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RESEARCH ARTICLE

APPLICATION OF THE GENERALIZED KUDRYASHOV METHOD TO THE KOLMOGOROV-PETROVSKII-PISKUNOV EQUATION

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Abstract

In this paper, we investigate the general solutions to the Kolmogorov-Petrovskii-Piskunov equation using the generalized Kudryasov method. It was demonstrated that all produced answers are supplied by exponential function solutions using the symbolic computer program Maple. These solutions are helpful for fluid dynamics, optics, and other fields. Lastly, we have presented some graphs for exact solutions of these equations with special parameter values. For the development of this method, the versatility and dependability of programming offer eclectic applicability to high-dimensional nonlinear evolution equations. The obtained results provided us with valuable insights on the suitability of the novel Kudryashov technique.

Keywords

Exact solution,
Generalized Kudryashov method,
Differential equations

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1. INTRODUCTION

Partial differential equations are an important part of applied mathematics. Many physical, social and natural events have been solved by modeling with non-linear partial differential equations. Examples of these include plasma physics, optical communications, laser technology, signal processing, and others, representing a significant challenge. Applied mathematicians have developed some methods to solve these models. Some of these methods; symmetry method [1], (G’/G)-expansion method [2], tanh-coth method [3], Baclund transformation method [4], exp-function method [5], first integral method [6], Kudryashov method [7] and so on.

Generalized Kudryashov method is one of the method to solve nonlinear partial differential equations. Also the generalized Kudryashov method is a more useful and effective approach to study consistent solutions of NLEEs. Therefore, we used this method to study the Kolmogorov-Petrovskii-Piskunov equation. The aim of this research is to create new analytical solutions for the Kolmogorov-Petrovskii-Piskunov equation using the method mentioned above.

$$u_t - u_{xx} + \mu u + \nu u^2 + \delta u^3 = 0. \quad (1)$$

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This equation, the process we will examine in this article, first appeared in the examination of genetic models. It was used in some physics, biological and physical models. The KPP equation includes various nonlinear equations well known in mathematical physics; the Newell-Whitehead equation for $\mu = -1$, $v = 0$, $\delta = 1$ values, the FitzHugh-Nagumo equation for $\mu = a$, $v = -(a+1)$, $\delta = 1$ values, and it is a special case of Fisher equation $\mu = -1$, $v = 1$, $\delta = 0$ [8].

In this study, firstly the steps of the generalized Kudryashov method will be defined. Then, by applying the generalized Kudryashov method to the Kolmogorov-Petrovskii-Piskunov equation, traveling wave solutions will be obtained for six different cases. In addition, 2D and 3D graphics of these solutions drawn with Maple will be given according to the special selected values. The results obtained will be shared in the last section.

2. GENERALIZED KUDRYASHOV METHOD

In this section, the working algorithm of the generalized Kudryashov method will be examined. Firstly, let's consider a nonlinear partial differential equation as follows.

$$\Omega(u, u_t, u_x, u_{tt}, u_{xy}, u_{xx}, \dots) = 0 \tag{2}$$

where Ω, represents to a polynomial containig derivates of $u_i(i = x, y, \dots., t)$.

Step 1: To reduce this partial differential equation to an ordinary differential equation, we substitute the equivalents of the partial derivatives in the equation using the $u(x, y, \dots, t) = U(\vartheta)$, $\vartheta = z_1 x + z_2 y + \dots + z_n t$ travelling wave equation, where $z_k (k = 1,2,3, \dots n)$ are constants. Then we obtain the following ordinary differential equation

$$G(U, U_\vartheta, U_{\vartheta\vartheta}, U_{\vartheta\vartheta\vartheta}, \dots) = 0. \tag{3}$$

Step 2: In the generalized Kudryashov method, the solution of the ordinary differential equation seek in rational form as follows.

$$U(\vartheta) = \frac{\sum_{i=0}^{N} a_i Q^i(\vartheta)}{\sum_{i=0}^{M} b_j Q^j(\vartheta)} \tag{4}$$

where $a_i (i = 0,1, \dots, N)$, $b_j (j = 0,1, \dots, M)$ are different coefficients of $Q^i(\vartheta), Q^j(\vartheta)$ that are not zero. $Q(\vartheta)$ is the following function which is the solution of the equation $Q'(\vartheta) = Q^2(\vartheta) - Q(\vartheta)$

$$Q(\vartheta) = \frac{1}{1 + Ae^{\vartheta}} \tag{5}$$

here, A is the integral constant and is a non-zero and positive real number to be determined later.

Step 3: We need M and N values to find the $U(\vartheta)$ solution given in (4). To find these values, we use homogenous balance between the highest degree nonlinear term and the highest order linear term in the ordinary differential equation.

Step 4: After substituting Solution (4) into Equation (2), the left-hand side of Equation (2) can be converted into a polynomial in powers of $Q(\vartheta)$. Collecting the terms that include the same power of $Q(\vartheta)$ and equating each coefficient equal to zero, one obtains an algebraic equation system for unknown values.

Step 5: We solve the algebraic equations in the Step 4 with the help of Maple. Substituting the obtained values into Solution (4) by considering Equation (5), the solutions of the NLPE in equation (1) can be obtained.

3. APPLICATION OF THE GENERALIZED KUDRYASHOV METHOD TO THE KOLMOGOROV-PETROVSKİİ-PİSKUNOV EQUATION

In this section, the exact solutions of the KPP equation will be found by the generalized Kudryashov method.

If the wave transformation $u(x,t) = U(\vartheta) = kx - wt$ is applied to the KPP equation in partial differential form given in (1), we obtain the following ordinary differential equation.

$$-wU_{\vartheta} - k^2 U_{\vartheta\vartheta} + \mu u + vU^2 + \delta U^3 = 0. \quad (6)$$

Considering the homogenous balance between the highest order derivative term $U_{\vartheta\vartheta}$ with the highest degree nonlinear term U^3,

$$3N - 3M = N - M + 2. \quad (7)$$

If we choose M=1, then N=2 and equation (4) takes the form

$$U(\vartheta) = \frac{a_0 + a_1 Q(\vartheta) + a_2 Q^2(\vartheta)}{b_0 + b_1 Q(\vartheta)}, \quad (8)$$

where a_0, a_1, a_2, b_0 and b_1 are constants to be determined later.

Substituting Solution (8) into the Equation (6) yields a polinamial in $Q(\vartheta)$. A system of algebraic equations is obtained by setting each coefficient of the equation to zero.

$$Q^6 = -2k^2 a_2 b_1^2 + \delta a_2^3,$$

$$Q^5 = -6k^2 a_2 b_0 b_1 + 3k^2 a_2 b_1^2 + 3\delta a_1 a_2^2 + va_2^2 b_1 - wa_2 b_1^2,$$

$$Q^4 = -6k^2 a_2 b_0^2 + 9k^2 a_2 b_0 b_1 - k^2 a_2 b_1^2 + 3\delta a_0 a_2^2 + 3\delta a_1^2 a_2 + \mu a_2 b_1^2 + 2va_1 a_2 b_1 + va_2^2 b_0 - 3wa_2 b_0 b_1 + wa_2 b_1^2,$$

$$Q^3 = 2k^2 a_0 b_0 b_1 + k^2 a_0 b_1^2 - 2k^2 a_1 b_0^2 - k^2 a_1 b_0 b_1 + 10k^2 a_2 b_0^2 - 3k^2 a_2 b_0 b_1 + 6\delta a_0 a_1 a_2 + \delta a_1^3 + \mu a_1 b_1^2 + 2\mu a_2 b_0 b_1 + 2va_0 a_2 b_1 + va_1^2 b_1 + 2va_1 a_2 b_0 + wa_0 b_1^2 - wa_1 b_0 b_1 - 2wa_2 b_0^2 + 3wa_2 b_0 b_1,$$

$$Q^2 = -3k^2 a_0 b_0 b_1 - k^2 a_0 b_1^2 + 3k^2 a_1 b_0^2 + k^2 a_1 b_0 b_1 - 4k^2 a_2 b_0^2 + 3\delta a_0^2 a_2 + 3\delta a_0 a_1^2 + \mu a_0 b_1^2 + 2\mu a_1 b_0 b_1 + \mu a_2 b_0^2 + 2va_0 a_1 b_1 + 2va_0 a_2 b_0 + va_1^2 b_0 + wa_0 b_0 b_1 - wa_0 b_1^2 - wa_1 b_0^2 + wa_1 b_0 b_1 + 2wa_2 b_0^2,$$

$$Q^1 = k^2 a_0 b_0 b_1 - k^2 a_1 b_0^2 + 3\delta a_0^2 a_1 + 2\mu a_0 b_0 b_1 + \mu a_1 b_0^2 + va_0^2 b_1 + 2va_0 a_1 b_0 - wa_0 b_0 b_1 + wa_1 b_0^2,$$

$$Q^0 = \delta a_0^3 + \mu a_0 b_0^2 + va_0^2 b_0.$$

Using Maple to solve the aforementioned system of algebraic equations yields the following cases.

Case 1:

$v = -\frac{(8k^2+\mu)\sqrt{2}}{4\sqrt{\frac{1}{\delta}}k}$,
$w = -2k^2 + \frac{\mu}{2}$,
$a_0 = 2\sqrt{2}\sqrt{\frac{1}{\delta}}b_0 k$,
$a_1 = -4kb_0\frac{\sqrt{2}}{\sqrt{\frac{1}{\delta}}\delta}$,
$a_2 = 2\sqrt{2}\sqrt{\frac{1}{\delta}}b_0 k$,
$b_1 = -2b_0$.

When the found values are substituted in the $U(\vartheta)$ solutions with using Equation (5) in Equation (8), the following solution $U_{1.1}(\vartheta)$ is obtained.

$$U_{1.1}(\vartheta) = \frac{2k\sqrt{2}A^2e^{2\vartheta}}{\sqrt{\frac{1}{\delta}}\delta\left(A^2e^{2\vartheta}-1\right)}. \quad (9)$$

After converting solution $U_{1.1}(\vartheta)$ from an exponential function to a hyperbolic function, we obtained at the following solution.

$$U_{1.2}(\vartheta) = 2k\sqrt{2}A^2\frac{cosh(\vartheta)+sinh(\vartheta)}{\sqrt{\frac{1}{\delta}}\delta(A^2\,cosh(\vartheta)+A^2\,sinh(\vartheta)-cosh(\vartheta)+sinh(\vartheta))} \quad (10)$$

where

$$\vartheta = kx - t\left(-2k^2+\frac{\mu}{2}\right). \quad (11)$$

Since A is free constant, we can choose the values of A randomly. If we take A=1 in solution $U_{1.2}(\vartheta)$, the Equation (6) has the following singular soliton solution

$$U_{1.3}(\vartheta) = k\sqrt{2}\frac{\left(coth\left(kx-t\left(-2k^2+\frac{\mu}{2}\right)\right)+1\right)\sqrt{\delta}}{\delta}. \quad (12)$$

Solution $U_{1.1}(\vartheta)$ contains unknown values of δ, A, k. and μ. When we choose these values as $\delta = 1, A = 5, k = 1$ and $\mu = 1$, we obtain the 3 and 2-dimensional graphics in Figure 1.

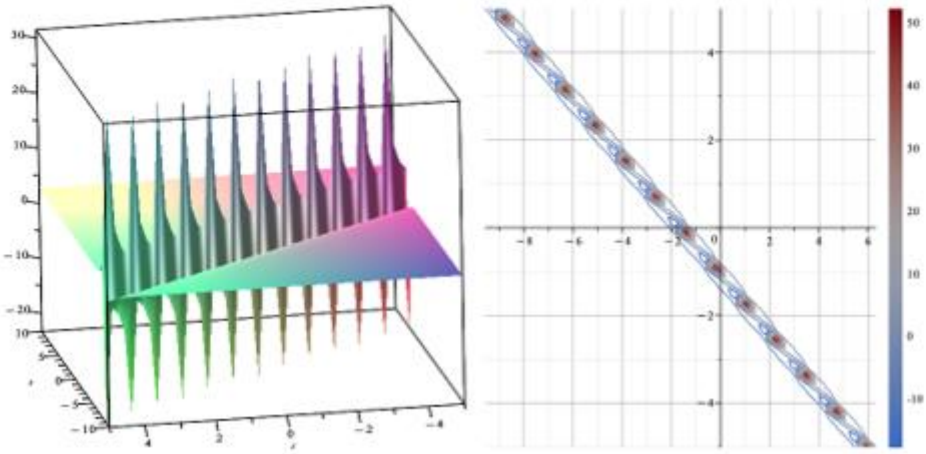

Figure 1. Solution $\mathbf{U_{1.1}}(\vartheta)$ when $\boldsymbol{\delta = 1, A = 5, k = 1, \mu = 1}$; in the range of x=[-10,10], t=[-5,5].

Case 2:

$$\mu = 2k\frac{\sqrt{2}\sqrt{\delta}\frac{v}{2} - k\delta}{\delta},$$
$$w = -k\frac{\sqrt{2}\sqrt{\delta}v - 3k\delta}{\delta},$$
$$a_0 = 0,$$
$$a_1 = -\sqrt{2}b_0\frac{k}{\sqrt{\delta}},$$
$$a_2 = -9k^2\frac{b_0}{v},$$
$$b_1 = \frac{9\sqrt{2}\sqrt{\delta}b_0k}{2v}.$$

When the found values are substituted in the $U(\vartheta)$ solutions in Equation (8), the following solution $U_2(\vartheta)$ is obtained.

$$U_2(\vartheta) = 2v\frac{-a_2\sqrt{\delta} + \sqrt{2}b_0k(1 + Ae^{\vartheta})}{\sqrt{\delta}\left(9\sqrt{2}b_0k(1 + Ae^{\vartheta})\sqrt{\delta} + 2v(1 + Ae^{\vartheta})^2 b_0\right)} \tag{13}$$

where

$$\vartheta = kx + tk\frac{\sqrt{2}\sqrt{\delta}v - 3k\delta}{\delta}. \tag{14}$$

When we choose the values as $\delta = 5, A = 5, k = 1$ and $v = 1$ in solution $U_2(\vartheta)$, we obtain the 3 and 2-dimensional graphics in Figure 2.

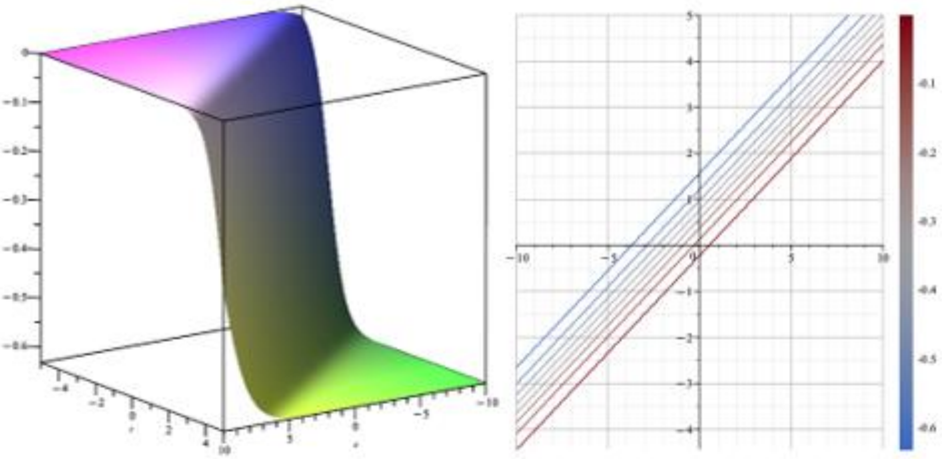

Figure 2. Solution $\mathbf{U_2(\vartheta)}$ when $\boldsymbol{\delta = 5, A = 5, k = 1, v = 1}$; in the range of x=[-10,10], t=[-5,5].

Case 3:

$$v = -2\delta \frac{2k^2+\sqrt{2}\sqrt{(2k^4+k^2\mu)}+\mu}{\sqrt{\left(\left(4k^2+2\sqrt{2}\sqrt{(2k^4+k^2\mu)}+\mu\right)\right)}},$$

$$w = \left(4\left(4k^2+2\sqrt{4k^4+2k^2\mu}+\mu\right)\delta\frac{k^2}{\mu^2}+\left(4k^2+2\sqrt{4k^4+2k^2\mu}+\mu\right)\frac{\delta}{\mu}-\delta\right)\frac{\mu^2}{2\left(4k^2+2\sqrt{4k^4+2k^2\mu}+\mu\right)}$$

$$a_1 = -2a_0\,,$$

$$a_2 = -32\left(\frac{\left(k^2-\frac{\mu}{8}\right)\sqrt{2}\sqrt{(2k^4+k^2\mu)}}{2}+k^2\left(k^2+\frac{\mu}{8}\right)\right)\frac{a_0}{\mu\left(4k^2+2\sqrt{2}\sqrt{(2k^4+k^2\mu)}-\mu\right)},$$

$$b_0 = \sqrt{4k^2+2\sqrt{4\sqrt{k^4+2k^2\mu}+\mu})\delta\frac{a_0}{\mu}},$$

$$b_1 = -2\sqrt{4k^2+2\sqrt{4\sqrt{k^4+2k^2\mu}+\mu})\delta\frac{a_0}{\mu}}.$$

When the found values are substituted in the $U(\vartheta)$ solutions in Equation (3.3), the following $U_3(\vartheta)$ solution is obtained.

$$U_3(\vartheta) = \frac{4A^2e^{2\vartheta}k^2\mu+2A^2e^{2\vartheta}\sqrt{2k^4+k^2\mu}\sqrt{2}\mu-A^2e^{2\vartheta}\mu^2-32k^4-16\sqrt{2}\sqrt{2k^4+k^2\mu}k^2-8k^2\mu+\mu^2}{\sqrt{\left(4k^2+2\sqrt{2}\sqrt{2k^4+k^2\mu}+\mu\right)\delta}\left(4k^2+2\sqrt{2}\sqrt{2k^4+k^2\mu}-\mu\right)\left(A^2e^{2\vartheta}-1\right)} \quad (15)$$

where

$$\vartheta = kx - t\frac{\left(4\left(4k^2+2\sqrt{4k^4+2k^2\mu}+\mu\right)\delta\frac{k^2}{\mu^2}+\left(4k^2+2\sqrt{4k^4+2k^2\mu}+\mu\right)\frac{\delta}{\mu}-\delta\right)\mu^2}{2\left(4k^2+2\sqrt{4k^4+2k^2\mu}+\mu\right)}. \quad (16)$$

Solution $U_3(\vartheta)$ contains unknown values of a_0, δ, A, k and μ. When we choose these values as $a_0 = 1, \delta = 0.1, A = 2, k = 1$ *and* $\mu = 0.01$, we obtain the 3 and 2-dimensional graphics in Figure 3.

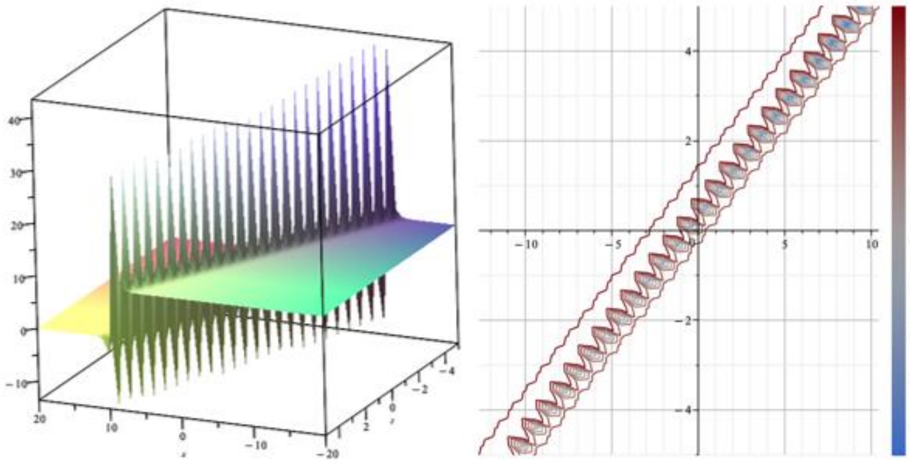

Figure 3. Solution $U_3(\vartheta)$ when $a_0 = 1,\ \delta = 0.1, A = 2, k = 1, \mu = 0.01$; in the range of x=[-20,20], t=[-5,5].

Case 4:

$$\delta = v^2 \frac{\sqrt{(\mu^2+4w^2}-2w}{2\mu\left(-\mu+\sqrt{(\mu^2+4w^2)}\right)-4w\left(-\mu+\sqrt{(\mu^2+4w^2}\right)+4\mu^2-8\mu w+8w^2},$$

$$k = \frac{\sqrt{-\mu+\sqrt{\mu^2+4w^2}}}{2},$$

$$a_1 = -2a_0\ ,$$

$$a_2 = a_0 \frac{3\mu-3\sqrt{(\mu^2+4w^2)}+2w}{\mu-2\sqrt{(\mu^2+4w^2)}+4w},$$

$$b_0 = -v \frac{a_0}{\mu+\sqrt{\mu^2+4w^2}-2w}\ ,$$

$$b_1 = 2v \frac{a_0}{\mu+\sqrt{\mu^2+4w^2}-2w}.$$

When the found values are substituted in the $U(\vartheta)$ solutions in Equation (8), the following $U_4(\vartheta)$ solution is obtained.

$$U_4(\vartheta) = -2\frac{\left(A^2\left(\sqrt{\mu^2+4w^2}-2w-\frac{\mu}{2}\right)e^{2\vartheta}+\frac{\sqrt{\mu^2+4w^2}}{2}+w-\mu\right)\left(\mu+\sqrt{\mu^2+4w^2}-2w\right)}{v\left(-\mu+2\sqrt{\mu^2+4w^2}-4w\right)\left(A^2e^{2\vartheta}-1\right)}, \tag{17}$$

where

$$\vartheta = \sqrt{-\mu+\sqrt{\mu^2+4w^2}}\,\frac{x}{2} - wt. \tag{18}$$

When we choose the values as $a_0, A = 5, w = 1, v = 1$ and $\mu = 1$ in solution $U_4(\vartheta)$, we obtain the 3 and 2-dimensional graphics in Figure 4.

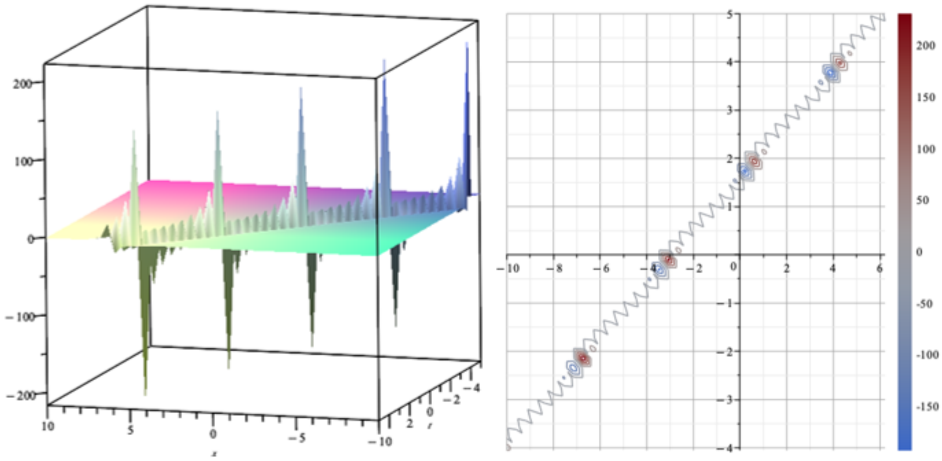

Figure 4. Solution $U_4(\vartheta)$ when $a_0 = 1, A = 5, w = 1, v = 1, \mu = 1$; in the range of x=[-10,10], t=[-5,5].

Case 5:

$$k = \sqrt{2}\sqrt{\delta}\frac{w}{v},$$
$$\mu = -\frac{16\delta^2 w^2 - v^4}{4\delta v^2},$$
$$a_1 = -2a_0,$$
$$a_2 = \frac{8w\delta a_0}{4\delta w - v^2},$$
$$b_0 = \frac{2\delta v a_0}{4\delta w - v^2},$$
$$b_1 = -\frac{4\delta v a_0}{4\delta w - v^2}.$$

When the found values are substituted in the $U(\vartheta)$ solutions in Equation (3.3), the following $U_5(\vartheta)$ solution is obtained.

$$U_{5.1}(\vartheta) = \frac{-A^2(-4\delta w + v^2)e^{2\vartheta} + v^2 + 4w\delta}{2\delta v(A^2 e^{2\vartheta} - 1)}. \tag{19}$$

After converting solution $U_{5.1}(\vartheta)$ from an exponential function to a hyperbolic function, we obtained at the following solution.

$$U_{5.2}(\vartheta) = \frac{-A^2(-4\delta w + v^2)(cosh(2\vartheta) + sinh(2\vartheta)) + v^2 + 4\delta w}{2v\delta(A^2(cosh(2\vartheta) + sinh(2\vartheta)) - 1)} \tag{20}$$

where

$$\vartheta = -\sqrt{2}\sqrt{\delta}\frac{w}{v}\,x - wt. \tag{21}$$

Solution $U_{5.1}(\vartheta)$ contains unknown values of a_0, A, w, v and δ. When we choose these values as $a_0 = 1, A = 5, w = 1, v = 1$ *and* $\delta = 9$, we obtain the 3 and 2-dimensional graphics in Figure 5.

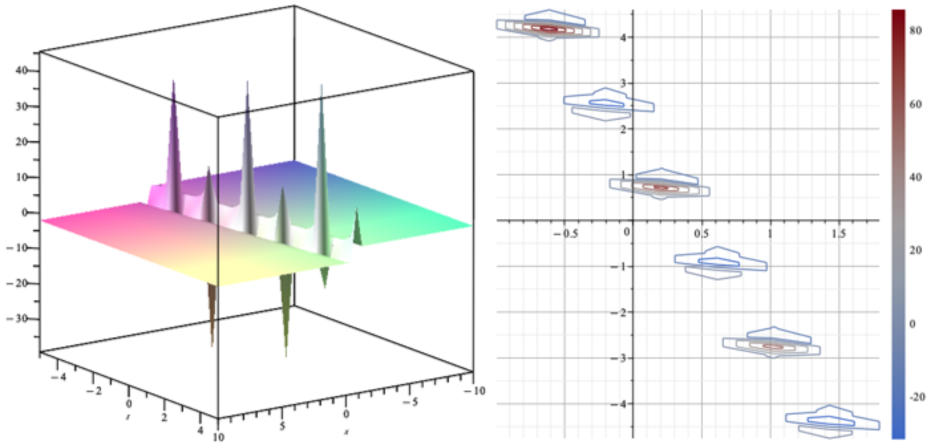

Figure 5. Solution $\mathbf{U_5(\vartheta)}$ when $\mathbf{a_0 = 1, A = 5, w = 1, v = 1, \ \delta = 9}$; in the range of x=[-10,10], t=[-5,5].

Case 6:

$$k = \frac{\sqrt{-\mu+\sqrt{\mu^2+4w^2}}}{2},$$

$$v = \left(-\mu - \sqrt{\mu^2 + 4w^2} - 2w\right)\frac{\delta}{\sqrt{\delta(\sqrt{\mu^2+4w^2}+2w}},$$

$$a_1 = -2a_0,$$

$$a_2 = 2a_0\frac{-\left(-9\mu^3 + 27\mu^2 w - 36\mu w^2 + 16w^3\right)\sqrt{\mu^2+4w^2} - 32w^4 + 72w^3\mu - 58w^2\mu^2 + 27\mu^3 w - 9\mu^4}{\left(9\mu^3 - 16\mu^2 w + 8\mu w^2\right)\sqrt{\mu^2+4w^2} + 16w^3\mu - 32w^2\mu^2 + 20\mu^3 w - 9\mu^4},$$

$$b_0 = \sqrt{\delta\sqrt{\mu^2 + 4w^2} + 2\delta w}\ \frac{a_0}{\mu},$$

$$b_1 =$$

$$\frac{4\delta a_0\left(27\mu^4 - 90\mu^3 w + 146\mu^2 w^2 - 112\mu w^3 + 32w^4\right)\sqrt{\mu^2+4w^2} + 64w^5 - 224w^4\mu + 300w^3\mu^2 - 208w^2\mu^3 + 90\mu^4 w - 27\mu^5}{\sqrt{\delta\sqrt{\mu^2+4w^2}+2w}\mu\left(9\mu^2\sqrt{\mu^2+4w^2} - 16\mu w\sqrt{\mu^2+4w^2} + 8w^2\sqrt{\mu^2+4w^2} - 9\mu^3 + 20\mu^2 w - 32\mu w^2 + 16w^3\right)\left(3\mu - 3\sqrt{\mu^2+4w^2} + 4w\right)}.$$

When the found values are substituted in the $U(\vartheta)$ solutions in Equation (8), the following $U_5(\vartheta)$ solution is obtained.

$$U_6(\vartheta) =$$

$$-\frac{\left(\left(\frac{\left(w^2 - 2\mu w + \frac{9}{8}\mu^2\right)\mu A^2 e^{2\vartheta}}{2} - 2w^3 + 4\mu w^2 - \frac{19\mu^2 w}{8} + \frac{9\mu^3}{16}\right)\sqrt{\mu^2+4w^2} + A^2\mu\left(w^3 - 2\mu w^2 + \frac{5}{4}\mu^2 w - \frac{9}{16}\mu^3\right)e^{2\vartheta} - 4w^4 + 8w^3\mu - \frac{21w^2\mu^2}{4} + \frac{17\mu^3 w}{8} - \frac{9\mu^4}{16}\right)\left(\frac{3\mu}{4} - \frac{3\sqrt{\mu^2+4w^2}}{4} + w\right)\sqrt{\left(\delta\left(\sqrt{\mu^2+4w^2} + 2w\right)\right)}}{2\left(1 + Ae^{\vartheta}\right)\delta\left(\left(\frac{1}{2}w^4 - \frac{7}{4}w^3\mu + \frac{73}{32}w^2\mu^2 - \frac{45}{32}\mu^3 w + \frac{27}{64}\mu^4\right)\sqrt{\mu^2+4w^2} + w^5 - \frac{7w^4\mu}{2} + \frac{75w^3\mu^2}{16} - \frac{13w^2\mu^3}{4} + \frac{45\mu^4 w}{32} - \frac{27\mu^5}{64}\right)\left(Ae^{\vartheta} - 1\right)} \quad (22)$$

where

$$\vartheta = \sqrt{-\mu + \sqrt{\mu^2 + 4w^2}}\,\frac{x}{2} - wt. \quad (23)$$

When we choose the values as $\mu = 5, A = 5, w = 1\ and\ \ \delta = 5$ in solution $U_6(\vartheta)$, we obtain the 3 and 2-dimensional graphics in Figure 6.

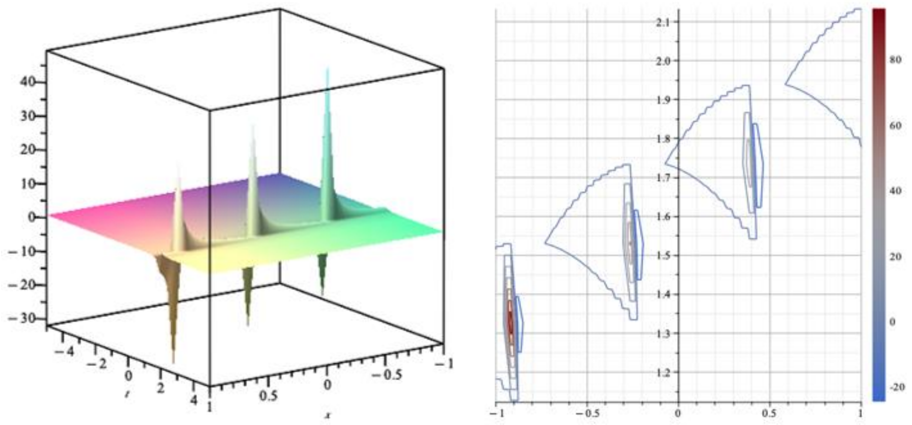

Figure 6. Solution $\mathbf{U_6(\vartheta)}$ when $\boldsymbol{\mu = 5, A = 5, w = 1,\ \delta = 5}$; in the range of x=[-1,1], t=[-5,5].

4. CONCLUSION

In this study, we examined some exact solutions of the Kolmogorov-Petrovskii-Piskunov equation using the generalized Kudryashov method. We reduced this partial differential equation to an ordinary differential equation with the traveling wave equation. We investigated the solution form in accordance with the steps of the generalized Kudryashov method. As a result, we found six cases. We obtained six solutions from these six cases, and in addition to these solutions, we found two singular soliton solutions. For our solutions, we had 3D and 2D graphics drawn with the help of Maple. The newly found hyperbolic function solutions may be particularly useful in understanding long-wave propagation, shallow water wave dynamics, and physical phenomena in plasma fluid. The results showed us the applicability of the generalized Kudryashov method to the KPP equation.

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CONFLICT OF INTEREST

The author(s) stated that there are no conflicts of interest regarding the publication of this article.

CRediT AUTHOR STATEMENT

Zeynep Aydın: Software, Validation, Investigation, Resources, Writing–Original Draft, Visualization
Filiz Tascan: Conceptualization, Writing–Review & Editing, Supervision, Project administration

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