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RESEARCH ARTICLE / ARAŞTIRMA MAKALESI

Investigation of Crack Effects on the Nonlinear Vibrations of Microbeams with a Tip Mass in a Magnetic Field

Uç Kütlesi Bulunan Mikrokirişlerin Manyetik Alan Altında Lineer Olmayan Titreşimlerinde Çatlakların Etkilerinin İncelenmesi

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Abstract

Microelectromechanical systems (MEMS) are critical members of modern technological devices, due to their applications in various industrial fields. In the physical applications of MEMS, cracks are a common structural problem, affecting the static and dynamic behavior of the system. In this paper, the effects of cracks on microbeams with a tip mass under the influence of a magnetic field have been investigated. The micro-size effect of the beam has been involved into the model by using the modified couple stress theory. The crack has been modeled by using a torsional spring, with the spring coefficient corresponds to the severity of the crack. Thus, the beam has been modeled as consisting of two segments connected by a torsional spring. The equations of motion have been formulated using Hamilton's principle. The obtained equations have been solved by using the method of multiple scales, a perturbation technique. Frequencies regarding both linear and nonlinear vibrations of the microbeams have been examined. The results obtained in this study have been validated by using available numerical results in the literature. The effects of parameters such as crack severity, crack location, tip mass and the magnetic field force on linear and nonlinear vibrations have been presented. The results indicate a significant decrease in the natural frequencies and nonlinear frequencies of microbeams with increasing crack severity.

Keywords: *Nonlinear Vibrations, Cracked Microbeams, Perturbation Techniques, Modified Couple Stress Theory, Method of Multiple Scales*

Öz

Mikroelektromekanik sistemler (MEMS), birçok endüstriyel alana uygulanabilmeleri sayesinde, modern teknolojik cihazların önemli elemanlarından biri haline gelmiştir. MEMS' in fiziksel uygulamalarında sıklıkla ortaya çıkan çatlaklar, sistemin statik ve dinamik davranışlarını etkilemektedir. Bu makalede, manyetik alan etkisi altındaki, uç kütleye sahip mikrokirişler üzerindeki çatlakların etkileri incelenmiştir. Kirişin mikro boyut etkisi, değiştirilmiş çift gerilme teorisi kullanılarak modele dahil edilmiştir. Çatlak, bir burulma yayı kullanılarak modellenmiştir ve burulma yay katsayısı çatlak şiddetine karşılık gelmektedir. Böylece kiriş, burulma yayı aracılığı ile birbirine bağlı iki kısımdan oluşacak şekilde modellenmiştir. Hareket denklemleri Hamilton prensibi uygulanarak oluşturulmuştur. Elde edilen denklemler, bir perturbasyon yöntemi olan çok ölçekli metot kullanılarak çözülmüştür. Mikrokirişlerin hem lineer hem de nonlineer titreşimlerine ilişkin frekansları incelenmiştir. Bu çalışmada elde edilen sonuçlar, literatürde bulunan mevcut sayısal sonuçlar kullanılarak doğrulanmıştır. Çatlak şiddeti, çatlak konumu, uç kütle ve manyetik alan kuvveti gibi parametrelerin lineer ve nonlineer titreşimler üzerindeki etkileri sunulmuştur. Elde edilen sonuçlar, mikrokirişlerin doğal frekansları ve nonlineer frekanslarında, çatlak şiddetinin artmasıyla birlikte önemli ölçüde düşüş olduğunu göstermektedir.

Anahtar Kelimeler: *Nonlineer Titreşimler, Çatlak İçeren Mikrokirişler, Perturbasyon Teknikleri, Değiştirilmiş Çift Gerilme Teorisi, Çok Ölçekli Metot*

1. Introduction

Microelectromechanical systems (MEMS) are significant component in numerous modern technological devices across various industries. Due to their ability to integrate mechanical and electrical functionalities on a micro scale structure, these systems have found applications in fields such as aerospace, automotive engineering, biomedical and health industries. Beam type structures are generally used as the main components of MEMS. Understanding the static and dynamic behavior of these systems is crucial for their reliable operation.

The mechanical characteristics of small sized systems have different properties than macro sized structures [1-2]. Hence, additional elasticity theories have been derived to consider the effects of small size properties on static and dynamic responses. Yang et al. [3] have firstly introduced a couple stress based strain gradient theory, which includes only one additional material length scale parameter. Park and Gao [4] and Ma et al. [5] have employed this theory to the Bernoulli-Euler beam and to Timoshenko beams, respectively. They have predicted decreasing deflections and increasing bending rigidity as the beam size decreases. Kong et al. [6] have investigated the vibrational characteristics of Bernoulli-Euler microbeams. They observed the significant decrease in natural frequencies as the size of the beam increases. Reddy [7] have presented static deflection, free vibration and buckling analyses of functionally graded beams. They have obtained smaller static deflections and increased natural frequencies as the beam thickness increases. Asghari et al. [8] have implemented the modified couple stress theory to static and vibration problems of inhomogeneous

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composite beams, revealing a significant difference between the results of the aforementioned theory and the classical beam theory. Şimşek et al. [9] have investigated the size dependent vibrations of microbeams and predicted the significant effects of the scale parameter on plates.

Nonlinear phenomena also have a profound impact on the dynamics of microstructures as they have complex dynamic responses. Wang et al. [10] have studied the nonlinear free vibrations of microbeam. The predicted nonlinear frequencies obtained by the modified couple stress theory are larger than those obtained from the classical beam model. Ghayesh et al. [11] have investigated nonlinear resonant dynamics of microbeams. In recent years, the researchers have studied through the linear and nonlinear vibrational characteristics of micro [12-13] and nano beams [14-15] by using the modified couple stress theory.

Cracks have been one of the structural problems on small sized structures, which affects the mechanical behaviors significantly. The previous studies in the literature [16-18] have predicted decreasing natural frequencies of the small sized structures, as the crack severity increases. Vibrations of cracked nanobeams have been analyzed by using modified couple stress theory [19] and nonlocal elasticity theory [20]. Larkin et al. [21] have investigated the effects of a crack on microgyroscopes. Free vibration analysis of multi-cracked microbeams [22], functionally graded microbeams [23] and piezoelectric nanobeams [24] have been studied, presenting the significant effects of the cracks on natural frequencies.

In this study, nonlinear vibrations of cracked microbeams under magnetic field force have been investigated. Micro size property of the beam has been involved into the model by using the modified couple stress theory. The crack has been modelled by introducing a torsional spring located at the crack position, where the spring coefficient corresponding to the crack severity. To obtain the approximate solutions for linear and nonlinear transverse vibrations, method of multiple scales, which is a perturbation technique [25] has been employed. The numerical results have been validated by the available results of the literature. The effects of crack severity, the crack location, tip mass parameter and the magnetic field force on natural frequencies and on nonlinear frequencies have been presented. Frequency-response curves have been plotted to demonstrate the nonlinear behavior affected by the crack and the system parameters.

2. Method

Mechanical properties of structures at micro-scale differ from those at the large scale [Fleck-Lam et al]. Modified couple stress theory [Yang et al] is employed in this study to involve the effects of small size of the beam. One additional parameter, which is used to characterize the couple stress is introduced, in addition to two classical Lame parameters. The strain energy of a Bernoulli-Euler beam under small deformation is written as follows [3]:

$$U = \frac{1}{2}\int_0^L (\mathrm{EI} + GAl^2)\, w''^2 dx \tag{1}$$

E is the modulus of elasticity, I is the area moment of inertia, G is the shear modulus and A is the cross-sectional area. w is the transverse displacements of the beam along z-axis, where x is the longitudinal axis. Here, l is the material length scale parameter, introduced by the modified couple stress theory and represents the small size-dependency of the structure. The energy expression regarding the classical beam theory is obtained by setting the parameter l to zero.

2.1. The model of the cracked microbeam with tip mass

In this paper, cantilever microbeams with tip mass under magnetic field force are investigated. The schematic figure of the model, including the crack, is given in Figure 1.

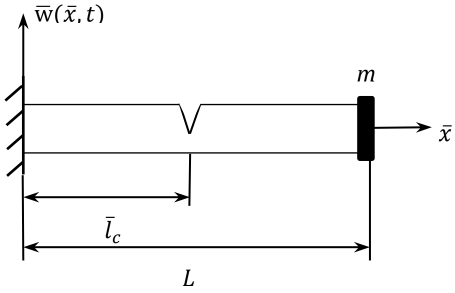


Figure 1. Schematic figure of the cantilever microbeam with tip mass.

Kinetic energy (T) and the strain energy (U) of the cracked microbeam are expressed as:

$$T = \frac{1}{2}\int_0^{\bar{l}_c} \rho A\, \dot{\bar{w}}_1^{\,2} d\bar{x} + \frac{1}{2}\int_{\bar{l}_c}^{L} \rho A\, \dot{\bar{w}}_2^{\,2} d\bar{x} + \frac{1}{2} m \dot{\bar{w}}_2(L)^2 \tag{2}$$

$$\begin{aligned} U = &\frac{1}{2}\int_0^{\bar{l}_c} (EI + GAl^2)\bar{w}_1''^2 d\bar{x} + \frac{1}{2}\int_{\bar{l}_c}^{L} (EI + GAl^2)\bar{w}_2''^2 d\bar{x} \\ &+ \frac{1}{2}EA\int_0^{\bar{l}_c}\left(\bar{u}_1' + \frac{1}{2}\bar{w}_1'^2\right) d\bar{x} \\ &+ \frac{1}{2}EA\int_{\bar{l}_c}^{L}\left(\bar{u}_2' + \frac{1}{2}\bar{w}_2'^2\right) d\bar{x} \end{aligned} \tag{3}$$

The beam is considered in two segments that are divided by the crack. $\bar{w}_1$ and $\bar{w}_2$ are the transverse displacements, $\bar{u}_1$ and $\bar{u}_2$ are the longitudinal displacements of the two segments, respectively. ρ is the mass density, m is the quantity of the tip mass, L is the length of the microbeam, $\bar{l}_c$ is the position of the crack measured from the left end of the beam. The strain energy of the beam includes the size-dependent deformation energy, which is given in Eq. (1).

The microbeam is subjected to an axial magnetic field, leading the Lorentz force expressed as follows [12]:

$$f_m = \eta A {H_x}^2 \frac{\partial^2 w}{\partial x^2} \tag{4}$$

where η is the magnetic permeability and H_x is the longitudinal magnetic field vector along x-axis.

The kinetic and strain energies of the system satisfy the below expression according to the Hamilton' s principle:

$$\int_{t_1}^{t_2} (\delta T - \delta U)\, dt = 0 \tag{5}$$

Equations of motion of the two segments of the microbeam becomes:

$$\begin{aligned} (EI + \mu A l^2)\overline{w_1}^{iv} + \rho A \ddot{\bar{w}}_1 \\ = \eta A {H_x}^2 \bar{w}_1'' + \frac{EA}{2L}\int_0^{\bar{l}_c} \bar{w}_1'^2\, d\bar{x}.\bar{w}_1' \end{aligned} \tag{6}$$

$$(EI + \mu A l^2)\overline{w_2}^{iv} + \rho A \ddot{\overline{w}}_2 = \eta A H_x{}^2 \overline{w}_2{}'' + \frac{EA}{2L}\int_{\bar{l}_c}^{L} \overline{w}_2{}'^2\, d\bar{x}.\overline{w}_2{}' \tag{7}$$

In order to express the equations in a convenient form, the following non-dimensional parameters are introduced and listed in Table 1.:

Table 1. The boundary and the compatibility conditions.

Non-dimensional parameters	
$w = \frac{\overline{w}}{d}$	Transverse displacement
$x = \frac{\bar{x}}{L}$	Longitudinal axis parameter
$t = \frac{\bar{t}}{L^2}\sqrt{\frac{EI}{\rho A}}$	Non-dimensional time parameter
$\gamma = \frac{\mu A l^2}{EI}$	Micro size parameter
$H = \frac{\eta A H_x{}^2 L^2}{EI}$	Magnetic force parameter
$\alpha_2 = \frac{Ad^2}{I}$	Slenderness of the beam
$l_c = \frac{\overline{l_c}}{L}$	Crack location parameter
$M = \frac{m}{\rho AL}$	Tip mas parameter
$\omega_n = \bar{\omega}_n L^2\sqrt{\frac{\rho A}{EI}}$	Non-dimensional natural frequency

Non-dimensional form of the equations are obtained as follows:

$$(1+\gamma)w_1{}^{iv} + \ddot{w}_1 + \mu\dot{w}_1 = H w_1{}'' + \frac{1}{2}\alpha_2\int_0^{l_c} w_1{}'^2\, dx.w_1{}'' + F\cos\Omega t \tag{8}$$

$$(1+\gamma)w_2{}^{iv} + \ddot{w}_2 + \mu\dot{w}_2 = H w_2{}'' + \frac{1}{2}\alpha_2\int_{l_c}^{1} w_2{}'^2\, dx.w_2{}'' + F\cos\Omega t \tag{9}$$

Transverse excitation with the amplitude F and the frequency Ω and damping effect with the damping constant μ are inserted into the equations additionally.

2.2 Method of Solution

The equations of motion of the microbeam are solved according to the method of multiple scales, a perturbation technique [25]. Time parameter is divided into slow ($T_0 = t$) and fast ($T_1 = \varepsilon t$) scales, where ε is a perturbation parameter in small order. Time derivatives are expressed in terms of these new time parameters as follows:

$$\frac{\partial}{\partial t} = \mathrm{D}_0 + \varepsilon D_1 \quad \frac{\partial^2}{\partial t^2} = \mathrm{D}_0{}^2 + 2\varepsilon D_0 D_1 + .. \quad D_n = \frac{\partial}{\partial T_n} \tag{10}$$

The solutions of the two segments of the microbeam are expanded as:

$$w_1(x,t:\varepsilon) = w_{01}(x,T_0,T_1) + \varepsilon w_{11}(x,T_0,T_1) + O(\varepsilon^2) \tag{11}$$

$$w_2(x,t:\varepsilon) = w_{02}(x,T_0,T_1) + \varepsilon w_{12}(x,T_0,T_1) + O(\varepsilon^2) \tag{12}$$

$O(\varepsilon^2)$ represents the smaller order parameters. Substituting the expanded solutions into Eqs. (8) and (9), the equations can be expressed separately for two orders. Eqs. (13-14) involve the parameters at $O(1)$ and present the linear problem. Eqs. (15-16), presenting the nonlinear problem, includes the $O(\varepsilon)$ parameters and excitation and damping effects which are assumed to be at $O(\varepsilon)$:0

$$(1+\gamma)w_{01}{}^{iv} + D_0{}^2 w_{01} - H w_{01}{}'' = 0 \tag{13}$$

$$(1+\gamma)w_{02}{}^{iv} + D_0{}^2 w_{02} - H w_{02}{}'' = 0 \tag{14}$$

$$(1+\gamma)w_{11}{}^{iv} + D_0{}^2 w_{11} - H w_{11}{}'' = -2D_0 D_1 w_{01} + \frac{1}{2}\alpha_2\int_0^{l_c} w_{01}{}'^2 dx\,.w_{01}'' - \mu D_0 w_{01} + F\cos\Omega t \tag{15}$$

$$(1+\gamma)w_{12}{}^{iv} + D_0{}^2 w_{12} - H w_{12}{}'' = -2D_0 D_1 w_{02} + \frac{1}{2}\alpha_2\int_{l_c}^{1} w_{02}{}'^2 dx\,.w_{02}'' - \mu D_0 w_{02} + F\cos\Omega t \tag{16}$$

The solution of the linear problem can be assumed as $w_{0i}(x,T_0,T_1) = A(T_1)e^{i\omega T_0}Y_i(x) + cc$, for i=1,2. Here, ω is the frequency of vibration and cc refers the complex conjugates. This assumption is substituted into the Eqs. (13-14), resulting the expression:

$$(1+\gamma)Y_i(x)^{iv} - \omega^2 Y_i(x) - HY_i(x)'' = 0 \quad i = 1{,}2. \tag{17}$$

$Y_i(x)$ can be expressed as the shape functions of the microbeam as follows:

$$\begin{aligned} Y_1(x) &= c_1 e^{ir_1 x} + c_2 e^{ir_2 x} + c_3 e^{ir_3 x} + c_4 e^{ir_4 x} \quad 0 < x < l_c \\ Y_2(x) &= c_5 e^{ir_1 x} + c_6 e^{ir_2 x} + c_7 e^{ir_3 x} + c_8 e^{ir_4 x} \quad l_c < x < 1 \end{aligned} \tag{18}$$

Eq. (18) is substituted into Eq. (17) and obtained that:

$$r_n{}^4(1+\gamma) - \omega^2 - Hr_n{}^2 = 0 \qquad n = 1 \ldots 4. \tag{19}$$

The unknown coefficients r_1- r_4 are found as:

$$r_n = \mp\sqrt{\frac{-H^2 \mp \sqrt{H^4 - 4(1+\gamma)\omega^2}}{2(1+\gamma)}} \qquad n = 1..4 \tag{20}$$

The boundary conditions of the microbeam with clamped-free ends and the compatibility conditions of the crack location are listed in Table 2. K_t is the torsional spring coefficient, which represents the crack severity.

Eq. (18) is subjected to the boundary and the compatibility conditions. A set of linear equations are obtained as a result. Coefficient matrix of the parameters $c_1 \ldots c_8$ are formed from the set of equations and determinant of the matrix is set to zero to calculate the natural frequencies.

Table 2. The boundary and the compatibility conditions.

Left end	Crack location	Right end
$Y_1(0)=0$	$Y_1(l_c)=Y_2(l_c)$	$Y_2''(1)=0$
$Y_1'(0)=0$	$Y_2'(l_c)-Y_1'(l_c)-K_tY_1''(l_c)=0$	$Y_2'''(1)-M\omega^2Y_2(1)=0$
	$Y_1''(l_c)=Y_2''(l_c)$	
	$Y_1'''(l_c)=Y_2'''(l_c)$	

The nonlinear equations that are given in Eqs. (15-16) have the solution in the following form:

$$w_{1i}(x,T_0,T_1;\varepsilon)=\phi(x,T_1)e^{i\omega T_0}+W(x,T_0,T_1)+cc \qquad i=1,2 \tag{21}$$

The assumed solution and the solutions of the linear problem, $w_{01}(x,T_0,T_1)$ and $w_{02}(x,T_0,T_1)$ are inserted into the nonlinear problem. W includes the non-secular parameters. The substitution results the following expressions for the two segments of the microbeam:

$$\begin{aligned}(1+\gamma)\phi_1^{iv}-\phi_1\omega^2-H\phi_1''&=-2i\omega D_1AY_1(x)\\&+\frac{1}{2}\alpha_2A^2\bar{A}\left[\bar{Y}_1''(x)\int_0^{l_c}Y_1'(x)^2dx\right.\\&\left.+2Y_1''(x)+\int_0^{l_c}Y_1'(x)\bar{Y}_1'(x)dx\right]\\&-\mu Ai\omega Y_1(x)+\frac{F}{2}e^{i\sigma T_1}+NST+cc\end{aligned} \tag{22}$$

$$\begin{aligned}(1+\gamma)\phi_2^{iv}-\phi_2\omega^2-H\phi_2''&=-2i\omega D_1AY_1(x)\\&+\frac{1}{2}\alpha_2A^2\bar{A}\left[\bar{Y}_1''(x)\int_{l_c}^{1}Y_1'(x)^2dx\right.\\&\left.+2Y_1''(x)+\int_{l_c}^{1}Y_1'(x)\bar{Y}_1'(x)dx\right]\\&-\mu Ai\omega Y_1(x)+\frac{F}{2}e^{i\sigma T_1}+NST+cc\end{aligned} \tag{23}$$

NST denotes the non-secular term of the equation. The frequency of excitation is taken as $\Omega=\omega+\varepsilon\sigma$ to ensure the primary resonance, where σ is the tuning parameter. The solvability condition, which eliminates the secular terms, is written as follows:

$$2i\omega D_1A-k_1A^2\bar{A}+\mu Ai\omega-f_1e^{i\sigma T_1}=0 \tag{24}$$

where k_1 and f_1 are obtained as:

$$\begin{aligned}k_1&=\frac{\alpha_2}{4i\omega S}\left[\int_0^{l_c}\bar{Y}_1''(x)\bar{Y}_1(x)\int_0^{l_c}Y_1'(x)^2dx\,dx\right.\\&+\int_{l_c}^{1}\bar{Y}_2''(x)\bar{Y}_2(x)\int_{l_c}^{1}Y_2'(x)^2dx\,dx\\&+2\int_0^{l_c}Y_1''(x)\bar{Y}_1(x)\int_0^{l_c}Y_1'(x)\bar{Y}_1'(x)dx\,dx\\&\left.+2\int_{l_c}^{1}Y_2''(x)\bar{Y}_2(x)\int_{l_c}^{1}Y_2'(x)\bar{Y}_2'(x)dx\,dx\right]\end{aligned} \tag{25}$$

$$f_1=\frac{F}{4i\omega S}\left[\int_0^{l_c}\bar{Y}_1(x)dx+\int_{l_c}^{1}\bar{Y}_2(x)dx\right] \tag{26}$$

where $S=\int_0^{l_c}Y_1(x)\bar{Y}_1(x)dx+\int_{l_c}^{1}Y_2(x)\bar{Y}_2(x)dx$.

Phase-amplitude equations are obtained by substituting the amplitudes as $A=\frac{1}{2}ae^{i\theta}$ and $\bar{A}=\frac{1}{2}ae^{-i\theta}$ into Eq. (24):

$$D_1a=k_{1R}\frac{1}{4}a^3-\frac{\mu}{2}a+2f_{1R}cos\alpha-2f_{1I}sin\alpha \tag{27}$$

$$D_1\alpha=\sigma-k_{1I}\frac{1}{4}a^2-2\frac{f_{1R}}{a}sin\alpha-2\frac{f_{1I}}{a}cos\alpha \tag{28}$$

where $\alpha=\sigma T_1-\theta$. The subscripts R and I denote the real and imaginary components of the terms, respectively. Derivatives of the amplitude and the phase, D_1a and $D_1\alpha$ respectively, converges to zero for a steady-state solution. The tuning parameter σ is solved by using Eq. (28) as follows:

$$\sigma=\frac{1}{4}k_{1I}a^2\pm\sqrt{\frac{4}{a^2}\left(f_{1R}{}^2+f_{1I}{}^2\right)-\frac{\mu^2}{4}} \tag{29}$$

The nonlinear frequency of the microbeam is calculated by considering the free and undamped vibrations and setting $f=\mu=0$ in Eq. (29). The obtained frequency is:

$$\omega_{nl}=\omega_n+\frac{1}{4}k_{1I}a_0{}^2 \tag{30}$$

3. Results and Dicussion

In this paper, numerical results of cracked cantilever microbeam with a tip mass are presented. Firstly, obtained results are verified by comparing with the results of the available literature. Then, the results of linear and nonlinear frequencies of the cracked microbeam are given in the following sections.

3.1. Verification of the results

In order to ensure the accuracy of the numerical results of the cracked beam model, non-dimensional frequency parameters, which are obtained in the present study, are compared with the ones of Loya et al. [16]. A simply-supported microbeam is considered with the properties of the beam: $L=100\,h$, $b=10h$, $\rho=8166\,kg/m^3$, $E=207\,GPa$, $\nu=0.3$. Nonlocal parameter is set to zero. The results for the first four modes of vibration are given in the Table 3.

Table 3. Verification of the results. (M=0, H=0, l_c=0.5)

Non-dimensional frequency parameters	K_t=0		K_t=0.065	
	Loya et al [16]	Present study	Loya et al [16]	Present study
1	3.1416	3.14159	3.0469	3.04691
2	6.2832	6.28319	6.2832	6.28319
3	9.4248	9.42478	9.1669	9.16691
4	12.5664	12.5664	12.5664	12.5664

Table 3 (continued)

Non-dimensional frequency parameters	K_t=0.35		K_t=2	
	Loya et al [16]	Present study	Loya et al [16]	Present study
1	2.7496	2.74957	2.0960	2.09598
2	6.2832	6.28319	6.2832	6.28319
3	8.6129	8.61288	8.0730	8.07304
4	12.5664	12.5664	12.5664	12.5664

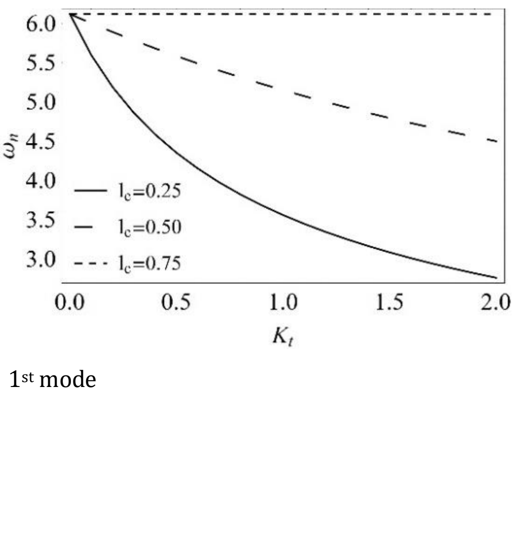
3.2 Numerical results of the cantilever microbeam with tip mass

In order to investigate the effects of the crack, the microbeam with tip mass under clamped-free boundary conditions are considered. Other parameters regarding the material and size properties are listed as:

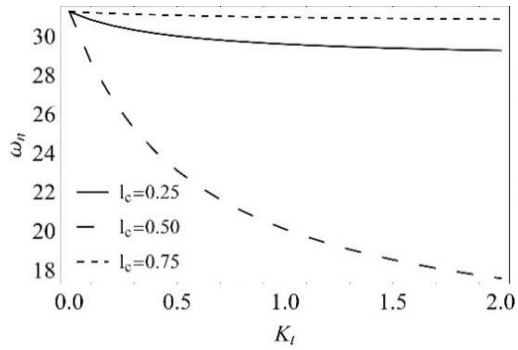
$$h=100\ \mu m\ b=3h,\ L=10\ h,\ E=1.44\ GPa,\\ \rho=1600\ kg/m^3,\ \nu=0.38,\ l=17.6\ \mu m \tag{31}$$

Non-dimensional natural frequencies regarding the first three modes of vibration with respect to the crack severity are presented in Figure 2. Varying crack positions for cantilever microbeams with M=0.1, H=10 are considered. For each mode, natural frequencies decrease with increasing crack severity. For the first mode, the decrease in natural frequency is more significant when the crack is located at l_c=0.25. The effect of crack severity becomes insignificant when the crack location is l_c=0.75, which is close to the free end boundary of the microbeam. In the second mode, the decrease in natural frequencies is more pronounced. Specifically, when the crack is at l_c=0.50, the decrease is greater than when the crack is at l_c=0.25. This is related to the mode shape of the second mode, which gives larger slope at the crack location. For the third mode, the decrease in natural frequency is more remarkable for l_c=0.25 and becomes insignificant as the crack location approaches to the free end of the microbeam.

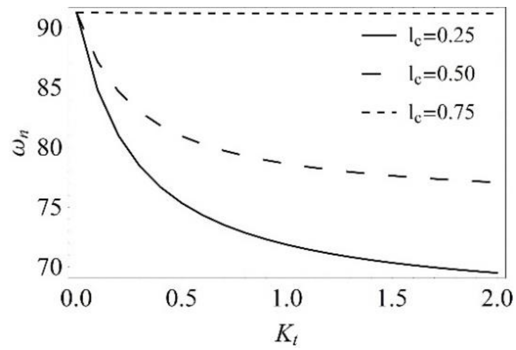
In Figure 3, the results predicting the effects of tip mass on the first mode non-dimensional natural frequencies are shown. The crack is taken at the midpoint of the beam and the magnetic force parameter is H=10. The results predict a significant increase in natural frequencies as the tip mass parameter increases. Moreover, the effect of the crack is seen more clearly for the microbeam without a tip mass. The presence of the crack becomes less significant as the tip mass parameter increases.



a. 1st mode



b. 2nd mode



c. 3rd mode

Figure 2. Non-dimensional natural frequencies of cantilever microbeams with varying crack positions.

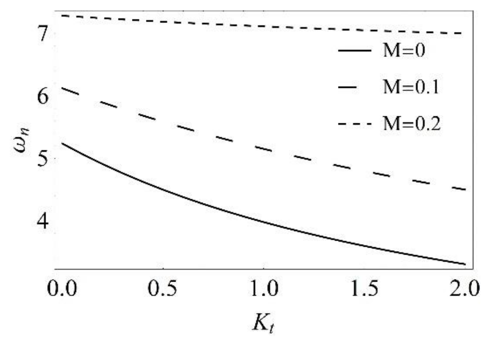


Figure 3. Non-dimensional natural frequencies of cantilever microbeams with varying tip mass values.

Figure 4 shows the non-dimensional natural frequencies with respect to the crack severity for different magnetic force parameters. The microbeams with the crack location l_c=0.5 and with the tip mass parameter M=0.1 are investigated. Increasing natural frequencies are obtained as the magnetic force parameter increases. The effect of the crack severity leads decreasing natural frequencies for all magnetic force parameters. However, the decrease becomes more pronounced when the force parameter increases.

By considering the steady-state free vibration without damping and excitation, nonlinear frequencies are plotted with respect to the amplitude in Figure 5. Cantilever microbeams with M=0.1 and l_c=0.5 and varying crack severity parameters are taken. The nonlinear frequencies decrease significantly as the crack severity increases.

In Figure 6, effects of varying crack location on the first mode nonlinear frequencies are illustrated, by considering the crack severity as K_t=1 and the tip mass parameter M=0.1. The nonlinear frequency increases as the crack location is closer to the free end of the microbeam. Moreover, the nonlinear behavior alters with the increasing amplitudes for different crack locations.

Figure 7 represents the effects of the tip mass of the cracked cantilever microbeam. Nonlinear frequencies are plotted with respect to the amplitude for varying tip mass parameters and for l_c=0.5 and K_t=1. As the tip mass parameter increases, the nonlinear frequencies increases. This result is related to the increasing stiffness of the microbeams with tip mass.

The frequency-response curves of the cracked microbeam having a M=0.1 tip mass are given in Figure 8. The curves corresponding to the different crack severity parameters are obtained by taking l_c=0.5, α_2=3, F=5 and μ=0.1. Increasing nonlinear behavior is observed as the crack severity increases.

Effects of the crack location and the tip mass parameter are demonstrated in Figures 9 and 10, respectively. Frequency-response curves of the first mode vibrations are plotted. In Figure 9, the microbeam has a crack with K_t=1 and tip mass parameter of M=0.1. The curve shows softening behavior for l_c=0.25 and l_c=0.75, whereas a hardening behavior is observed for l_c=0.5. In Figure 10, the beam has a crack at the midpoint with K_t=1. Softening nonlinear behavior is observed for the microbeam without tip mass. However, hardening effect is observed when the tip mass parameter increases.

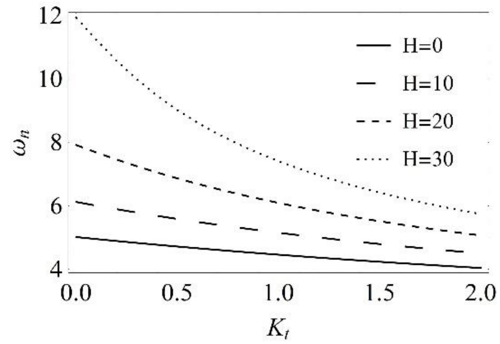


Figure 4. Non-dimensional natural frequencies of cantilever microbeams with varying magnetic force parameters.

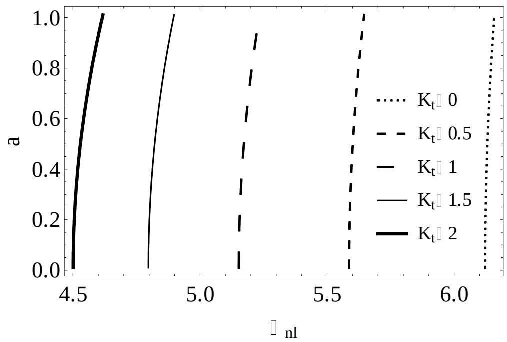


Figure 5. Nonlinear frequencies with respect to the amplitude for varying crack severities.

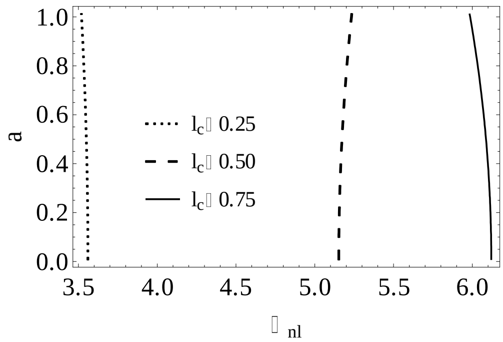


Figure 6. Nonlinear frequencies with respect to the amplitude for varying crack locations.

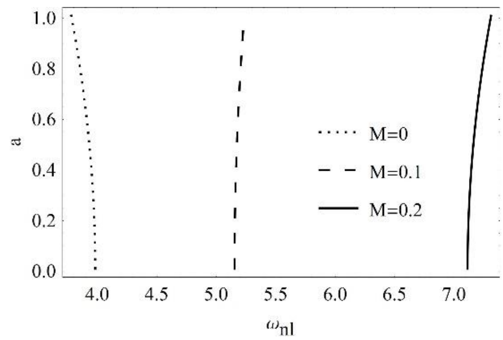


Figure 7. Nonlinear frequencies with respect to the amplitude for varying tip mass parameters.

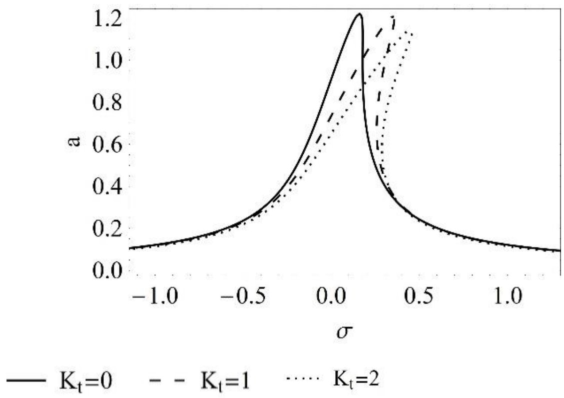


Figure 8. Frequency-response curves of microbeams for varying crack severities.

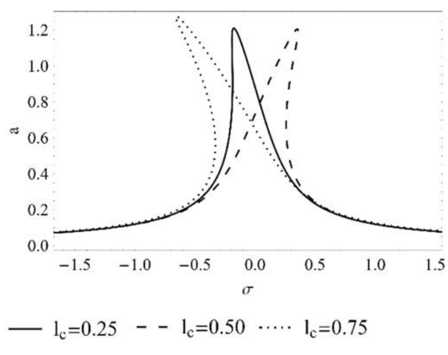


Figure 9. Frequency-response curves of microbeams for varying crack severities.

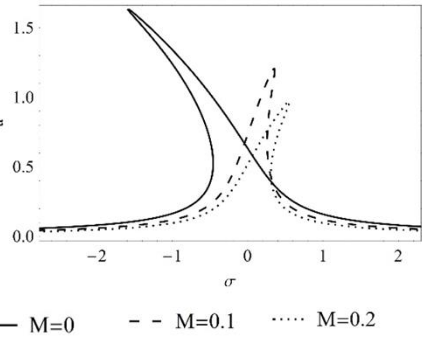


Figure 10. Frequency-response curves of microbeams for varying tip mass parameters.

4. Conclusions

The effects of cracks on nonlinear vibrations of microbeams under a magnetic field have been investigated in this paper. Cantilever microbeams with tip mass attached to the free end have been included. The micro size effect of the beam has been involved into the model by using modified couple stress theory. A material length scale parameter has been included in the strain energy of the microbeam, which describes the small size property. The crack has been modelled by using a torsional spring and the spring coefficient corresponds to the crack severity. The derived equations of motion have been solved by employing the method of multiple scales, which is a perturbation technique. The approximate numerical results regarding both linear and nonlinear vibrations of the cracked microbeams have been obtained. To ensure the accuracy of the results, natural frequencies obtained from this study have been compared with the ones obtained from the literature.

The effects of the crack severity, crack location, magnetic field force, tip mass have been investigated. It is concluded that natural frequencies and nonlinear frequencies decrease significantly as the crack severity increases. The frequency-response curves of the cracked microbeams reveals a change in the nonlinear behavior. As the crack severity increases, hardening characteristic of the curves increases. Additionally, the peak amplitudes decrease with increasing crack severities.

The crack location has been observed as a major parameter, which alters the effects of cracks. For different mode shapes of vibration, sensitivity to crack severity of natural frequencies varies, as the crack location varies. For the first mode, the decrease amount of natural frequencies increases as the crack location is closer to the fixed end of the microbeam. The presence of the crack becomes more critical and effective in such microbeams. On the other hand, for the second mode, the decrease is more pronounced for the microbeam having a crack at the midpoint. This is related to the mode shape of the second mode, resulting larger slope at the crack location. Nonlinear frequencies have also been investigated and it is concluded that the nonlinear frequency decreases as the crack location becomes closer to the fixed end of the beam, in the first mode vibration. Moreover, the frequency-response curves reveal altering nonlinear behaviors with different crack locations.

The presence of the tip mass on the cantilever microbeams leads an increasing effect on the natural frequencies and on the nonlinear frequencies. There is a consistent increase in frequencies with the increasing tip mass parameter. The results show a softening nonlinear behavior for the microbeam without tip mass. However, the frequency-response curves predict a hardening behavior when the tip mass is added. The hardening behavior has an increasing effect as the tip mass parameter is increased. In addition, the peak amplitude of the response decreases as the tip mass parameter increases. This results re related with the increasing stiffness of the microbeam as the tip mass parameter increases.

The influence of the magnetic field has been investigated. As the magnetic force parameter increases, the natural frequencies increases. Specifically, the effect of the crack severity becomes more significant as the magnetic force parameter increases. The findings of this study can provide insights for the real-world applications of MEMS, to estimate the health and performance and to design the micro components of these systems.

Ethics committee approval and conflict of interest statement

This article does not require ethics committee approval. This article has no conflicts of interest with any individual or institution.

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