152 154 159 162 165 171 176 198 200 202 204 209 213 219 223 243 248 250 253 260 261 262 267],
math.(5,5).4=[18 26 27 29 33 35 40 47 49 50 60 64 69 74 77 80 101 102 107 111 115 118 120 124 129 130 140 144 145 147 152 159 162 170 171 173 195 198 200 204 213 214 219 223 245 250 253 256 260 261 262 267],
math.(5,5).5=[19 21 24 32 33 42 45 47 50 54 63 64 66 67 72 75 97 101 107 108 118 122 124 125 130 134 143 144 145 154 157 159 163 165 168 176 198 202 204 205 209 213 219 220 242 243 248 251 260 261 262 267],
math.(6,7).1=[2 12 15 16 17 24 29 32 39 41 42 45 49 56 57 60 66 70 72 77 97 99 103 108 134 135 138 143 147 151 152 153 162 163 170 175 177 178 182 192 214 217 221 223 243 250 252 256 263 265 270 273],
math.(6,7).2=[4 5 9 15 37 45 46 48 51 57 61 62 65 66 69 79 81 82 84 89 115 117 121 128 131 132 135 141 146 147 148 155 177 183 187 190 194 199 203 205 231 238 239 240 241 251 255 256 263 265 270 273].

Previously, no disjoint maximal (52, 4)-arcs were known to exist in JOHN plane. However, our computations show that there are such sets in this plane as well. Details of some statistics for the known disjoint maximal (52, 4)-arcs in JOHN plane can be found in Table 2:

Table 2. Known disjoint maximal (52,4)-arcs in Johnson plane

i\j	1	2	3	4
1			$16^2(1)$	$16^2(1)$
2			$16^2(1)$	$16^2(1)$
3				
4				

Representatives of the new disjoint maximal (52, 4)-arcs in JOHN plane are

john.(1,3).1=[1 7 10 21 24 28 33 39 45 48 51 62 70 75 88 89 100 109 114 127 193 194 195 197 204 206 207 208 214 216

217 219 227 229 231 232 233 234 236 238 242 244 253 255 258 261 264 266 268 269 270 273],

john.(1,4).1=[1 24 39 45 65 66 69 76 79 80 83 84 87 90 93 94 103 106 117 124 129 131 134 139 142 144 145 160 162 175 178 180 181 188 189 191 198 199 202 203 210 211 222 223 228 230 235 237 243 245 252 254], *john.(2,3).1*=[1 4 13 18 24 31 38 39 43 45 56 57 70 75 88 89 100 109 114 127 193 200 201 208 209 212 215 216 217 218 221 224 226 231 234 239 241 242 246 247 250 251 255 256 257 260 262 263 265 267 271 272], *john.(2,4).1*=[1 24 39 45 65 80 83 94 99 103 104 105 106 110 113 117 118 123 124 128 136 137 147 150 152 153 155 158 164 165 167 170 172 173 183 186 196 198 203 205 211 213 220 222 225 228 237 240 245 248 249 252].

These results show that *thirty-seven* new disjoint maximal arcs of degree 4 are discovered: *four* in Johnson plane

Table 3. Binary codes associated with the known 104-sets of type (4,8)

Plane	104-set	Group order	$[k, d]_2$	$d_2^{\perp}$	Equivalent to?
MATH	(1,4).1	4	[94,2]	18	
	(1,5).1	4	[93,2]	18	
	(2,2).1	16	[90,2]	20	
	(2,2).2	16	[90,4]	20	
	(2,2).3	16	[89,2]	20	
	(2,3).1	8	[90,2]	20	
	(2,3).2	8	[90,2]	20	
	(2,6).1	8	[91,2]	20	
	(2,6).2	8	[89,2]	20	
	(3,3).1	16	[90,4]	20	*math.(3,3).2*
	(3,3).2	32	[90,4]	20	*math.(3,3).1*
	(3,3).3	32	[90,4]	20	*math.(3,3).4*
	(3,3).4	32	[90,4]	20	*math.(3,3).3*
	(3,3).5	32	[90,4]	20	
	(3,7).1	8	[90,2]	20	
	(3,7).2	8	[91,2]	20	
	(4,4).1	8	[91,2]	18	
	(4,4).2	8	[90,2]	18	
	(4,4).3	16	[89,2]	18	
	(4,4).4	16	[88,2]	18	
	(4,4).5	32	[90,2]	18	*math.(4,4).6*
	(4,4).6	32	[90,2]	18	*math.(4,4).5*
	(4,5).1	8	[89,2]	18	
	(4,5).2	8	[90,2]	18	
	(4,5).3	8	[91,2]	18	
	(4,5).4	8	[88,2]	18	
	(5,5).1	4	[93,2]	18	
	(5,5).2	8	[91,2]	18	
	(5,5).3	8	[91,2]	18	
	(5,5).4	16	[90,2]	18	
	(5,5).5	16	[92,2]	18	
	(6,7).1	4	[94,2]	20	
	(6,7).2	4	[93,2]	20	
JOHN	(1,3).1	16	[90,2]	20	
	(1,4).1	16	[90,4]	20	
	(2,3).1	16	[89,2]	20	
	(2,4).1	16	[90,4]	20	

and *thirty-three* in Mathon plane, *eighteen* of which provide examples of 104-sets of type (4,8) coming from non-isomorphic pairs of maximal (52, 4)-arcs, giving first examples for such sets. Combining this with the results previously known, we have

Theorem 3.2 *Up to isomorphism, the number of 104-sets of type (4,8) coming from disjoint maximal (52, 4)-arcs in the known planes of order 16 is ≥ 74.*

Our computations show that none of the 1-designs associated with the 104-sets of type (4,8) presented in this section are isomorphic. We check this with the sets previously found in [9], but they are not isomorphic either. We end this section with the following fact:

Theorem 3.3 *None of the 1-designs associated with the known 104-sets of type (4,8) coming from disjoint maximal 52-arcs in the known planes of order 16 are isomorphic.*

Codes Associated with 104-Sets of Type (4,8)

In this part of the study, we report some properties of the binary and ternary codes of the 1-designs associated with the 104-sets of type (4,8) presented in the previous section. In Table 3, Columns 1 and 2 presents the name of the plane and the name of the 104-set, respectively, Column 3 gives the automorphism group of the 1-design associated with 104-set, Columns 4 and 5 indicates the parameters of the binary code and the minimum distance of the dual code of the design, respectively, and the last column shows equivalencies of the binary codes considered.

Table 3 shows that even though the 1-designs coming from some of the disjoint sets are not isomorphic, the binary codes of some are equivalent. An equivalency check was performed between the binary codes associated with the 104-sets presented in this study and the binary codes associated with the ones already known in [9], no equivalent codes were found.

Our computations show that most of the ternary codes associated with the 104-sets of type (4,8) are equivalent to the 104-length code with codimension 1 (i.e., a ternary [104, 103, 2] code). The ternary codes associated with *math.(2,2).1*, *math.(3,3).5* and *math.(4,4).2* are equivalent, and the ternary code associated with *john. (2,3).1* is equivalent to the ternary code associated with *jowk.(2,2).2*. The ternary code associated with *jowk.(1,1).1* and the ternary code associated with *math.(1,1).3* are not equivalent to any ternary code associated with the known 104-sets. These observations show that the number of inequivalent ternary codes associated with the known 104-sets of type (4,8) (including the ones presented in this paper) is *five*. Parameters of these codes are given in Table 4, where a ∗ indicates the rest of the known 104-sets of type (4,8) not presented in Table 4.

Table 4. Ternary codes associated with the known 104-sets of type (4,8)

104-set	$[k, d]_3$	Equivalent to?
math.(1,1).3	[102,2]	-
math.(2,2).1	[101,2]	*math.(3,3).5, math.(4,4).2*
john.(2,3).1	[101,2]	*jowk.(2,2).2*
jowk.(1,1).1	[101,2]	-
*	[103,2]	*

CONCLUSION

This paper summarizes details and results of computer searches related to disjoint maximal (52, 4)-arcs in the known planes of order 16. It is shown that pairs of disjoint maximal arcs come not only from isomorphic copies of maximal arcs but also from non-isomorphic copies of such sets as well (previously, no such examples were known to exist).

Disjoint maximal arcs may be of interest in the study of partial geometries [11, 12], unitals [13], and code words for projective planes [14]. Disjoint maximal (52, 4)-arcs provide 104-sets of type (4, 8). A new lower bound on the number of 104-sets of type (4,8) associated with disjoint maximal (52, 4)-arcs in the known planes of order 16 is obtained. The 104-length binary and ternary linear codes generated by the blocks of 1-designs associated with the known 104-sets of type (4,8) are classified. Our computations show that 1-designs studied in this paper are unique (up to isomorphism).

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AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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