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A Novel Approach for Portfolio Optimization Using Fuzzy AHP Based on Gustafson Kessel Clustering Algorithm *

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Abstract

Portfolio management involves modeling risk-return relationships. However, the diverse factors impacting financial markets introduce uncertainty into future portfolio selection. The aim of this study is to propose a portfolio selection model to assist investors in creating the most suitable investment plan in the financial market uncertainty. In this context, a preliminary reduction step is applied to the stocks using the Gustafson-Kessel (GK) algorithm, a fuzzy clustering method, to select portfolio stocks. Later, trapezoidal fuzzy numbers (TrFNs) were defined instead of triangular fuzzy numbers (TFNs) used in the Constrained Fuzzy Analytic Hierarchy Process (AHP) for portfolio selection problems. By using new fuzzy numbers, the weights of the criteria were obtained as TrFNs. Then, a linear programming problem was modeled using the weights of the obtained criteria as a TrFN. For this purpose, a method available in the literature was used that uses price variables in the objective function as TFNs. In this study, a linear programming model that uses these variables as TrFNs is proposed as an alternative to the method that uses the price variables in the objective function as TFNs. In this proposed model, the weights obtained from the Constrained Fuzzy AHP using TrFNs are used as price variables in the objective function of the created linear programming problem. Proposed model then applied to the 48-month return data set of stocks in the Istanbul Stock Exchange 100 (ISE-100) index to determine which stocks the investor should choose and the investment rates investor should make in these stocks. In addition, in order to examine the effectiveness of the proposed model within the scope of the study, portfolio distributions were obtained with different portfolio optimization methods using the same data set and the results were compared.

Keywords: Fuzzy AHP, Gustafson-Kessel algorithm, Portfolio selection, Trapezoidal fuzzy numbers.

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1. INTRODUCTION

By means of technological developments in financial markets, new financial instruments are being developed and the number of markets where investors can trade is increasing. While this situation means new opportunities for investors, it also makes the investor's decision-making process more difficult by offering more tools and markets to the investor. In this case, both individual and institutional investors need analyzes to decide which market and which financial instrument to invest in. For this reason, investors often want to benefit from portfolio selection methods to gain profit. Modeling the relationship between risk and return is very important in portfolio management. In addition, financial markets are immediately affected by social and political changes, causing uncertainty in the portfolio selection process. In the field of portfolio management, studies are frequently carried out on both measuring and managing risk, which represents uncertainty. Various portfolio theories have been put forward to reduce risk. Traditional Portfolio Theory, which was accepted until the 1950s, made recommendations based on the principle that risk can be reduced by diversifying with a large number of financial instruments. Modern Portfolio Theory, proposed by H. M. Markowitz in 1952, brought additional approaches to Traditional Portfolio Theory and made important contributions to reducing risk in portfolio management (Markowitz, 1952). Developed theories and studies reveal a common view that risk management is an important factor on returns.

Financial markets are instantly affected by economic, social and political events, causing these markets to have an uncertain structure. This uncertainty in financial markets also causes uncertainty in the risk and return criteria that are effective in portfolio selection. Therefore, this uncertainty situation needs to be taken into consideration in portfolio optimization problems. In cases of such uncertainty, the fuzzy logic approach, which is an effective method, is preferred. Since many decisions made today involve uncertainty or cannot be represented with precise numerical expressions, the concept of fuzzy logic has been applied to decision-making methods. Fuzzy logic, first introduced by Zadeh in 1965, is used to control processes in situations where complex and information is uncertain (Zadeh, 1965). The fact that fuzzy logic can be applied to both qualitative and quantitative decision problems due to the use of linguistic variables has been one of the biggest reasons why Fuzzy AHP is preferred in decision-making problems. In the Fuzzy AHP, verbal expressions generally characterized by fuzzy numbers are used to show the evaluation values of all alternatives according to subjective and objective criteria. The most important advantage of Fuzzy AHP is the convenience it provides when considering multiple criteria. Since the preferences in AHP are perception-based judgments of decision makers, the fuzzy approach can define a more accurate decision-making process. The earliest work in the field of fuzzy AHP was seen in 1983, when Van Laarhoven and Pedrycz compared fuzzy ratios defined with triangular membership functions (van Laarhoven and Pedrycz, 1983). In 1985, Buckley stated the priorities of fuzzy comparison ratios with trapezoidal membership functions (Buckley, 1985). In 1996, Chang proposed a new approach for the fuzzy AHP by using TFNs in comparisons (Chang, 1996). Stam et al.

have studied how artificial intelligence can be used in the approach of determining the priority values of the AHP (Stam et al., 1996). Shapiro and Koissi, examined and compared the models in three articles by van Laarhoven and Pedrycz (van Laarhoven and Pedrycz, 1983), Buckley (Buckley, 1985) and Chang (Chang, 1996), which form the basis of Fuzzy AHP (Shapiro and Koissi, 2017). In 1997, Weck et al. used the fuzzy AHP when choosing between different production cycle alternatives (Weck et al., 1997). In 1999, Cheng, Yang and Hwang proposed a new method using the fuzzy AHP, to evaluate weapon systems in their studies (Cheng et al., 1999). Fuzzy AHP is widely used by researchers in many fields in the literature.

Trying to simplify complex world problems has been one of the main goals of scientists. In this context, one of the methods aimed at classifying complex events or elements is cluster analysis. The most widely known cluster analysis method is the classical clustering method. In the classical clustering method, each element defined in the cluster must be assigned to a cluster. This assignment is made with a membership degree of zero or one, depending on whether the item belongs to the set or not. In the classical clustering method, the membership values of the elements in the cluster are only 0 or 1. However, in order to eliminate the problem that this situation is not valid for real life problems, Zadeh proposed the fuzzy set theory, in which membership values can take not only 0 or 1, but also values between 0 and 1 (Zadeh, 1965). Bezdek et al. developed the Fuzzy C Means (FCM) algorithm, one of the popular clustering algorithms, inspired by Zadeh's fuzzy set theory (Bezdek et al., 1984). Unlike the classical clustering algorithm, the FCM algorithm allows data points to be assigned to more than one cluster. In the FCM algorithm, Euclidean distance is used to determine data classes and the clusters must be spherical. The GK algorithm was developed to detect clusters of data sets that show scattering in different geometric shapes. In the GK clustering algorithm, Mahalanobis distance is used to determine the classes to which the data belong, and there is no restriction on the clusters being spherical. There are many clustering algorithms in the literature that work similarly to GK (Gath-Geva (GG) (Gath and Geva, 1989), Entropy Weighted Fuzzy C-Means (EWFCM) (Cardone and Martino, 2020), Kernel-Based Fuzzy C-Means (KFCM) (Zhang and Chen, 2003).

There are many studies in the literature where clustering algorithms are used in portfolio selection. In his study, Huang aimed to create an automatic stock market forecaster and portfolio selection mechanism. In his proposed mechanism, financial data is automatically collected every three months to predict trends in the next quarter or six months. The mechanism proposed in his study is based on the FCM clustering algorithm and fuzzy theories (Huang, 2009). Khedmati and Azin's work consists of two stages. First, they used clustering algorithms in sample selection. They then used four different portfolio optimization algorithms to create the optimal portfolio (Khedmati and Azin, 2020). Chen and Huang proposed a portfolio selection model in which future return and risk rates are represented by TFNs. They also proposed a cluster analysis to divide equity investment funds into several groups based on different evaluation index (Chen and Huang, 2009).

The aim of this study is to propose a portfolio selection model for investors to make the most appropriate investment under uncertainties in financial markets. In this context, a preliminary reduction step is first applied to the stocks in the data set in order to determine the stocks that will be included in the portfolio and the ratio of these stocks to be included in the portfolio. This reduction was carried out using the GK algorithm, one of the fuzzy clustering algorithms. Later, trapezoidal fuzzy numbers were defined instead of TFNs used in the Restricted Fuzzy AHP proposed by Enea and Piazza in the literature for portfolio selection problems (Enea and Piazza, 2004). By using new fuzzy numbers, the weights of the criteria were obtained as TrFNs. Then, a linear programming problem was modeled using the weights of the obtained criteria as a trapezoidal fuzzy number. For this purpose, a method available in the literature was used. In this method, a linear programming model that uses TrFNs instead of price variables used as TFN in the objective function is proposed.

The rest of the paper is as follows. GK clustering algorithm is given in Section 2. In Section 3, Fuzzy Inference System (FIS) is briefly mentioned. The proposed algorithm for portfolio selection is given step by step in Section 4. Section 5 provides application and comparison results. Conclusion is given in Section 6.

2. GUSTAFSON-KESSEL CLUSTERING ALGORITHM

GK clustering algorithm was proposed by Gustafson and Kessel in 1979 (Gustafson and Kessel, 1979). The GK clustering algorithm is an extension of the standard FCM clustering (Serir et al., 2012) to enable the use of Mahalanobis distance to detect data classes. While there is a restriction for clusters to be spherical in the FCM approach, there is no such restriction in the GK approach. In order to identify different geometric shapes in the data set, this algorithm uses the covariance matrices of the relevant clusters in distance measurement instead of the Euclidean distance in the FCM clustering approach (Abdullah et al., 2017). When studies conducted by many researchers in different fields are examined in the literature, it is seen that the GK clustering algorithm performs better than the FCM approach (Abdullah et al., 2017; Ghosh et al., 2011; Hadiloo et al., 2018; Miller et al., 2009). The objective function of the GK clustering algorithm is as given by;

$$J(X,\mu,v)=\sum_{j=1}^{C}\sum_{i=1}^{N}\left(\mu_{ij}\right)^{m}D^{2}(x_i,v_j)\,, \tag{1}$$

where, m denotes the fuzziness index, x_i is the i^{th} observation value, v_j is the center of j^{th} cluster, $D^2\left(x_i,v_j\right)$ is the Mahalanobis distance between x_i and v_j.

Step 1. Initial membership degrees $\mu^0=\left[\mu_{ij}\right]$ are determined randomly from a uniform distribution in the range [0,1).

Step 2. Fuzzy cluster centers are calculated for each cluster as in Eq. (2).

$$v_j = \frac{\sum_{i=1}^{N} \mu_{ij}^m x_i}{\sum_{i=1}^{N} \mu_{ij}^m}, \quad j = 1, \ldots, C \tag{2}$$

Step 3. The fuzzy covariance matrix (F_j), is calculated for each cluster using Eq. (3).

$$F_j = \frac{\sum_{i=1}^{N} \mu_{ij}^m \left(x_i - v_j\right)\left(x_i - v_j\right)^t}{\sum_{i=1}^{N} \mu_{ij}^m}, \quad j = 1, \ldots, C \tag{3}$$

Step 4. Mahalanobis distances for each data are calculated with Eq. (4).

$$D^2\left(x_i, v_j\right) = \left(x_i - v_j\right)^t A_j (x_i - v_j), \quad i = 1, \ldots, N, \ j = 1, \ldots, C \tag{4}$$

Here;

$$A_j = \left(\det\left(F_j\right)\right)^{\frac{1}{s}} * F_j^{-1}, \quad j = 1, \ldots, C \tag{5}$$

A_j in Eq.5 is a positive definite $s \times s$ matrix.

Step 5. The membership matrix is updated with Eq. (6).

$$\mu_{ij} = \frac{1}{\sum_{k=1}^{C} \left(\frac{D\left(x_i, v_j\right)}{D\left(x_i, v_k\right)}\right)^{\frac{2}{m-1}}}, \quad j = 1, \ldots, C \quad i = 1, \ldots, N \tag{6}$$

Step 6. Updated membership degrees are compared with previous membership degrees and the process of obtaining optimal membership degrees is stopped when the condition $\left|\left|\mu^t - \mu^{t-1}\right|\right| < \varepsilon$ is provided, where ε is the termination tolerance.

3. FUZZY INFERENCE SYSTEM

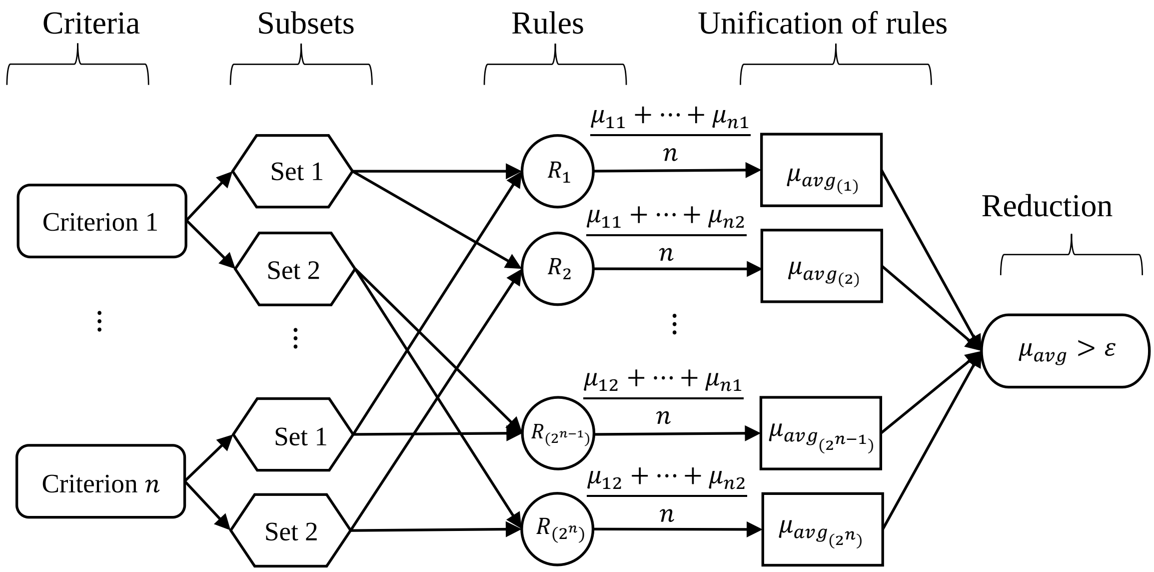
Fuzzy Inference System (FIS) is called a system that includes a collection of operations that enable the system to behave with a single output by banding together the relationships between input and output fuzzy sets in the fuzzy rule base. Using IF-THEN rules, fuzzy logic, can model qualitative aspects of human expressions and reasoning processes (Jang, 1993), formulate actions with high levels of uncertainty and make decisions, without using precise quantitative information (Cox, 1994). FIS consists of three basic conceptual components: A rule base that includes the selection of fuzzy rules, database used to define membership functions in rules and a reasoning mechanism that performs inference (Jang et al., 1997). Although there are different FIS approaches in the literature, Mamdani type FIS and Sugeno type FIS are the most important. After Fuzzy rule-based systems were proposed by Mamdani (Mamdani, 1974), Mamdani and Assilian (Mamdani and Assilian, 1975) developed Mamdani type FIS including Zadeh's (Zadeh, 1975) concept of linguistic variables.

Adaptive Neuro Fuzzy Inference System (ANFIS) is a fuzzy technique that incorporates the learning function of neural networks into fuzzy inference systems. The Takagi-Sugeno-Kang (Sugeno and Kang, 1988; Takagi and Sugeno, 1985) graded inference system is the most common fuzzy inference system. Adaptive networks consist of interconnected nodes, and these nodes contain functions with fixed or variable parameters. With the learning algorithm, the adaptive network determines how the change in these parameters will occur in order to minimize the error size.

4. A NEW ALGORITHM FOR PORTFOLIO SELECTION

The aim of this study is to carry out the portfolio selection process under uncertainty from a data set containing many stocks with the proposed hybrid algorithm. In many portfolio selection processes in the literature, financial indicators such as return and risk of stocks are mostly used, but it is observed that expert opinions are not included. This study proposes a portfolio selection process in which, in addition to return and risk, financial indicators such as price / earnings, market value / book value and net profit / stockholder's equity are used as criteria and expert opinions are also included in the process. The proposed portfolio selection process takes place in three stages. In the first stage, the stocks that are weak in terms of financial ratios belonging to the stocks determined as the criteria are eliminated by using the GK algorithm. The diagram of the fuzzy inference system designed for the elimination process is as shown in Figure 1.

Figure 1. Reduction diagram based on fuzzy inference system



The ANFIS diagram given in Figure 1 is a five-layer architecture. These 5 layers consist of input variables, subsets, created rules, combining the rules by taking their average, and the reduction process, respectively. In Fig. 1, n is the number of criteria, μ_{ij} is the membership degree of i^{th} criterion to j^{th} subset ($i = 1,..,n,\ j = 1,2$), R_k is the rule obtained from the membership degrees of criteria to subsets and $\mu_{avg_{(k)}}$ is the average membership value obtained from the rules ($k = 1, \ldots 2^n$). After the elimination, the portfolio selection process continues with the remaining stocks. The second stage is the determination of the weights of the criteria in the process using Fuzzy AHP. The FLP problem to be used to determine the portfolio distribution is created with the weights obtained in the second stage (Zimmermann, 1978). Finally, the method suggested by Lai and Hwang in the literature was used to solve the FLP problem created with the obtained weights (Lai and Hwang, 1992). TFNs are used in modeling FLP problems in the literature. In this study, mathematical models were created in which the theoretical structure that takes TFN into account was adapted to use TrFNs (Akbaş and Erbay Dalkılıç, 2021). The stages of the hybrid portfolio selection algorithm based on TrFNs, where the adapted methods are used together, are as follows.

Step 1: Criteria are determined to make a portfolio distribution on stocks. These criteria are the criteria required to be maximized or minimized when selecting a portfolio. Financial data of stocks are given in Table 1. In Table 1, the number of criteria is given as n, the number of assets is m and D_{mn} represents the financial value of the m^{th} asset for the n^{th} criterion.

Table 1. Financial data of assets

Assets	Financial Data				
	C_1	$\cdots$	C_j	$\cdots$	C_n
1	D_{11}	$\cdots$	D_{1j}	$\cdots$	D_{1n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	D_{m1}	$\cdots$	D_{mj}	$\cdots$	D_{mn}

Step 2: With the GK algorithm, the membership degrees of asset data to the "Subset 1 (S_1)" and "Subset 2 (S_2)" subsets created for the criteria are determined. Thus, since there will be n criteria and two subsets of each criterion, a total of 2^n different rules will be created. In line with the created rules, the membership degrees of the entities in the subsets of each criterion are given in Table 2.

Table 2. Membership degrees of assets in the subsets belonging to each criterion

Assets	Membership Degree									
	C_1		$\cdots$		C_j		$\cdots$		C_n	
	Subset 1	Subset 2	$\cdots$		Subset 1	Subset 2	$\cdots$		Subset 1	Subset 2
1	$\mu_{11}(S_1)$	$\mu_{11}(S_2)$	$\cdots$	$\cdots$	$\mu_{1j}(S_1)$	$\mu_{1j}(S_2)$	$\cdots$	$\cdots$	$\mu_{1n}(S_1)$	$\mu_{1n}(S_2)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	$\mu_{m1}(S_1)$	$\mu_{m1}(S_2)$	$\cdots$	$\cdots$	$\mu_{mj}(S_1)$	$\mu_{mj}(S_2)$	$\cdots$	$\cdots$	$\mu_{mn}(S_1)$	$\mu_{mn}(S_2)$

Step 3: From the 2^n different rules created, the membership degrees belonging to the "Subset 1" cluster are determined for each criterion that is desired to be maximized, and the membership degrees belonging to the "Subset 2" cluster are determined for each criterion that is desired to be minimum. Then, the average of the membership degrees (μ_{avg}) of the rules created in line with the criteria is calculated. The calculation of the average membership degrees is shown in Table 3.

Table 3. The calculation of the average membership degrees

Assets	Membership Degree					μ_{avg}
	Subset1/Subset2 C_1	⋯	Subset1/Subset2 C_j	⋯	Subset 1/Subset2 C_n	
1	$\mu_{11}(S_1/S_2)$	⋯	$\mu_{1j}(S_1/S_2)$	⋯	$\mu_{1n}(S_1/S_2)$	$(\mu_{11}(S_1/S_2)+\cdots+\mu_{1j}(S_1/S_2)+\cdots+\mu_{1n}(S_1/S_2))/n$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
m	$\mu_{m1}(S_1/S_2)$	⋯	$\mu_{mj}(S_1/S_2)$	⋯	$\mu_{mn}(S_1/S_2)$	$(\mu_{m1}(S_1/S_2)+\cdots+\mu_{mj}(S_1/S_2)+\cdots+\mu_{mn}(S_1/S_2))/n$

Step 4: According to the rule created from the membership degrees obtained by the GK algorithm, the average of the weights calculated is greater than a determined ε value ($\mu_{avg} > \varepsilon$), and the stock reduction process is carried out.

Step 5: Comparison values (b_{ijp}) determined by decision makers regarding the criteria are given in Table 4.

Table 4. Criteria comparison matrix.

	C_1	⋯	C_j	⋯	C_n
C_1	(1 1 1 1) ⋮ (1 1 1 1)	⋯	b_{1j1} ⋮ b_{1jP}	⋯	b_{1n1} ⋮ b_{1nP}
⋮	⋮	⋱	⋮		⋮
C_i	b_{i11} ⋮ b_{i1P}	⋯	b_{ij1} ⋮ b_{ijP}	⋯	b_{in1} ⋮ b_{inP}
⋮	⋮		⋮	⋱	⋮
C_n	b_{n11} ⋮ b_{n1P}	⋯	b_{nj1} ⋮ b_{njP}	⋯	(1 1 1 1) ⋮ (1 1 1 1)

A TrFN consist of four parameters and is expressed with:

$$b_{ijp} = \left(b_{ijp}^{l} \;\; b_{ijp}^{m_1} \;\; b_{ijp}^{m_2} \;\; b_{ijp}^{u}\right),\ i = 1, \dots, n;\ j = 1, \dots, n;\ \ p = 1, \dots, P. \qquad (7)$$

here, b_{ijp} is the importance value of i^{th} criteria corresponding to j^{th}criteria, according to p^{th} decision maker.

Step 6: $a_{ij} = [a_{ij}^{l} \;\; a_{ij}^{m_1} \;\; a_{ij}^{m_2} \;\; a_{ij}^{u}]$ TrFNs obtained by taking the geometric mean of the b_{ijp} values for each criterion in Table 4 represent an average decision maker's opinion and given Table 5. To preserve the symmetric structure of the comparison matrix $A = [a_{ij}]$, we must have; $\forall i \neq j,\ a_{ji} = (1/a_{ij}^{u}, 1/a_{ij}^{m_2}, 1/a_{ij}^{m_1}, 1/a_{ij}^{l})$ and $\forall i = j,\ a_{ij} = (1,1,1,1)$.

Table 5. The comparison matrix consisting of TrFNs

	C_1	$\cdots$	C_j	$\cdots$	C_n
C_1	$(1\ 1\ 1\ 1)$	$\cdots$	a_{1j}	$\cdots$	a_{1n}
$\vdots$	$\vdots$	$\ddots$	$\vdots$		$\vdots$
C_i	a_{i1}	$\cdots$	a_{ij}	$\cdots$	a_{in}
$\vdots$	$\vdots$		$\vdots$	$\ddots$	$\vdots$
C_n	a_{n1}	$\cdots$	a_{nj}	$\cdots$	$(1\ 1\ 1\ 1)$

Step 7: Let $S_i = (S_i^l, S_i^{m_1}, S_i^{m_2}, S_i^u)$ be the fuzzy score for the i^{th} criterion of comparison matrix, where the indices l, m_1, m_2 and u denote its lower, medium1,medium2 and upper respectively. Here, S_i^l and S_i^u are obtained as given by Eqs. (8-9):

$$S_i^l = \min\left[\left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}\right] \Big/ \sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} a_{kj}\right)^{\frac{1}{n}}\right], i = 1, \ldots, n, \tag{8}$$

$$S_i^u = \max\left[\left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}\right] \Big/ \sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} a_{kj}\right)^{\frac{1}{n}}\right], i = 1, \ldots, n,$$

subject to the constraints: (9)

$$a_{kj} \in \left[a_{ij}^l, a_{ij}^u\right] \ \forall j > k,$$

$$a_{jk} = \frac{1}{a_{jk}} \ \forall j < k,$$

$$a_{jj} = 1.$$

Moreover, $S_i^{m_t}$ calculated by

$$S_i^{m_t} = \left[\left(\prod_{j=1}^{n} a_{ij}^{m_t}\right)^{\frac{1}{n}}\right] \Big/ \sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} a_{kj}^{m_t}\right)^{\frac{1}{n}}\right], t = 1,2;\ i = 1, \ldots, n, \tag{10}$$

Step 8: Using the weights determined in Step 7, the fuzzy linear programming problem is modeled as given in Eq. (11).

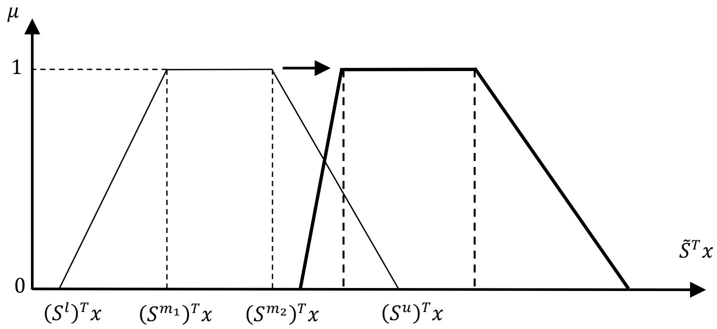
$$\begin{aligned}
&\max \sum_{l=1}^{L} w_l \lambda_l + \sum_{t=1}^{T} \beta_t \gamma_t,\\
&\lambda_l \le \mu_{Z_l}(x),\ \ l = 1, \ldots, L,\\
&\gamma_t \le \mu_{g_t}(x), \qquad t = 1, \ldots, T,\\
&\sum_{l=1}^{L} w_l + \sum_{t=1}^{T} \beta_t = 1,\\
&g_k(x) \le b_k, k = 1, \ldots, K,\\
&\lambda_l \in [0,1], \gamma_t \in [0,1],\ w_l \ge 0, \beta_t \ge 0,
\end{aligned} \tag{11}$$

$x_m \geq 0, m = 1, \ldots, M.$

Here, the coefficients corresponding to fuzzy goals w_l and the coefficients corresponding to fuzzy constraints β_t are obtained from TrFNs. λ_l is the fuzzy targets, γ_t is the fuzzy constraint parameters, $\mu_{Z_l}(x)$ is the membership function for each target and $\mu_{g_t}(x)$ is the membership functions for each fuzzy constraint.

Step 9: In order to maximize the fuzzy goal, the parameters of the weights are shifted as in Figure 2 and new objective functions are created as in Eq. (12).

Figure 2. The strategy to solve $\boldsymbol{max\tilde{S}^T x}$



$$\begin{aligned} &\min Z_1 = \left(S^{m_1} - S^l\right)^T x, \\ &\max Z_2 = (S^{m_1})^T x, \\ &\max Z_3 = (S^{m_2})^T x, \\ &\max Z_4 = (S^u - S^{m_2})^T x, x \in X. \end{aligned} \tag{12}$$

Step 10: The fuzzy programming method proposed by Zimmerman was used in the normalization process to solve Eq. (12) (Zimmermann, 1978). The Positive Ideal Solution (PIS) and the Negative Ideal Solution (NIS) of the four objective functions are obtained by Eqs. (13-16):

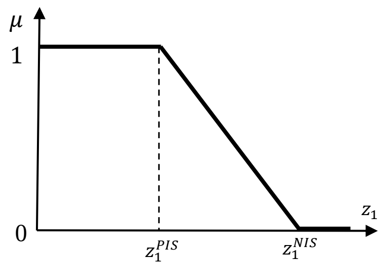
$$Z_1^{PIS} = \min_{x \in X}\left(S^{m_1} - S^l\right)^T x, \qquad Z_1^{NIS} = \max_{x \in X}\left(S^{m_1} - S^l\right)^T x, \tag{13}$$

$$Z_2^{PIS} = \max_{x \in X} (S^{m_1})^T x, \qquad Z_2^{NIS} = \min_{x \in X}(S^{m_1})^T x, \tag{14}$$

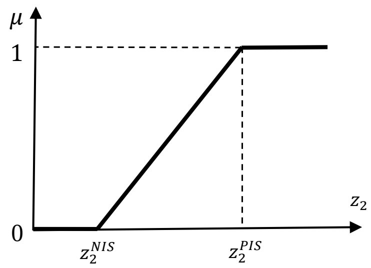
$$Z_3^{PIS} = \max_{x \in X} (S^{m_2})^T x, \qquad Z_3^{NIS} = \min_{x \in X}(S^{m_2})^T x, \tag{15}$$

$$Z_4^{PIS} = \max_{x \in X}(S^u - S^{m_2})^T x, \qquad Z_4^{NIS} = \min_{x \in X}(S^u - S^{m_2})^T x. \tag{16}$$

Step 11: The linear membership functions of the objective functions given in Eqs. (13-16) are obtained from Eqs. (17-18).



$$\mu_{z_1} = \begin{cases} 1, & z_1 < z_1^{PIS}, \\ \dfrac{z_1^{NIS} - z_1}{z_1^{NIS} - z_1^{PIS}}, & z_1^{PIS} \le z_1 \le z_1^{NIS}, \\ 0, & z_1 > z_1^{NIS}, \end{cases} \tag{17}$$

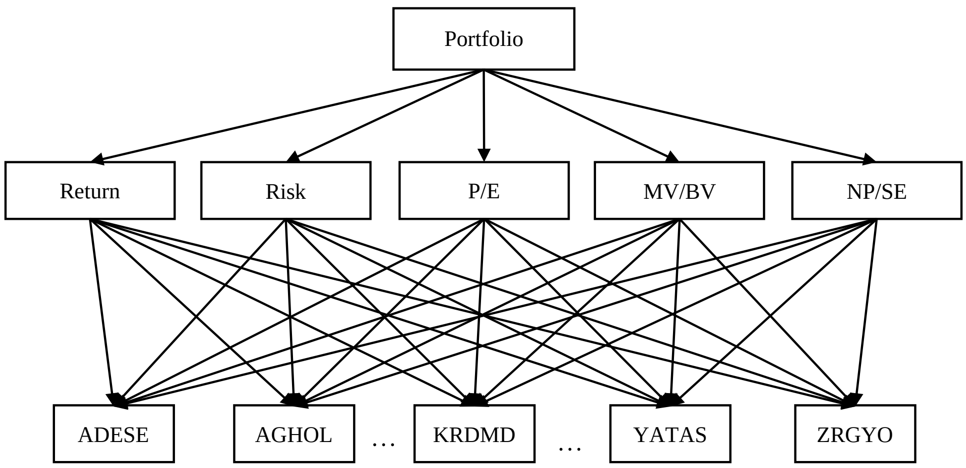


$$\mu_{z_2} = \begin{cases} 1, & z_2 < z_2^{PIS}, \\ \dfrac{z_2 - z_2^{NIS}}{z_2^{PIS} - z_2^{NIS}}, & z_2^{PIS} \le z_2 \le z_2^{NIS}, \\ 0, & z_2 > z_2^{NIS}, \end{cases} \tag{18}$$

5. APPLICATIONS

In this section, the portfolio distribution of stocks traded in the ISE-100 index is obtained using the proposed method. To achieve the optimal portfolio distribution, five criteria were determined in the form of Return, Risk, Price/Earnings (P/E), Market Value/Book Value (MV/BV) and Net Profit/Equity (NP/SE). In this study, monthly return rates of stocks are taken as the return criterion and it is desired to be maximized. The risk criterion is calculated from the return rates of stocks and is desired to be minimized. Price / Earnings ratio is used to measure the price of a stock according to the profit it makes, Market Value / Book Value ratio is used to measure the price of a stock according to its equity capital, Net Profit / Stockholder's Equity ratio, also known as return on equity, is used to measure how much profit shareholders make in return for their unit investment. These ratios are desired to be minimized, maximized and maximized respectively. The hierarchical structure of the problem is given in Figure 3.

Figure 3. Hierarchical structure of the application



ISE-100 data was sourced from https://tr.investing.com/. Between January 1, 2018, and December 31, 2021, this dataset comprises the 48-month Return, Risk, P/E, MV/BV, NP/SE ratios of stocks. Since there were 85 stocks in the ISE-100 index between the mentioned dates, the study was carried out on these 85 stocks. In line with the determined criteria, data on stocks are given in Table 6.

Using the GK algorithm, the degrees of membership of each criterion determined within the scope of the study into two clusters designated as "High" and "Low" was calculated. Thus, two sets are formed for each criterion. Since there were 5 criteria within the scope of the study and two rules for each criterion, low and high, a total of 32 rules were created. With the knowledge that the criteria determined in this study will be maximized or minimized; Membership degrees of the "high return - low risk - low P/E - high MV/BV and high NP/SE" rule and the average μ_{avg}weights of the five membership degrees that can decide which stocks will be included in these rules are given in Table 7.

Table 6. Stocks and financial ratios

	Stocks	Return	Risk	P/E	MV/BV	NP/SE
1	ADESE	0.043	0.364	1.86	0.39	9.09
2	AGHOL	0.023	0.154	16.26	0.95	1.32
3	AKBNK	0.005	0.115	2.33	0.47	13.86
4	AKSA	0.047	0.117	9.16	4.43	29.08
5	AKSEN	0.040	0.123	8.64	2.09	17.92
6	ALGYO	0.040	0.134	1.65	0.67	32.9
7	ALARK	0.036	0.127	5.21	2.53	28.49
8	ALBRK	0.013	0.132	12.64	0.64	6.55
9	ALKIM	0.038	0.083	10.27	4.15	38.13
⋮	⋮	⋮	⋮	⋮	⋮	⋮
78	TURSG	0.036	0.148	5.19	1.34	27.62
79	VAKBN	-0.006	0.117	4.39	0.38	14.03
80	VERUS	0.026	0.134	20.48	4.18	24.87
81	VESBE	0.057	0.120	8.31	2.7	39.78
82	VESTL	0.041	0.165	3.95	0.83	20.26
83	YKBNK	0.012	0.121	2.27	0.47	13.54
84	YATAS	0.018	0.148	6.75	1.82	32.56
85	ZRGYO	0.064	0.112	9.88	1.69	14.65

In accordance with this rule, stocks with high returns, low risks, low P/E ratios, high MV/BV ratios and high NP/SE ratios were determined.

Table 7. Membership degrees of stocks

	Stocks	Membership Degrees					μ_{avg}
		Return High	Risk Low	P/E Low	MV/BV High	NP/SE High	
1	ADESE	0.994	0.036	0.807	0.090	0.013	0.388
2	AGHOL	0.198	0.714	0.065	0.017	0.109	0.220
3	AKBNK	0.074	0.982	0.836	0.078	0.012	0.396
4	AKSA	0.999	0.989	0.751	0.917	0.880	0.907
5	AKSEN	0.973	0.999	0.822	0.132	0.134	0.612
6	ALGYO	0.971	0.967	0.794	0.049	0.978	0.752
7	ALARK	0.895	0.997	0.988	0.312	0.855	0.810
8	ALBRK	0.000	0.980	0.280	0.053	0.040	0.271
9	ALKIM	0.932	0.790	0.584	0.875	0.998	0.836
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
78	TURSG	0.898	0.811	0.988	0.000	0.813	0.702
79	VAKBN	0.188	0.987	0.956	0.092	0.015	0.448
80	VERUS	0.406	0.966	0.005	0.880	0.636	0.579
81	VESBE	0.960	0.995	0.863	0.391	0.992	0.840
82	VESTL	0.980	0.515	0.934	0.029	0.277	0.547
83	YKBNK	0.006	0.997	0.832	0.078	0.009	0.385
84	YATAS	0.040	0.811	0.986	0.056	0.973	0.573
85	ZRGYO	0.926	0.971	0.643	0.031	0.025	0.519

According to the rule created from the membership degrees obtained by the GK algorithm, the weights calculated as greater than 0.7 ($\mu_{avg} > 0.7$) were selected and the stock elimination process was carried out. The 22 stocks remaining after the stock elimination process and the average weights of the stocks are given in Table 8.

Table 8. Stocks with $\boldsymbol{\mu_{avg} > 0.7}$

	Stocks	$\mu_{avg} > 0.7$
1	BRISA	0.9532
2	EGEEN	0.9273
3	AKSA	0.9074
4	SARKY	0.8833
5	OTKAR	0.8820
6	FROTO	0.8764
7	TTRAK	0.8739
8	TOASO	0.8711
9	VESBE	0.8402
10	ALKIM	0.8358
11	ISMEN	0.8280
12	DEVA	0.8276
13	ALARK	0.8096
14	DOAS	0.7632
15	ERBOS	0.7583
16	ALGYO	0.7520

(Table 8 cont.)

	Stocks	$\mu_{avg} > 0.7$
17	LOGO	0.7438
18	KARTN	0.7409
19	TKNSA	0.7332
20	KOZAL	0.7291
21	HEKTS	0.7213
22	TURSG	0.7021

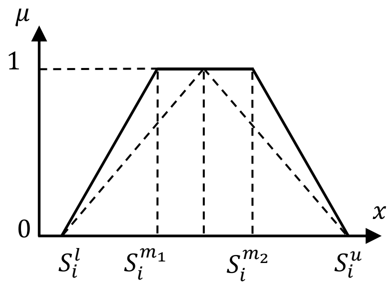
After the stock elimination process, the algorithm proposed within the scope of the study is run to make portfolio distribution. The importance levels of the criteria are given in Table 9. These levels are determined as TFNs, by four decision makers who are securities and investment analyst.

Table 9. Importance levels of criteria

	Return	Risk	P/E	MV/BV	NP/SE
Decision Maker 1	(9 10 10)	(5 7 9)	(3 5 7)	(5 7 9)	(7 9 10)
Decision Maker 2	(7 9 10)	(7 9 10)	(5 7 9)	(5 7 9)	(3 5 7)
Decision Maker 3	(9 10 10)	(9 10 10)	(7 9 10)	(7 9 10)	(5 7 9)
Decision Maker 4	(7 9 10)	(7 9 10)	(3 5 7)	(5 7 9)	(7 9 10)

TFNs in the importance scale were converted into TrFNs by keeping their right and left distributions constant and expanding the point at which they had the highest membership degree to a certain range, as shown in Figure 4.

Figure 4. Transformation of TFN into TrFN



The importance levels of the criteria are obtained by taking the geometric average of the decision maker's opinions are given in Table 10.

Table 10. Representation of the importance degrees given by decision makers as TrFNs

Criteria	Importance degree
Return	(7.94 9.10 9.62 10.00)
Risk	(6.85 8.22 8.95 9.74)
P/E	(4.21 5.78 6.76 8.15)
MV/BV	(5.44 6.95 7.90 9.24)
NP/SE	(5.21 6.78 7.70 8.91)

The fuzzy pairwise comparison matrix obtained by using decision maker opinions for the five criteria of the problem is given in Table 11.

Table 11. Matrix of pairwise comparisons of criteria

	Return	Risk	P/E	MV/BV	NP/SE
Return	(1 1 1 1)	(0.82 1.02 1.17 1.46)	(0.97 1.35 1.66 2.38)	(0.86 1.15 1.38 1.84)	(0.89 1.18 1.42 1.92)
Risk	(0.68 0.85 0.98 1.22)	(1 1 1 1)	(0.84 1.22 1.55 2.31)	(0.74 1.04 1.29 1.79)	(0.77 1.07 1.32 1.87)
P/E	(0.42 0.60 0.74 1.03)	(0.43 0.65 0.82 1.19)	(1 1 1 1)	(0.46 0.73 0.97 1.50)	(0.47 0.75 1.00 1.56)
MV/BV	(0.54 0.72 0.87 1.16)	(0.56 0.78 0.96 1.35)	(0.67 1.03 1.37 2.17)	(1 1 1 1)	(0.61 0.90 1.17 1.77)
NP/SE	(0.52 0.70 0.85 1.12)	(0.53 0.76 0.93 1.30)	(0.64 1.00 1.33 2.13)	(0.56 0.85 1.11 1.64)	(1 1 1 1)

The fuzzy weight of each criterion was calculated by applying the Step (7-8) part of the algorithm for the Constrained Fuzzy AHP, where TrFNs are used in portfolio selection, and is given in Table 12.

Table 12. Fuzzy weights of the criteria in the problem

Criteria	Weights
Return	(0.17 0.23 0.26 0.32)
Risk	(0.15 0.20 0.24 0.30)
P/E	(0.10 0.15 0.18 0.25)
MV/BV	(0.12 0.17 0.21 0.28)
NP/SE	(0.12 0.17 0.20 0.27)

The linear programming problem was modeled as in P_1, using the values of the stocks related to the criteria.

$$P_1:\quad Z_{Re(max)} = 0.038x_1 + 0.050x_2 + \cdots + 0.075x_{21} + 0.036x_{22},$$

$$Z_{Ri(min)} = 0.134x_1 + 0.146x_2 + \cdots + 0.150x_{21} + 0.147x_{22},$$

$$Z_{P/E(min)} = 7.42x_1 + 7.27x_2 + \cdots + 49.15x_{21} + 5.19x_{22},$$

$$Z_{MV/BV(max)} = 4.51x_1 + 4.85x_2 + \cdots + 14.09x_{21} + 1.34x_{22},$$

$$Z_{NP/SE(max)} = 34.38x_1 + 54.63x_2 + \cdots + 33.29x_{21} + 27.62x_{22},$$

$$x_1 + x_2 + \cdots + x_{21} + x_{22} = 1,$$

$$x_i \geq 0, i = 1, \ldots, 22.$$

Here x_i, represents the percentage of investment to be made in the ith stock. PIS and NIS values of each objective function were determined using the MATLAB program and are given in Table 13.

Table 13. PIS and NIS values of each objective function

Objective Functions	PIS	NIS
$Z_{Re(max)}$	0.08	0.03
$Z_{Ri(min)}$	0.08	0.27
$Z_{P/E(min)}$	1.65	49.15
$Z_{MV/BV(max)}$	14.09	0.67
$Z_{NP/SE(max)}$	71.37	27.12

A new fuzzy multi-objective linear programming model (P_2) is created using the fuzzy weights.

$$P_2: \ max \ (0.17 \ \ 0.23 \ \ 0.26 \ \ 0.32)\lambda_1 + (0.15 \ \ 0.20 \ \ 0.24 \ \ 0.30)\lambda_2 + (0.10 \ \ 0.15 \ \ 0.18 \ \ 0.25)\lambda_3 + (0.12 \ \ 0.17 \ \ 0.21 \ \ 0.28)\lambda_4 + (0.12 \ \ 0.17 \ \ 0.20 \ \ 0.27)\lambda_5$$

$$\lambda_1 \leq \frac{(0.038x_1+0.050x_2+\cdots+0.075x_{21}+0.036x_{22})-0.03}{0.05},$$

$$\lambda_2 \leq \frac{0.27-(0.134x_1+0.146x_2+\cdots+0.150x_{21}+0.147x_{22})}{0.19},$$

$$\lambda_3 \leq \frac{49.15-(7.42x_1+7.27x_2+\cdots+49.15x_{21}+5.19x_{22})}{47.50},$$

$$\lambda_4 \leq \frac{(4.51x_1+4.85x_2+\cdots+14.09x_{21}+1.34x_{22})-0.67}{13{,}42},$$

$$\lambda_5 \leq \frac{(34.38x_1+54.63x_2+\cdots+33.29x_{21}+27.62x_{22})-27.12}{44.25},$$

$$x_1 + x_2 + \cdots + x_{21} + x_{22} = 1, \quad x_m \geq 0, \ \ m = 1, \ldots, 22,$$

$$\lambda_i \in [0,1], i = 1, \ldots, 5.$$

In the P_2 model, $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and λ_5 refer to the membership functions for the Return, Risk, Price/Earnings, Market Value/Book Value and Net Profit/ Stockholder's Equity criteria, respectively. Problem P_2 is created using the values in Table 13.

The problem P_2 also includes the constraint that will ensure that the sum of the stock distribution ratios is 1. The objective functions created for the solution of the P_2 problem are given in Eq. (20).

$$\begin{aligned} &min \ \ 0.06\lambda_1 + 0.05\lambda_2 + 0.05\lambda_3 + 0.05\lambda_4 + 0.05\lambda_5 \ (Z_1) \\ &max \ \ 0.23\lambda_1 + 0.20\lambda_2 + 0.15\lambda_3 + 0.17\lambda_4 + 0.17\lambda_5 \ (Z_2) \\ &max \ \ 0.26\lambda_1 + 0.24\lambda_2 + 0.18\lambda_3 + 0.21\lambda_4 + 0.20\lambda_5 \ (Z_3) \\ &max \ \ 0.06\lambda_1 + 0.06\lambda_2 + 0.07\lambda_3 + 0.07\lambda_4 + 0.07\lambda_5 \ (Z_4) \end{aligned} \qquad (20)$$

The PIS and NIS of each objective function given by Eq. (20) were solved within the constraints of the P_2 model and the results obtained are given in Table 14.

Table 14. PIS and NIS of the four objective functions created for the solution of the P_2 model

	PIS	NIS
Z1	0	0.1835
Z2	0.6372	0
Z3	0.7621	0
Z4	0.2437	0

Membership functions for each objective function are obtained by Eq. (21-24).

$$\mu_{z_1} = \begin{cases} 1 & z_1 < 0 \\ \dfrac{0.1835 - z_1}{0.1835} & 0 \leq z_1 \leq 0.1835 \\ 0 & z_1 > 0.1835 \end{cases} \qquad (21)$$

$$\mu_{z_2} = \begin{cases} 1 & z_2 < 0.6372 \\ \dfrac{z_2}{0.6372} & 0 \le z_2 \le 0.6372 \\ 0 & z_2 > 0 \end{cases} \quad (22)$$

$$\mu_{z_3} = \begin{cases} 1 & z_3 < 0.7621 \\ \dfrac{z_3}{0.7621} & 0 \le z_3 \le 0.7621 \\ 0 & z_3 > 0 \end{cases} \quad (23)$$

$$\mu_{z_4} = \begin{cases} 1 & z_4 < 0.2437 \\ \dfrac{z_4}{0.2437} & 0 \le z_4 \le 0.2437 \\ 0 & z_4 > 0 \end{cases} \quad (24)$$

Then, the P_3 model is created using the obtained membership degrees.

P_3: $max \ \ \alpha$

$$\begin{aligned}
&0.1835\alpha + (0.06\lambda_1 + 0.05\lambda_2 + 0.05 + 0.05\lambda_4 + 0.05\lambda_5) \le 0.1835 \\
&0.6372\alpha - (0.23\lambda_1 + 0.20\lambda_2 + 0.15\lambda_3 + 0.17\lambda_4 + 0.17\lambda_5) \le 0 \\
&0.7621\alpha - (0.26\lambda_1 + 0.24\lambda_2 + 0.18\lambda_3 + 0.21\lambda_4 + 0.20\lambda_5) \le 0 \\
&0.2437\alpha - (0.06\lambda_1 + 0.06\lambda_2 + 0.07\lambda_3 + 0.07\lambda_4 + 0.07\lambda_5) \le 0 \\
&0.05\lambda_1 - (0.038x_1 + 0.050x_2 + \cdots + 0.075x_{21} + 0.036x_{22}) \le -0.03, \\
&0.19\lambda_2 + (0.134x_1 + 0.146x_2 + \cdots + 0.150x_{21} + 0.147x_{22}) \le 0.27, \\
&47.50\lambda_3 + (7.42x_1 + 7.27x_2 + \cdots + 49.15x_{21} + 5.19x_{22}) \le 19.15, \\
&13.42\lambda_4 - (4.51x_1 + 4.85x_2 + \cdots + 14.09x_{21} + 1.34x_{22}) \le -0.67, \\
&44.25\lambda_5 - (34.38x_1 + 54.63x_2 + \cdots + 33.29x_{21} + 27.62x_{22}) \le -27.12, \\
&x_1 + x_2 + \cdots + x_{21} + x_{22} = 1 \\
&x_m \ge 0, \ m = 1, \dots 22, \ \alpha \in [0,1]; \ \lambda_j \in [0,1], \ j = 1, \dots, 5.
\end{aligned}$$

By solving the P_3 problem, the portfolio distribution was obtained as given in Table 15.

Table 15. The ratios to be invested in the stocks by solving the P_3 model.

Stocks	Percentage of stocks
OTKAR	0.9567
TKNSA	0.0433

As can be seen in Table 15, using the proposed method, it was determined that for optimum investment, the investor fund should be allocated to OTKAR and TKNSA stocks at rates of 95.67% and 4.33%, respectively.

In order to examine the effectiveness of the proposed model, portfolio distributions were obtained using the mean-variance method and Conditional Value at Risk (CVAR) methods. The Mean Variance method (Markowitz, 1952) is used to reduce risk without reducing expected return by using the correlation between investment assets. Results regarding the Mean Variance method were obtained using MATLAB. Additionally, in this numerical application, the asset distributions of the portfolio were obtained using CVaR method (Rockafellar and Uryasev, 2000) at the α = 0.01 significance level with MATLAB. Finally, the portfolio distribution was made in January and February 2022 according to the results obtained from the applied methods and compared according to the return rates.

Table 16. Results obtained from the methods used in the study

	Stocks	Investment Rate (IR)	Return Rate (%) (RR) (Monthly)		Investment Return (%) (IR × RR)	
			January 2022	February 2022	January 2022	February 2022
Results obtained from the proposed method	OTKAR	0.9567	0.0718	0.0612	6.87	5.86
	TKNSA	0.0433	0.1820	-0.1874	0.79	-0.81
				Total	**7.66**	**5.04**
Results obtained from the Mean-Variance method	ADESE	0.0254	0	-0.0533	0	-0.14
	ALKIM	0.0076	0.1737	-0.1081	0.13	-0.08
	AEFES	0.0235	-0.0196	-0.168	-0.05	-0.40
	BIMAS	0.2046	0.1476	0.0391	3.02	0.80
	CCOLA	0.1006	0.2874	-0.0465	2.89	-0.47
	ERBOS	0.0227	0.0563	-0.1133	0.13	-0.26
	EREGL	0.0586	-0.0312	0.1393	-0.18	0.82
	ISFIN	0.0069	0.0194	-0.1234	0.01	-0.08
	OYAKC	0.0151	0.1456	-0.1978	0.22	-0.30
	RTALB	0.0044	-0.121	-0.1706	-0.05	-0.07
	TSKB	0.0105	0.0556	-0.1513	0.06	-0.16
	VERUS	0.0381	-0.0859	0.0089	-0.33	0.03
	VESTL	0.0171	-0.0343	-0.0687	-0.06	-0.12
	ZRGYO	0.4648	-0.1029	0.055	-4.78	2.56
				Total	**1.01**	**2.13**
CVaR $\alpha = 0.01$	ADESE	0.0241	0	-0.0533	0	-0.13
	ALKIM	0.0759	0.1737	-0.1081	1.32	-0.82
	BIMAS	0.1866	0.1476	0.0391	2.75	0.73
	CCOLA	0.0840	0.2874	-0.0465	2.41	-0.39
	DEVA	0.0155	0.0786	-0.0935	0.12	-0.14
	ERBOS	0.0207	0.0563	-0.1133	0.12	-0.23
	EREGL	0.0686	-0.0312	0.1393	-0.21	0.96
	ISFIN	0.0123	0.0194	-0.1234	0.02	-0.15
	ISMEN	0.0488	-0.1114	-0.1543	-0.54	-0.75
	OYAKC	0.0065	0.1456	-0.1978	0.09	-0.13
	RTALB	0.0100	-0.121	-0.1706	-0.12	-0.17
	TSKB	0.0160	0.0556	-0.1513	0.09	-0.24
	VERUS	0.0070	-0.0859	0.0089	-0.06	0.01
	VESTL	0.0325	-0.0343	-0.0687	-0.11	-0.22
	ZRGYO	0.3916	-0.1029	0.055	-4.03	2.15
				Total	**1.85**	**0.46**

Table 16 shows the portfolio distributions obtained from the methods applied in this study and the return rates of the stocks in these distributions for January and February 2022. Additionally, the table gives a comparison of the total return rates obtained when invested in January and February 2022, according to the results obtained from all methods.

6. CONCLUSION

With the advancement of technology, investors have started to manage their investments using computer-based software. However, it is observed that these software programs do not take expert opinions into account during the portfolio selection process and rely solely on financial ratios based on stock returns. This results in the same expected return for all investors at the same level of risk. To overcome this issue, incorporating both financial ratios and expert opinions based on experience into the model has made the proposed model more effective. Additionally, in the proposed portfolio selection model, during the process of selecting from a set of stocks under uncertainty, a pre-selection was made for indices with a large number of stocks to reduce the number of stocks in the index and efficiently the selection process. In this reduction process, two levels were determined for each criterion within the scope of the study and the membership degrees of the "high return - low risk - low P/E - high MV/BV and high NP/SE" rule suitable for optimal investment selection from 32 rules obtained for 5 criteria were calculated using the GK algorithm from fuzzy cluster methods. The reduction process was carried out with the weight values obtained in the form of the average of the calculated membership degrees. To determine the most suitable investment based on the reduced data set, expert opinions were included in the model, and a linear programming problem using trapezoidal fuzzy numbers was formulated to incorporate uncertainty into the process. In this study, the 48-month return rates of the stocks in the ISE-100 index between January 1, 2018 and December 31, 2021 financial rates of these stocks and decision-making opinions were used. After the stock reduction process using GK Cluster Algorithm, portfolio distribution was made with the proposed model. As can be seen from Table 16, according to the portfolio distributions obtained from the method proposed within the scope of the study, if invested in OTKAR and TKNSA stocks, total profit of 7.66% in January 2022 and 5.04% in February 2022 is obtained. When investing according to the January and February 2022 return rates of stocks obtained using the Mean-Variance method, a total profit of 1.01% and 2.13% is obtained, respectively. When investing according to the January and February 2022 return rates of stocks obtained using the CVaR method ($\alpha = 0.01$), a total profit of 1.85% and 0.46% is obtained, respectively. As can be seen from the results, the return percentage as a result of the investment method determined with the proposed method is greater than the return percentage obtained from other methods available in the literature. This can be stated that the solution process, in which the opinions of investment experts, criteria other than return and risk, and uncertainty are included, is a usable model for portfolio selection.

In further studies, the proposed algorithm can be used in portfolio selection by performing stock reduction in different indices using different fuzzy clustering algorithms (GG, EWFCM, KFCM …).

The study does not necessitate Ethics Committee permission.

The study has been crafted in adherence to the principles of research and publication ethics.

The authors declare that there exists no financial conflict of interest involving any institution, organization, or individual(s) associated with the article. Furthermore, there are no conflicts of interest among the authors themselves.

The authors declare that they all equally contributed to all processes of the research.

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