Turkish Journal of Engineering

Turkish Journal of Engineering (TUJE)
Vol. 2, Issue 2, pp. 60-72, May 2018
ISSN 2587-1366, Turkey
DOI: 10.31127/tuje.345153
Research Article

MODULAR APPROACH TO THE DESIGN OF PATH GENERATING PLANAR MECHANISMS

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Received: 19/10/2017 Accepted: 14/12/2017

ABSTRACT
A novel approach, called modular approach, is presented in this paper making possible the closed-form solutions of the planar path-generating multi-link mechanisms with lower pairs. In this approach, the mechanism is viewed as a suitable combination of some simpler components called "modules". The design of the modules is realized by applying the so-called Precision-Point, Subdomain and Galerkin methods. The approach is illustrated on 4-bar, slider-crank, double-slider, 5-bar and 6-link mechanisms. Numerical results prove the effectiveness of the approach.

Keywords: *Mechanism, Path Generating, Multi-link*

1. INTRODUCTION

Analytical methods for synthesizing mechanisms can be classified mainly into two categories. One of them is the numerical iterative approach and the other is the closed form solution approach. Out of a desire to use high speed computing abilities of the computers, the tendency towards utilizing numerical iterative approach has usually been strong. For instance, Roth and Freudenstein (Roth *et al.* 1963) have applied the so called Newton Raphson method to solve the synthesis of geared five bar mechanism to pass through nine path points. Kramer and Sandor (Kramer *et al.* 1975) have referred to direct search techniques for minimizing a penalty function against violation of what they call "accuracy neighborhoods" around selected path points. A similar approach has been shown by Bakthavachalam and Kimbrell (Bakthavachalam *et al.* 1975) in path generation involving clearances and manufacturing tolerances. There are well-known limitations of the numerical iterative approaches. First of all, these methods are crippled by serious convergence difficulties. No assurance regarding the convergence of an arbitrary starting design to a final one exists. Thus, they highly depend upon the suitable selection of an initial solution. Although they require a large computation time, they finally provide one single solution. On the other hand, the closed form solution does not possess the aforementioned undesirable features associated with the numerical iterative techniques. However, one important drawback of the closed form solution is the fact that the number of parameters, which can be taken into account in the synthesis procedure, is generally very limited. This indicates that most mechanisms, especially those which are of multi-link structure, display a situation where closed-form approach is not directly applicable due to the abundant number of parameters. Nevertheless, if multi-link mechanisms can be decomposed into simpler components whereby closed form solution is possible, then the whole mechanism can be synthesized by bringing together in a suitable manner the design of simpler structural units called modules. This is the underlying idea behind this work. From this angle, the mechanisms are viewed as an appropriate combination of "modules". Thus, this approach is conveniently termed as a "modular approach".

This paper presents the application of this approach on the design of path generating planar mechanisms with lower pairs. Illustrations of the approach have been shown on four bars, slider crank, double slider, 5-bar and six link mechanisms. The applications have been put in the form of computer programs. The numerical results indicate that the approach is an effective and efficient one.

2. MODULES

Planar mechanisms with lower pairs can be thought of as being made up of simpler components, referred to as "modules" hereafter in this paper, whose motion relationships exhibit compact forms for handling the design equations within the framework of the closed form solution. Modules which compose most planar constrained mechanisms can be considered as three basic types; namely, a dyad, a crank rocker, and a slider. Dyad and crank rocker have the similar property involving two links connected by only revolute pairs in the plane. They basically differ from one another in that dyad can perform only partial rotation whereas a crank rocker module involves a full revolution. Thus, where a crank drive is needed, this module is supplemented to another assemblage of links supposed to generate a given path. Slider module consists of a prismatic pair connected to a link through a revolute pair. Now formulation and solution of the design equations governing each module have been developed as follows:

2.1. Dyad Module

Dyad, as shown in Fig.1, is a two-member assembly with two degrees of freedom. Its end point C is supposed to move on the given path y=f(x), x0≤x≤xn.. From the geometry of Fig.1, The following is written for the coordinate of point C within the reference frame xoy:

$$x = x_7 + x_1 \operatorname{Cos}\psi + d_{45} \operatorname{Cos}\delta \tag{1}$$

$$y = x_8 + x_1 \operatorname{Sin}\psi + d_{45} \operatorname{Sin}\delta \tag{2}$$

By eliminating angle δ from equations (1) and (2), the displacement function (G) governing the motion in the dyad is obtained as follows:

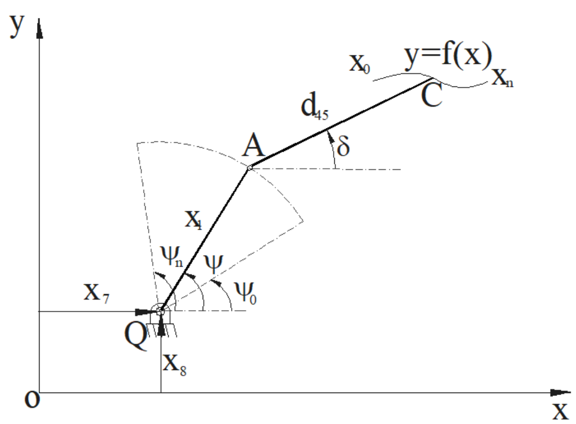

Fig. 1. Dyad Module

$$G(x_1, d_{45}, x_7, x_8, \psi_0, x, y) = x^2 + y^2 - 2xx_7 - 2yx_8 - x_1(x\operatorname{Cos}\psi + y\operatorname{Sin}\psi - x_7\operatorname{Cos}\psi - x_8 Sin\psi) + t_1 = 0 \quad (3)$$

$$t_1 = x_1^{\ 2} + x_7^{\ 2} + x_8^{\ 2} - d_{45}^{\ 2} \quad (4)$$

where $\psi = \psi_0 + h(x)$ and ψ'=h(x) represents the well-defined the motion of the arm QA against the independent variable x. As can be seen from equation (3), the number of available system parameters is five. Therefore, the number of design equations that can be written is limited by five.

The problem of formulating the design can be approached from three different points of view. In one of them, the displacement function is set equal to zero at a number of collocation points called precision or accuracy points. This approach referred to in the literature as Collocation, Precision or Accuracy point approach (F 1955; SH 1956; Kao *et al.* 2006), yields the following:

$$G(x_1, d_{45}, x_7, x_8, \psi_0, x_i, y_i) = 0; \quad i = 1,2,3,4,5 \quad (5)$$

where (xi,yi) defines the coordinates of the precision points.
Another point of view in formulating the design is to make the average of the displacement equation (G) zero over selected subintervals of the function interval (x_0,xn). The number of subintervals will be equal to the number of available system parameters. This constitutes the so called Subdomain method (F 1955; SH 1956; Hartenberg *et al.* 1964; Akcali *et al.* 1979; Akcali 1983; Akcali 1987; Akcali *et al.* 1989), which, in the case under consideration, leads to the following equation set:

$$\int_{x_{i-1}}^{x_i} G(x_1, d_{45}, x_7, x_8, \psi_0, x_i, y_i)dx = 0 \quad i = 1,2,3,4,5 \quad (6)$$

where (x_{i-1},x_i) are subintervals belonging to the interval (x0,xn).
The design of the dyad to generate the given path can also be formulated by making the displacement equation orthogonal to a set of weighting functions W_i(x) defined on the same interval (x_0,x_n) as the given path y(x). The number of weighting functions will be equal to that of the parameters, thus leading to the following design equations; in accordance with the so-called Galerkin method (SH 1956; Akcali 1987):

$$\int_{x_0}^{x_n} G(x_1, d_{45}, x_7, x_8, \psi_0, x, y)W_i dx = 0; \quad i = 1,2,3,4,5 \quad (7)$$

Now all the design equations resulting from Precision point, Subdomain and Galerkin methods can be represented by the following set:

$$A_i - B_i x_7 - C_i x_8 - D_i x_1 + F_i t_1 = 0; \ i = 1,2,3,4,5 \quad (8)$$

where:

$$D_i = V_{ci} Cos\,\psi_0 + V_{si} Sin\,\psi_0 - D_{ci} x_7 Cos\,\psi_0 + D_{si} x_7 Sin\,\psi_0 - D_{ci} x_8 Sin\,\psi_0 - D_{si} x_8 Cos\,\psi_0; \ i = 1,2,3,4,5 \quad (9)$$

The coefficients A_i,B_i,C_i,F_i,V_{ci},V_{si},D_{ci},D_{si} are calculated according to each method as follows:
In Precision-Point approach:

$$\begin{matrix} A_i = x_i^2 + y_i^2; B_i = 2x_i; C_i = 2y_i; F_i = 1; V_{ci} = 2(x_i Cos\psi' + y_i Sin\psi'); \\ V_{si} = 2(-x_i Sin\psi' + y_i Cos\psi'); D_{ci} = 2Cos\psi'; D_{si} = 2Sin\psi'; \end{matrix} \ i = 1,2,3,4,5 \quad (10)$$

In Subdomain method:

$$\begin{matrix} A_i = \int_{x_{i-1}}^{x_i} (x^2 + y^2)dx; \ B_i = 2\int_{x_{i-1}}^{x_i} xdx; \ C_i = 2\int_{x_{i-1}}^{x_i} ydx; \ F_i = \int_{x_{i-1}}^{x_i} dx; \ V_{ci} = 2\int_{x_{i-1}}^{x_i} (xCos\psi' + y\operatorname{Sin}\psi')dx; \\ V_{si} = 2\int_{x_{i-1}}^{x_i} (-x\operatorname{Sin}\psi' + y\operatorname{Cos}\psi')dx; \ D_{ci} = 2\int_{x_{i-1}}^{x_i} \operatorname{Cos}\psi' dx; \ D_{si} = 2\int_{x_{i-1}}^{x_i} \operatorname{Sin}\psi' dx; \ i = 1,2,3,4,5 \end{matrix} \quad (11)$$

In Galerkin methods:

$$\begin{matrix} A_i = \int_{x_0}^{x_n} (x^2 + y^2)W_i dx; B_i = 2\int_{x_0}^{x_n} xW_i dx; C_i = 2\int_{x_0}^{x_n} yW_i dx; F_i = \int_{x_0}^{x_n} W_i dx; V_{ci} = 2\int_{x_0}^{x_n} (x\operatorname{Cos}\psi' + y\operatorname{Sin}\psi')W_i dx; \\ V_{si} = 2\int_{x_0}^{x_n} (-x\operatorname{Sin}\psi' + y\operatorname{Cos}\psi')W_i dx; D_{ci} = 2\int_{x_0}^{x_n} \operatorname{Cos}\psi' W_i dx; D_{si} = 2\int_{x_0}^{x_n} \operatorname{Sin}\psi' W_i dx; i = 1,2,3,4,5 \end{matrix} \quad (12)$$

In order to solve equation set (8), first t1 and x1 and then ψ_0 are eliminated, thus reducing the set to the following form:

$$E_{1k}x_7^4+(E_{2k}x_8+E_{3k})x_7^3+(E_{4k}x_8^2+E_{5k}x_8+E_{6k})x_7^2+(E_{7k}x_8^3+E_{8k}x_8^2+E_{9k}x_8+E_{10k})x_7 \\ +(E_{11k}x_8^4+E_{12k}x_8^3+E_{13k}x_8^2+E_{14k}x_8+E_{15k})=0 \quad ;k=1,2 \tag{13}$$

where (E_{jk} ,j=1,..,15,k=1,2) are constants. The solution of (13) is realized for all possible sets of (x_7,x_8) by first eliminating the term x_7^4 from the set and drawing x_7 from the resultant equation to be substituted back into one of the equations. Based on the solutions of (13) for x_7,x_8 , it is now a simple matter to solve for the rest of the unknowns, namely, ψ_0 , x_1 and d_{45}.

The resulting dyad designs can be analyzed first by solving δ from eq'n (1) for a given value of x_{th} , then substituting it in eq'n (2) to determine y_{ac} with the purpose of computing structural error $e=y(x_{th})-y_{ac}$.

2.2. Slider Module

One of the lower pairs to be found in planar mechanisms is the sliding or prismatic pair. Thus, in order to accomplish the designs of planar mechanisms involving prismatic pairs, development of the design scheme of a new type of module, referred to as the slider module here, is needed.

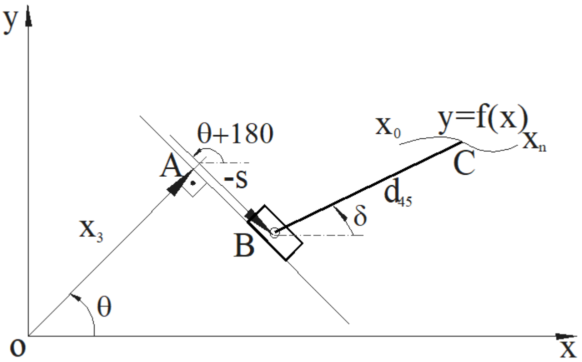

Fig. 2. Slider Module

The problem to be formulated here is to find the suitable dimensions of the slider module shown in Fig.2 such that a point (C) on the floating link BC trace the given curve y=f(x) $x_0 \le x \le x_n$ as close as possible, while the sliding link moves along a line between points characterized by s_0 and s_n
Writing out the x and y co-ordinate of the floating-point C of the module in terms of the variables and parameters shown in Fig.2 and then eliminating the angle δ from the two equations will yield the displacement function (H) of the slider module:

$$H(\theta,s_0,d_{45},x_3,x,y)=x^2+y^2+s'^2-x_3(2xC\text{os}\,\theta+2yS\text{in}\,\theta)+(s'+s_0)(2xS\text{in}\,\theta-2yC\text{os}\,\theta)+2s_0s'+P=0 \tag{14}$$

where:

$$P=x_3^2+s_0^2-d_{45}^2 \tag{15}$$

Here, it is assumed the motion ($s'=s-s_0$) of the slider is defined in the form of specified g(x) functional relationship against x. An examination of Fig.2 as well as of equation (14) will explain that there are four unknown parameters, namely, (θ,s_0,d_{45},x_3) and thus treatment of the displacement function according to the requirements of the Precision-Point, Subdomain and Galerkin methods will yield four design equations in the following form:

$$k_{ai}+2k_{ei}s_0+k_{di}P-x_3(k_{bi}\text{Cos}\,\theta+k_{ci}\text{Sin}\,\theta)+(s_0+k_{ei})(k_{bi}\text{Sin}\,\theta-k_{ci}\text{Cos}\,\theta)=0;\ i=1,2,3,4 \tag{16}$$

where coefficients in equations (16) are defined according to each method as follows:
In Precision-Point method:

$$k_{ai}=x_i^2+y_i^2+s_i'^2;\ k_{bi}=2x_i;\ k_{ci}=2y_i;\ k_{di}=1;\ k_{ei}=s_i';\ i=1,2,3,4 \tag{17}$$

In Subdomain method:

$$k_{ai}=\int_{x_{i-1}}^{x_i}(x^2+y^2+s'^2)dx;\ k_{bi}=2\int_{x_{i-1}}^{x_i}xdx;\ k_{ci}=2\int_{x_{i-1}}^{x_i}ydx;\ k_{di}=\int_{x_{i-1}}^{x_i}dx;\ k_{ei}=\int_{x_{i-1}}^{x_i}s'dx;\ i=1,2,3,4 \tag{18}$$

In Galerkin method:

$$k_{ai} = \int_{x_0}^{x_n} (x^2 + y^2 + s'^2) W_i \, dx; \; k_{bi} = 2\int_{x_0}^{x_n} x W_i \, dx; \; k_{ci} = 2\int_{x_0}^{x_n} y W_i \, dx; \; k_{di} = \int_{x_0}^{x_n} W_i dx; \; k_{ei} = \int_{x_0}^{x_n} s' W_i \, dx; i = 1,2,3,4 \tag{19}$$

For the solution of the set (16), first P then x_3 and s_0 are eliminated leaving the final equation in one single unknown (θ):

$$l_1 \mathrm{Cos}^3\theta + l_2 \mathrm{Sin}^3\theta + l_3 \mathrm{Cos}^2\theta + l_4 \mathrm{Sin}^2\theta + l_5 \mathrm{Cos}\theta + l_6 \mathrm{Sin}\,\theta + l_7 \mathrm{Cos}^2\theta \mathrm{Sin}\,\theta + l_8 \mathrm{Sin}^2\theta \mathrm{Cos}\theta + l_9 \mathrm{Cos}\theta \mathrm{Sin}\,\theta + l_{10} = 0 \tag{20}$$

After solving the equation (20) for θ between 0 and 2π, it is not difficult to determine the other unknowns (s_0,d_{45},x_3) from equation set (16), thus completing the design of the slider module.
The motion analysis of the resulting module design can be performed first by computing the floating link angle (δ) from the abscissa ($x=x_{th}$) expression to be inserted into the ordinate (y_{ac}) expression for the evaluation of structural error, $e=y(x_{th})-y_{ac}$.

2.3. Crank-rocker Module

In the resulting path-generating mechanisms, usually the coupler point is required to trace only a portion of a closed curve very nearly and there is no guarantee that a member of the resulting mechanism will make a full revolution, a necessary condition for bringing the whole assembly in motion by means of a rotary actuator. In that case, there arises the need to add the crank-rocker module to the path-generating assembly in order to operate the system. Frictionless engine, grass-cutter and oil pump drivers, (Dittrich *et al.* 1978) can be cited as well-known examples in this regard.

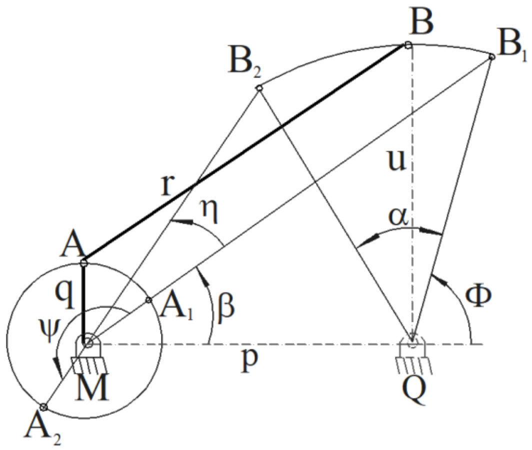

Fig. 3. Crank-rocker module

Crank-rocker module is, in fact, a dyad, Fig.3, with the difference that the crank MA rotates 360° degrees while the endpoint (B) of the floating link BA is constrained to move on a circular path with center at Q and radius BQ. In designing the crank-rocker module, it is a fundamental condition that the assembly assumes the limiting configurations shown in Fig.3. Sine Law is written for triangles ΔMB_1Q and ΔMB_2Q in Fig.3. Then supposing that rocker swing angle (α), time ratio (t_R) or angle between dead-centers (η) and initial crank angle (β) are specified, the following relations are deduced to determine the relative dimensions (p,q,r,u) of the crank-rocker module:

$$\frac{u}{p} = \frac{Sin\beta}{Sin(\Phi-\beta)}; \frac{r}{p} = \frac{1}{2}\left[\frac{\mathrm{Sin}\,\Phi}{\mathrm{Sin}(\Phi - \beta)} + \frac{\mathrm{Sin}\,\Phi + \alpha}{\mathrm{Sin}(\Phi + \alpha - \beta - \eta)}\right]$$
$$\frac{q}{p} = \frac{1}{2}\left[\frac{\mathrm{Sin}\,\Phi}{\mathrm{Sin}(\Phi - \beta)} - \frac{\mathrm{Sin}\,\Phi + \alpha}{\mathrm{Sin}(\Phi + \alpha - \beta - \eta)}\right]$$
$$with\ \Phi = \tan^{-1}\left[\tan\beta \frac{\mathrm{Sin}(\beta+\eta) + \mathrm{Sin}(\alpha - \beta - \eta)}{\mathrm{Sin}(\beta+\eta) - \tan\beta \mathrm{Cos}(\alpha - \beta - \eta)}\right] \tag{21}$$

Depending on the design situation, several specifications from among the parameters defined on Fig.3, namely, (p,q,r,u,α,β,η,Φ), equation set (21) may be rearranged with respect to the given situation. Thus, in this way module design can be adapted to the designer's needs and conditions.

3. CIRCLE AND LINE POINTS OF A GENERAL COUPLER PLANE

In order to transform modules into mechanisms those points of the coupler plane which lie on a circle and those on a line should be searched, consistent with the modules developed previously. Firstly, the case of circle-points will be handled. Referring to Fig.4, point D of the plane is required to follow a circular path as point C of the same plane traces the given path y=f(x) $x_0 \le x \le x_n$. The co-ordinates (x,y) of the point is written parametrically relative to A in terms of in the same variable (t) xoy-system of Fig.4:

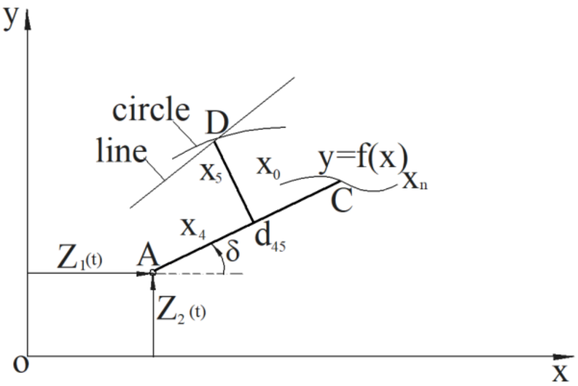

Fig. 4. General coupler motion

$$x = Z_1(t) + k_1 x_4 - k_2 x_5 \quad (22)$$
$$y = Z_2(t) + k_2 x_4 + k_1 x_5 \quad (23)$$
$$k_1 = Cos\delta(t) \quad ; \quad k_2 = Sin\delta(t) \quad (24)$$

For D to be on the circle, the following should be satisfied:

$$(x - x_m)^2 + (y - y_m)^2 = R^2 \quad (25)$$

Substituting (22), (23) in (25) and arranging yields the displacement function F as follows:

$$F(x_4, x_5, x_m, y_m, R, t) = P_1 - 2k_1 x_m x_4 + 2k_2 x_m x_5 - 2k_2 y_m x_4 - 2k_1 y_m x_5 + 2(Z_1 k_1 + Z_2 k_2) x_5 - 2(Z_1 k_2 - Z_2 k_1) x_5 - 2Z_1 x_m - 2Z_2 y_m + Z_1^2 + Z_2^2 = 0; \quad (26)$$

where $P_1 = x_4^2 + x_5^2 + x_m^2 + y_m^2 - R^2$

Applying Precision-Point, Subdomain and Galerkin methods on function F, the following equation set will result:

$$U_i P_1 - Z_{ai} x_m x_4 + Z_{bi} x_m x_5 - Z_{bi} y_m x_4 - Z_{ai} y_m x_5 + Z_{ci} x_4 - Z_{di} x_5 - Z_{ei} x_m + Z_{fi} y_m + Z_{gi} = 0; \; i = 1,2,3,4,5 \quad (27)$$

The coefficients in the set (27) are defined according to each method as follows:

In Precision-Point method:

$$Z_{ai} = 2k_{1i}; \; Z_{bi} = 2k_{2i}; \; Z_{ci} = 2(Z_{1i} + Z_{2i} k_{2i}); \; Z_{di} = 2(Z_{1i} k_{2i} - Z_{2i} k_{1i}); \; Z_{ei} = 2Z_{1i}; \; Z_{fi} = 2Z_{2i}; \; Z_{gi} = Z_{1i}^2 + Z_{2i}^2; \; U_i = 1; \; i = 1,2,3,4,5 \quad (28)$$

In Subdomain method:

$$Z_{ai} = 2\int_{t_{i-1}}^{t_i} k_1 dt; \; Z_{bi} = 2\int_{t_{i-1}}^{t_i} k_2 dt; \; Z_{ci} = 2\int_{t_{i-1}}^{t_i} (Z_1 k_1 + Z_2 k_2) dt; \; Z_{di} = 2\int_{t_{i-1}}^{t_i} (Z_1 k_2 - Z_2 k_1) dt; \; Z_{ei} = 2\int_{t_{i-1}}^{t_i} Z_1 dt; \; Z_{fi} = 2\int_{t_{i-1}}^{t_i} Z_2 dt; \; Z_{gi} = \int_{t_{i-1}}^{t_i} (Z_1^2 + Z_2^2) dt; \; U_i = \int_{t_{i-1}}^{t_i} dt; \; i = 1,2,3,4,5 \quad (29)$$

In Galerkin method:

$$Z_{ai} = 2\int_{t_0}^{t_n} k_1 W_i dt; \; Z_{bi} = 2\int_{t_0}^{t_n} k_2 W_i dt; \; Z_{ci} = 2\int_{t_0}^{t_n} (Z_1 k_1 + Z_2 k_2) W_i dt; \; Z_{di} = 2\int_{t_0}^{t_n} (Z_1 k_2 - Z_2 k_1) W_i dt; \; Z_{ei} = 2\int_{t_0}^{t_n} Z_1 W_i dt; \; Z_{fi} = 2\int_{t_0}^{t_n} Z_2 W_i dt; \; Z_{gi} = \int_{t_0}^{t_n} (Z_1^2 + Z_2^2) W_i dt; \; U_i = \int_{t_0}^{t_n} W_i dt; \; i = 1,2,3,4,5 \quad (30)$$

After eliminating parameters P_1, x_m and y_m in (27), equations containing only unknowns x_4 and x_5 are obtained:

$$L_{mi}x_4^3 + (K_{mi}x_5 + H_{mi})x_4^2 + (G_{mi}x_5^2 + F_{mi}x_5 + E_{mi})x_4 + (D_{mi}x_5^3 + C_{mi}x_5^2 + B_{mi}x_5 + A_{mi}) = 0\,;\; i = 1,2 \tag{31}$$

Coefficients L_{mi},K_{mi},H_{mi},G_{mi},F_{mi},E_{mi},D_{mi},C_{mi},B_{mi} and A_{mi} (i=1,2) are all computable constants. In solving (31) the cubic terms of x_4 are eliminated and the resulting single quadratic equation can be used to express x_4 as a function of x_5, which is then substituted in one of (31) to yield all possible solutions.

In the case of line points of the coupler plane, point D in Fig.4 is to satisfy the equation of a line in the form:

$$y = mx + n \tag{32}$$

In (32) (m) is the slope and n is the intercept of the line. Then the following displacement function Q involving the system parameters (x_4,x_5,m,n) will result as such:

$$Q(x_4, x_5, m, n, t) = k_2x_4 + k_1x_5 - k_1mx_4 + k_2mx_5 - Z_1m + Z_2 - n = 0 \tag{33}$$

If the function Q is evaluated under the criteria of Precision-Point, Subdomain and Galerkin methods, then the following equation set is found:

$$h_{ai}x_4 + h_{bi}x_5 - h_{bi}mx_4 + h_{ai}mx_5 - h_{ci}m - h_{di}n + h_{ei} = 0\;\; ; i = 1,2,3,4 \tag{34}$$

The coefficients in (34) are calculated according to each method in question as follows:
In Precision-point method:

$$h_{ai} = k_{2i}\,; h_{bi} = k_{1i}\,; h_{ci} = Z_{1i}\,; h_{di} = 1\,; h_{ei} = Z_{2i}\,;\;\; i = 1,2,3,4 \tag{35}$$

In Subdomain method:

$$h_{ai} = \int_{t_{i-1}}^{t_i} k_2 dt\,; h_{bi} = \int_{t_{i-1}}^{t_i} k_1 dt\,; h_{ci} = \int_{t_{i-1}}^{t_i} Z_1 dt\,; h_{di} = \int_{t_{i-1}}^{t_i} dt\,; h_{ei} = \int_{t_{i-1}}^{t_i} Z_2 dt; i = 1,2,3,4 \tag{36}$$

In Galerkin method:

$$h_{ai} = \int_{t_0}^{t_n} k_2W_i dt\,; h_{bi} = \int_{t_0}^{t_n} k_1W_i dt\,; h_{ci} = \int_{t_0}^{t_n} Z_1W_i dt\,; h_{di} = \int_{t_0}^{t_n} W_i dt\,; h_{ei} = \int_{t_0}^{t_n} Z_2W_i dt;\quad i = 1,2,3,4 \tag{37}$$

(34) can be reduced to a single equation in the unknown (m):

$$S_{A5}m^4 + S_{A4}m^3 + S_{A3}m^2 + S_{A2}m + S_{A1} = 0 \tag{38}$$

S_{A1} to S_{A5} are simply constants. After determining m from (38) all possible solution of the unknown set (x_4,x_5,m,n) is easily drawn from (34).

3.1. Application to Coupler Plane of Modules

Now the theory developed for a general coupler plane motion will be applied to modules. In doing so, it is sufficient to specify the co-ordinates Z_1,Z_2 and angle δ as a function of some variable t. First the dyad module will be considered. Referring to Fig.5, Z_1,Z_2 co-ordinates which will be substituted in equation sets (27) and (34) will be represented in terms of the input angle ψ, as follows:

$$Z_1 = x_7 + x_1 \operatorname{Cos}\psi$$

(39)

$$Z_2 = x_8 + x_1 \operatorname{Sin}\psi \tag{40}$$

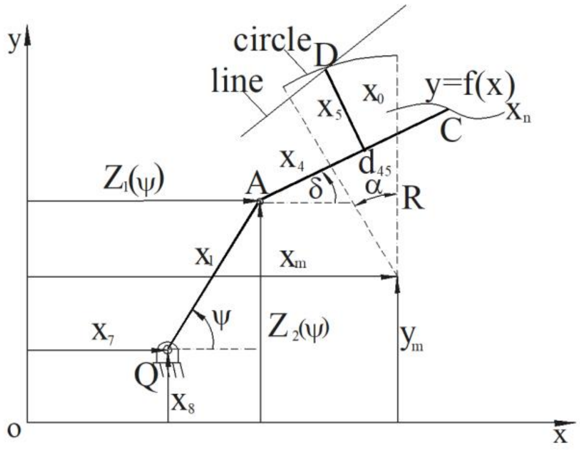

Fig. 5. Coupler motion of dyad module

Additionally, coupler angle δ will be determined as a function of ψ from relations (1),(2) after realizing the inverse transformation $x_{th}=h^{-1}(\psi-\psi_0)$. Knowing δ, k_1 and k_2 can be calculated from relation (24). Thus circle points of the dyad coupler are now found out by evaluating equation set (27) to lead to the values of the unknown parameters (x_4,x_5,R,x_m,y_m). Similarly, (x_4, x_5, m, n) parameters are obtained by solving equation set (34) for the line points of the dyad coupler.

In the case of slider module, Fig.6, Z_1, Z_2 co-ordinate functions are expressed as a function of the linear variable s, as given below:

$$Z_1 = x_3 \operatorname{Cos}\theta - s \operatorname{Sin}\theta \tag{41}$$

$$Z_2 = x_3 \operatorname{Sin}\theta + s \operatorname{Cos}\theta \tag{42}$$

Coupler angle δ is computed together with the inverse transformation $x_{th}=g^{-1}(s-s_0)$. Then the rest of the procedure is identical with that of the dyad module, thus defining the circle and line points of the slider coupler.

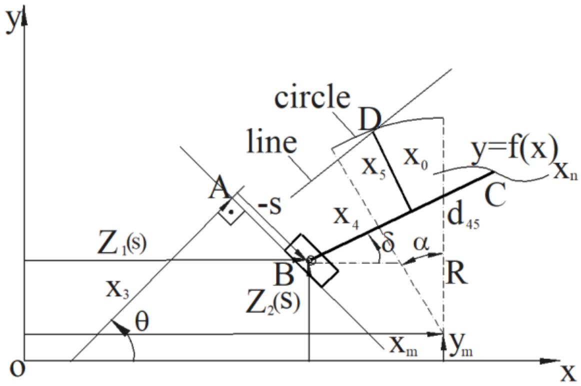

Fig. 6. Coupler motion of slider module

4. CONSTRUCTION OF MECHANISMS VIA MODULAR APPROACH

Four-bar and slider-crank mechanisms which are constructed using modular approach are shown in Fig 7(a),(b). In designing the 4-bar, first, the dyad module to generate the curve y=f(x) $x_0 \le x \le x_n$ at point C is synthesized as explained earlier, indicating that (x_7,x_8,x_1,ψ_0,d_{45}) are found out.

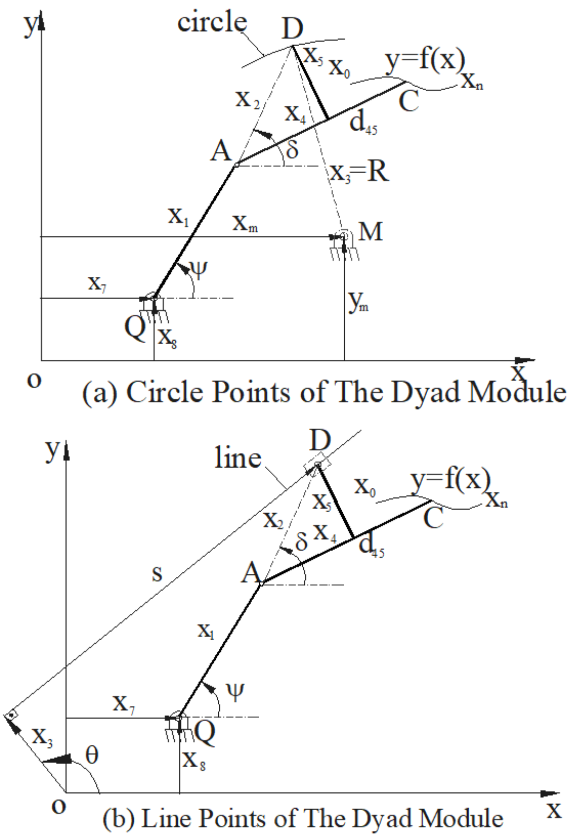

Fig. 7. Module coupler points

Then, circle points of the dyad coupler are determined, which are characterized by the parameter set (x_4,x_5,x_m,y_m,R), Fig.7(a). Now the circle point D is joined physically by the center point M, the relative location of which is defined by x_6 and x_9, by a link of length x_3=R, completing the construction of the four-bar QADM generating the given path at point C, where the connecting link length (x_2) is defined by AD=$(x_4^2+x_5^2)^{1/2}$. In order to estimate the quality of the designed 4-bar, the structural error (e) has to be calculated. To that end, the coupler angle (δ) in the resulting 4-bar is computed in accordance with the following relation:

$$\delta_{\mp} = 2\tan^{-1}\left[\frac{M_c \mp \sqrt{M_c^2 + L_c^2 - N_c^2}}{L_c + N_c}\right] \tag{43}$$

where

$$L_c = 2(x_1 \operatorname{Cos}\psi - x_6)x_2\ ;\ M_c = 2(x_1 \operatorname{Sin}\psi - x_9)x_2\ ;$$
$$N_c = x_3^2 - [x_1^2 + x_6^2 + x_9^2 + x_2^2 - 2x_1(x_6 \operatorname{Cos}\psi + x_9 \operatorname{Sin}\psi)] \tag{44}$$

Of the same dyad module (QAC) utilized in 4-bar design, line points (D) are determined to signify the estimation of the parameter set (x_4,x_5,m,n), Fig. 7(b). A slider is inserted at point D, which is then joined to point A of the dyad module, thus forming the slider-crank mechanism (QAD), Fig.7(b). For the structural error (e) analysis in the resulting slider-crank mechanism, connecting link angle (δ) is to be found by the following relation:

$$\delta_{\mp} = 2\tan^{-1}\left[\frac{M_s \mp \sqrt{M_s^2 + L_s^2 - N_s^2}}{L_s + N_s}\right] \tag{45}$$

where

$$L_s = x_2 \operatorname{Cos}\theta\ ;\ M_s = x_2 \operatorname{Sin}\theta\ ;\ N_s = x_3 - x_1(\operatorname{Cos}\theta \operatorname{Cos}\psi + \operatorname{Sin}\theta \operatorname{Sin}\psi) \tag{46}$$

Now the modular approach will be applied to design a path-generating double-slider. First, a slider-module is synthesized by means of the parameter set (θ,s_0,d_{45},x_3) to trace a given path at point C, Fig. 8. Then line points of the slider coupler are

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searched to conclude on the values of the parameter set (x_4, x_5,m,n). Now placing sliders at points D and B, and joining them together physically to obtain a length of $x_2=(x_4^2+x_5^2)^{1/2}$ will produce the double slider with determinate movement directions, to generate the given path. In the structural error analysis of the resulting mechanism, the angular position of the connecting link will be needed, which is simply as follows:

$$\delta_{\mp} = 2\tan^{-1}\left[\frac{M_d \mp \sqrt{M_d^2 + L_d^2 - N_d^2}}{L_d + N_d}\right] \tag{47}$$

where

$$L_d = \frac{x_2}{Sin\theta'}; \quad M_d = \frac{x_2}{Cos\theta'}$$

$$N_d = \frac{1}{Sin\theta' Cos\theta'}\left[x_3' - x_3(Cos\theta Cos\theta' + Sin\theta Sin\theta') + s(Sin\theta Cos\theta' - Cos\theta Sin\theta')\right] \tag{48}$$

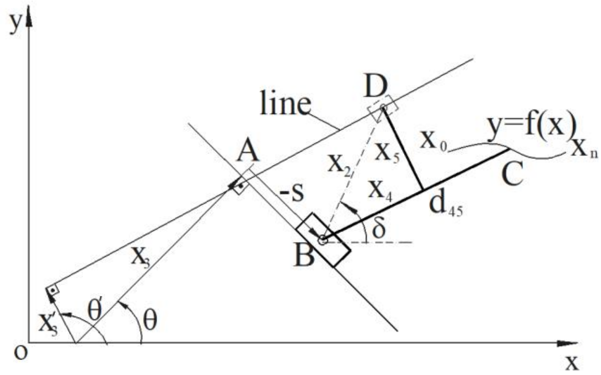

Fig. 8. Line points of the slider module

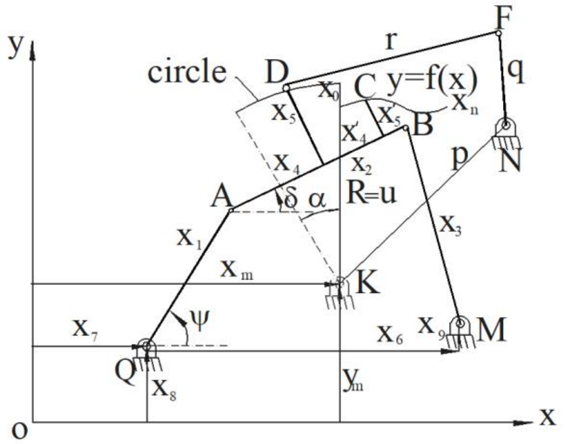

Fig. 9. Construction of a Six Bar Mechanism

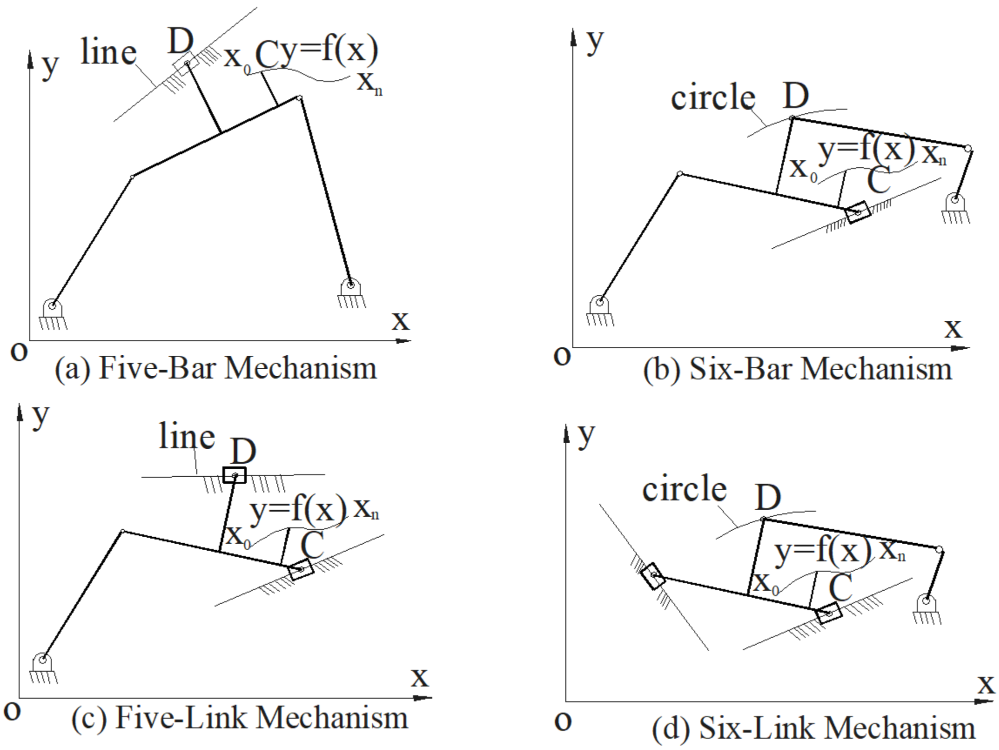

Fig. 10. Applications of the modular approach

The modular approach will be illustrated further on a six-bar mechanism, taking into account the four-bar and the crank-rocker module. To this end, the four-bar shown in Fig.9, characterized by the parameter set (x_7,x_8,ψ_0,x_2,x_3,x_6,x_9,x_4',x_5') is synthesized through the modular approach as described earlier, in the first place. Later the circle points of the coupler plane of the designed four-bar QABM generating the given path at point C, are searched. This establishes the values of the parameter set (x_4,x_5,x_m,y_m,R) associated with the circle point D, the center of which is located at point K, Fig.9. In this way, the length of the circular arc becomes known hence the angle (α) from the center K. Now using the outcome concerning the rocker swing angle (α) and rocker length (u) being equal to the radius of the circle (R), and also assuming one other criterion such as time ratio (t_R) ar angle (η) or initial crank angle (β), crank-rocker module is designed, based on the equation set (21). It should be pointed out that K is an imaginary joint and DK is an imaginary link whereas D,F and N signify physically existent revolute joints, and DF,FN physical links. This concludes the design of the whole six-bar mechanism QADBFN, in Fig. 9, such that the given path is generated as the crank FN drives the system through 360° rotations.

The modular approach explained in detail on specific examples can be easily applied to many other lower-paired mechanisms, some of which are shown in Fig.10. For instance, in Fig.10 (a), the line points of the four-bar coupler plane can be evaluated by inserting a linear slider, forming a five-bar to generate the specified path. Similarly, this technique can be utilized in designing the six-link mechanism seen on Fig.10 (b), which is constructed from circle-points of the slider-crank coupler plane together with a crank-rocker module. An extension of the line-points of the slider-crank coupler plane covers the five-link mechanism, whereby a linear slider is attached, Fig.10 (c). One final example may be given for the case, where circle-points of the double-slider can be assessed as a joint for combining it with a crank-rocker module, thus leading to a 6-bar mechanism which can be driven by a rotary power source, Fig.10(d).

5. Numerical Results and Discussion

The modular approach developed here has been put in the form of program packages on the personal computer for numerical applications. Wherever numerical integration is needed, for instance in Subdomain and Galerkin methods, the so-called Simpson's rule is used. To test the approach, the following specific examples have been taken as a basis for assessment. Although all possible solutions in each example have been obtained, only one typical solution is included here.

Example 1: A dyad module is to be designed to generate the path y=x $0\le x\le 1$ for a 90 degree clockwise crank rotation. Some numerical results, with reference to the previous notation, are given according to each method as follows:

In Precision-point method: Precision points(x(i),i=1,..,5)=0.00,0.20,0.70,0.80,1.00;
Solution: x_1=0.4645; x_7=3.0064; x_8=-2.0064;d_{45}=4.0091;ψ_0= 0.0000°; e_{max}=0.000005;
In Subdomain method: Subdomains(x(i),i=1,..,6)=0.00,0.01,0.40,0.60,0.99,1.00;
Solution: x_1=0.4636; x_7=3.0089; x_8=-2.0089; d_{45}=4.01180;ψ_0= 0.00°;e_{max}=0.000005;
In Galerkin method: Weighting function(W_i,i=1,5)= $x, 2x^2-1, x^3-1, 8x^4-8x^2+1, 16x^5-20x^3+5x$

Solution: x_1=0.4642; x_7=3.0074; x_8=-2.0074;d_{45}=4.0101;ψ_0= 0.00°;e_{max}=0.000004;

Example 2: A slider module is to be synthesized for generating the path y=x 0≤x≤1 within one unit of slider displacement. The resulting solutions and the corresponding data are as follows:
In Precision-point method: Precision points(x(i),i=1,..,4)=0.00,0.50,0.80,1.00;
Solution: d_{45}=0.8073; x_3=-0.1105; θ=-22.92°;s_0=0.7997;e_{max}=0.003813;
In Subdomain method: Subdomains(x(i),i=1,..,5)=0,0.20,0.5,0.80,1.00;
Solution: d_{45}=0.6890; x_3=-0.1888; θ=-27.45°;s_0=0.6647;e_{max}=0.003626;
In Galerkin method : Weighting function(W_i,i=1,4) $x, 2x^2-1, 4x^3-3x, 8x^4-8x^2+1$
Solution: d_{45}=0.7108; x_3=-0.1678; θ=-26.38°;s_0=0.6931;e_{max}=0.002515;

Example 3: A four-bar design is to be obtained such that the coupler point trace the path y=x 0≤x≤1 within 90 degrees clockwise crank rotation. In the solution, the circle points of the dyad coupler of Example 1 are referred to, thus leading to the numerical results given below in accordance with each method.
In Precision-point method: Precision points(ψ_p(i),i=1,..,5)=0.00°,-0.10°,-0.20°,-0.30°,-90.00°
Solution:
x_1=0.4642;x_2=3.0468;x_3=2.8626;x_4=2.0164;x_5=3.4662;x_6=-0.7778;x_7=3.0074;
x_8=-2.0074;x_9=0.6015;ψ_0=0.00°;e_{max}=0.0002682;
In Subdomain method: Subdomains (ψ_s(i),i=1,..,6)=0.00°,-0.10°,-0.20°,-0.30°,-89.00°,-90.00°;
Solution:
x_1=0.4642;x_2=4.9697;x_3=4.8188;x_4=1.1231;x_5=3.8496;x_6=-0.6695;x_7=3.0074;
x_8=-2.0074;x_9=0.4636;ψ_0=0.00°;e_{max}=0.000268;
In Galerkin method: Weighting function(W_i,i=1,5) $1, \sin\psi, \cos\psi, \sin^2\psi, \cos^2\psi$
Solution:
x_1=0.4642;x_2=1.4668;x_3=0.6528;x_4=3.5875;x_5=1.7919;x_6=-0.6340;x_7=3.0074;
x_8=-2.0074;x_9=0.6341;ψ_0=0.00°;e_{max}=0.000863;

Example 4: A slider-crank is required to generate the path y=x 0≤x≤1 for a 90 degree clockwise input rotation. In solving this problem, the line points of the dyad coupler of Example 1 are searched, producing the following results by each method.
In Precision-point method: Precision points(ψ_p(i),i=1,..,4)=0°,-50°,-65°,-90°;
Solution:
x_1=0.4642;x_2=1.7442;x_3=1.1094;x_4=3.4259;x_5=-2.0843;x_7=3.0074;
x_8=-2.0074;θ=-148.48°;ψ_0=0°;e_{max}=0.010061;

In Subdomain method: Subdomains (ψ_s(i),i=1,..,5)= 0°,-10°,-55°,-75°,-90°;
Solution:
x_1=0.4642;x_2=1.8768;x_3=1.2711;x_4=3.3144;x_5=-2.2573;x_7=3.0074;
x_8=-2.0074;θ=-148.56°;ψ_0=0°;e_{max}=0.009973;

In Galerkin method: Weighting function(W_i,i=1,4) $e^{\psi}, \sin\psi\cos\psi, \cos\psi, \sin\psi$
Solution:
x_1=0.4642;x_2=2.7238;x_3=2.2345;x_4=3.4925;x_5=-1.9707;x_7=3.0074;
x_8=-2.0074;θ=-173.45°;ψ_0=0°;e_{max}=0.0074223;

Example 5: A double-slider is sought for the generation of the path y=x 0≤x≤1 for a unit input displacement. The line points of the slider module of Example 2 will be the answer of this problem, in the following form:
In Precision-point method: Precision points (x_p(i),i=1,..,4)=0,0.01,0.011,1;
Solution: x_2=1.3435; x_4=-0.3300; x_5=0.66243; x_3=-0.1679; x_3'=0.4767; θ=-26.38°; θ'=-43.61°; s_0=0.6932; e_{max}=0.003176;
In Subdomain method: Subdomains (x_s(i),i=1,..,5)=0,0.01,0.011,0.012,1;
Solution: x_2=1.4220; x_4=-0.4453; x_5=0.5540; x_3=-0.1679; x_3'=0.6404; θ=-26.38°; θ'=-40.99°; s_0=0.6932; e_{max}=0.042500;
In Galerkin method: Weighting function(W_i,i=1,4)= $x-e^{-x^2}, x-e^{-x^3}, x-e^{-x^4}, x-e^{-x^5}$
Solution: x_2=0.9093; x_4=-0.5209; x_5=0.4837; x_3=-0.1679; x_3'=0.3652; θ=-26.38°; θ'=177.49°; s_0=0.6932; e_{max}=0.015030;

Example 6: A slider-crank is constructed by slider module that generate the path y=x 0≤x≤1 for a unit input displacement .The circle points of the slider module of Example 2 will be the answer of this problem, in the following form:
In Precision-point method: Precision points (x_p(i),i=1,..,4)=0,0.01,0.03,0.999,1;
Solution: x_1=0.7965; x_2=1.2373; x_4=1.6724; x_5=-0.5621; x_3=0.5134; x_7=0.1476; x_8 =1.8308;
θ=-26.38°; s_0=0.6932; e_{max}=0.008103;
In Subdomain method: Subdomains (x_s(i),i=1,..,5)=0,0.01,0.05,0.994,0.996,1;
Solution: x_1=1.1690; x_2=1.2288; x_4=1.6670; x_5=-0.5597; x_3=0.2106; x_7=0.5226; x_8 =1.9056;
θ=-26.38°;s_0=0.6932; e_{max}=0.002516;
In Galerkin method: Weighting function(W_i,i=1,4)= $1, e^x, \sin x, \cos x, \sin x\cos x$
Solution: x_1=1.3348; x_2=1.2565; x_4=1.6845; x_5=-0.5675; x_3=0.1128; x_7=0.6505; x_8 =1.9431;
θ=-26.38°; s_0=0.6932; e_{max}=0.002319;

Example 7: A six-bar of Fig.9 is required to trace the given path y=x 0≤x≤1 as one input link drives the whole mechanism through a 360 degree rotation. Applying the modular approach, circle points of the four-bar coupler of Example 3 with Galerkin method are determined first, thus yielding the length and swing angle of rocker. Then, the matching crank-rocker module is designed to be supplemented to the four-bar with a revolute joint at the circle point. The process described leads to the following numerical results:
In Precision-point method: Precision points(ψ_p(i),i=1,..,5)=0°,-20°,-45°,-85°,-90°;
Solution: x_4=2.6823; x_5=1.4993;x_m=-1.6625;y_m=2.0322;R=3.5657; α=15.08°; η=0.00°; β=30.00°; ϕ=-67.54°; u/p=-0.5044; r/p=0.8660;q/p=0.0662; e_{max}=0.001027;

In Subdomain method: Subdomains (ψ_s(i),i=1,..,6)=0,-20°,-45°,-80°,-85°,-90°;
Solution: x_4=2.3670;x_5=1.1354;x_m=0.5972;y_m=0.5024; R=1.3182; α=30.91°; η=0.00°; β=30.00°; ϕ=-75.46°; u/p=-0.5188; r/p=0.8660;q/p=0.1383; e_{max}=0.003378;
In Galerkin method: Weighting function(W_i,i=1,5) $e^{-\psi}, e^{-\psi^2}, e^{-\psi^3}, e^{-\psi^4}, e^{-\psi^5}$
Solution: x_4=2.4016; x_5=1.1546 ;x_m=0.4867;y_m=0.6101;R:=1.4329;α=29.23°; η=0.00°; β=30.00°; ϕ=-74.61°;u/p=-0.5167; r/p=0.8660;q/p=0.13035; e_{max}=0.003207;

Example 8: Five-link mechanism of Fig. 10(a) is to be designed to generate the path y=x 0≤x≤1 with in an input clockwise rotation of 90°. In accordance with the modular approach, the solution lies in finding the line points of the four-bar coupler of Example 3 with Galerkin method. The following outcome will define the sought design:
In Precision-point method: Precision points(ψ_p(i),i=1,..,4)=0°,-20°,-80°,-90°;
Solution: x_4=3.4292;x_5=2.0461;m=1.2588;n=-0.1204; e_{max}=-0.0022637;
In Subdomain method: Subdomains (ψ_s(i),i=1,..,5)=0°,-10°,-45°,-85°,-90°;
Solution: x_4=3.5415;x_5=1.8451;m=1.0597;n=-0.0524; e_{max}=-0.0008472;
In Galerkin method: Weighting function(W_i,i=1,4) $e^{-\psi}, e^{-\psi^2}, e^{-\psi^3}, e^{-\psi^4}$
Solution: x_4=3.5081;x_5=1.8999;m=1.1115;n=-0.07930; e_{max}=0.0006162;

Example 9: Mechanisms which have an input link driving the whole mechanism through a 360-degree rotation with circular and linear paths are widely used in practice. In this example, applying the modular approach, a crank-rocker module is designed with circle and line points on its coupler. The following results are obtained:
Selected dimensions of crank-rocker: q=0.2682; p=1; R=2.7902; u=2.2487;
Four-bar mechanisms: x_1=q; x_2=R; x_3=u; x_6=p; x_7=0; x_8=0; x_9=0;
The initial angle (ψ_0) and the crank rotation ($\Delta\psi$) of the input link are specified by designer. Here, ψ_0=30°; $\Delta\psi$=90° (counter clockwise) are selected. The numerical results of design for circle points in Galerkin method are the following:
x_4=-1.9750; x_5=-1.5291; x_m=0.5586; y_m=-0.8751; R=1.8226; e_{max}=0.001833;
The numerical results of design for line points in Galerkin method are the following:
x_4=2.2515; x_5=-1.5646;m=-1.5602; n=4.4946; e_{max}=0.035378;

In view of the numerical results displayed above, it can be said that as the number of links and joints increases the degree of precision in governing the desired path improves, as is to be expected. This justifies also the possible cost of using more elements in the preferred mechanism with respect to very fine errors.

5. CONCLUSION

A novel approach, termed as a modular approach, has been devised to design path-generating mechanisms with lower pairs. The essence of the approach lies in constructing the multi-link mechanisms out of two-link assemblies, called modules, which provide closed-form solutions for the design equations. Thus, there is always solution assurance for any physically feasible problem of path generation. Furthermore, there is the possibility of choosing the most appropriate one from among many possible solutions that may result from the process.
This approach can easily be extended to other areas of mechanism synthesis too.

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