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A Fuzzy Modelling Approach to Robust Design via Loss Functions

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A R T I C L E I N F O

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A B S T R A C T

Especially in a world where industrial development is reinforced by globalization tendencies, competitive companies know that satisfying customers' needs and running a successful operation requires a process that is reliable, predictable and robust. Therefore, many of quality improvement techniques focus on reducing process variation in line with the “loss to society” concept. The upside-down normal loss function is a weighted loss function that has the ability to evaluate losses with a more reasonable risk assessment. In this study, we introduce a fuzzy modelling approach based on expected upside-down normal loss function where the mean and standard deviation responses are fitted by response surface models. The proposed method aims to identify a set of operating conditions to maximize the degree of satisfaction with respect to the expected loss. Additionally, the proposed approach provides a more informative and realistic approach for comparing competing sets of conditions depending upon how much better or worse a process is. We demonstrate the proposed approach in a well-known design of experiment by comparing it with existing methods.

1. Introduction

In the early 1980s, Japanese quality engineer G. Taguchi coined the term robust parameter design (RPD), and his approach was popularized by many statisticians and quality engineers. The RPD can be defined as an experimental procedure based upon the factors that affect the quality performance of a system. Although the Taguchi’s RPD approach and his significant contributions have been adopted by most influential scientists, his experimental methodologies and analysis techniques have been criticized mostly by the statistical communities. Consequently, new methodologies based on Taguchi’s important engineering ideas have been proposed. The response surface methodology (RSM), first developed by [1] has revisited around the early of the 1990s and then got popularized. RSM can be defined as a collection of statistical and mathematical techniques for characterizing the relationship between a quality performance of a system, i.e. the response variable, and a set of independent factors, and uses stochastic models to explore this relationship. RSM enables engineers to gain more insights about the system of interest and provides a simplified relationship which can be used for practical engineering purposes especially when a complicated analysis and high-cost designs are not desirable.

The use of response surface optimization is a natural tool for the engineers since one of the main objectives of the RSM is the determination of the optimum settings of the control factors that optimize the response over a certain region of interest. [2] discussed the treatment procedure for the mean and variance responses via a constrained optimization technique called Dual Response Surface (DRS) which is based on fitting separate response surfaces for the system mean and the system variance. They optimize one fitted response subject to a constraint on the value of the other fitted response. [3] suggested the use of nonlinear programming to solve a similar optimization problem

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replacing the equality constraints by inequalities. Their proposed approach constrains the mean at the target and searches over the constrained control factor space to determine the optimal setting which minimizes the system variance. Criticizing the use of equality constraints, [4] proposed a procedure based on the mean squared error criterion (MSE). This approach takes into account the distance from the target and the variability of the response and involves finding the control factor setting which minimizes the estimated MSE. [5] proposed a novel mathematical programming formulation for the DRS problem based on fuzzy optimization methodology. They introduce a fuzzy modelling approach which considers both the deviation of the mean from the target and the magnitude of standard deviation. Following these articles, several procedures have been proposed for the DRS problems; see, for example [6], [7], [8], [9], and [10].

Traditionally, the role played by loss functions is fundamental in every quality engineering and management approach. In statistics, a loss function represents the monetary loss associated with variation about, and deviations from, either desired or target value concerning a quality characteristic of interest. Poorly operated manufacturing facilities and poorly designed products result in major incidents involving financial and social losses. A loss may occur even when the product is shipped within the specification limits, and this includes both company costs such as rework, repair, scrap and administrative costs, and any loss to the customer through unsatisfactory product performance and customer service. The whole concept of 'loss' usually refers to the 'loss to society'. A sound loss function is the upside-down normal loss function (UDNLF) of [11]. The UDNLF, which is essentially a normal density function flipped upside down, provides a more reasonable risk assessment to the losses of being off-target in a product engineering context. In fact, the UDNLF with a finite maximum loss is often effective in accurately modelling losses and leads to optimal decisions in real manufacturing environments. [12] used expected UDNLF as an objective function where the mean and standard deviation responses are fitted by quadratic response surface models and aim to find the best operating condition by minimizing the expected loss of UDNLF. [13] generalized their idea to more than one response under possible correlations and co-movement effects of responses. They adapted the RSM, and minimize the estimated the expected multivariate UDNLF to find the optimal control factor settings of a given problem.

In this article, we introduce a fuzzy modelling approach to optimize the expected loss function for a given system. The proposed approach aims to identify a set of process parameter conditions to maximize the degree of satisfaction with respect to the expected loss fitted by mean and standard deviation response surfaces. Additionally, the proposed approach provides a more informative and realistic approach for comparing competing sets of conditions depending upon how much better or worse a process is. The remainder of this manuscript is divided into four sections. Section 2 reviews UDNLF, while Section 3 provides the proposed optimization technique. All findings are illustrated in an example in Section 4 before the manuscript finally ends with a conclusion.

2. Review of the UDNLF

[11] proposed the UDNLF as a weighted loss function that has the ability to evaluate losses with a more reasonable risk:

$$L_{UDN}(y|\tau) = 1 - exp\left(-\frac{(y-\tau)^2}{2\lambda^2}\right) \quad (1)$$

where y indicates the process measurements, τ is the target, and λ is a scale parameter. The UDNLF is zero at the target and asymptotically approaches one. The scale parameter λ adjusts the penalty associated with any deviation from the target. Lacking better information, a choice is to set λ to 42.5% of the specification range. A larger value of λ signifies that relatively large deviations from the target can be tolerated. Figure 1 illustrates the UDNLF when $\tau = 0$, and the specifications are $(-3, 3)$, $(-2, 2)$ and $(-1, 1)$. As shown from Figure 1, as the specification range increases, the value of scale parameter increases. As a result, the loss function tends to its maximum with a slow rate.

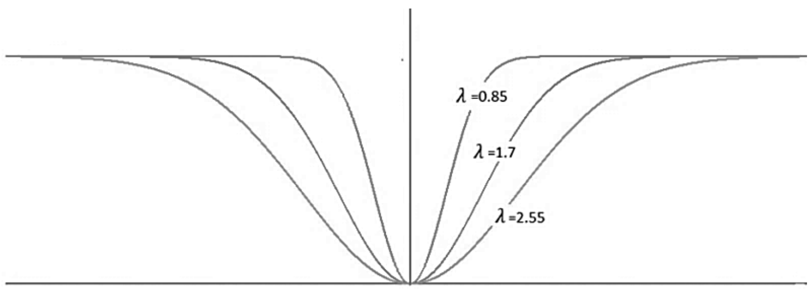

Figure 1. UDNL when $\lambda = 2.55, 1.7, 0.85$

Given the expectation of the UDNLF with respect to a normal density function with the mean μ and the variance σ^2, [11] present a simple analytical formula for the expected loss:

$$EL_{UDN} = 1 - \frac{\lambda}{\sqrt{\sigma^2+\lambda^2}} exp\left(\frac{-(\mu-\tau)^2}{2(\sigma^2+\lambda^2)}\right) \quad (2)$$

This formula quantifies the economy-wise impacts of process changes by combining the company view, i.e., the information about the systems, and the customer feedback, i.e., the unsatisfactory product performance caused by a deviation from the target.

3. Proposed Optimization Scheme

The response surface model can take many forms, but we are interested in the case that k control variables, x, influence the response. Following [2]'s notations, the process mean and standard deviation can be modelled by the second order response surfaces as follows,

$$\hat{\mu}(x) = \hat{\alpha}_0 + \sum_{j=1}^{k} \hat{\alpha}_j x_j + \sum_{j=1}^{k} \hat{\alpha}_{jj} x_j{}^2 + \sum_{j<t}^{k} \sum \hat{\alpha}_{jt} x_j x_t \quad (3)$$

$$\hat{\sigma}(x) = \hat{\beta}_0 + \sum_{j=1}^{k} \hat{\beta}_j x_j + \sum_{j=1}^{k} \hat{\beta}_{jj} x_j{}^2 + \sum_{j<t}^{k} \sum \hat{\beta}_{jt} x_j x_t \quad (4)$$

Using Equation (3) and (4), the estimator of Equation (2); see, [12], is as follows:

$$\hat{E}L_{UDN} = 1 - \frac{\lambda}{\sqrt{\hat{\sigma}^2(x)+\lambda^2}} exp\left(\frac{-(\hat{\mu}(x)-\tau)^2}{2(\hat{\sigma}^2(x)+\lambda^2)}\right) \quad (5)$$

It is assumed that the degree of satisfaction of the decision maker with respect to the Equation (5) is maximized when $\hat{\mu}(x) = \tau$ and decreases as $\hat{\mu}(x)$ moves away from the target. Thus the satisfaction level with respect to the expected loss can be modelled by a function which decreases monotonically from 1 to 0. As [5] noted, such a function can be referred as a membership function as in fuzzy set theory. The membership function reflects the decision maker's belief and has been viewed analogously to a utility function in decision analysis; see, [14]. [5] also refers that, a nonlinear membership function offers potential benefits in terms of realism and are chosen with varying perception of the decision maker. In view of this, following [5]'s notations, we suggest using of an exponential function and the degree of satisfaction with respect to the estimated expected loss given by Equation (5) can be modelled in the form,

$$m(\hat{E}L_{UDN}) = \begin{cases} \frac{e^d - e^{d|z|}}{e^d - 1} & , \quad if\ d \neq 0 \\ 1 - |z| & , \quad if\ d = 0 \end{cases} \quad (6)$$

where d is constant ($-\infty < d < \infty$), called the exponential constant. z is a standardized parameter and can be defined as,

$$z = \frac{\hat{E}L_{UDN} - EL_{UDN_{min}}}{EL_{UDN_{max}} - EL_{UDN_{min}}} \quad (7)$$

where $EL_{UDN_{min}}$ and $EL_{UDN_{max}}$ represent the acceptable minimum and maximum loss. In nature, in DRS problems, the desired value of the expected loss is zero; however, the capability of technology, economic factors, customer satisfactions etc., can affect the tolerable quality loss. This indicates that the results of process optimization vary depending on the practitioner's choice.

The membership function $m(\hat{E}L_{UDN})$ can represent many different shapes depending upon d. It can be convex, linear and concave when $d < 0, d = 0, and\ d > 0$, respectively. As a result, as [15] and [16] noted, the function $m(\hat{E}L_{UDN})$ given in Equation (6) has been proven to provide a reasonable and flexible representation of human perception.

Finally, the optimization problem can be stated as,

$$\text{Maximize} \quad \delta$$
$$\text{Subject to} \quad m(\hat{E}L_{UDN}) \geq \delta$$
$$x \in \Omega$$

where Ω defines the feasible region of x. This optimization problem aims to identify x^* which would maximize the minimum degree of satisfaction, δ, with respect to the expected loss within feasible region.

4. Example: Printing Process Study

This example borrows a case study from [17], which is revisited by [2], [4], and [5]. This experiment was conducted to find the optimum combination of the effects of speed (x_1), pressure (x_2) and distance (x_3) factors on the quality of a printing process (y). Three design factors thought to be potentially important are listed in Table 1, along with the levels of each factor, denoted by -1 and +1. A 3^3 factorial design with three replicates was used to fit the responses.

Table 1. The printing process study data

u	x_1	x_2	x_3	y_1	y_2	y_3	$\bar{y}$	s
1	-1	-1	-1	34	10	28	24	12.5
2	0	-1	-1	115	116	130	120.3	8.4
3	1	-1	-1	192	186	263	213.7	42.8
4	-1	0	-1	82	88	88	86	3.5
5	0	0	-1	44	178	188	136.7	80.4
6	1	0	-1	322	350	350	340.7	16.2
7	-1	1	-1	141	110	86	112.3	27.6
8	0	1	-1	259	251	259	256.3	4.6
9	1	1	-1	290	280	245	271.7	23.6
10	-1	-1	0	81	81	81	81	0.0
11	0	-1	0	90	122	93	101.7	17.7
12	1	-1	0	319	376	376	357	32.9
13	-1	0	0	180	180	154	171.3	15
14	0	0	0	372	372	372	372	0.0
15	1	0	0	541	568	396	501.7	92.5
16	-1	1	0	288	192	312	264	63.5
17	0	1	0	432	336	513	427	88.6
18	1	1	0	713	725	754	730.7	21.1
19	-1	-1	1	364	99	199	220.7	133.8
20	0	-1	1	232	221	266	239.7	23.5
21	1	-1	1	408	415	443	422	18.5
22	-1	0	1	182	233	182	199	29.4
23	0	0	1	507	515	434	485.3	44.6
24	1	0	1	846	535	640	673.7	158.2
25	-1	1	1	236	126	168	176.7	55.5
26	0	1	1	660	440	403	501	138.9
27	1	1	1	878	991	1161	1010	142.5

For illustrated purposes, the estimator of expected loss function and the exponential membership function is obtained with specification bound is $(490,510)$, the target is 500, and also, the acceptable minimum and maximum losses are 0 and 1, $EL_{UDN_{min}} = 0$ and $EL_{UDN_{max}} = 1$.

The fitted response surfaces from [2] are as follows,

$$\hat{\mu}(x) = 327.6 + 177.0x_1 + 109.4x_2 + 131.5x_3 + 32.0{x_1}^2 - 22.4{x_2}^2 - 29.1{x_3}^2 + 66.0x_1x_2 + 75.5x_1x_3 + 43.6x_2x_3 \quad (8)$$

$$\hat{\sigma}(x) = 34.9 + 11.5x_1 + 15.3x_2 + 29.2x_3 + 4.2{x_1}^2 - 1.3{x_2}^2 + 16.8{x_3}^2 + 7.7x_1x_2 + 5.1x_1x_3 + 14.1x_2x_3 \quad (9)$$

and the estimator of expected loss is,

$$\hat{E}L_{UDN} = 1 - \frac{8.5}{\sqrt{\hat{\sigma}^2(x) + 8.5^2}} exp\left(\frac{-(\hat{\mu}(x) - 500)^2}{2(\hat{\sigma}^2(x) + 8.5^2)}\right) \quad (10)$$

where $\hat{\mu}(x)$ and $\hat{\sigma}(x)$ are defined by Equations (8) and (9), respectively. The scale parameter of loss function is obtained as 8.5 by setting 0.425 times of the defined specification range.

A convex-shaped exponential membership function with $d = -4.39$ (corresponding the study of [5]) is chosen with respect to the Equation (10), where $z = \hat{E}L_{UDN}$ from Equation (7), that is,

$$m(\hat{E}L_{UDN}) = \frac{e^{-4.39} - e^{-4.39\hat{E}L_{UDN}}}{e^{-4.39} - 1} \quad (11)$$

The complete formulation for the printing process study problem is as follows,

$$\text{Maximize} \quad \delta$$

$$\text{Subject to} \quad \frac{e^{-4.39} - e^{-4.39\hat{E}L_{UDN}}}{e^{-4.39} - 1} \geq \delta$$

$$-1 \leq x \leq 1$$

The optimal design point for the proposed approach turns out to be $x^* = (1.00, 0.076, -0.252)$ where $\hat{\mu}(x) = 494.84$ and $\hat{\sigma}(x) = 44.48$ and the resulting $\delta = 0.015$. When $d = -4.39$, the membership function is highly convex, which indicates that the high stringency. As a result, the proposed approach results are better both estimated mean and standard deviation. Figure 2 shows an illustration of the relationship between membership function given by Equation (11) and estimated expected process loss.

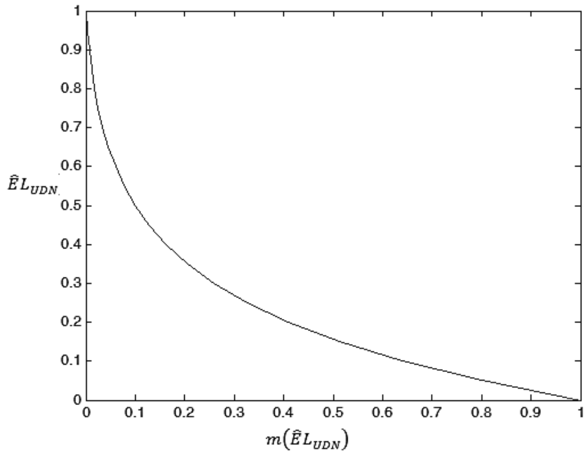

Figure 2. The relationship between $m(\hat{E}L_{UDN})$ and $\hat{E}L_{UDN}$

The results of the proposed approach are summarized in Table 2 and compared with those of [2], [4] and [5].

Table 2. Comparison of optimal settings for the printing process study.

Method	Optimal setting	$\hat{\mu}(x)$	$\hat{\sigma}(x)$	MSE	$\hat{E}L_{UDN}$
Vining and Myers (1990)	(0.62,0.23,0.10)	500.00	51.77	2679.70	Unknown
Lin and Tu (1995)	(1.00,0.07,-0.25)	494.44	44.43	2005.14	Unknown
Kim and Lin (1998)	(1.00,0.086,-0.254)	496.08	44.63	2007.07	Unknown
Proposed approach	(1.00,0.076,-0.252)	494.84	44.48	2005.10	0.81

The solution of the proposed approach is much closer to [4] and [5] ($\delta = 0.17$) in terms of the estimated mean and standard deviation. On the other hand, [2]'s optimal setting allows for the target to be hit with a larger standard deviation. However, when the obtained values of MSE's are compared, it is obvious that the proposed approach has the smallest MSE. Furthermore, the proposed method provides an additional information compared the existing methods as $\hat{E}L_{UDN} = 0.81$. However, it should be pointed out that, [12] noted as the results from different approaches cannot be compared in a straightforward manner since the methods differ in terms of their optimization criteria.

5. Conclusion

A fuzzy modelling approach is presented to optimize the expected loss function for a given system. The proposed approach aims to identify a set of operating conditions to maximize the degree of satisfaction with respect to the expected loss fitted by mean and standard deviation response surfaces. In using a well-known design of the experiment, printing data, it was shown that the proposed method can model the decision maker's preference on the estimated expected loss very flexible and achieves a better balance between bias and variability. Additionally, the proposed approach provides a more informative and realistic approach for comparing competing sets of conditions depending upon how much better or worse a process is.

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References

[1] G.E.P. Box and K.B. Wilson, On the Experimental Attainment of Optimum Conditions, J. Roy. Statist. Soc. Ser. B Metho. 13 (1951) 1-45.

[2] G.G. Vining and R.H. Myers, Combining Taguchi and Response Surface Philosophies: A Dual Response Approach, J. Qual. Technol. 22 (1990) 38-45.

[3] E. Del Castillo and D.C. Montgomery, A Nonlinear Programming Solution to the Dual Response Problem, J. Qual. Technol. 25 (1993) 199-204.

[4] D.K.J. Lin and W. Tu, Dual Response Surface Optimization, J. Qual. Technol. 27 (1995) 34-39.

[5] K. Kim and D.K.J. Lin, Dual Response Surface Optimization: A Fuzzy Modeling Approach, J. Qual. Technol. 30 (1998) 1-10.

[6] A.C. Shoemaker, K.L, Tsui, and C.F.J. Wu, Economical Experimentation Methods for Robust Parameter Design, Technometrics. 33 (1991) 415-427.

[7] K.A. Copeland and P.R. Nelson, Dual Response Optimization via Direct Function Minimization, J. Qual. Technol. 28 (1996) 331-336.

[8] J.M. Lucas, How to Achieve a Robust Process Using Response Surface Methodology, J. Qual. Technol. 26 (1994). 248-260.

[9] O. Köksoy and N. Doganaksoy, Joint Optimization of Mean and Standard Deviation in Response Surface Experimentation, J. Qual. Technol. 35 (2003) 239-252.

[10] O. Köksoy, Multiresponse Robust Design: Mean Square Error (MSE criterion), Appl. Math. Comput. 175 (2006) 1716-1729.

[11] D.C. Drain and A.M. Gough, Applications of the Upside-down Normal Loss Function, IEEE T. Semıconduct M. 9 (1996) 143–145.

[12] O. Köksoy and S.S. Fan, An Upside-down Normal Loss Function-based Method for Quality Improvement, Eng. Optimiz. 44 (2012) 935-945.

[13] M. Zeybek and O. Köksoy, Optimization of Correlated Multi-response Quality Engineering by the Upside-down Normal Loss Function, Eng. Optimiz. 48 (2016) 1419-1431.

[14] H.J. Zimmermann, Fuzzy Sets, Decision Making, and Expert Systems, Kluwer Academic Publishers, Boston, M.A. (1987).

[15] C.W. Kirkwood, Strategic Decision Making, Duxbury Press, Belmont, C.A. (1996).

[16] H. Moskowitz and K. Kim, On Assessing the H Value in Fuzzy Linear Regression, Fuzzy Sets Syst. 58 (1993) 303-327.

[17] G.E.P. Box and N.R. Draper, Empirical Model Building and Response Surfaces, John&Sons, New York, N.Y. (1987).