

Nonlinear Dynamics Induced by Coil Heat in the PMDC Motor and Control

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ABSTRACT In this paper, the interesting dynamics of chaos induced by the effect of the variation of internal average heat during operation in the DC motor control by the full bridge drive are analyzed. By using simple powerful tools of analyzing nonlinear dynamical systems like phase portraits, time traces and frequency spectrum in the MATLAB-SIMULINK environment, we showed that under certain conditions, the PMDC motor develops different behaviors as periodic limit cycles, and chaotic attractors, when the motor drive different form of external load torque and the windings resistance variation. This paper presents the first studies on the variation of the average heat of the motor and the amplitude of the triangular load torque to produce the strange phenomena like chaos as far as our knowledge go. A chaos control of the unstable regime is proposed to stabilize the PMDC motor in a desire regime. This contribution is very important in industry because some unexplained dynamical behaviors of the DC motor driven by a full bridge now can be avoided.

KEYWORDS

PMDC motor Triangular load torque Windings coil heat Phase portraits Periodic oscillations **Torus** Chaos Chaos control

IN[T](#page-0-0)RODUCTION

Th[e](#page-0-1) [e](#page-0-2)l[e](#page-0-3)[ct](#page-0-4)ric motor (EM) is the most used actuator in industry [\(Yu](#page-8-0) *[et al.](#page-8-0)* [2011;](#page-8-0) [Dalcalı](#page-7-0) [2018\)](#page-7-0) . The electromotive force (EMF) powers most of the electro-mechanical actuators in the industry. In these systems the torque is created by using the Laplace laws. Among various type of these EM actuators, there is the permanent magnet DC electrical motors that are sufficiently used in industry today due to its linear torque characteristics, adjustable speed simplicity of control less noise operation and longer durability [\(Klein and](#page-7-1) [Kenyon](#page-7-1) [1984;](#page-7-1) [Arat](#page-7-2) [2018;](#page-7-2) [Gieras](#page-7-3) [2009\)](#page-7-3). Generally, The actuator in this contribution is a type of DC motor that uses a permanent magnet (PM) to create the electromotive force (EMF) useful for its operation[\(Öztürk](#page-7-4) [2020;](#page-7-4) [Pillay and Krishnan](#page-7-5) [1989\)](#page-7-5). This electromechanical actuator is useful in applications like automotive industrial robots, production automated system, agriculture, aerospace just to name some sample contributions [\(Liao](#page-7-6) *et al.* [1995\)](#page-7-6).

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In order to fulfill their industrial requirements, a bridge controlled is widely used in this purpose. In PMDC electrical motor drive, power transistors provide nonlinearity in the overall system due to their on/off state [\(Parsa and Toliyat](#page-7-7) [2005\)](#page-7-7). Some nonlinear dynamical contributions have been intensively studied and have proven to exhibit stable, regular, oscillation behaviors and chaos [\(Öztürk](#page-7-4) [2020\)](#page-7-4). Some studies show that nonlinear system can entered chaos regime via complex behavior while one or two parameters are varied [\(Poliashenko and Aidun](#page-7-8) [1995;](#page-7-8) [Ayan and](#page-7-9) [Kurt](#page-7-9) [2018\)](#page-7-9). Some simple condition to entering chaos in systems including few fundamental electrical drive systems are described by [\(Chau and Wang](#page-7-10) [2011\)](#page-7-10) .These chaos regimes are usually unwanted because they could be harmful to the industrial application [\(Tahir](#page-8-1) *[et al.](#page-8-1)* [2017\)](#page-8-1). Among these parameters are the load the DC source, the control speed, the duty ratio, or the parameter of the feedback control system .

[Okafor](#page-7-11) *et al.* [\(2010b\)](#page-7-11) reported the multistability of P-1 and P-3 attracting sets and fractal basin boundaries in dc drives controlling a PMDC motor by using a 4 quadrants DC/DC converter circuit. Using the well-known Filippov condition, the stability nature of P-1 and P-3 attractors was computed to explain the presence of the competing limit cycles. His contribution aimed to prevent the occurrence of this behavior by adding a controller that extend the parameter range for safe limit cycle operation. [Öztürk](#page-7-4) [\(2020\)](#page-7-4) shows

by using MATLAB/Simulink model, that under chaotic load the asynchronous machine develops chaos. As far as concerned the chaotic effect in the motor, we note that other authors described the behaviors of dynamics and control of chaos in the IFOC of 3-Φ IM. They show simple criterion for the motor to develops periodic and chaotic behavior [\(Tsafack](#page-8-2) *et al.* [2020\)](#page-8-2). [Okafor](#page-7-12) *et al.* [\(2010a,](#page-7-12) [2015\)](#page-7-13) analyzed and controlled period cascade to chaos in DC/DC converter employing a full-bridge using Filippov's algorithm. The authors showed that the monodromy matrix and its eigenvalues could be computed by the periodic orbit stability analyses. The previous result can be modified by the appropriate computation of the saltation matrix of the system. Hence, they designed a controller to stabilize the behavior to period-l operation of the PMDC motor.

[Abdullah](#page-7-14) *et al.* [\(2016\)](#page-7-14) and [Tahir](#page-8-1) *et al.* [\(2017\)](#page-8-1) suppressed the chaos regime by using the sliding mode control method. Recently, [Moustafa](#page-7-15) *et al.* [\(2021\)](#page-7-15) study the Floquet theory to control the system, they experienced oscillations when varying the load torque without qualitative variation of the dynamics. The control the speed of the driven load and obtained strong stability of the system. [Tsafack](#page-8-2) *et al.* [\(2020\)](#page-8-2) suppressed chaos by applying a self-feedback controller in 3phases IFOC IM. In other hand, it is well known that different losses (friction, Joule effect, copper losses. . .) contributed to motor heat increasing and reduce the motor efficiency, (speed, torque . . .) resulting in a thermal aging process and eventually destroy the motor. Therefore, reducing thermal effect and increasing motor cooling systems result to the longevity and reliability of the EM [\(Bonnett](#page-7-16) [2001\)](#page-7-16). Some good standing papers have focused on the thermal effects on the torque speed performance of a Brushless Direct Current Motor (BLDCM) [\(Fussell](#page-7-17) [1993\)](#page-7-17). Another contribution analyzed the transient thermal network model that could be used to predict some sensitivity of the model towards design variables [\(Junak](#page-7-18) *et al.* [2008;](#page-7-18) [Minghui and Weiguo](#page-7-19) [2010\)](#page-7-19).

In summary of the previous contributions and as far as out knowledge goes, no contribution has been interested on windings currents heat consequences behavior on the nonlinear effect of the PMDC motor that could explained some industrial chaotic behavior of the driven process. This paper tries to bridge this gap. We observed that till now, no-contribution has focused on the influence of the thermal behavior of the rotor resistance to the dynamic behavior of the PMDC motor: we set this as the main objective of this paper. The chaotic behavior due to effect of the internal heat in the motor when the motor drive different type of load torque is proposed and some interesting dynamics like limit cycles, torus, chaos due to the heat or resistance or the variation of inductance are shown. These results are encouraging because some unexplained dynamical behaviors of the DC motor driven by a full bridge now can be explained and avoided [\(Pisarchik and](#page-7-20) [Feudel](#page-7-20) [2014\)](#page-7-20).

The main highlights of this contributions are as follows:

- The variation of the amplitude value of a triangle load is investigated and showed some chaotic regime in the PMDC Motor.
- We discovered that the thermal heat on the winding's resistance induced interesting nonlinear behaviors like limit cycles, period doubling route to chaos, torus, and chaotic attractors.
- We showed the existence of the nonlinear dynamics induced by the heat on the magnet field not yet explained.
- A chaos control of the unstable regime due to these novel dynamics is proposed to stabilize the PMDC motor in a desire regime.

The outline of this article is as follows: the next section 2 introduces the description of mathematical model of PMDC motor. In section 3, the dynamical behaviors of the motor induced by the load torque are presented. The section 4 highlights the effect of the winding resistance variation due to heat with different variation. The section 5 exhibits the control of the chaos using the load in the PMDC motor. Finally, the section 6 concludes this paper.

MATHEMATICAL MODEL OF THE PMDC DRIVE

The mathematical model of the PMDC motor driven by a full bridge converter is well documented [\(Okafor](#page-7-11) *et al.* [2010b;](#page-7-11) [Okafor](#page-7-21) [2013;](#page-7-21) [Abdullah](#page-7-14) *et al.* [2016\)](#page-7-14) and the schematic diagram is recalled in Figure [\(1\)](#page-1-0).

Figure 1 The PMDC driven by a full bridge converter.

The system is built by connecting the power converter bridge, a control electronic and permanent magnet DC motor. After a long-term running, the system in Figure [\(1\)](#page-1-0) toggled between 2 subsystems in accordance with the state of the output of the second operational amplifier A2 used in switched mode. In the Figure [\(1\)](#page-1-0), *V* represent the supply voltage *T*1 , *T*2, *T*3, and *T*4 are the semiconductor controlled rectifier, (SCR) *L* is the inductance, R is the resistance, i(t) is the current, *E* is the voltage across the motor, Vramp(t) is the sawtooth signal, *ω*(*t*) is the speed of the motor, Vref represents the value of the reference speed of the motor, $Vcomp(t)$ is the difference between the reference speed and the actual speed developed by the motor; *PWM* signal Pulse Width Modulated voltage.

Operational amplifier Switch A2 is high

The differential equations governing the evolution of the current and the speed of the motor are given in equations (1).

$$
\begin{cases}\n\frac{dw(t)}{dt} = \frac{T_e - T_L - bw(t)}{J} \\
\frac{di(t)}{dt} = \frac{-Ri(t) - E_b - Vin}{L}\n\end{cases}
$$
\n(1)

where we have

$$
T_e = k_t i(t), \ E_b = k_e w(t) \tag{2}
$$

After replacing the equation (2) in equations (1), we have the equations (3).

$$
\begin{cases}\n\frac{dw(t)}{dt} = \frac{k_t i(t) - T_L - bw(t)}{J} \\
\frac{di(t)}{dt} = \frac{-Ri(t) - k_e w(t) + Vin}{L}\n\end{cases}
$$
\n(3)

Note that $i(t)$ is the current absorbed by the motor, $w(t)$ is the motor output speed, *R* is the sum of resistance of the armature and the field coil inductances. *L* L is the sum of the armature and field coil inductances, k_e and k_t are the back electromotive force (EMF) constant and the torque constant, respectively; *b* is the friction coefficient, J is the moment of inertia; T_L is the load torque. In this sub-part of the work, the system settings are chosen as follows: $R = 7.2 \Omega$, $L = 0.0917H$, $T_L = 0.2 N_m$, $T = 10 ms$, $k_e =$ $k_t = 0$, 1236 N .*m*/ A^2 , $J = 7$, 046 e^{-4} kg . m^2 , $b = 4e^{-4}N$ m.rads.s , $V_L = 0V$, $V_u = 8V$ and $w_{ref} = 100$ *rad*/*s* the expression of $V_{com}(t) = A1(w(t) - w_{ref})$ [\(Abdullah](#page-7-14) *et al.* [2016\)](#page-7-14).

Operational amplifier Switch A2 is low

In this subsection, the switch is off the generator signal is equal to zero ($V_{in} = 0V$). Equations. (3) becomes equations (4) as shown as follows.

$$
\begin{cases}\n\frac{dw(t)}{dt} = \frac{k_t i(t) - T_L - bw(t)}{J} \\
\frac{di(t)}{dt} = \frac{-Ri(t) - k_e w(t)}{L}\n\end{cases}
$$
\n(4)

The parameters values of equations (4) are the same as in section 1. This is the model that will be used throughout the paper.

DYNAMICAL BEHAVIORS ON LOAD VARIATION OF PMDC OPERATION

This section is devoted to the numerical study of the PMDC motor in normal operation driving an external load and a triangular load.

Chaos behavior induced by an external load torque

The investigation of the external load torque effect of the general behavior of the PMDC motor is given as follows:

Firstly, when the motor drives the constant load torque (the normal driving) the behavior of the system dynamics quality changes as TL varies. The system in Figure [\(1\)](#page-1-0) is run in Matlab-Simulink for a long time and the transient phase is omitted. The current *i*(*t*) and the output speed $\omega(t)$ are recorded and plotted in Figure [\(2\)](#page-2-0). The Figure [\(2\)](#page-2-0) shows the different dynamics observed. We considered that the general parameter of the motor is unchanged. We recalled the parameters values: $R=50\Omega$; $J=0$,000010388 $kg m^2$; $b = 0,00025N$ *m.s*; $K_t = 1,8N$ *m*/*A*; $L = 1,2m$ *H*; The external load torque is varied between 0 to 5*N*.*m*.

Figure 2 Phase portraits (*i*) of different dynamical behaviors of the PMDC motor driven by a full bridge converter and the corresponding Power Spectral Density (PSD) of the speed $(\omega(t))$ (*ii*) under variation of the amplitude of the external linear*T* . The values of *T* are given as (*a*) $T = 0,00001N$ *m* period-1; (*b*) $T =$ 0, 0125*N*.*m* periodic bursting; (*c*) $T = 0.5N$.*m* periodic bursting; (*d*) $T = 1,95N$ *m* chaotic motion. The rest of the parameters of the system are the ones of Figure [\(1\)](#page-1-0). The Figure [\(2\)](#page-2-0) shows that the behavior of the motor changes from the period limit cycle, to periodic bursting, to chaotic behaviors. The PSD plots highlighted the different kinds of the observed attractors.

Effect of the amplitude variation of the triangular T

In the industry, some load applied to the PMDC motor are nonconstant, but varies continuously. To mimic this type of load, we introduced here a triangular load. The triangular signal is periodic linear in pieces and continuous, it contains odd harmonics. It can be seen as the absolute value of the saw-tooth signal and for a period of time as follows. $x(t) = 2\left[\frac{t}{a} - \left[\frac{t}{a} + \frac{1}{2}\right]\right]$. In this case, we used the signal equation as: $x(t) = A * [sawtooth(2.\pi.50.t, 1/2)]$. available in Matlab-Simulink. The triangular load torque and the corresponding PSD graph are shown in Figure [\(3\)](#page-3-0).

Figure 3 Triangular waveform T (*a*); and the corresponding PSD (*b*). The amplitude Ttri=1N.m; and the frequency is $F = 50H_z$. Sub-harmonics are visible. $A = 1$.

The effects of the amplitude variation of the triangular external load torque are investigated in this sub-section. For this purpose, the system in Figure [\(1\)](#page-1-0) is run in Matlab-Simulink with load torque T replaced by the triangular one *Tri*(*t*) for a long time and only the steady state data are recorded. The parameters of the motor are given as: *R* = 50Ω; *J* = 0, 000010388*kgm*² ; *b* = 0, 00025*N*.*m*.*s*; $K_t = 1,8N \cdot m/A$; $L = 1,2mH$; $T = 2N \cdot m$; We plotted different current $i(t)$ and speed $\omega(t)$ recorded as in sub-section 3.1 to describe the situation. The variation of T highlights some unobserved streaking dynamical phenomena.

In the light of Figure (4) , it is easy to see that when the total load torque driving by the PMDC motor have a triangle form, the motor develops different behavior like periodic, torus and chaos behaviors.

Dynamical behaviors induced by the windings heat during PMDC operation

In the industry the load driven by the system (1) transforms electrical energy into mechanical energy for the manufacturing process. Energy laws states that this transformation cannot occurred without loss of energy. In this paper we focused on the Joule energy effects dissipated by the winding's coils heat due to current flowing in these elements.

VARIATION OF THE WINDING'S COIL RESISTANCE OF THE PMDC MOTOR

When the motor drive different type of load torques, the general temperature of the systems increases, and some parameters are affected such as resistance and reactance winding coil of the motor. The general equation of the resistance as function of the temperature is give as $R = R_0 \left(1 + A \left(\theta - \theta_0 \right) + B (\theta - \theta_0)^2 + \right)$

Figure 4 Phase portraits (*i*) and the corresponding PSD (*ii*) describing the dynamics of the system under variation of the amplitude of the external triangular load torque *A*. The values of *A* for each case are given as: $(a(i))$ $A = 0,00001N$ *m*, period-1; $(b(i))$ $A = 0,025N.m$, torus; $(c(i))$: $A = 5N.m$, chaos the initial condition are $(i(0) = 0, 01; \omega(0) = 0, 01)$.

[\(Malvino](#page-7-22) [1993\)](#page-7-22) and the simplified form of this equation can be deduced as:

$$
R = R_0(1 + \alpha \theta) \tag{5}
$$

Where R_0 is a value of the resistance when $\theta = \theta_0$, A et B are the thermal coefficients depending of the type of the motor resistance coil used. In the case the temperature is varying between 0 ◦*C* to 100◦*C* and when the reference temperature is equal to zero, the thermal coefficient is replaced by the average thermal coefficient *α*.

Using equation (5), we replaced the resistor *R* in equations (1) and we derived the new equations of system in Figure [\(1\)](#page-1-0), depending on the internal heat of the motor give in equations (6) and (7) as follows:

$$
\begin{cases}\n\frac{d\omega(t)}{dt} = \frac{(K_t i(t) - T_L - b\omega(t))}{J} \\
\frac{di(t)}{dt} = \frac{(-i(t)R_0(1 + \alpha\theta) - K_e\omega(t) + Vin)}{L}\n\end{cases}
$$
\n(6)

when *A*2 is in the high and

.

$$
\begin{cases}\n\frac{d\omega(t)}{dt} = \frac{(K_t i(t) - T_L - b\omega(t))}{J} \\
\frac{di(t)}{dt} = \frac{(-i(t)R_0(1 + \alpha\theta) - K_e\omega(t))}{L}\n\end{cases}
$$
\n(7)

When the *A*2 output is low.

By defining the state vector $X(t) = [x_1(t), x_2(t)]^t =$ $\left[\omega(t), i(t)\right]^t$, The DC drive + full bridge can be expressed in standard form as in Equation. (8).

$$
\begin{cases}\n\dot{X} = A_{on}X(t) + Von \\
\dot{X} = A_{off}X(t) + Voff\n\end{cases}; where\nX(t) = \begin{bmatrix}\nx_1(t) \\
x_2(t)\n\end{bmatrix} = \begin{bmatrix}\n\omega(t) \\
i(t) \\
i(0)\n\end{bmatrix}
$$
\n(8)

$$
A_{on} = A_{off} = A = \begin{bmatrix} \frac{-b}{f} & \frac{k_t}{f} \\ \frac{-k_e}{L} & \frac{-R_0(1+\alpha\theta)}{L} \end{bmatrix};
$$

$$
V_{on} = \begin{bmatrix} \frac{-T_L}{f} \\ \frac{V_{in}}{L} \end{bmatrix}; V_{off} = \begin{bmatrix} \frac{-T_L}{f} \\ 0 \end{bmatrix}
$$
(9)

Where the system parameters are the same as in section 1.

Chaos behavior induced by the resistance variation

The system (1) is used in Matlab-Simulink with the resistor as in equation (5) and the motor is powered by the inverter that drives a linear load. The curves in the following Figures are obtained by varying the total resistance of the machine as a function of temperature. It is noted that in an electric motor driving a load torque, the energy is dissipated by thermal effect in the form $J = RI^2$ This dissipation of heat acts at the level of the different resistances of the motor windings coil due to the modification of temperature according to law of equation (5). (the resistance at an initial temperature which is the ambient temperature in our case it is 37◦ C) We remarked that this variation of the temperature in an electric motor driving a load can be at the origin of strange phenomena such as periodic limit cycles, torus and chaos as illustrated in section 3. Noted that the general form of the total impedance of the motor is give as: $Z = R + jX$ where *R* is the total resistance

and *X* is the total inductive part. In this special case we suppose firstly that the total reactance of the motor is unchanged and the total resistance of the motor is varied by the different degree of the temperature when the system drive the linear load torque as shown in equation (5) assuming that the parameters of the motor in this study is given as: $L = 1, 2mH; J = 0,000010388kgm^2;$ $b = 0,00025N$ *m.s;* $K_t = 1,8N$ *m*/ A ;*T* = 2*Nm*; and *R* vary between 1, 2Ω*to*3000Ω. After long-term running of system (1) in the MATLAB-Simulink, we obtained some phase portraits showing different form of attractors of dynamical the behaviors of the motor in the plane $(w(t), i(t))$ in the Figure [\(5\)](#page-4-0).

Figure 5 Phase portraits describing the different behaviors of the motor and the corresponding PSD when the resistance *R* is varied between 1.2Ω to 3000Ω due to heat: $(a(i))$ $R = 135Ω$;($a(ii)$); $R = 100\Omega$:($b(i)$) $R = 50\Omega$; ($b(ii)$) $R = 30\Omega$; ($c(i)$) $R =$ 10Ω ; (*c*(*ii*)) *R* = 1, 2Ω; (*d*(*i*)) PSD for *R* = 1, 2Ω ; (*d*(*ii*)) PSD for *R* = 100Ω the initial conditions are: $(i(0) = 0, 01; ω(0) = 0, 01)$;

In the light of the curves in Figure [\(5\)](#page-4-0), the reader can see that

when the DC motor + full bridge drives some external load torque, their internal temperature changes and affects the global resistance of the system and induced some interesting dynamics like period limit cycles and chaotic attractors. These interesting behaviors are found in many physical systems but not yet observed in this system caused by windings coil heat. This contribution falls in the second criteria of publishing novel contribution of chaotic system [\(Sprott](#page-8-3) [2011\)](#page-8-3) and important to shared.

Chaos behavior induced by the reactance variation

In this section, we set the resistance constant to $R = 100\Omega$ (no longer influenced by the coil heat) and then varying the reactance L. For the simplicity of this paper the phenomena causing the variation of *L*is not illustrated. The permanent magnet is a magnetic material (strong) is characterised by the large hysteresis cycle the variation of the reducing induction as a function of the temperature is governed by the following equation; This equation is valid in a certain temperature range. In this equation is the remnant induction at temperature *T* , B_{r0} is the induction at temperature T_0 and β is the temperature coefficient. It should be noted that this coefficient is negative, which is why the induction decreases when the temperature increases The greater this coefficient (in absolute value), the greater the drop in induction *B*.

$$
B_r = B_{r0} (1 + \beta (T - T_0))
$$
 (10)

where $B = L = Xw$ in this case we have:

$$
\begin{cases}\n\frac{d\omega(t)}{dt} = \frac{(K_t i(t) - T_L - b\omega(t))}{J} \\
\frac{di(t)}{dt} = \frac{(-i(t)R_0 - K_e\omega(t) + Vin)}{L(1 + \alpha\theta)}\n\end{cases}
$$
\n(11)

when the *A*2 output is hiegh, and

$$
\begin{cases}\n\frac{d\omega(t)}{dt} = \frac{(K_t i(t) - T_L - b\omega(t))}{J} \\
\frac{di(t)}{dt} = \frac{(-i(t)R_0 - K_e\omega(t))}{L(1 + \alpha\theta)}\n\end{cases}
$$
\n(12)

when the *A*2 output is low. By defining the state vector $X(t) =$ $[x_1(t), x_2(t)]^t = [\omega(t), i(t)]^t$. the DC drive + full bridge can be expressed in matrix form as given in equation (13)

$$
\begin{cases}\n\dot{X} = A_{on}X(t) + Von \\
\dot{X} = A_{off}X(t) + Voff \\
\text{where } X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \omega(t) \\ i(t) \end{bmatrix}\n\end{cases}
$$
\n(13)

$$
A_{on} = A_{off} = A = \begin{bmatrix} \frac{-b}{f} & \frac{k_t}{f} \\ \frac{-k_e}{L(1+\alpha\theta)} & \frac{-R_0}{L(1+\alpha\theta)} \end{bmatrix};
$$

$$
V_{on} = \begin{bmatrix} \frac{-T_L}{f} \\ \frac{V_{in}}{L(1+\alpha\theta)} \end{bmatrix}; V_{off} = \begin{bmatrix} \frac{-T_L}{f} \\ 0 \end{bmatrix};
$$
(14)

The phase portraits describing the different behaviors of the motor when the inductance varied are given in the Figure [\(6\)](#page-5-0) .

Figure 6 phase portraits in plane $(w(t), i(t))$ and the PSD describing the different behaviors of the motor when the inductance value *L* is varied between 0,042 mH to 10,2 mH ($a(i)$) $L = 0.042mH$ period-1,($a(ii)$) $L = 0.085mH$ period-2, ($b(i)$) $L = 0, 5m$ *H* period-8, $(b(ii)) L = 0, 7m$ *H* period-16, $(c(i))$ $L = 0,85m$ *H* period-32, $(c(ii)) L = 1,952395m$ *H* chaos PSD of speed $\Omega(t)$ ($d(i)$) $L = 1,952395mH$ chaos and ($d(ii)$) $L = 0$, 085*mH* periodic, $(e(i))$ the time series of the speed $w(t)$, the initials conditions are $(i(0) = 0, 01; \omega(0) = 0, 01;$ and the rest of parameters are $R = 100\Omega$; $J = 0,000010388kgm^2$; $b = 0,00025N.m.s; K_t = 1,8N.m/A; T = 2N.m;$.
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In the light of Figure (6) , it is easy to see that when the inductance value changes, some interesting dynamics occur in the system (1) such as limit cycle, torus and chaos.

CHAOS SUPPRESSION IN THE PMDC MOTOR

This section is focus on the chaos suppression in the studied system. Note that in the literature, different type of controllers to suppressed chaos have been presented. For example, the time delay controller, sliding mode controller, indirect field controller and so on; a new control structure for applications such as electric traction without mechanical sensors studied is the Direct Torque Control. This technique has many advantages over classic oriented flow vector control (FOC) structure. Indeed, while this typically requires three control loops, a PWM current generator and coordinate transformations, DTC requires only one pair of comparators to hysteresis to perform dynamic flow and torque control. The objective of this part of work is the study the effect of this controller in electric drive.

The Direct Torque Control (DTC, or DTFC) method from appointment The Anglo-Saxon "Direct Torque (and Flux) Control" was developed in 1985 by Takahashi and Depenbrock especially for asynchronous machines [\(Chergui](#page-7-23) *et al.* [2020\)](#page-7-23) In this technique, it is no longer needed to use the position of the rotor to choose the voltage vector, this particularity defines DTC as a well-suited method for the Mechanical sensorless control of AC machines. The DTC control of an asynchronous machine is based on the direct determination of the Control sequences applied to switches in a voltage inverter from the calculated values of the stator flux and the torque . [\(Toufouti](#page-8-4) *et al.* [2007\)](#page-8-4) So the state of the switches is linked directly to the evolution of the electromagnetic state of the motor. Direct torque control of the machine provides a satisfactory solution robustness problems encountered in conventional control technology based on the orientation of the rotor flow.

On the other hand, DTC, is as simple, interesting given its simplicity; in particular, by the fact that it does not requires no realtime speed measurement or complex modulation control Pulse Width (PWM) close to the inverter. Under certain condition, the motor drive exhibit chaotic phenomena and the chaos suppression from the system is to be forcing the system from chaos regime to periodic oscillation. In this situation, we use the gain of the different between electromagnetic torque produced by the motor and the resistive external torque. The block diagram described the form of the controller apply in this system in our cases is shown in the Figure [\(7\)](#page-6-0).

Figure 7 Block diagram of the PMDC drive with a controller parameter

The general form of the mechanical equation of the motor is giving by apply the second low of newton as follows:

$$
j\frac{dw}{dt} + fw = T_{em} - T_r \tag{15}
$$

where *j* being the inertia moment of the motor, *Tem* is the electromagnetic torque produced by the motor, *f* is the coefficient viscous of friction, T_r is the external torque . we chose the load torque equation as: $T = T_{em} - T_r$ in our case; we use the gain of the different between the load torque to force the system from chaotic regime to the periodic oscillations. The equation describing this situation is given as $T = g(T_{em} - T_r)$ where *g* is the gain. From Matlab-Simulink environment when the system (1) is ran for long times, the controlled parameter varying here between 0 to 1.5 .We realized that when $g = [0.0001; 0.05]$ the system is under control; the special case is when $g = 0.0025$ after 0.398*s* the total load torque is equal to zero and the behavior of the motor changes from chaotic to periodic regime. the time series in the Figure [\(8\)](#page-6-1) bellow describes the situation where in Figure $(8(b))$ $(8(b))$ the chaos is suppressed and the system is periodic.

Figure 8 Phase portrait of chaotic behavior a(i) and the time series of the electromagnetic torque +with external torque superimposed $g = 1.02$ Periodic phase portrait after tuning parameter g b(i) and the corresponding time traces of *Tem* and *Tr* superimposed (ci) is the error between the too different load torque when the gain *g* is $g = 0.00025$. the initials conditions of the system are $(i(0) = 0, 01; w(0) = 0, 01).$

In the light of Figure (8) , it is easy to see that the proposed controller is effective and therefore, the system is forced to exit from chaotic regime and settle in the periodic oscillations after some small time of 0.395*s*.

CONCLUSION

In this paper the full bridge converter driving the PMDC motor with load is analysed under particular situation where the temperature affects the resistance of the inductive windings of the engine. We used simple power full tool of analysing nonlinear dynamical systems like phase portrait, time traces and power density spectrum diagram. In Matlab-Simulink environment, numerical simulations of the PMDC motor show that under certain conditions of the temperature of the internal resistance and of some value of the inductance and the value of the amplitude of triangular and linear load torque, the PMDC motor and the full bridge drive under investigation shows periodic limit cycle and chaotic attractors. A chaos control of the unstable regime is proposed to stabilize the PMDC motor in a desire periodic regime. These results mean that the variation of the average internal temperature and the variation of the amplitude of different load torque influence the general behavior of the motor drive. These results would have industrial application of the PMDC motor driven by a full bridge converter because the application is widely used.

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Authors' contributions

A.N.T and J.R.M.P. developed the theoretical formalism. A.N.T and A.S.K.T contributed to sample preparation. A.N.T and A.C planned and carried out the simulations. A.N.T and A.S.K.T. contributed to the interpretation of the results. J.R.M.P. and A.N.T took the lead in writing the manuscript. All authors provided critical feedback and helped shape the research, analysis and manuscript.

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Data availability statement

The data generated during this study will be made available at reasonable request.

Conflicts of interest

The authors declare that they have no conflict interest regarding the publication of this paper.

LITERATURE CITED

- Abdullah, M. A., F. R. Tahir, and K. M. Abdul-Hassan, 2016 Sliding mode control-based chaos stabilization in pm dc motor drive. Iraqi Journal for Electrical & Electronic Engineering **12**.
- Arat, H. T., 2018 Numerical comparison of different electric motors (im and pm) effects on a hybrid electric vehicle. Avrupa Bilim ve Teknoloji Dergisi pp. 378–387.
- Ayan, S. Ş. and A. Kurt, 2018 Computational bifurcation analysis to find dynamic transitions of the corticotroph model. Communications Faculty of Sciences University of Ankara Series A2-A3 Physical Sciences and Engineering **60**: 41–64.
- Bonnett, A. H., 2001 Operating temperature considerations and performance characteristics for ieee 841 motors. IEEE Transactions on Industry Applications **37**: 1120–1131.
- Chau, K. and Z. Wang, 2011 Introduction to chaos theory and electric drive systems .
- Chergui, H. *et al.*, 2020 *Commande Directe Du Couple d'un Moteur Asynchrone avec la Reconstitution de l'Algorithme Des Commutations par la Logique Floue*. Ph.D. thesis.
- Dalcalı, A., 2018 Optimal design of high performance interior pm motor for electric vehicle. The International Journal of Energy and Engineering Sciences **3**: 46–54.
- Fussell, B., 1993 Thermal effects on the torque-speed performance of a brushless dc motor. In *Proceedings of Electrical/Electronics Insulation Conference*, pp. 403–411, IEEE.
- Gieras, J. F., 2009 *Permanent magnet motor technology: design and applications*. CRC press.
- Junak, J., G. Ombach, and D. Staton, 2008 Permanent magnet dc motor brush transient thermal analysis. In *Proceedings of the 2008 International Conference on Electrical Machines*, pp. 978–1.
- Klein, F. N. and M. E. Kenyon, 1984 Permanent magnet dc motors design criteria and operation advantages. IEEE transactions on industry applications pp. 1525–1531.
- Liao, Y., F. Liang, and T. A. Lipo, 1995 A novel permanent magnet motor with doubly salient structure. IEEE transactions on industry applications **31**: 1069–1078.
- Malvino, A., 1993 Electronic principles.
- Minghui, Z. and L. Weiguo, 2010 Transient coupled electromagnetic thermal analysis of a permanent magnet brushless dc motor. In *2010 International Conference on Computer, Mechatronics, Control and Electronic Engineering*, volume 4, pp. 221–224, IEEE.
- Moustafa, E., A.-A. Sobaih, B. Abozalam, and A. S. A. Mahmoud, 2021 Period-doubling bifurcation analysis and chaos control for load torque using flc. Complex & Intelligent Systems **7**: 1381– 1389.
- Okafor, N., 2013 *Analysis and control of nonlinear phenomena in electrical drives*. Ph.D. thesis, Newcastle University.
- Okafor, N., D. Giaouris, and B. Zahawi, 2015 Analysis and control of fast scale bifurcation in series connected dc drive operating in continuous conduction mode. In *IECON 2015-41st Annual Conference of the IEEE Industrial Electronics Society*, pp. 000667– 000671, IEEE.
- Okafor, N., D. Giaouris, B. Zahawi, and S. Banerjee, 2010a Analysis of fast-scale instability in dc drives with full-bridge converter using filippov's method .
- Okafor, N., B. Zahawi, D. Giaouris, and S. Banerjee, 2010b Chaos, coexisting attractors, and fractal basin boundaries in dc drives with full-bridge converter. In *Proceedings of 2010 IEEE International Symposium on Circuits and Systems*, pp. 129–132, IEEE.
- Öztürk, H., 2020 A novel chaos application to observe performance of asynchronous machine under chaotic load. Chaos Theory and Applications **2**: 90–97.
- Parsa, L. and H. A. Toliyat, 2005 Five-phase permanent-magnet motor drives. IEEE Transactions on Industry applications **41**: 30–37.
- Pillay, P. and R. Krishnan, 1989 Modeling, simulation, and analysis of permanent-magnet motor drives. i. the permanent-magnet synchronous motor drive. IEEE Transactions on industry applications **25**: 265–273.
- Pisarchik, A. N. and U. Feudel, 2014 Control of multistability. Physics Reports **540**: 167–218.
- Poliashenko, M. and C. K. Aidun, 1995 A direct method for compu-

tation of simple bifurcations. Journal of Computational Physics **121**: 246–260.

- Sprott, J. C., 2011 A proposed standard for the publication of new chaotic systems. International Journal of Bifurcation and Chaos **21**: 2391–2394.
- Tahir, F. R., K. M. Abdul-Hassan, M. A. Abdullah, V.-T. Pham, T. M. Hoang, *et al.*, 2017 Analysis and stabilization of chaos in permanent magnet dc motor driver. International Journal of Bifurcation and Chaos **27**: 1750173.
- Toufouti, R., S. Meziane, and H. Benalla, 2007 Direct torque control strategy of induction motors. Acta Electrotechnica et Informatica **1**: 1–7.
- Tsafack, A. S. K., R. Kengne, A. Cheukem, J. R. M. Pone, and G. Kenne, 2020 Chaos control using self-feedback delay controller and electronic implementation in ifoc of 3-phase induction motor. Chaos Theory and Applications **2**: 40–48.
- Yu, J., T. Zhang, and J. Qian, 2011 *Electrical motor products: international energy-efficiency standards and testing methods*. Elsevier.

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