GU J Sci 30(4): 232-251 (2017)

Gazi University

Journal of Science

http://dergipark.gov.tr/gujs

A New feedback gain matrix based LQR PI Controller for Integrator Time Delay process

K Harshavardhana REDDY[1], Prabhu RAMANATHAN[1*]

[1]School of Electrical Engineering, VIT University, Vellore, India

Article Info

Received: 11/05/2017
Accepted: 18/08/2017

Keywords

LQR
Feedback Matrix
IPTD Process
Control Matrix
PI Controller

Abstract

In this paper, a Linear Quadratic Regulator (LQR) based PI controller is proposed for the Integrator plus Time Delay Process (IPTD). In LQR PI controller design process the selection of weight matrices 'Q' plays a key role in order to minimize settling time, peak overshoot and Integral Errors. So, In order to improve the system performance, a new feedback gain matrix is proposed for the optimal selection of weight matrices in the controller design. First, the LQR PI controller is designed for First order Time Delay Process (FOPDT) and then grabbed the formulas for IPTD process by assuming the state variable as zero. The simulation results are presented to validate the proposed method by using different IPTD process. The proposed controller is also experimentally validated on a temperature control process.

1. INTRODUCTION

Integrating plus time delay (IPTD) processes, in a system have at least one pole at the origin combined with time delay [1-2]. Some examples of these type integrating processes are liquid storage tank, bioreactors, distillation column, pressure flowing to a turbine generator in power plant, isothermal copolymerization [3-5] etc. This type of integrating systems has open loop stability and unbounded output for a bounded input. Thus controlling of integrating time delay systems is a really difficult task. In recent times, many researchers are focused on the integrating systems to achieve the better performance with advanced controlling techniques [6-8]. The PI/PID controller tuning was first proposed by Ziegler-Nichols [9], and has been improved by many researchers. The overview of various methods to tune the PI/PID controller gain values for the integrator, first and second order time delay process which is in recent literature are presented in the introduction. Some noticeable methods are nonlinear PD controller [1], IMC methods [10-14], Direct synthesis method [15-19], Smith predictor controller [20-24], sliding mode controller [25], 2 Degree of Freedom controller [26-28], time optimal plug & control [29,30] and frequency domain method [31,32].

In recent trends, researchers also following the optimization technique to optimize the controller gain values using various algorithms. In [33], authors proposed the online tuning PI controller based on the minimization of Integral Square Errors. In [34], authors applied the genetic algorithm to tune the PID controllers for Integrator and unstable process. Here the authors taken the objective function subjected to the minimization Integral Square Error (ISE), Integral Time Square Error (ITSE), Integral Square Time error (ISTE). In [35,36], authors proposed the optimal tuning of PID controllers using a genetic algorithm with objective function as Integral Absolute Error (IAE), ISE and constraints of maximum sensitivity, set-point tracking, and disturbance rejection. But disadvantages of [34-36], is genetic optimization algorithm purely depends on the size of the population. In [37], authors used the advanced optimization algorithm

*Corresponding author, e-mail: ishupa@gmail.com

called Bacterial foraging criteria to get optimal PI/PID controllers. In these, the authors considered the objective function as minimization of ISTE, IST2E, and IST3E to get optimal gain values. In [38-40], authors used the differential evaluation algorithm to tune the optimal values of PID controller. The differential evolution algorithm is advanced than a genetic algorithm, which is used to minimize the IAE and ISE errors as the objective function.

The disadvantages of sequential quadratic model programming have heavy computational time and it purely depends on the size of the population. The design techniques based on LQR are well recognized in modern control theory. To overcome above problems authors proposed the LQR solution to obtain optimal gain values. However, two main constraints of LQR problem have been the subject of investigation since the 1960s and the constraints are not only the choice of Q and R weighting matrices but also solutions of Algebraic Riccati Equation (ARE) [41]. The two problems are strongly time dependent, under certain operational conditions. Even if all of the control strategies are optimal in nature, different values of Q and R will ultimately end up with a different system response, which indicates that the response is non-optimal in the true sense [41-42]. In [43] authors proposed the LQR PI controller for first order time delay process. The normal feedback gain matrix is used to find the optimal weight matrices. In [44] the authors proposed the LQR PID controller for second order time delay process by considering the dominant pole placement method. In this paper we proposed a new feedback gain matrix based to obtain the optimal values of Q and R. The organization of paper is as follows section-1 deals the introduction and literature survey of a paper, section-2 describes the design of LQR PI controller for integrator time delay process, section-3 explains the effect of weight matrices, proposed feedback gain matrix based optimal selection of weight matrices, section 4 shows the simulation and experimental results of proposed technique and finally section 5 gives the conclusions of the paper.

2. LQR BASED OPTIMAL PI CONTROLLER FOR PURE INTEGRATOR TIME DELAY PROCESS

In this section, the design of the LQR based PI controller is presented. In general, industrial process is modeled by transfer function with integrator time delay process. In order to design an LQR based PI controller, a pure integrator time delay process is considered. The transfer function model G(s) of IPTD is shown in Eq. (1) [1].

$$G(s) = \frac{b}{s} e^{-Ls} \tag{1}$$

First, we designed the LQR PI controller for first order time delay process, from the formulas of FOPDT, derive the tuning formulas for IPTD process.

A linear plant with time delay can be represented as

$$\dot{x}(t) = Ax(t) + Bu(t-L) \quad t \geq 0 \tag{2}$$

Where 'A' is state transition matrix, 'B' is control matrix, 'X' is state matrix and 'L' represents time delay.

When $0 \leq t < L$, u(t-L) =0; There is no input signal to process

When, t≥L; the eq. (2) has to the valid non-zero input signal.

Then the two cases are derived from Eq.(2), as obtained as Eq.(3) and Eq.(4)

$$\dot{x}(t) = Ax(t), \text{when } 0 \leq t < L \tag{3}$$

$$\dot{x}(t) = Ax(t) + Bu^n(t), \text{when } t \geq L \tag{4}$$

When $u^n(t) = u(t-L)$; Eq. (3) and Eq. (4) are free dead time, standard LQR method could be easily applied.

To find the optimal control vector $u^n(t)$ subjected to minimize the cost function, 'J' given in Eq. (5).

$$J = \int_0^\infty (x^T(t)Qx(t) + u^T(t)Ru(t))dt \tag{5}$$

Where 'Q' is semi-positive definite state weight matrix and 'R' is positive definite control weight matrix.

The LQR solution for the cost function is given in Eq. (6)

$$u^n(t) = -R^{-1}B^T Px(t) \tag{6}$$

Where 'P' is the positive symmetric definite solution of the Algebraic Riccati equation [42], which is shown in Eq. (7)

$$A^T P + PA + Q - PBR^{-1}B^T P = 0 \tag{7}$$

By Converting $u^n(t)$ in Eq. (6) back to u(t), we obtain the LQR solution to the original process Eq. (3) with the index Eq. (5) as $u(t) = u^n(t+L) = -R^{-1}B^T Px(t+L)$.

Here u(t) gives the control signal in the overall time bound $t \geq 0$. However x(t+L) is not directly known. Using the Eqs. (3), (4) and (6) the x(t+L) is expressed as x(t) [43]. The optimal control vector of the x(t+L) can be expressed in following Eqs. (8) and (9).

$$u(t) = -R^{-1}B^T Pe^{A_c(t)}e^{A(L-T)}X(t)\ ; \qquad 0 \leq t \leq L \tag{8}$$

$$u(t) = -R^{-1}B^T Pe^{A_c L}X(t)\ ; \qquad t \geq L \tag{9}$$

Where, $A_c = A - BR^{-1}B^T P$ i.e. feedback gain matrix.

The order of the weight matrices depends on the order of the system. In the design of PI controller, the selection of weight matrices in the diagonal form will not affect the performance of LQR based PI controller [45]. The Eqs.(10) and (11) shows the diagonal form of R value and weight matrices.

$$Q = diag(q_1, q_2, \ldots\ldots\ldots q_n) \tag{10}$$

$$R = diag(r_1, r_2, \ldots\ldots\ldots r_n) \tag{11}$$

2.1. PI Controller Tuning for Integrator Time Delay Process

A Systematic block diagram of closed loop system with the PI Controller for FOPDT is shown in Figure 1.

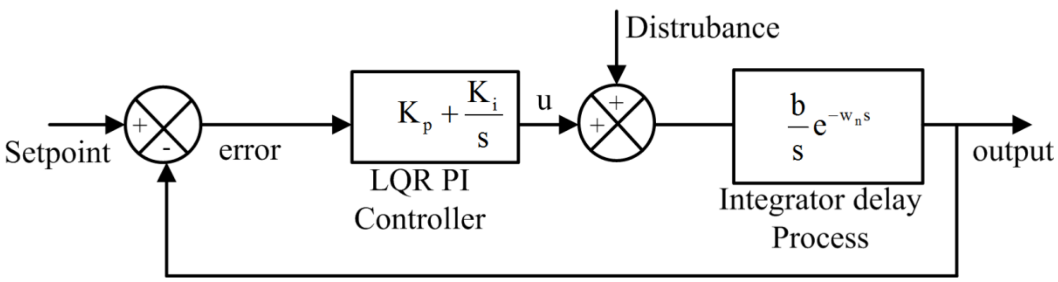

Figure 1. *LQR based PI Controller for Pure Integrator time delay process*

The PI Controller has two variables known as proportional gain k_p and integral gain k_i. The transfer function of PI controller is stated in Eq. (12).

$$C(s) = k_p + \frac{k_i}{s} \tag{12}$$

The two controller gains are considered as state variables to design the LQR PI controller for delay process. The state variables are given in Eq. (13).

$$u(t) = k_p e(t) + k_i \int e(t)dt \tag{13}$$

The state model of the first order process is given by the Eq. (14).

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & -a \end{bmatrix} x + \begin{bmatrix} 0 \\ -b \end{bmatrix} u(t-L) \tag{14}$$

The model emphasized that both the variables are available and state feedback matrix is simplified by $k_i \int_0^t e\,dt + k_P\,e$.

To derive the gain value of the PI controller, the control matrix, state transition matrix, weight matrices and P matrix is substituted in Riccati Equation is as shown in Eq. (7).

$$\begin{bmatrix} 0 & 0 \\ 1 & -a \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} + \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & -a \end{bmatrix} + \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix} - \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}\begin{bmatrix} 0 \\ -b \end{bmatrix} R^{-1} \begin{bmatrix} 0 & -b \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} = 0 \tag{15}$$

Where

A = state transition matrix i.e. $\begin{bmatrix} 0 & 1 \\ 0 & -a \end{bmatrix}$

B = Control Matrix i.e $\begin{bmatrix} 0 \\ -b \end{bmatrix}$

Q = Weight matrices considering diagonal matrix i.e. $\begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}$

P = $\begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}$

From Eq. (16), P_{11}, P_{12} and P_{22} are derived and is shown in Eqs. (16) - (18).

$$P_{12} = \frac{\sqrt{q_1 R}}{b} \tag{16}$$

$$P_{22} = \frac{\left(-Ra + \sqrt{R^2 a^2 + Rb^2 (2P_{12} + q_2)}\right)}{b^2} \tag{17}$$

$$P_{11} = aP_{12} + R^{-1} b^2 P_{12} P_{22} \tag{18}$$

For IPTD process the constant ‘a’ = 0 then the modified formulas of P_{11}, P_{12} and P_{22} is given as shown in Eqs. (19)- (22)

$$P_{12} = \frac{\sqrt{q_1 R}}{b} \tag{19}$$

$$P_{22} = \frac{\sqrt{(2Rb^2 p_{12} + Rb^2 q_2)}}{b^2} \tag{20}$$

$$P_{11} = R^{-1} b^2 P_{12} P_{22} \tag{21}$$

The feedback gain matrix of the LQR solution in Eq. (7) is given in Eq. (22).

$$A_c = A - BF \tag{22}$$

Where A = $\begin{bmatrix} 0 & 1 \\ 0 & -a \end{bmatrix}$; B = $\begin{bmatrix} 0 \\ -b \end{bmatrix}$ and F is the feedback matrix, shown in Eq. (23).

$$F = -R^{-1}\begin{bmatrix} P_{12} & P_{22} \end{bmatrix} \tag{23}$$

Matrices A, B, and F are substituted in Eq. (22) to get Eq. (24)

$$A_c = \begin{bmatrix} 0 & 1 \\ 0 & -a \end{bmatrix} + \begin{bmatrix} 0 \\ -b \end{bmatrix} R^{-1} b \begin{bmatrix} P_{12} & P_{22} \end{bmatrix} \tag{24}$$

Solving the above equation with substituting P_{22} shown in Eq. (17) then Eq. (25) is obtained

$$A_c = \begin{bmatrix} 0 & 1 \\ -R^{-1}b^2 p_{12} & -\sqrt{a^2 + R^{-1}b^2(2p_{12} + q_2)} \end{bmatrix} \tag{25}$$

For the IPTD process, the modified feedback matrix is given in Eq. (25) considering ‘a’ value as zero.

$$A_c = \begin{bmatrix} 0 & 1 \\ -R^{-1}b^2 P_{12} & -\sqrt{R^{-1}b^2(2P_{12} + q_2)} \end{bmatrix}$$

For the time delay process, the tuning formulae for the LQR based PI controller [45] is given as

$$U(t) = \begin{cases} -Fe^{A_c t} e^{A(L-T)} x(t); & 0 \le t \le L \\ -Fe^{A_c t} x(t); & t \ge L \end{cases} \tag{26}$$

Here $e^{(A(L-T))} = L^{-1}(SI - A)^{-1}\big|_{(L-T)} = \begin{bmatrix} 1 & \dfrac{(1 - e^{(-a(L-T))})}{a} \\ 0 & e^{(-a(L-T))} \end{bmatrix}$ (27)

And $e^{A_c t} = L^{-1}(SI - A_c)^{-1} = \begin{bmatrix} Z_{11}(t) & Z_{12}(t) \\ Z_{21}(t) & Z_{22}(t) \end{bmatrix}$ (28)

Where

$$Z_{11}(t) = \frac{1}{\alpha_1 - \alpha_2}\left[(\alpha_1 + \hat{a}_1)e^{\alpha_1 t} - (\alpha_2 + \hat{a}_1)e^{\alpha_2 t}\right]$$

$$Z_{12}(t) = \frac{1}{\alpha_1 - \alpha_2}\left[e^{\alpha_1 t} - e^{\alpha_2 t}\right]$$

$$Z_{21}(t) = \frac{-\hat{a}_2}{\alpha_1 - \alpha_2}\left[e^{\alpha_1 t} - e^{\alpha_2 t}\right]$$

$$Z_{22}(t) = \frac{1}{\alpha_1 - \alpha_2}\left[\alpha_1 e^{\alpha_1 t} - \alpha_2 e^{\alpha_2 t}\right]$$

To set optimal controller gains stated in Eq. (12), then Eqs. (27),(28) are substituted in Eq. (25).
For the interval $0 \le t \le L$,

$$k_i(t) = R^{-1}b\left[P_{12}Z_{11}(t) + P_{22}Z_{21}(t)\right] \tag{29a}$$

$$k_p(t) = R^{-1}b\left\{\frac{1}{a}P_{12}Z_{11}(t) + \frac{1}{a}P_{22}Z_{21}(t) + \left[P_{12}Z_{12}(t) - \frac{1}{a}P_{12}Z_{11}(t) + P_{22}Z_{22}(t) - \frac{1}{a}Z_{21}(t)\right]e^{-a(L-T)}\right\}$$

(29b)

For the interval $t \ge L$,

$$k_i(t) = R^{-1}b(P_{12}Z_{11}(L) + P_{22}Z_{21}(L)) \tag{30a}$$

$$k_p(t) = R^{-1}b(P_{12}Z_{21}(L) + P_{22}Z_{22}(L)) \tag{30b}$$

Where the constants P_{12} and P_{22} are given in Eq. (19) and Eq. (20). From Eq. (19) and Eq.(20) , LQR based PI controller performance depends on the weight matrices Q and R. It would be computationally simple If 'R' value is assumed as 1[43]. Finally, the PI controller parameter for $0 \le t < L$ obtained using the Eqs. (29a) and (29b). The PI parameters for $t \ge L$, calculated from Eqs. (30a) and (30b). At the interval between $0 \le t < L$ the PI gain parameters is time varying. This leads to larger control efforts. It may cause actuator saturation. Hence, for time varying samples, implementing PI controller is difficult. Therefore, PI parameters are constant throughout the performance time. For the ease of practical implementation, we consider only for all the time $t \ge L$ for the examples. The output response for condition t ≥ L only the optimal response will be obtained. The output performance of the process depends upon the natural frequency and damping ratio. If these parameters are not selected properly the performance of the controller will be poor.

3. SELECTION OF WEIGHT MATRICES USING NEW FEEDBACK GAIN MATRIX

In this section, the effect of weight matrices on the response of the time delay process is analyzed. The weight matrices Q and R will affect the output performance of the time delay system to a larger extent. If weight matrices 'Q' values are selected randomly, the system illustrates a poor performance, it shows the higher settling time, which necessitates the selection of Q values in the context of the defined system. For better understanding, let us consider an example with the performance of different weight matrices as shown in Figure.2.

The output response for q1= q2=1 settles at 7 sec with IAE of 1.174. When q1= q2=2, the response settles at 6.7 sec, error IAE is 1.043. For q1=3 and q2= 1, settling time is 18 sec and error IAE is 2.4, It has an oscillatory response when it is settled. By observing these variations of output response for different weight matrices, it is evident that the Integral error IAE, other time domain specifications of the system will solely depend on the selection of weight matrices.

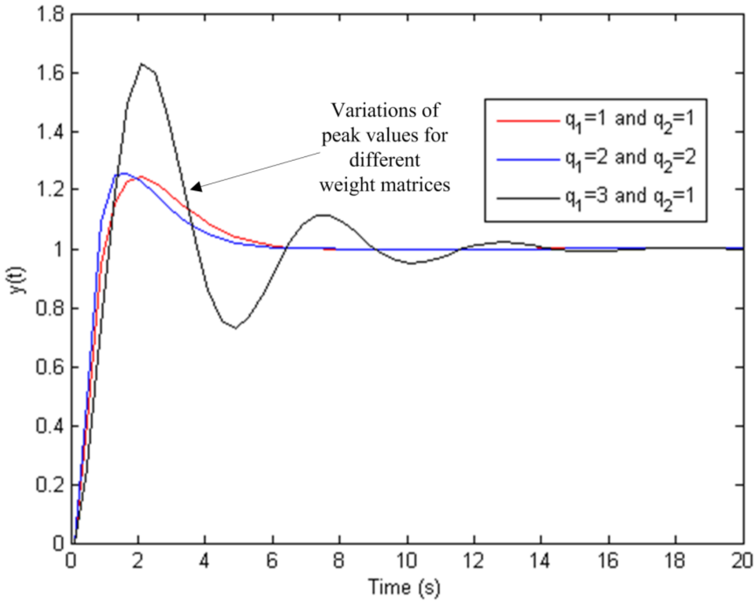

Figure 2. *Output response of the time delay system for different weight matrixes*

As both Q and R both influence the system, it would be computationally simple, if R value is assumed as 1[43].

3.1. Selection of weight matrices for Pure Integrator time delay process

To find the selection of weight matrices the modified feedback gain matrix is used. The general formula for the feedback gain matrix is given in following Eq.(31).

$$A_c = A - \dot{B}\dot{F} \tag{31}$$

Here consider the value of BF is modified as 2(A-BF) to get optimal weight matrices for integrator time delay process.

$$\dot{B}\dot{F} = 2(A - BF) = \begin{bmatrix} 0 & 2 \\ -b^4 P_{12} & -2\sqrt{a^2 + b^2(2P_{12} + q_2)} \end{bmatrix} \tag{32}$$

So the final modified equation is given in Eq.(33).

$$A\text{-}(2(A\text{-}BF)) \tag{33}$$

Then substitute the values of A and B,F values in the above equations.

$$A_c = \begin{bmatrix} 0 & 1 \\ 0 & -a \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ -bP_{12} & -2\sqrt{a^2 + b^2(2P_{12} + q_2)} \end{bmatrix} \tag{34}$$

$$A_c = \begin{bmatrix} 0 & -1 \\ 2b^2 P_{12} & -a - 2\sqrt{a^2 + b^2(2P_{12} + q_2)} \end{bmatrix} \tag{35}$$

The characteristic equation of a matrix is given in Eq. (36).

$$\Delta(s) = |SI - A_c| = s^2 + 2\xi_p \omega_p s + \omega_p^2 \tag{36}$$

To find the characteristic equation for the modified feedback matrix is given Eq. (35) is shown in Eq.(37)

$$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 2b^2 P_{12} & -a - 2\sqrt{a^2 + b^2(2P_{12} + q_2)} \end{bmatrix} \tag{37}$$

$$\begin{bmatrix} s & -1 \\ -2b^2 P_{12} & -a + 2\sqrt{a^2 + b^2(2P_{12} + q_2)} \end{bmatrix} \tag{38}$$

$$|SI - A_c| = s^2 + s(-a - 2(\sqrt{a^2 + b^2(2P_{12} + q_2)})_{12} + b^2 P_{12} \tag{39}$$

The Eq. (39) is compared to the general characteristics equation of Eq. (36) to get the suitable values of weight matrixes for time delay models derived in Eqs. (40) and (41) by substituting the values of P_{12} and $P_{22.}$

$$q_1 = \frac{\omega_p^4}{4b^2} \tag{36}$$

$$q_2 = \frac{(2 * \xi_p * \omega_p - a)^2 - 4 * a^2 - 2 * \omega_p^4}{4b^2} \tag{37}$$

Here for integrator time delay systems, the value 'an' is zero then the modified weight matrices 'q_1' and 'q_2' are written in Eq. (32) and Eq. (33).

$$q_1 = \frac{\omega_m^4}{4b^2} \tag{38}$$

$$q_2 = \frac{(4\xi_m^2 \omega_p^2 - 2 * \omega_p^4)}{4b^2} \tag{39}$$

The parameters of weight matrices are obtained in the form of damping ratio (ξ_p) and natural frequency (ω_p). The selection of these parameters are specified in the range of ξ_p = [0.7 to 1.0] and $\omega_p L$ = [0.1 to 0.4] [43] & [44]. Here, done the simulations for various natural frequency and damping ratio values, then we consider the best value of W_pL and ξ_p to get the PI controller values. The proposed method is used to tune only the PI controller.

4. SIMULATION RESULTS

The proposed design formulas are tested with various process models, and in each case study observed that the proposed controller gives the better performance than the existing methods. In order to validate the proposed controller method, three examples of IPTD process systems are considered and shows the mathematical and simulation results. The proposed controller is also implemented in the practical temperature process station to validate the real time implementation.

Example 1

In Example 1 the transfer function G_1 (s) is considered with process (k) = 1, delay time (L) = 1. Then the pure integrator time delay process is obtained as shown in Eq. (40).

$$G_1(s) = \frac{1}{s} e^{-s} \tag{40}$$

For the proposed method we considered the desired closed loop parameters i.e damping ratio $\xi_p = 0.91$, the natural frequency with delay $\omega_p L = 0.4$. The simulation results of proposed method for this example are compared with Ali et.al [32] stated in the literature. The gain values of the proposed and Ali et.al [32] methods are listed in Table.1. To validate the performance of the controller the settling time, IAE, Integral Square Error(ISE), Integral Absolute Error (ITAE) are tabulated in Table.2

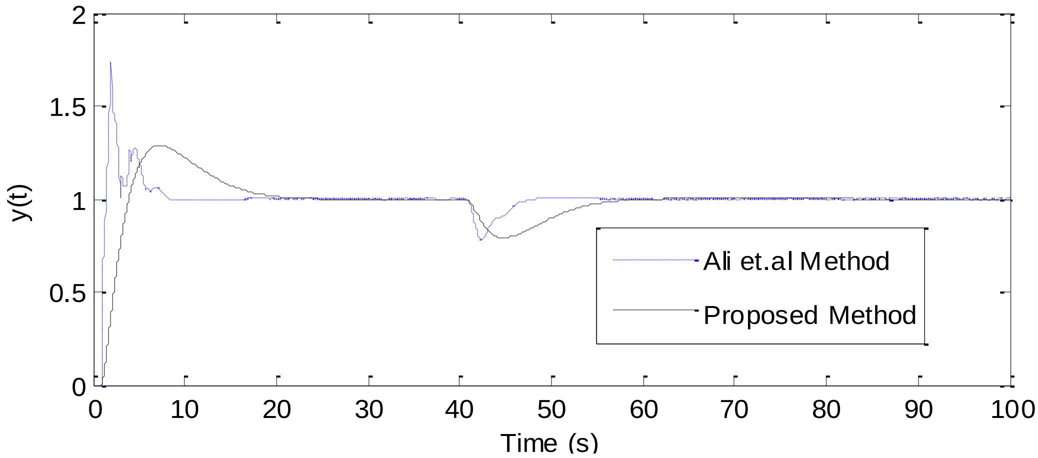

Figure 3. *Closed loop response for nominal plant in Example 1*

The simulation results of the plant are shown in Figure 3. The input disturbance of the magnitude -0.2 is applied at 30 sec to evaluate the performance of the system.

Robustness Analysis with a perturbation of 10% in the process gain (k), delay time (L) and both k and L are shown in Figure 4, 5 and 6 respectively. The existing method Ali et.al [32] has more rise time than proposed method. When 10% mismatch occurs in the process then the existing method shows the drastic improvement in rise time but proposed method rise time quite same as a normal response. And also using the proposed method we achieved the good tracking response than Ali et.al [32] method which is more oscillations. This result shows the proposed method yields the better robustness property than the existing method with respect to a mismatch in the process

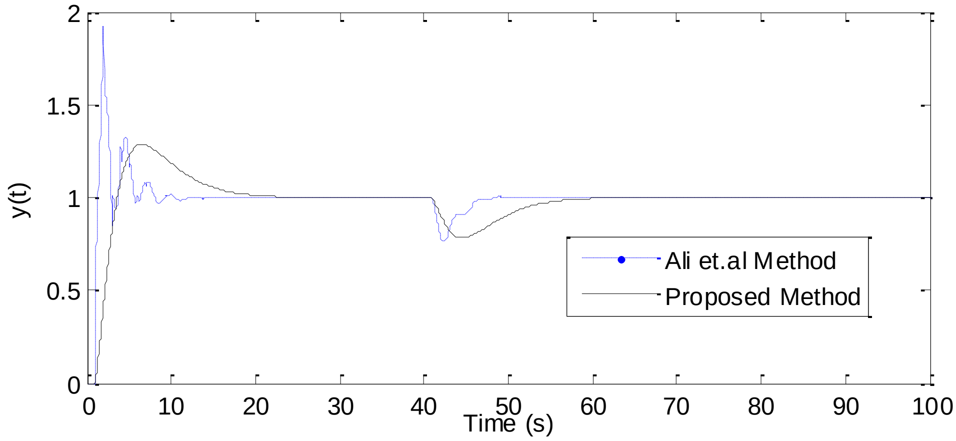

Figure 4. *Closed loop response for example 1 with +10% perturbation in process gain*

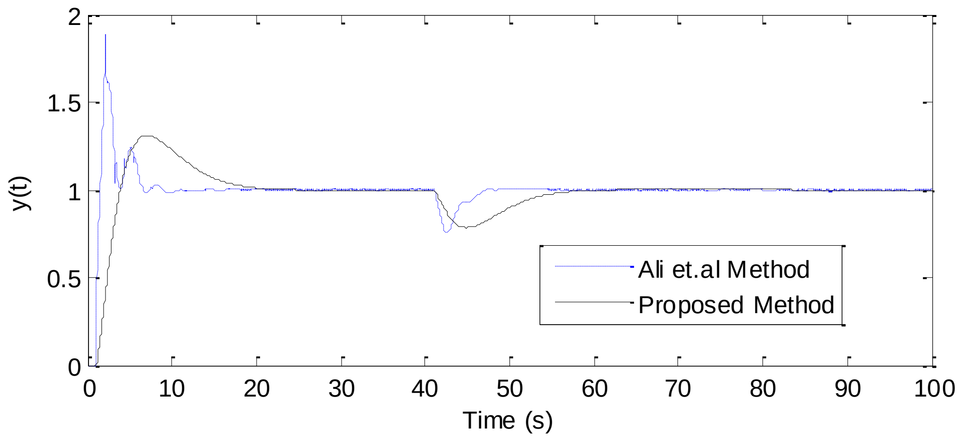

Figure 5. *Closed loop response for example 1 with +10% perturbation in delay time*

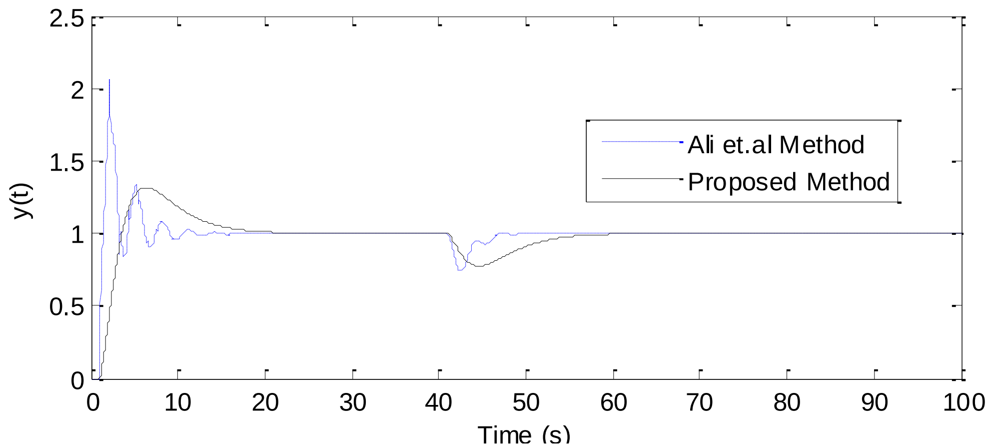

Figure 6. *Closed loop performance for example 1 with +10% perturbation in both process gain and delay time*

The figure 7 shows the closed loop responses for different values of natural frequency and damping ratio. Here, done the simulations for various of natural frequency values and we consider the best value of W_pL and ξ_p to get the PI controller values.

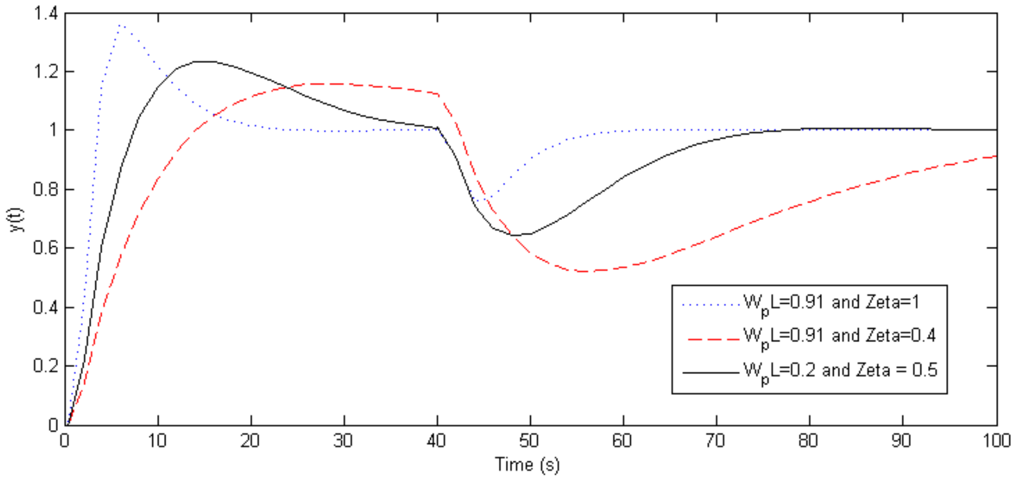

Figure 7. *Closed loop performance for example 1 with different values of $\omega_p L$ and ξ_p*

Example 2

For Example 1 the transfer function $G_2(s)$ is considered with process (k) = 1, delay time (L) = 5. Then the integrator time delay process is obtained as shown in Eq. (41).

$$G_1(s) = \frac{1}{s} e^{-5s} \tag{41}$$

For the proposed method we considered the desired closed loop parameters, damping ratio $\xi_p = 0.91$, the natural frequency with delay $\omega_p L = 0.1$. The simulation results of proposed method for this example are compared with M. Chidambaram et.al [36] stated in the literature. The gain values of the proposed and M. Chidambaram et.al [36] methods are listed in Table.1. To validate the performance of the controller the settling time and performance indices are tabulated in Table.2. The simulation results of the plant are shown in Figure 8. The input disturbance of the magnitude -0.2 is applied at 150 sec to evaluate performance.

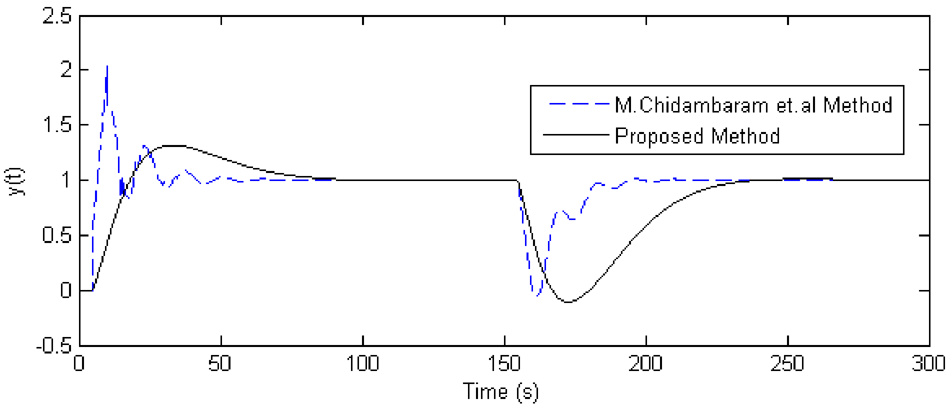

Figure 8. *Closed loop response for nominal plant in Example 2*

Robustness Analysis with a perturbation of 10% in the process gain (k), delay time (L) and both k and L are shown in Figure 9, 10 and 11 respectively. The existing method M. Chidambaram et.al [36] has more rise time than proposed method. When 10% mismatch occurs in the process the existing method shows the high rise time, but proposed method rise time has quite same as a normal response. The existing method also has the oscillatory waveform and its take long time to settle down. The proposed method has achieved the good tracking response than Chidambaram et.al [36] method. The existing method has more oscillations when set to mismatch the process. This result shows the proposed method yields the better robustness property than the existing method with respect to a mismatch in the process.

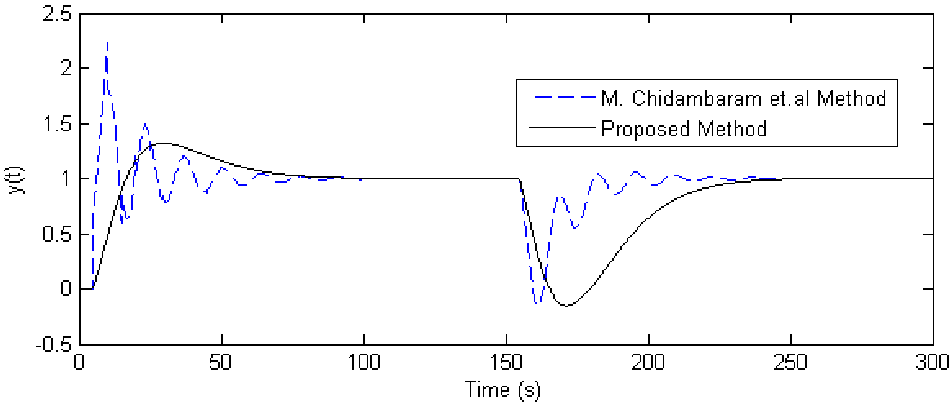

Figure 9. *Closed loop response for example 2 with +10% perturbation in process gai*

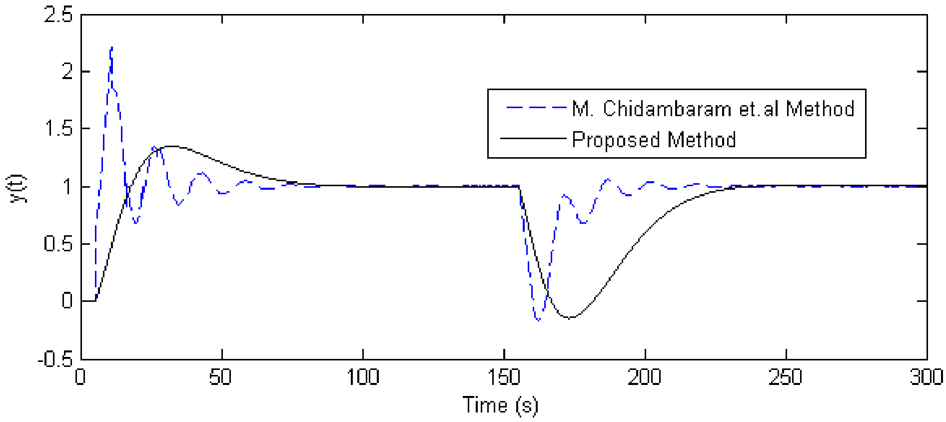

Figure 10. *Closed loop response for example 2 with +10% perturbation in delay time*

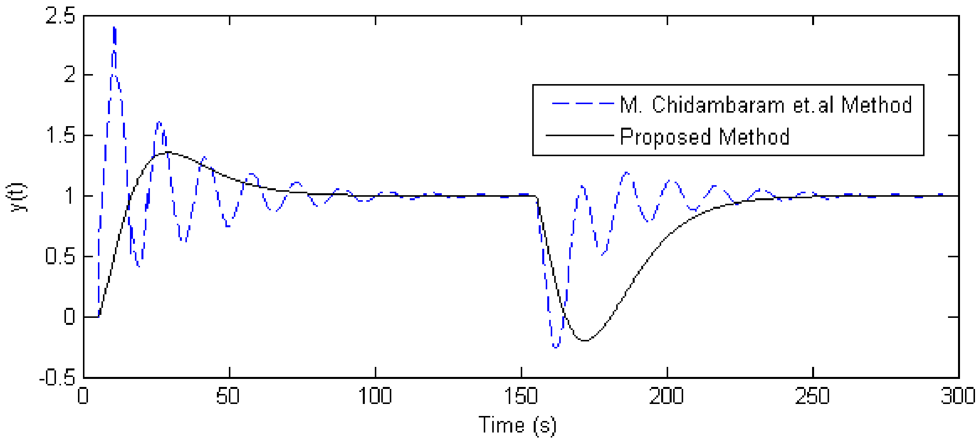

Figure 11. *Closed loop response for example2 with +10% perturbation in both process gain and delay time*

Example 3

Next, for Example 3 the transfer function $G_3(s)$ is considered with considered as process gain (k) = 1, delay time (L) = 4. Then the integrator time delay process is obtained as shown in Eq (42).

$$G_1(s)=\frac{0.01}{s}e^{-4s} \tag{42}$$

For the proposed method we considered the desired closed loop parameters i.e damping ratio $\xi_p = 0.97$, the natural frequency with delay $\omega_p L = 0.15$. The simulation results of proposed method for this example are compared with M. Chidambaram et.al [36] and Ali et.al [32] methods stated in the literature. The gain values of the proposed and existing methods are listed in Table.1. To validate the performance of the controller, the settling time and performance indices are tabulated in Table.2.

Table 1. *Controller parameters for Example 1-4*

System	**Transfer Function**	Method	Controller Parameters		
			k_p	k_i	k_d
1.	$G_1(s)=\frac{1}{s}e^{-1s}$	Ali et.al [32]	1.03	0.3280	0.5047
		Proposed Method	0.3671	0.0052	-
2.	$G_2(s)=\frac{1}{s}e^{-5s}$	M. Chidambaram et.al [36]	0.25	0.0152	0.6
		Proposed Method	0.0782	0.0025	-
3.	$G_4(s)=\frac{0.01}{s}e^{-4s}$	Ali et.al Method [32]	25.76	2.03	50.489
		M. Chidambaram et.al [36]	30.86	1.7144	55.548
		Proposed Method	8.7	0.32	-
4.	Temperature Process Station	Z.N Method	2	0.01	-
		Proposed Method	1.24	0.016	-

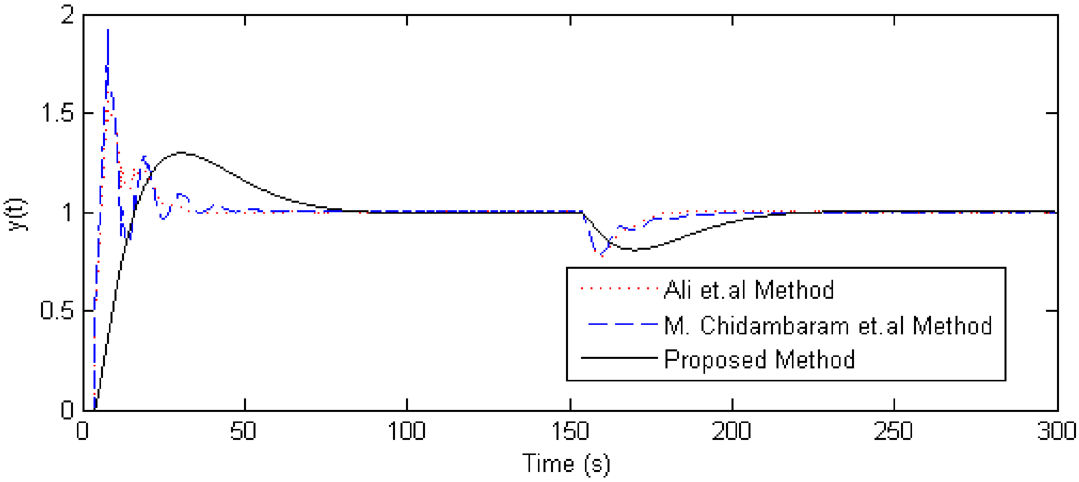

Figure12. *Closed loop response for nominal plant in Example 3*

The simulation results of nominal plant are shown in Figure 12. The input disturbance of the magnitude -2 is applied at 150 sec to evaluate performance. Robustness analysis with a perturbation of 20% in the process gain (k), delay time (L) and both k and L are shown in Figure 13, 14 and 15 respectively. The existing method M. Chidambaram et.al [36] has more rise time than proposed method.

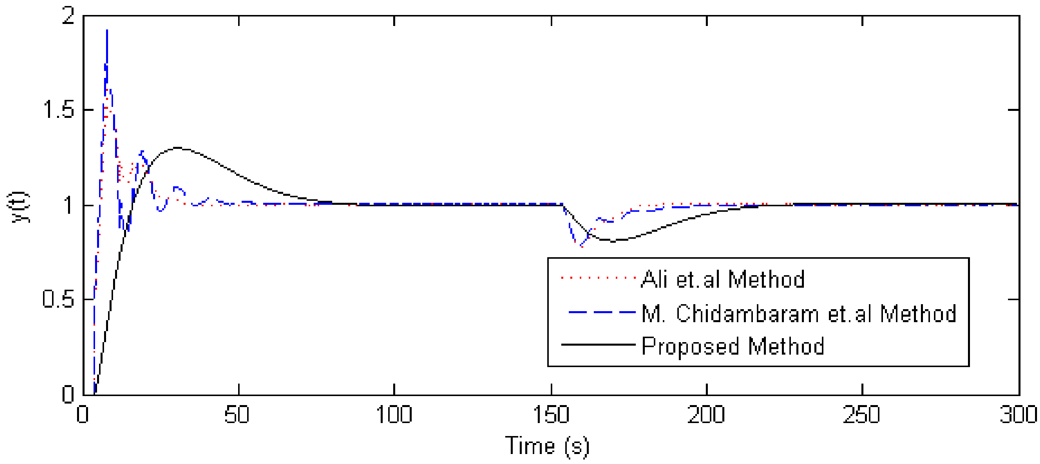

Figure13. *Closed loop responses for Example 3 with +20% perturbation in process gain*

When 20% mismatch occurs in the process the existing method shows the very high rise time but proposed method output response has less rise time and quite similar to the nominal performances. The existing method also has the oscillatory wave form and it's take a long time to settle down. The proposed method has achieved the good tracking response than Chidambaram et.al [36] & Ali [32] et.al methods. For the condition, +20 perturbation in both process and time delay the existing method Chidambaram et.al [36] has more oscillatory which becomes to nearly critically damped response, But the proposed method as good minimal response for this condition. This result shows the proposed method yields the better robustness property than the existing method with respect to a mismatch in the process.

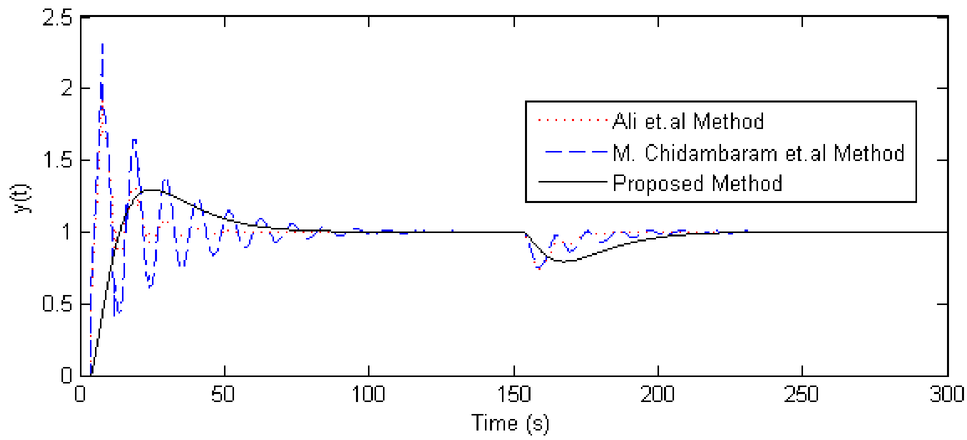

Figure14. *Closed loop responses for Example 3 with +20% perturbation in time delay*

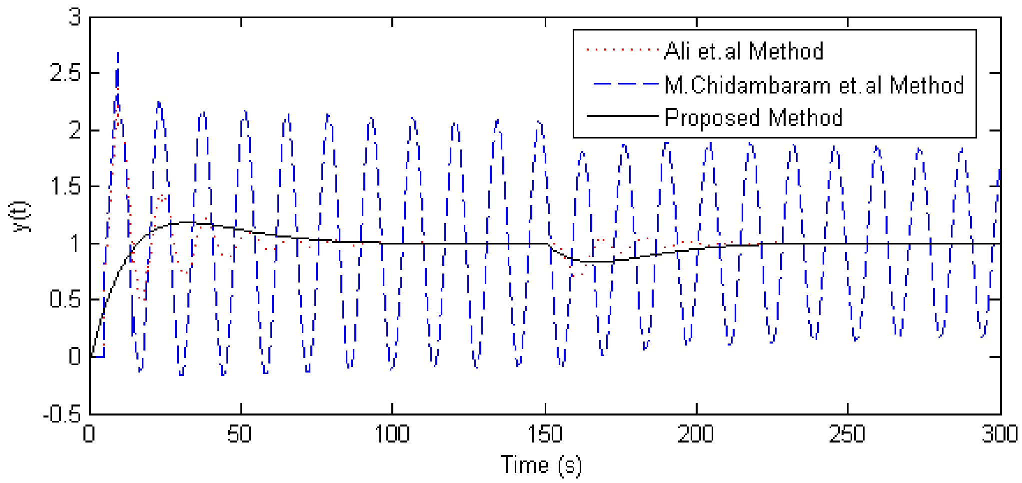

Figure 15. *Closed loop Response for Example 3 with +20% perturbation in both process gain and delay time*

Example 4 – Temperature Process Station

To show the effectiveness of the proposed controller a real time experiment has been carried out on temperature control station. The hardware setup of pressure process station is available in process control laboratory in VIT University, Vellore, India. The diagram of the process description are shown in Figure 16. The technical specifications of the temperature process are shown in Table 3. The process setup consists heating tank fitted with thyristor controlled heated for on-line heating of the water. The flow of the water can be manipulated and measured by rotameter. The temperature transmitter (RTD) type sensor and transmits the signals (4-20mA) to unit/control module. The PPI diagram of the temperature process station are shown in Figure 17.

Here the control objective is to maintain a constant temperature to regulate the hot water flow. The system identification tool is used to derive the integrator delay process model for temperature control plant. Initially, constant step input is applied to the process to get the input and output data.
Then the obtained data is loaded in the system identification tool in MATLAB to obtain the process model. The temperature process is a very slow process, it is modeled from input-output data is shown in Eq. (43).

$$G(s)=\frac{0.028767}{s}e^{-10.127} \tag{43}$$

Table. 2. *Performance indices for proposed and existing methods of Example 1-4.*

Sl.No	Process Model	Controller Method	Time Domain Specifications			
			$T_s(s)$	ISE	ITAE	IAE
1. Example 1 [32]	$G_1(s)=\frac{1}{s}e^{-s}$	Ali et.al [32]	24.2	4.92	65.58	3.652
		Proposed Method	25.05	4.782	68.42	2.456
2. Example 2 [32]	$G_2(s)=\frac{1}{s}e^{-5s}$	M. Chidambaram et.al [36]	175.2	40.26	3343	7.524
		Proposed Method	145.2	39.5	3335	6.534
3. Example 3	$G_3(s)=\frac{0.01}{s}e^{-4s}$	Ali et.al Method [32]	73.76	7376	9609	6.169
		M. Chidambaram et.al [36]	60.5	7258	9525	8.521
		Proposed Method	56.2	7215	9517	6.2456
4. Example-4	Temperature Process Station	Z.N Method	550	7.89e05	6.27e05	8257
		Proposed Method	280	5.678e05	5.7e05	7583

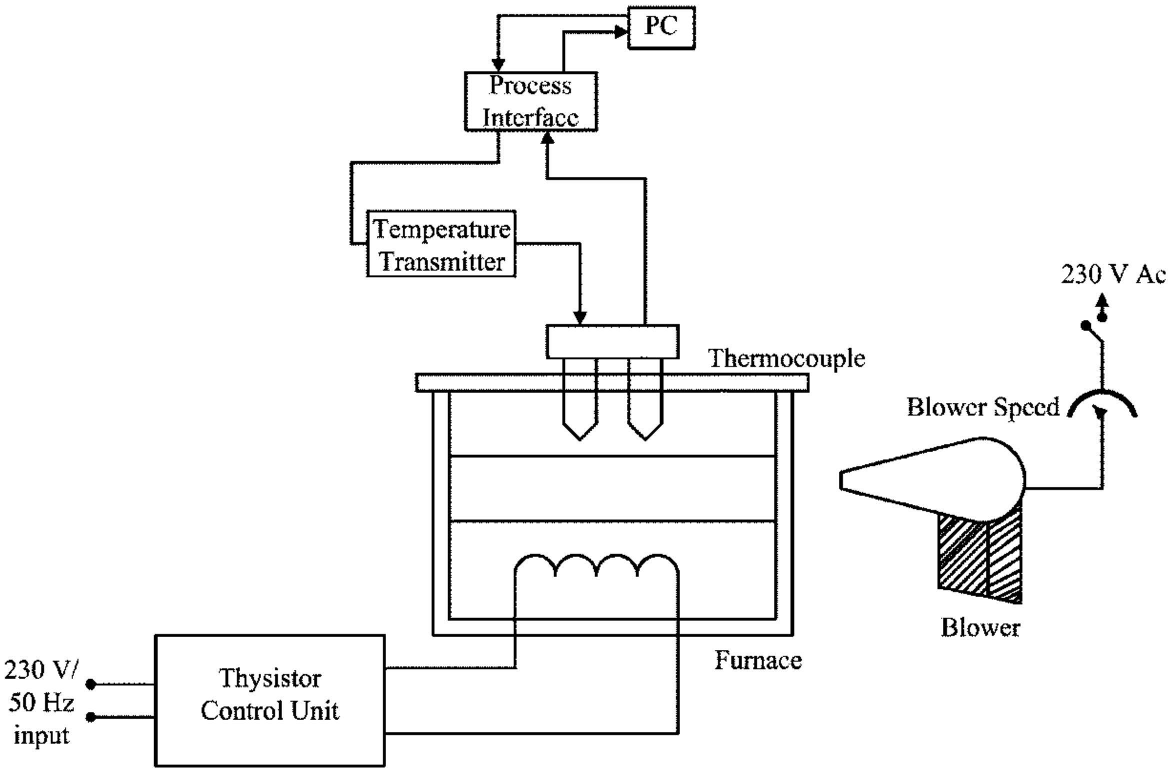

Figure 16. *Temperature process description*

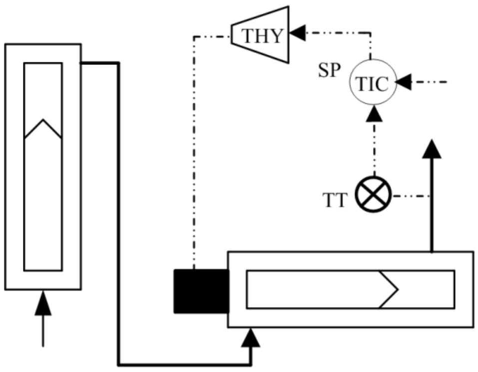

Figure 17. *PPI Diagram of Temperature Process*

The open loop response for the temperature process for both simulation and experiment are shown in Figure 18. From the observation of Figure 18, the results of process model simulation and piratical are closely match each other. As per process model from Eq. (37), the proposed LQR PI controller is designed for the temperature process. The obtained PI controller gain values are K_p is 1.24 and K_i is 0.0164. For the validation of the results of proposed controller is compared with the Manual tuning called Z-N method. The controller gain values of the Z-N method is K_p is 2 and K_i is 0.01. These gain values of both methods are tabulated in Table.1.

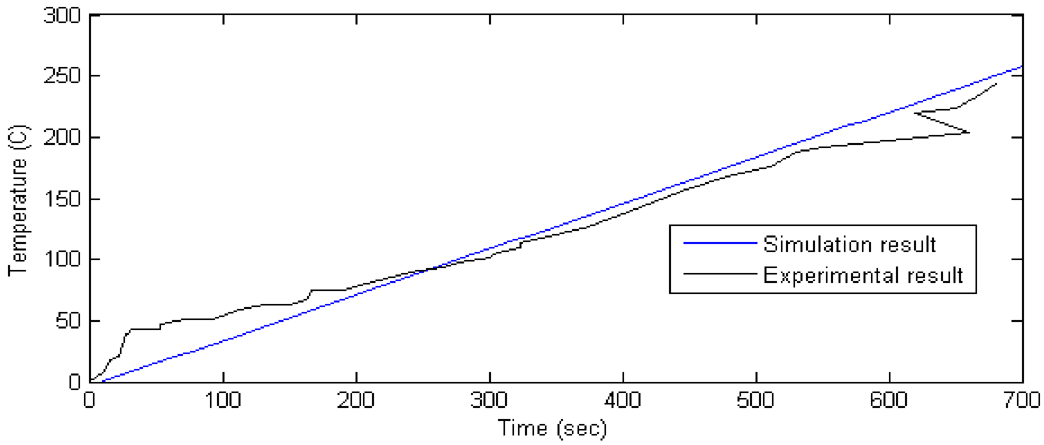

Figure 18. *Open loop response of the temperature process at 75% valve opening.*

The experimental results are shown in Figure 19 with the setpoint 200 C. To validation of results the control value position is fixed at 75%. The settling time and Integral errors of the both the methods are tabulated in Table 2.

Table 3*. Technical Specifications of Temperature Process*

Part name	**Details**
Process tank	SS, 0.5 litre capacity, insulated Electrically heated 2 x 1.5 kW heaters for on-line water heating. Make Rixy/ Bajaj, Type-3.0 kW, 2 coil, size 1.25 BSPx10" length
Temperature Transmitter	Input 2 wire PT100 RTD, output 4 -20mA for 0 -100 deg C
Rotameter	Range 10 -100 LPH
Thyristor	Capacity 2kW
Interfacing Unit	one input, one output with RS232Cfacility computer Programmable p controller with RS 485- RS 232 converter.

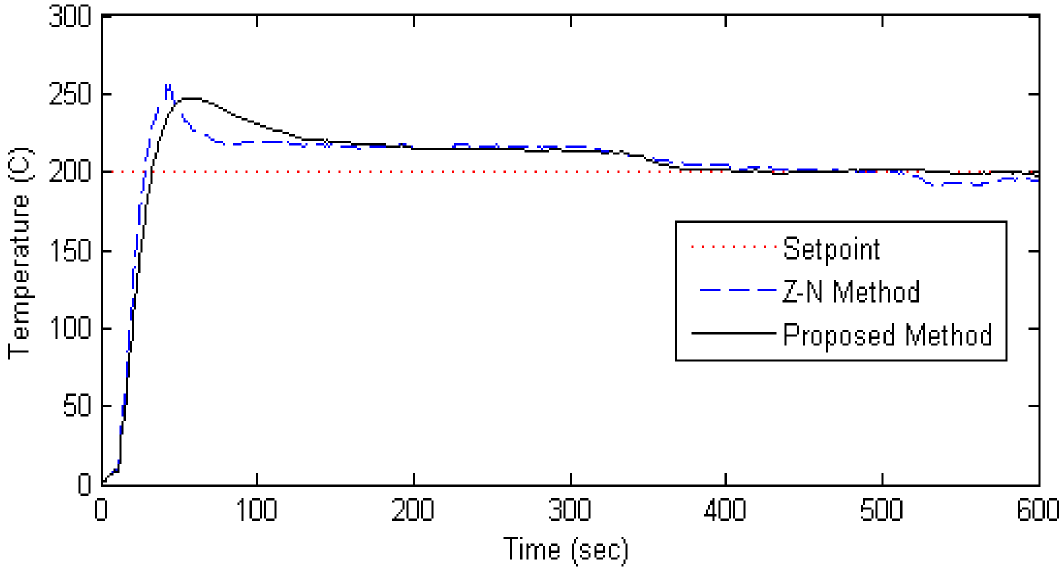

Figure 19. *Experimental Results for Temperature Control Process*

From comparison of performance output of both the controllers the proposed controller as settled at 350 sec, existing method settles at 55 sec. The IAE and ISE are also very lower than the existing Z-N Method.

4.1 Performance indices

It is important to note that IAE, ITAE, ISE and settling time are used as performance indices for comparisons of proposed study with existing methods. The definitions of parameters are given as follows.

Integral Absolute error (IAE)

To evaluate closed–loop performance, the IAE criterion is considered here for both disturbance rejection and set point tracking it is defined as

$$IAE = \int_0^\infty |e(t)| dt \tag{38}$$

IAE value should be as small as possible.
Integral Square Error (ISE)

$$ISE = \int_0^\infty e^2(t) dt \tag{39}$$

Integral Time Absolute Error

$$ITAE = \int_0^\infty t|e(t)| dt \tag{40}$$

Settling time (T_s) : The time required for response to reach steady state value .

$$T_s = \frac{4}{\xi_n * \omega_n} \tag{41}$$

The performance indices of the proposed and existing methods for the examples are noted in the Table 2. From the Observation, the proposed control method yields better settling time, IAE, ISE and ITAE.

5. CONCLUSION

The LQR PI controller is designed for pure IPTD process is presented in this paper. The gain formulas for the LQR PI controller, are design in terms of state feedback gain matrix. The modified feedback gain matrix is used to find the optimal values of weight matrices. The proposed controller design is based on the natural frequency (ξ_p) and damping ratio (ω_p). A numerical example along with simulation results has been presented, and it shows the lesser settling time, low ISE and IAE values. The proposed controller also achieves the good robustness with respect to mismatching in processes and time dealy. The practical validation of proposed controller is done by considering the temp process station. From the observation of temperature process results, the proposed controller is applicability even in larger delay and small process gains. The proposed method is also applicable for second order time delay process by assuming larger pole of the process is consider as k_d. But the proposed method is not applicable for the higher order integrator time delay process because it will give worse outputs.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

REFERENCES

[1] Chakraborty, Sudipta., Sandip Ghosh., and Asim Kumar Naskar., “All-PD control of pure Integrating Plus Time-Delay processes with gain and phase-margin specifications”, ISA transactions (2017).

[2] Hamamci, Serdar Ethem., and Muhammet Koksal., “Calculation of all stabilizing fractional-order PD controllers for integrating time delay systems,” Computers & Mathematics with Applications, 59(5):1621-1629, (2010).

[3] Ogunnaike BA., Ray WH., “Process dynamics modeling and control”, New York: Oxford University Press, 666–7,(1997).

[4] Wang L., Cluette WR., “PID tuning controllers for integrating processes”, IEEE Proc – Control Theory Appl.,144:385,(1997).

[5] Srividya R., Chidambaram M., “Online controller tuning for integrator plus time delay processes”, Process Control Qual, 59:66,(1997).

[6] Padhan DG,, Majhi S., “Enhanced cascade control for a class of integrating processes with time delay,” ISA Transactions, 52(1):45–55,(2013).

[7] Jin Q., Liu Q., “Analytical IMC-PID design in terms of performance/robustness trade-off for integrating processes: from 2-Dof to 1-Dof”,Journal of Process Control,24(3):22-32,(2014).

[8] Rao AS., Chidambaram M., “PI/PID controllers design for integrating and unstable systems. In: PID control in the third millennium” Springer; London; 75-111,(2012).

[9] Ziegler, John G., Nathaniel B, Nichols., “Optimum settings for automatic controllers” Trans ASME, 64.(11),(1942).

[10] Kumar, DB Santosh., and R. Padma Sree., “Tuning of IMC based PID controllers for integrating systems with time delay.” ISA transactions, 63: 242-255,(2016).

[11] Rivera DE., Morari M., Skogested S., “Internal model control for PID controller design”. Ind Eng Chem Process Des, 25:252-265, (1986).

[12] Ali, Ahmad., and Somanath Majhi., “PI/PID controller design based on IMC and percentage overshoot specification to controller setpoint change.” ISA transactions, 48(1):10-15, (2009).

[13] Selvi, J. Arputha Vijaya., T. K. Radhakrishnan., and S. Sundaram., “Performance assessment of PID and IMC tuning methods for a mixing process with time delay”, ISA transactions, 46.(3),391-397, (2007).

[14] Wang, Qing., Changhou Lu., and Wei Pan., “IMC PID controller tuning for stable and unstable processes with time delay,” Chemical Engineering Research and Design, 105:120-129, (2016).

[15] Rao, A. Seshagiri., V. S. R. Rao., and M. Chidambaram., “Direct synthesis-based controller design for integrating processes with time delay”, Journal of the Franklin Institute, 346(1):38-56, (2009).

[16] Chen, Dan., and Dale E. Seborg., “PI/PID controller design based on direct synthesis and disturbance rejection”, Industrial & engineering chemistry research, 41(19):4807-4822, (2002).

[17] Seshagiri Rao A., Rao VSR., Chidambaram M., “Direct synthesis based controller design for integrating processes with time delay,” Journal of Franklin Institute, 346:38–56,(2009).

[18] Ajmeri M, Ali A., “Direct synthesis based tuning of parallel control structure of integrating systems”, Int J Syst Sci (2015). http://dx.doi.org/10.1080/00207721.2013.871369.

[19] Jeng, Jyh-Cheng., “A model-free direct synthesis method for PI/PID controller design based on disturbance rejection”, Chemometrics and Intelligent Laboratory Systems, 147:14-29, (2015).

[20] Rao, A. Seshagiri., V. S. R. Rao., and M. Chidambaram., “Simple analytical design of modified Smith predictor with improved performance for unstable first-order plus time delay (FOPTD) processes.” Industrial & engineering chemistry research, 46(13):4561-4571, (2007).

[21] Kaya, Ibrahim., “A new Smith predictor and controller for control of processes with long dead time”, ISA transactions 42(1):101-110,(2003).

[22] Padhan, Dola Gobinda., and Somanath Majhi., “Modified Smith predictor based cascade control of unstable time delay processes”, ISA transactions 51(1):95-104, (2012).

[23] Nortcliffe, Anne., and Jonathan Love., “Varying time delay Smith predictor process controller”, ISA Transactions, 43(1):61-71, (2004).

[24] Camacho, Oscar., and Francisco De la Cruz., “Smith predictor based-sliding mode controller for integrating processes with elevated deadtime.” ISA Transactions, 43(2),257-270, (2004).

[25] Wu, Ligang., and Wei Xing Zheng., “Passivity-based sliding mode control of uncertain singular time-delay systems.” Automatica, 45(9): 2120-2127,(2009).

[26] Zhong, Qing-Chang., and Han-Xiong Li., "2-Degree-of-Freedom Proportional− Integral−Derivative-Type Controller Incorporating the Smith Principle for Processes with Dead Time." Industrial & engineering chemistry research, 41(10):2448-2454, (2002).

[27] Wang, Dong, et al., "Discrete-time domain two-degree-of-freedom control design for integrating and unstable processes with time delay",ISA Transactions 63 :121-132,(2016).

[28] Liu, Tao, Weidong Zhang., and Furong Gao., "Analytical Two-Degrees-of-Freedom (2-DOF) Decoupling Control Scheme for Multiple-Input− Multiple-Output (MIMO) Processes with Time Delays",Industrial & engineering chemistry research, 46(20):6546-6557, (2007).

[29] Visioli, Antonio., "Experimental evaluation of a time-optimal plug & control strategy", ISA Transactions 46(4):519-525, (2007).

[30] Visioli, Antonio., "Time-optimal plug & control for integrating and FOPDT processes" Journal of Process Control 13(3):195-202. (2003).

[31] Srivastava, Saurabh., and V. S. Pandit., "A PI/PID controller for time delay systems with desired closed loop time response and guaranteed gain and phase margins", Journal of Process Control, 37:70-77, (2016).

[32] Ali A, Majhi S., "PID controller tuning for integrating processes", ISA Transactions",49(1):70–8,(2010).

[33] Kookos, I. K., A. I. Lygeros, and K. G. Arvanitis., "On-line PI controller tuning for integrator/dead time processes" European journal of control, 5(1):19-31, (1999).

[34] Visioli, A. "Optimal tuning of PID controllers for integral and unstable processes", IEE Proceedings-Control Theory and Applications, 148(2):180-184, (2001).

[35] Padula, F., and A. Visioli. "Optimal tuning rules for proportional-integral-derivative and fractional-order proportional-integral-derivative controllers for integral and unstable processes." IET Control Theory & Applications, 6(6) :776-786,(2012).

[36] Chidambaram M., Sree RP., "A simple method of tuning PID controllers for integrator/dead-time processes", Comput Chem Eng, 27(2):211–215,(2003).

[37] Ali, Ahmad., and Somanath Majhi., "Integral criteria for optimal tuning of PI/PID controllers for integrating processes." Asian Journal of Control, 13(2):328-337, (2011).

[38] Anil, Chitturi., and Ravi Padma Sree., "Design of optimal PID controllers for integrating systems", Indian Chemical Engineer 56(3):215-228, (2014).

[39] Anil, Ch, and R. Padma Sree., "Tuning of proportional integral derivative controllers for integrating systems using differential evolution technique", Indian Chemical Engineer, 53(4):239-260, (2011).

[40] Bingul, Zafer. "A new PID tuning technique using differential evolution for unstable and integrating processes with time delay." International Conference on Neural Information Processing, Springer Berlin Heidelberg, (2004).

[41] Kumar, Elumalai Vinodh., Ganapathy Subramanian Raaja., Jovitha Jerome. "Adaptive PSO for optimal LQR tracking control of 2 DoF laboratory helicopter". Applied Soft Computing,41:77-90,(2016).

[42] Desineni Subbaram. Naidu. Optimal Control Systems.CRC press;(2003).

[43] He, Jian-Bo., Qing-Guo Wang., Tong-Heng Lee. "PI/PID controller tuning via LQR approach". Chem Eng Sci, 55(13):2429-39, (2000).

[44] Saurabh Srivastava., Anuraag Misra., S.K. Thakur, V.S. Pandit, "An optimal PID controller via LQR for standard second order plus time delay systems". ISA Transactions, 244-53(2015).