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Research Article

The adaptive extended kalman filter approach for the Lotka-Volterra model and application to economic variables

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ABSTRACT

The main aim of this article is to extend on the application of the grey Lotka-Volterra model by Wu et al. [1] with a linear programming method. We used this method for estimating the parameters of behavioral variables under the criterion of the minimization of mean absolute percentage error (MAPE). Our empirical analysis indicates that the adaptive extended Kalman filter (EKF) approach performs far better compared to traditional Lotka-Volterra model in the prediction of the relevant parameters. Comparisons of empirical results with the linear programming method for parameter estimation of the grey Lotka-Volterra model demonstrate that the EKF approach has more powerful and efficient prediction performance.

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INTRODUCTION

Wu et al. [1] proposed particular version of grey Lotka-Volterra model by employing their own algorithm calibration technique in the form of a linear programming method to estimate the parameters of the grey Lotka-Volterra model under the criterion of minimization of mean absolute percentage error (MAPE). Theoretical and empirical improvements in the technological forecasting methods allow researchers to adopt more efficient ways to predict with a certain rate of precision rather than to conjecture the future values of variables in the social and economic system.

The general framework of the traditional grey system theory developed by [2] has its limitations for it is a single variable forecasting model. Hence alternative approaches are employed so as to overcome the drawbacks of Grey Model First Order One Variable types of estimation process. Since the grey system is set in between full information and null information assumptions of the social and economic environment, uncertainty requires more clear-cut criteria of error correction for model specification [2, 3].

The gray Lotka–Volterra model is applied in different areas Gatabazi et al. [4], Mao et al. [5] . The grey model GM(1,1) was proposed by [2]. It has been applied in various studies on forecasting such as electricity costs, integrated circuit industry, wafer fabrication, opto-electronics industry output value and fatality risk estimation measure Mao et al. [5], Tsai et al. [6], Zhang et al. [7]. However, GM(1,1) can

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work only as a single variable forecasting model. It cannot analyse the long-term relationship between the two variables and predict the values of two variables in social system or economic system Gatabazi et al. [4]. Lotka-Volterra models can also be used for short-term forecasting in the Stock Exchange Debnath et al. [8], Debnath et al. [9].

In this paper, we extend on the proposed application of grey Lotka-Volterra model developed by Wu et al. [1] to demonstrate that the adaptive extended Kalman filter approach is more efficient to use and predicts parameters in the Lotka-Volterra model with a higher degree of precision.

When the literature was reviewed, it was seen that EKF was not used to estimate the parameters in the Lotka-Volterra model. The use of EKF to estimate the parameters in the Lotka-Volterra model is the unique aspect of this study.

The plan of the paper is as follows. The next section examines the elements of Lotka-Volterra model in dynamic equilibrium analysis and makes assumptions for behavioral variables together with their relevant interactive parameters. The third section provides formal representation of state-space model and the adaptive extended Kalman filter. The fourth section discusses in detail the empirical experiments according to the results of alternative forecasting methods and models. The fifth section concludes with the critical comment and appraisal.

THE ELEMENTS OF LOTKA-VOLTERRA MODEL

The general functional form of the Lotka-Volterra model consists of the strategic interactions of two variables with their relevant parameters in a setting for social or economic environment [10] (Goel et al., 1971). The variables are sometimes defined as species when the functional form in differential equations with respect to time reflects growth patterns for predator-prey population and their dynamic mode of interaction Volterra [11]. The first variable $\{x\}$ is the growth equation for species (hare) of prey population

$$\frac{dx(t)}{dt} = ax(t) - bx(t)y(t) = \big(a - by(t)\big)x(t). \quad (1)$$

The second element is variable $\{y\}$ specifies the differential equation for the growth rate of species (lynx) predator population

$$\frac{dy(t)}{dt} = -my(t) + rx(t)y(t) = \big(-m + rx(t)\big)y(t) \quad (2)$$

where $\{t = 0\}$ denotes present instant in time for continuous sequences within infinitesimal units whilst it indicates present period in time for discrete sequences such that initial values of the variables relate to the present time $x(0) = x_0$ and $y(0) = y_0$. The element of time enters into the function in continuous as well as discrete form. The definitions of variables and their relevant parameters in the model specifications of equations (1) and (2) are given in the following section.

The Specific Definitions and Strategic Nature of Behavioral Variables and Interactive Parameters of Dynamic Lotka-Volterra Model

(i.) Behavioral Variables

$x(t)$: *Number of population of the prey with respect to time*. The size of prey population has positive effect on the predator population since predators are fed upon the preys (Shim et al. [12]). Preys coexist with predators in the ecosystem and the link between them cointegrates through movements of cyclical patterns Volterra [11].

$y(t)$: *Number of population of the predator with respect to time*. The size of predator population has negative effect of the prey population since it decreases as a result of predation process. Predators coexist with preys and their existence depends upon preys' availability for predation.

(ii.) Interactive Parameters

$a(a > 0)$: *Growth rate of prey population in the absence of predator*. This parameter is also defined as the birth rate of prey and it is set strictly above zero since any negative value signifies extinction of the species whilst any value exact to zero rests upon static equilibrium condition which violates dynamic equilibrium nature of the interactive strategic ecosystem modeling. Predators cease to exist unless preys are available and attainable for predation (Křivan et al. [13]).

$b(b > 0)$: *Predation rate by the predators on the prey*. This parameter is also defined as the death rate of the prey and it is set strictly above zero as a consequence of the nature of predation where preys are consumed by predators. If, for example, parameter b equals zero there will be no predation and the ecological system is stuck into static equilibrium condition.

$m(m > 0)$: *Decay rate of the predator in the absence of the prey*. This parameter shows the death rate of predators if they cannot find prey for food. The rate is above zero because the nature of the predation sometimes let predators die by starvation. In case the parameter m is set to zero there will be no decay of predators and the ecosystem is set into static equilibrium which is an unrealistic condition for ever changing true nature of ecosystem.

$r(r > 0)$: *Growth rate of predators depends upon the population of the prey*. The parameter r forms interactive component of equation (2) together with population variables $\{x\}$ and $\{y\}$ or numbers of preys and predators respectively. This parameter is also defined as the birth rate of predators and it is set strictly above zero for any negative value relates to the extinction of predator species and creates stationary state of disequilibrium in ecosystem (Křivan et al. [14]).

2.2. Factual and Analytical Assumptions of the Model Components with Strategic Interaction in Prey-Predatory Ecosystem

The strategic nature of interactive variables and parameters formed in the Lotka-Volterra model can be observed within the components of equation (3) and (4). The first

Table 1. Multi-mode competitive market structure identification with regards to the positive or negative signs of interactive parameters of the Lotka-Volterra model

b	*r*	**Market Structure**	**Behavioral Identification**
+	+	Perfect Competition	Both negatively affect each other
+	–	Predator-Prey Situation	One negatively affect the other
–	–	Strategic Complements	Both positively affect each other

Source: Wu et al. [1]

variable $\{x\}$ in the equation (3) denoting prey population is apt to grow exponentially under condition of isolation from predator population. In other words, the absence of predators in the ecosystem means that second component of the equation will be zero and only the first component $\frac{dx(t)}{dt} = \{ax(t)\}$ will grow exponentially provided that the parameter $a > 0$. In the same manner, if preys are unavailable for predation then the second component of the equation will become zero so that the first component $\frac{dy(t)}{dt} = \{-my(t)\}$ will decay exponentially provided that the ecosystem is neither at a static equilibrium nor disequilibrium which means that the parameter $m > 0$.

If, for example, parameters a and m are set to zero then the ecosystem will be at static equilibrium. If, on the other hand, parameters a and m are set to some values lower than zero then this time the ecosystem will be under disequilibrium condition which means that both species deteriorate and face the danger of extinction. Only positive values of behavioral variables provide dynamic equilibrium of the ecosystem. These natural conditions set factual and analytical assumptions for the equilibrium analysis of ecosystem.

The third and last case depends upon the condition of coexistence and strategic interaction for the behavioral variables and interactive parameters in the model. In this case, the decay rate in the prey population relates proportionally to the growth rate of the predator population for the second preys upon the first. The proportional rate is determined by the frequency of prey-predator interactions which are reflected in the behavioral variables and interactive parameters of the model. The second components of the equations (3) and (4) show the nonlinear interaction process.

We can also modify the model to transform its framework of time from continuous units to discrete units by rearranging equations (2.1) and (2.2) as follows

$$x(t + \Delta t) = x(t) + [a - by(t)]x(t)\Delta t \tag{3}$$

$$y(t + \Delta t) = y(t) + [-m - rx(t)]y(t)\Delta t \tag{4}$$

where Δ is numerical operator for incremental increase in discrete time intervals as opposed to infinitesimal increase in continuous time units. These equations with discrete time units are employed in state space model for adaptive extended Kalman filtering approach (Sun et al. [15]).

In a competitive market environment, besides the natural setting of an ecosystem which is constrained within the struggle to survive strategy for species, we can specify multi-mode competitive relationship between behavioral variables and their respective interactive parameters. The identification of market structure is possible according to the positive or negative signs of parameters b and r in the second components of equations (3) and (4).

As shown in the above table, perfect competition market structure provides an environment for agents to behave as strategic substitutes to each other. Coexistence is the only condition for full competition where interactive parameters have positive values. When interactive parameter b has positive value whilst the other interactive parameter r has negative value then predator-prey situation prevails in the market similar to that of the natural ecosystem in which major agent preys upon the minor. The third row of the table shows mutual solidarity as opposed to the first raw. Agents behave in a way to provide strategic complements to each other. This time, coexistence is the only condition for cooperation where both of the relevant interactive parameters have negative values.

STATE-SPACE MODEL AND KALMAN FILTER

Consider a general linear discrete-time stochastic system represented by the state and measurement models given by

$$\begin{aligned} x_{k+1} &= \Phi_k x_k + B_k u_k + w_k \\ z_k &= H_k x_k + v_k \end{aligned} \tag{5}$$

where, x_k is an $n \times 1$ system state vector, z_k is an $m \times 1$ observation vector, Φ_k is an $n \times n$ system matrix, u_k is a $p \times 1$ vector of the input forcing function, B_k is an $n \times p$ matrix, H_k is an $m \times n$ matrix, w_k an $n \times 1$ vector of zero mean white noise sequence and v_k is an $m \times 1$ measurement error vector assumed to be a zero mean white sequence uncorrelated with the w_k sequence. The matrices F_k, B_k, H_k, Q_k, R_k are assumed known at time k. The covariance matrices w_k and v_k are defined by $E(w_k w_k') = Q_k \delta_{kl}$, $E(v_k v_k) = R_k \delta_{kl}$,

$E(w_k v_k) = 0$ where δ_{kl} is the Kronecker delta function. The optimum Kalman filter update equations are

$$\hat{x}_{k/k-1} = \Phi_{k-1}\hat{x}_{k-1} + B_{k-1}u_{k-1}$$

$$\hat{x}_k = \hat{x}_{k/k-1} + K_k\left[z_k - H\hat{x}_{k/k-1}\right]$$

$$P_{k|k-1} = \Phi_{k-1}P_{k-1}\Phi'_{k-1} + Q_{k-1} \tag{6}$$

$$P_k = (I - K_k H_k)P_{k|k-1}$$

where K_k, the optimum Kalman gain is given by

$$K_k = P_{k|k-1}H'_k(H_k P_{k|k-1}H'_k + R_k)^{-1} \tag{7}$$

In the above equations $\hat{x}_{k/k-1}$ is the a priori and $\hat{x}_k$ is the a posteriori estimate of x_k. Also $P_{k|k-1}$ and P_k are the covariance of a priori and a posteriori estimates respectively. Ozbek et al. [16] as well as Ozbek et al. [17] proposed a scalar, namely a forgetting actor to the standard Kalman filtering that was introduced in the error covariance equation to limit the memory of the recursive least square.

$$P_{k|k-1} = \alpha(\Phi_{k-1}P_{k-1}\Phi'_{k-1} + Q_{k-1}) \tag{8}$$

This modification adds an adaptive nature to the standard filter which provides robustness to the filter when time varying parameters are to be estimated. For $\alpha = 1$ the resulting filter is the same as the standard Kalman filter, whereas for $\alpha > 1$ the filter has an adaptive nature via exponential data weighting. The idea behind using a forgetting factor is to artificially emphasize the effect of current data by exponentially weighting the observations. This modification expanded to the EKF in order to yield the adaptive extended Kalman filter (See Appendix).

EMPIRICAL EXPERIMENTS

Wu et al. [1] compares different models with respect to their linear programming method they used to estimate the parameters of the grey Lotka-Volterra model under the criterion of the minimization of mean absolute percentage error (MAPE). The models they compare the empirical results are original GM (1,1) as a traditional core theory of grey prediction method (Liu et al. [18]), the grey Verhulst model (Wang et al., [19]), the Lotka-Volterra model (Morris et al. [20]) and the grey Lotka-Volterra model (Wu et al. [21]). Empirical results of Wu et al. [1] are based on two case studies each of which measures a pair of variables in comply with the assumptions of grey Lotka-Volterra model of prey-predator scheme.

The first case is the research and development (R&D) and the gross domestic product (GDP) forecasting experiment. The second case is the fixed assets investment (FAI) versus the consumer price index (CPI) forecasting experiment. We follow Wu et al. for our empirical investigation and use these two pairs of variables but adopting adaptive extended Kalman filter approach which yields better estimates for the parameters of Lotka-Volterra model under which mean absolute percentage error is more minimized than the other estimation methods mentioned above.

Case Study 1. The Research and Development (R&D) Investment and the Gross Domestic Product (GDP) Forecasting Experiment

In the first case study, we examine the relationship between the R&D investment and real GDP. Economic interpretations of prey-predator model can be driven from data indicating R&D investment 'breeds' real GDP growth (Mao et al. [5]). So the movements of R&D and GDP variables resemble those of $\{x\}$ and $\{y\}$ behavioral variables of the traditional Lotka-Volterra model. We cover the same period and the same sample data from Chang et al. [3] in the same manner of Wu et al. [1] in order to compare empirical results of different estimations of relevant model parameters.

The total accumulation of R&D investment is defined as $\{x\}$ variable and the GDP as $\{y\}$ variable of the Lotka-Volterra model. The time unit is taken as year and the period is covered in between 1990-2004.

Table 2 presents actual values and forecasting values for the total accumulation of R&D investment in the period 1990-2004 with respect to three compared models. The grey Lotka-Volterra model reduces MAPE of GM (1,1) model from 35.65% to 5.30% whilst Lotka-Volterra model extended Kalman filter (EKF) approach reduces MAPE of the grey Lotka-Volterra model from 5.30% to 0.1076 which means EKF approach reaches the objective of minimizing forecast error better than the other two models.

Table 3 indicates actual values and estimation results for the total real GDP within the period 1990-2004 with respect to three compared models. The grey Lotka-Volterra model reduces MAPE of GM(1,1) model from 11.7% to 9.1% whilst Lotka-Volterra model extended Kalman filter (EKF) approach reduces MAPE of the grey Lotka-Volterra model from 9.1% to 0.0036 which means EKF approach reaches with a great precision the objective of minimizing forecast error far better than the other two models.

Case Study 2. The Fixed Assets Investment (FAI) and the Consumer Price Index (CPI) Forecasting Experiment

In the second case study, we examine the relationship between the fixed assets investment (FAI) and the consumer price index (CPI). The movement of fixed asset investments along with consumer price index indicates the pattern of mutual interaction. This feature corresponds to the market structure of strategic complements according the modes of identification of Table 1. The sign of the interaction parameters $b < 0$ and $r < 0$ of the Lotka-Volterra model verifies the

Table 2. Estimation results and MAPE values for the variable $\{x\}$ R&D

Year	Actual x	Lotka-Volterra Model Extended Kalman Filter Estimation	Grey Lotka-Volterra Model Estimation	GM (1,1) Model Estimation
1990	2.04	2.016494696		
1991	4.31	4.303685258	3.25	14.0
1992	7.77	7.750894544	6.80	18.6
1993	12.2	12.1840674	12.12	24.8
1994	20.08	20.07292244	18.71	33.0
1995	29.22	29.217743	30.17	43.9
1996	39.72	39.7192671	42.68	58.5
1997	54.91	54.90968841	56.16	77.9
1998	74.61	74.60987525	75.32	103.8
1999	101.66	101.6599677	99.63	138.2
2000	138.26	138.2600055	133.07	184.1
2001	183	183.0000065	178.54	245.2
2002	240.65	240.6500018	234.003	326.6
2003	318.41	318.4100012	306.74	435.1
2004	413.41	413.410001	408.85	579.5
MAPE		**0.1076**	**5.30**	**35.65**

MAPE = $\frac{1}{n}\sum_{k=1}^{n} |(A_k - \hat{p}_k)/A_k| \times 100\%$ where A_k is the actual value and $\hat{p}_k$ is the fitted value.

Table 3. Estimation results and MAPE values for the variable $\{y\}$ GDP

Year	Actual y	Lotka-Volterra Model Extended Kalman Filter Estimation	Grey Lotka-Volterra Model Estimation	GM (1,1) Model Estimation
1990	898.0	897.4956226		
1991	1081.8	1081.800757	541.4	1777.2
1992	1365.1	1365.101613	1676.9	2047.6
1993	1909.5	1909.502441	2041.8	2359.2
1994	2666.9	2666.902817	2710.6	2718.2
1995	3524.8	3524.80334	3592.1	3131.9
1996	4146.1	4146.10367	4332.3	3608.4
1997	4638.2	4638.204252	4594.6	4157.5
1998	4987.5	4987.505052	4824.6	4790.2
1999	5364.9	5364.905563	4958.1	5519.1
2000	6036.3	6036.304449	5406.0	6359.0
2001	6748.2	6748.200979	6342.2	7326.6
2002	7796.0	7795.99999	7295.0	8441.5
2003	9395.0	9394.999994	9084.7	9726.0
2004	11243.0	11243.00001	12443.8	11206.1
MAPE		**0.0036**	**9.1**	**11.7**

MAPE = $\frac{1}{n}\sum_{k=1}^{n} |(A_k - \hat{p}_k)/A_k| \times 100\%$ where A_k is the actual value and $\hat{p}_k$ is the fitted value.

strategic complementarity assumption. In literature, different approaches adopted for forecasting.

Recently, Liang et al. [23] conducts an empirical investigation with autoregressive model using times series data in between 1990-2016. Empirical results prove that the variables are synergistic. The link between fixed asset investments and inflation as measured by CPI is close. For our research, again we examine the same period and the same

Table 4. Estimation results and MAPE values for the variable $\{x\}$ FAI

Year	Actual x	Lotka-Volterra Model Extended Kalman Filter Estimation	Grey Lotka-Volterra Model Estimation	Grey Verhulst Model Estimation
1995	17.46968	17.43714846		
1996	14.45705	14.43357054	19.2691	18.1227
1997	8.848932	8.819456968	11.6807	18.7676
1998	13.89313	13.77499851	16.5463	19.4022
1999	5.099239	5.061881298	5.4512	20.0246
2000	10.25969	10.16215603	12.0179	20.6330
2001	13.05012	12.93654928	15.0684	21.2258
2002	16.89279	16.74903557	17.9527	21.8016
2003	27.7396	27.49462089	29.6778	22.3590
2004	26.83411	26.64613643	29.8685	22.8972
2005	25.96038	25.81279504	26.3789	23.4153
2006	23.90868	23.76670356	23.2950	23.9127
2007	24.8	24.64575634	27.3290	24.3888
2008	25.9	25.77858351	29.6701	24.8435
2009	30.0	29.87123279	23.1443	25.2767
2010	23.8	23.67208934	23.2051	25.6885
MAPE		**0.6087**	**12.6714**	**24.7623**

MAPE = $\frac{1}{n}\sum_{k=1}^{n} |(A_k - \hat{p}_k)/A_k| \times 100\%$ where A_k is the actual value and $\hat{p}_k$ is the fitted value.

Table 5. Estimation results and MAPE values for the variable $\{y\}$ CPI

Year	Actual y	Lotka-Volterra Model Extended Kalman Filter Estimation	Grey Lotka-Volterra Model Estimationl	Grey Verhulst Model Estimation
1995	17.1	17.11225607		
1996	8.3	8.333054891	7.6703	8.7573
1997	2.8	2.816459052	2.5466	5.7488
1998	−0.8	−0.784010088	−0.4168	4.2008
1999	−1.4	−1.389987086	−1.1148	3.2594
2000	0.4	0.394413806	0.5389	2.6277
2001	0.7	0.694561708	0.8603	2.1755
2002	−0.8	−0.788591094	−0.3754	1.8364
2003	1.2	1.184104491	1.6300	1.5734
2004	3.9	3.876498437	4.0318	1.3639
2005	1.8	1.82351333	2.1991	1.1934
2006	1.5	1.504717602	1.9244	1.0523
2007	4.8	4.767969485	4.7607	0.9339
2008	5.9	5.889543539	5.5805	0.8333
2009	−0.7	−0.63784973	0.4870	0.7470
2010	3.3	3.262971858	3.4788	0.6723
MAPE		**1.3683**	**38.6608**	**168.2882**

MAPE = $\frac{1}{n}\sum_{k=1}^{n} |(A_k - \hat{p}_k)/A_k| \times 100\%$ where A_k is the actual value and $\hat{p}_k$ is the fitted value.

sample data from Chang et al. [3] following of Wu et al. [1] in order to compare empirical results of different estimations of model parameters.

The growth rate of FAI is defined as $\{x\}$ variable on percentage (%) basis and the growth rate of the CPI as $\{y\}$ variable on percentage basis (%) of the Lotka-Volterra model.

The time unit is taken as year and the period is covered in between 1995-2010.

Table 4 depicts actual values and forecasting values for the growth rate of FAI defined as {x} variable on the percentage basis in the period 1995-2010 with respect to three alternative models in comparison. The grey Lotka-Volterra model reduces the MAPE of the grey Verhulst model from 24.7623% to 12.6714% whilst Lotka-Volterra model extended Kalman filter (EKF) approach reduces MAPE of the grey Lotka-Volterra model from 12.6714% to 0.6087 which means EKF approach reaches the objective of minimizing forecast error with the prediction performance far better than the other two models.

Table 5 shows actual values versus forecasting values for the growth rate of CPI defined as {y} variable on the percentage basis in the period 1995-2010 with respect to three alternative models in comparison. The grey Lotka-Volterra model reduces the MAPE of the grey Verhulst model from 168.2882% to 38.6608% whilst Lotka-Volterra model extended Kalman filter (EKF) approach reduces MAPE of the grey Lotka-Volterra model from 38.6608% to 1.3683 which means EKF approach reaches the objective of minimizing forecast error with a prediction performance more powerful than the other two models.

CONCLUSION

The relationship between two variables in different models in some cases indicates wide range of empirical results. Lotka-Volterra model for forecasting two variables in the ecosystem can be a useful tool to analyze economic variables in a competitive market mechanism. However forecast techniques have important effects on the improvement of prediction performance. In this article we've examined the behavioral relationship of two pairs of crucial economic variables by employing the adaptive extended Kalman filter approach for parameter estimation in the traditional Lotka-Volterra model in comparison to three alternative models. These alternative models in our comparative empirical analysis are GM(1,1) model, grey Lotka-Volterra model and grey Verhulst model.

As a extention on Wu et al. [1], we've conducted an empirical analysis to indicate that the adaptive extended Kalman filter approach as a forecasting technique is more powerful as well as efficient due to the succinct formulation instead of elaborate algorithm to use and predicts parameters in the Lotka-Volterra model with a higher degree of precision. The first case study is conducted to examine the relationship between the research and development (R&D) investment and real gross domestic product (GDP).

In the empirical experiment, the estimation results and mean absolute percentage error (MAPE) values for the variables R&D and GDP of the adaptive extended Kalman filter (EKF) approach is far better than the other two alternative models. The prediction performance and forecast accuracy of EKF approach for the variable R&D is almost 50% higher than the grey Lotka-Volterra model, and almost 300% higher than GM(1,1) model. On the other hand, the prediction performance and forecast accuracy of EKF approach for the variable GDP is almost 2.5‰ higher than the grey Lotka-Volterra model, and almost 3.25‰ higher than GM(1,1) model.

The second case study is conducted to analyze the relationship between the growth rate of total accumulation of fixed asset investments (FAI) and the inflation rate measured as consumer price index (CPI). These variables which exhibit strategic complementarity in terms of market structure identification have important economic implications for development policies. According to the empirical experiment, the estimation results and MAPE values for the relevant variables of the EKF approach is far better than the other two alternative models. The prediction performance and forecast accuracy of EKF approach for the FAI variable is almost 20% higher than the grey Lotka-Volterra model, and almost 40% higher than grey Verhulst model. In the same manner, the prediction performance and forecast accuracy on the basis of error minimization of EKF approach for the variable CPI are almost 28% higher than the grey Lotka-Volterra model, and almost 122% higher than grey Verhulst model.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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Appendix: Adaptive Extended Kalman Filter as State Estimator

The discrete time state space model of the Lotka-Volterra model (2.2.1)-(2.2.2) has been obtained in the following way. Let $x_k = [x_{1k}\ x_{2k}]'$ be the state vector containing the states to be estimated at time k, where the states are defined as $x_{1k} = x_k$ and $x_{2k} = y_k$

$$x_{k+1} = \begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \end{bmatrix} = \begin{bmatrix} ax_{1,k} - bx_{1,k}x_{2,k} \\ -mx_{2,k} + rx_{2,k}x_{2,k} \end{bmatrix} + w_k \quad \text{(A1)}$$

$$z_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_k + v_k \quad \text{(A2)}$$

Here, the term noise has been added to state and observation vectors. As can be seen from equations (A1) - (A2), this system is not linear and in this case KF cannot be applied. Extended Kalman Filter (EKF) is used to estimate the system state vector in nonlinear state-space models. In order to use EKF, the parameters included in the model must be known. When the parameters included in the model given with (A1) - (A2) equations are not known, the parameter vector created by the parameters must be considered as the system state and added to the state vector in order to use EKF. In this case, both the system state vector and the parameter vector can be estimated at the same time. This is called system identification. This situation is explained as follows. While this constitutes a good model to estimate the states it does not account for the unknown parameters a, b, m and r. Therefore, this model has to be extended to include the parameters in the estimation process. For this purpose, let $\Phi_k(\theta)$ be a known vector that is a function of some unknown vector defined as $\theta = [a\ b\ m\ r]^T$. Here, θ is treated as a random variable and the objective is to identify θ. Also, assume that the random variable θ evolves according to,

$$\theta_{k+1} = \theta_k + \zeta_k \quad \text{(A3)}$$

where ζ_k is any zero-mean white noise sequence uncorrelated with v_k and with pre-assigned positive definite variances $Cov(\zeta_k) = S_k$. In applications, S_k may be chosen $S_k = S > 0$ for all k. The system given by equations (A1) and (A2) together with the assumption given by equation (A3) can be re-formulated as the nonlinear model:

$$\begin{bmatrix} x_{k+1} \\ \theta_{k+1} \end{bmatrix} = \begin{bmatrix} \Phi_k(\theta_k)x_k \\ \theta_k \end{bmatrix} + \begin{bmatrix} w_k \\ \zeta_k \end{bmatrix} \quad \text{(A4)}$$

$$z_k = \begin{bmatrix} H_k & 0 \end{bmatrix} \begin{bmatrix} x_k \\ \theta_k \end{bmatrix} + v_k \quad \text{(A5)}$$

and the AEKF procedure can be applied to estimate the state and parameter vector. This procedure is called adaptive system identification. The extended Kalman filtering process can then be applied to adaptively estimate the states and parameters. The initial state and the corresponding covariance is defined as

$$\begin{bmatrix} \hat{x}_0 \\ \hat{\theta}_0 \end{bmatrix} = \begin{bmatrix} E(x_0) \\ E(\theta_0) \end{bmatrix}$$
$$P_0 = \begin{bmatrix} Cov(x_0) & 0 \\ 0 & S_0 \end{bmatrix} \quad \text{(A6)}$$

and the state prediction and predicted covariance, in the adaptive form, are given as follows:
for $k = 1,2,\ldots.$

$$\begin{bmatrix} \hat{x}_{k|k-1} \\ \theta_{k|k-1} \end{bmatrix} = \begin{bmatrix} \Phi_{k-1}(\hat{\theta}_{k-1})\hat{x}_{k-1} \\ \hat{\theta}_{k-1} \end{bmatrix}$$

$$P_{k|k-1} = \alpha \begin{bmatrix} \Phi_{k-1}(\hat{\theta}_{k-1}) & \frac{d}{d\theta}\left(\Phi_{k-1}(\hat{\theta}_{k-1})\right)\hat{x}_{k-1} \\ 0 & I \end{bmatrix} \times P_{k-1} \begin{bmatrix} \Phi_{k-1}(\hat{\theta}_{k-1}) & \frac{d}{d\theta}\left(\Phi_{k-1}(\hat{\theta}_{k-1})\right)\hat{x}_{k-1} \\ 0 & I \end{bmatrix}^T + \alpha \begin{bmatrix} Q_{k-1} & 0 \\ 0 & S_{k-1} \end{bmatrix}$$

then the estimated state and the associated covariance are given by

$$\begin{bmatrix} \hat{x}_k \\ \hat{\theta}_k \end{bmatrix} = \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{\theta}_{k|k-1} \end{bmatrix} + K_k \left\{ z_k - \left[H_k \hat{x}_{k|k-1} \right] \right\} \quad \text{(A7)}$$

$$P_k = [I - K_k \begin{bmatrix} H_k & 0 \end{bmatrix}] P_{k|k-1}$$

Where K_k is the Kalman gain defined as

$$K_k = P_{k|k-1} \begin{bmatrix} H_k & 0 \end{bmatrix}' \left[\begin{bmatrix} H_k & 0 \end{bmatrix} P_{k|k-1} \begin{bmatrix} H_k & 0 \end{bmatrix}' + R_k \right]^{-1}.$$

This concludes the derivation of the adaptive extended Kalman filter with forgetting factor α that is used to discount old measurements Özbek [24].