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Research Article

Hybrid Bee Colony Algorithm with Whale Algorithm

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ABSTRACT

In this paper, we introduce two hybrid meta-heuristic algorithms inspired by natural processes: Bee Colony Optimization (BCO) and Whale Optimization Algorithm (WOA). The BCO algorithm, first proposed by Karaboga in 2005, draws on the foraging behaviors of honeybees. It is known for its simplicity and effectiveness in solving various optimization problems. We will provide an overview of the BCO algorithm, including its principles and modifications within the context of swarm intelligence. This technique, which examines decentralized systems composed of numerous interacting elements, is particularly notable for its exploration capabilities. The Whale Optimization Algorithm, introduced by Mirjalili and Lewis in 2016, mimics the bubble-net hunting behavior of humpback whales. This algorithm employs swarm intelligence to avoid local optima and balance exploration and exploitation through a simulated fishing net approach. Its design helps achieve optimal solutions and effectively avoid local traps.We describe a hybridization of BCO and WOA into a new algorithm, named ABCWOA. This hybrid algorithm was tested across 16 optimization tasks with varying frequencies of (100, 200, 500, 1000). The results demonstrated that ABCWOA effectively reached optimal solutions, often outperforming traditional search algorithms by achieving lower minimum values (f_min) for most tasks.

1. Introduction

Several optimization issues dealing with different search areas of areas such as feature selection have been applied. Reduce dimensions. Mass transportation services. Recently, many Meta-Heuristic algorithms have been developed because of their advantages that help researchers propose a solution to complex computational problems. Such as Flock Amendment. inborn algorithms. Bat algorithm (BAT). Ant-Lion Algorithm (ALO). And other post intuitive algorithms because of their advantages of flexibility in dealing with different problems compared to traditional algorithms make optimization techniques in these algorithms popular among researchers [1]. Post-intuitive optimization algorithms have become more widely used in applications related to engineering because they are based on easy concepts that are plain to perform and do not demand tendency inclination and can also avoid falling into local solutions. Post-intuitive algorithms inspired by optimization problems are solved by simulating biological or materialistic events. They can be set into three major class: growth-founded methods. and materialistic based. And the list of the flock.[2]

In post intuitive algorithms, it relies on the random population principle in exploration and exploitation. Post-intuitive algorithms work on less computational Access time the optimal resolution, shun descent till solutions with faster convergence, and take out the mundial optimal resolution that makes it Adecuado for practical applications without modifications in

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the structure of the algorithm for a different solution that is beneficial or not useful in algorithm optimization problems.[3]

2. Research Problem

The research problem focused on finding the comprehensive optimal solution for unrestricted high-measurement optimization problems without falling into the trap of local solutions. and its characteristics to solve the problem of dropping into topical solutions.

3. Research Importance

The value of the discussion is in beneficent the showing of the algorithms by interbreeding them between the bee optimization algorithm and the Whale Optimization Algorithm and it was called (ABCWOA). It is a suggested method for solving unrestricted optimization problems with a high measurement, which is considered one of the difficult (NP-hard) problems, to help researchers and specialists to benefit from the proposed hybrid algorithm in solving this type of problem, as well as all the issues that fall within its framework.

4. Bee Colony Algorithm

4.1. Behavior of bees: Behavior Bee

Honeybees live in and maintain a densely populated colony. It is a uniquely explained about social organization. They are creatures of unexpected complexity capable of generating food. and explore food sources. And communicating through complex dances, the colony can be considered a unique swarm of social elements, the bees, who exchange information with them, leading the bees to form collective knowledge. A bee colony consists of approximately one queen, hundreds of drones and many thousands of workers. The miss lays false eggs and an alchemical plant. Her labor is to spread bleaches and onset a modern colony. She tampers conduct by sending (chemical messages) [4]. The Bee order is a gauge model of neat teamwork, coordination, interaction, division of labor, and continuous performance of tasks in the colony.[5]

Despite the large digit of diverse gregarious mealy bug host and the divergence in their types of behavior, individual insects can be described as able to carry out a diversity of synthesis tasks. The superior ideal is collecting and converting nectar, which is much more orderly. Each bee chooses to gauge the source of nectar by next a special system among them in distributing the tasks and duties required of them. The colony consists of [6] the queen and her responsibility are to lay eggs so that they form a new colony. The males are responsible for mating with the miss and this is their only turn in the hive. They are eliminated from the habitation over their downfall. The laborer bees are the feminine of the stand. They are the basic erection blocks of the stand that builds the melissa bee reed, cleans, maintains, protects and feed the talent and males. Aloof from this aspect of responsibilities, the prime act of the laborer bee is to seek for and converge opulent food. There are two forms of laborer bees, prospector bees and elemental feeder bees.[5]

Prospector bees take wing and food fountains randomly from flowers and nectar. They reversion to the hive after depleting their power and the edges of distance, and when they reversion to the hive, they portion the experiment of discovery from the remarkable datum with the nutrient element, the prospectors tell the foragers about the position of the opulent food roots that make up the orientation (nook) of the food root from the hive to the bask and the interval from the hive and it is done This is done through the use of a dance [5]. This dance changes with an increase in the distance to the food source, and different types of this behavior have been identified, including

- **Circular dance**
- **Waggle tail dance**

And each type of them can have degrees or differences that illustrate a certain behavior, as in Figure (1) the bee in the circular dance runs in a circular motion, where several circles are made by the bee with the making of bubbles when giving samples of nectar to the bees. Dance shows the location of a source within a certain distance. In the case of food close to the colony.[7] If the food is far from the colony, the circular dance is developed into the wagging dance or the wagging tail dance, which is in the form of the number 8 by imposing providing the beehive with information about the distance and direction of the food with running in a circle in one direction and then in a circular run in the other direction with shaking the body during a straight run . These dances are performed on a horizontal disc called the dance area, which is a specific place in the colony[7] .

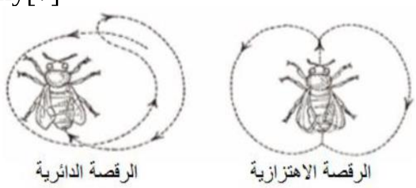

Figure 1 Illustration image of bee dances.

4.2. Bee Colony System

It is an inspiring exploratory method developed to find effective solutions to the difficulty of getting involved in optimization problems. The requisite motif backwards a bee colony system is to build a multi-component system (synthetic Bee habitation) that will look for perfect resolution for unlike improvements, and explore the principles that honey bees use while collecting nectar. A synthetic bee colony ordinarily depends on a few individuals. To find the best possible solutions in a given search area, artificial bees cooperate by exchanging information and food locations using" waggling dances.[8]

4.3. Synthetic Bee habitation Algorithm

The ABC synthetic bee-colony algorithm was suggested by Karaboga in 2005 for true preference criteria. It is an Optimization algorithm that affects the feeding manner of a bee colony [7]. Each bee is a simple component of the ABC. In the formation of a bee colony with these simple components, it can display complex and coherent behavior and to establish an integrated system for its detection and exploration of flower nectar. Each habitation has three brackets of prospector bee, sighting bees, and workers each bees with an onus. Prospector bees must search and find new sources of food. The search is random about the surrounding environment [9]. The top bisection of the habitation includes the used Synthetic bees and the latter bisection implicates the observer bees. For each source of nourishmto ent yonder is a singular bee working on the other hand, the numeral of worker bees equals the numeral of nourishment provenance. Worker bees determine the food provenance by those memories. Worker bees part their acquaintance with the spectator bees in the hive and the spectator bees choose a single source of food. The worker bee that has been derelict from this source becomes a prospector that else initiates a random search for new nourishment sources.[10]

Each research cycle includes three phases: transferring the spectators (watching bees) and working bees to food sources, in addition to calculating the quantities of their nectar, as well as marking the prospector bees, and then randomly transferring them to imaginable nourishment provenances. The nourishment provenance appears a scientific resolution to an issue improve it. The compensation of nectar of the nourishment provenance agrees to the fineness of the food source. Viewer bees are placed on foods using a dancing disc. Every hive has prospectors who are scouts for the habitation, scouts don't have any leading when manhole for habitation they are only interested in feedback on any type of nourishment origin. As a result of this manner, prospectors have lower search costs as well as lower average food source quality.

Sometimes scout bees can accidentally discover rich food sources[10] .

The manner of the BCO bee habitation is stagy The ABC synthetic bee colony algorithm differs from BCO because in ABC we use but prospectors and foragers in equal proportions as the inhabitance priority. The prime proceedings of the ABC algorithm are:

- config
- Repetition
- Send worker bees nourishment origin
- Toss off watch bees towards food sources
- Toss off prospector bees to explore modern nourishment origin
- obtaining the wished-for (appropriate) status[9]

step 1:

Bisection of the bees in the ABC Algorithm includes worker bees and the other bisection includes no-worker bees. There is lone singular cohort of worker bees for either food origin.

Step 2:

Use the following relationship. That a modern return was formed solution for either trouble

$$v_{ij} = x_{ij} + \varphi_{ij}(x_{ij} - x_{kj}) \qquad (1)$$

where

$i\ ,\ \ k\, \epsilon \{1.2 \ldots .BN\}\ \&\ k \neq i$

$j\, \epsilon \{1.2 \ldots .D\}$

$\varphi \epsilon [-1{,}1]$

The vicinity of the nourishment origin is performed by x_i, the nourishment origin v_i has to be a value of x_i, j an is an indiscriminate integer in the zone of $[1, D]$, D is the number of optimization coefficients, and K is a randomly selected index whose range is $[1, BN]$, BN is the issue of premier answers to the trouble, and unlike i , φ is a true random issue in the range $[-1{,}1]$.[9]

Step 3:

$$p_i = \frac{fit_i}{\sum_{n=1}^{sn} fit_n} \qquad (2)$$

The eventuality of receiving bees from each site is calculated using the following equation (minimization problem)

Where fit_i: is the fitness function and is the source of the fit, p_i is the eventuality of election by the observer bees, the numeral of bees is allocated pacting to the fit of any matter in this tread, and all bees may be assigned to a feeding place rule on the fitness function. next any origin

***Step 4**:*

The indiscriminate response will replace the no-optimized response, if the number of no-optimizing responses reaches a preset edge C_{max}. Moreover, the iteration finish events are investigated. At this scene the iterations will be terminated as long as the algorithm end events are specified, otherwise They will recur to another pace.[9]

5. Whale Optimization Algorithm (WOA)

The Whale Optimization Algorithm is a modern algorithm proposed by Mirjalil and Lewis (2016) that simulates the attitude of humpback whales in their discussion for nourishment and hunting. Whales are amazing world, and they are gazed the largest mammals in the cosmos, where a grown-up whale mastery magnifies up to 30 meters in length and 180 tons in metering. Whales are mostly predators, and they are animals that do not shut eye so that they have to respire from the flat of the peripheries. And that one of the largest baleen whales are humpback whales, where the size of a grown-up humpback whale is nearly the magnitude of a schoolmistress bus, and that their preferable victim is (Krill) and modicum flocks of fish. Figure (2) display these mammals[2] .

The generality enjoyable thing about humpback whales is their chasing style. This foraging demeanor is called bleb decoy nourishment. The humpback whale prefers to chase the aquatic herd of (Krill) or modicum fish close to the superficies, and it has been monitored that this for-aging behavior is well-done by creating distinct blebs over an area or path in the form of 9 as shown in the numbered figure 2[2](2) .

2

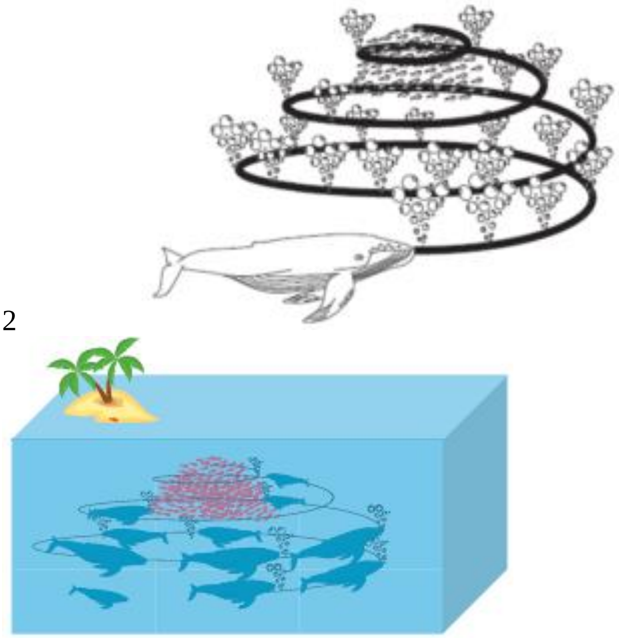

Figure 2 bleb decoy nourishment demeanor in humpback whales

The arithmetical example of the Whale-Optimization-Algorithm can be summarized in the following paragraphs:

5.1. Encircling prey

Humpback whales can meet the position of victim and surround it to hunt it, and this demeanor is mathematically expressed by the following poise:

Since:

$$\vec{D} = \left|\vec{C}.\vec{X}^*(t) - \vec{X}(t)\right| \qquad (3)$$

$$\vec{X}(t+1) = \vec{X}^*(t) - \vec{A}\vec{D} \qquad (4)$$

t denotes the current iteration, $\vec{X}^*$: is the position vector of the best solution obtained so far, X: is the position vector, and the symbol | | It refers to the absolute value, and the symbol (.) represents the product of multiplying an element by an element. In addition, A, C: refer to coefficient vectors and are calculated using the equations below 11]

$$\vec{A} = 2\vec{a}.\vec{r} - \vec{a} \qquad (5)$$

$$\vec{C} = 2\vec{r} \qquad (6)$$

As it $\vec{a}$ decreases linearly from 2 to 0 over the iterations (in both stages of exploration and exploitation) and that it is a random vector that takes values [0,1]. It is worth noting that X ⃗^* should be updated every iteration if there is a better solution [12].

5.2. Bubble-net attacking method

In tidy to arithmetically perform the demeanor of humpback whales in bubble decoy hunting, a hybrid was designed:

. Shrinking encircling mechanism

This behavior $\vec{a}$ is achieved by decreasing the value $\vec{a}$ in equation (5) noting that the fluctuation $\vec{A}$ is also decreasing by $\vec{a}$. And be calculated by $\vec{a}$ the following equation:

$$a = 2 - t\frac{2}{MaxIter} \qquad (7)$$

Where: t: indicates the current iteration and $MaxIter$: represents the largest number of iterations allowed.

. Spiral updating position

Figure 9 or the shape of the helix is simulated using the following equation:

$$\vec{X}(t+1) = \vec{D}.e^{bi}\cos(2\pi) + \vec{X}^{*}(t) \qquad (8)$$

Since:

$\vec{D}$:denotes the distance *i* from the whale to the prey (the best solution obtained so far), b is a constant for determining the shape of the logarithmic vortex, l is a random number in the interval [-1,1], and the sign (.) is the product of two components. Figure (3)

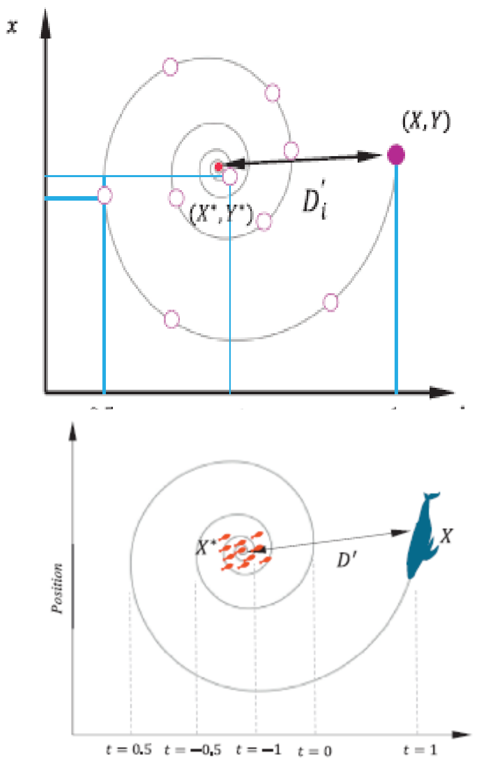

Figure 3 The spiral site

It has been observed that humpback whales bathe about the victim into a contraction ring and straight a cochleate-shaped track altogether. To paradigm this demeanor altogether, we suppose a 50% eventuality of choosing amidst either a contraction encircling technicality or a winding paradigm to update the whales' location midst optimization. The arithmetical paradigm is as follow:

$$(t+1) = \begin{cases} \vec{X}^{*}(t) - \vec{A}\vec{D} & if\ p < 0.5 \\ \vec{D}.e^{bi}\cos(2\pi) + \vec{X}^{*}(t) & if\ p \geq 0.5 \end{cases} \qquad (9)$$

Where: p is a random number in the interval.[0,1]

5.3. Search for prey

Humpback whales, as well as bubble decoys, search for prey randomly according to their respective locations, so we randomly update the finder location in the exploration phase instead of the best finder found so far. So we use a vector $\vec{A}$ with indiscriminate merits maximal than 1 or not so much than -1 to impact the cast around the element to walk from here from the hint whale. This mechanism and $\left|\vec{A}\right| > 1$ emphasizes the discovery process and allows the Whale Optimization Algorithm (WOA) to carry out a comprehensive scout out. The arithmetical paradigm is as follows up:[2]

$$\vec{D} = \left|\vec{C}.\vec{X}^{*}(t) - \vec{X}(t)\right| \qquad (10)$$

$$\vec{X}(t+1) = \overrightarrow{X_{rand}} - \vec{A}\vec{D} \qquad (11)$$

Since

$\overrightarrow{X_{rand}}$ is the random vector (the random whale) that was chosen from the current population.

6. Hybrid Algorithm:

Swarm intelligence is a section of synthetic cleverness that projects the mass demeanor and emerging characteristics of multiplex, Self-organizing and decentralized frameworks with a friendly frame. This system consists of modest interactive factors marshaled in small communities called (flocks), even though each element has a specific area to work and there is no centric observation, and the accumulated demeanor is one of the characteristics of imposed in the swarm. They can respond to ecological variations and rebate making abilities.

In this section, I proposed a new hybrid algorithm by linking post-intuitive ideas inspired by nature. Generally, the synthetic Bee settlement (ABC)

Algorithm was hybridized with Whale Optimization (WOA),

which uses swarm intelligence to get it the overall optimal resolution to optimization troubles. The new proposed algorithm was named (ABCWOA).

7.Practical side

The qualification of the proposition new algorithm (ABCWOA) was examined by using it to solve (16) high-measurement numerical optimization problems.

The new IWOMFO algorithm was compared with the synthetic Bee settlement algorithm (ABC) and the Whale Optimization Algorithm (WOA). The table shows 1) specifics of the trial functions, as well as the special domain, the minimum value (fmin) for each function, the slash and celestial limits for each function, the number of iterations (100, 200, 500, and 1000) reestablishment were used, and the remoteness that set forth the numeral of changeable in the styling. Elements. The proposition new algorithm (ABCWOA) is an amalgamation of two algorithms synthetic Bee settlement (ABC) and Whale Optimization Algorithm (WOA). Tables numbered from (2) to (4) show the values of the outputs.

Table 1 Details of the parameters and the lower and upper limits

Function	Dim	Range	*Fmin*
$F_1(x) = \sum_{i=1}^{n} x_i^2$	30	[−100,100]	0
$F_2(x) = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$	30	[−10,10]	0
$F_3(x) = \sum_{i=1}^{n} (\sum_{j-1}^{i} x_i)^2$	30	[−100,100]	0
$F_4(x) = \max_i\{\lvert x_i \rvert . 1 \le i \le n\}$	30	[−100,100]	0
$F_5(x) = \sum (100 * (x_{i+1}:\dim\ ) - (x_i:\dim\ )^2 + ((x_{i+1}:dim - 1) - 1)^2$	10	[−30,30]	
$F_6(x) = \sum_{i=1}^{n} ([x_i\ + 05])^2$	30	[−100,100]	0
$F_7(x) = \sum_{i=1}^{n} i x_i^4 + random[0,1)$	30	[−1.28,1.28]	0
$F_8(x) = \sum_{i=1}^{n} -x_i \sin(\sqrt{\lvert x_i \rvert})$	30	[−500,500]	-418.9829
$F_9(x) = \sum_{i=1}^{n} \lvert x_i^2 - 10\cos(2\pi x_i) + 10 \rvert$	30	[−5.12,5.12]	0
$F_{10}(x) = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right) + 20 + e$	30	[−32,32]	0
$F_{11}(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}}) + 1$	30	[−600,600]	0
$F_{12}(x) = \frac{\pi}{n}\{10\sin(\pi y_i) + \sum_{i=1}^{n-1}(y_i - 1)^2[1 + 10\sin^2(\pi y_{i+1}) + (y_n - 1)^2\} + \sum_{i=1}^{n} u(x_i, 10, 100, 4)$	30	[−50,50]	0
$y_i = 1 + \frac{x_i + 1}{4} u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m & x_i > a \\ 0 & -a < x_i < a \\ k(-x_i + a)^m & x_i < a \end{cases}$			
$F_{13}(x) = 1.0\{\sin^2(3\pi x_i) + \sum_{i=1}^{n-1}(x_i - 1)^2[1 + \sin^2(\pi x_{i+1}) + (x_n - 1)^2[1 + \sin^2(2\pi x_n)]\} + \sum_{i=1}^{n} u(x_i, 5, 100, 4)\}$	30	[−50,50]	0
$F_{14}(x) = \sum_{i=1}^{11}\left[a_i - \frac{x_1 - (b_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4}\right]^4$	4	[−5,5]	0.00030
$F_{15}(x) = 4x_i^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1 x_2 - 4x_2^2 + 4x_2^4$	2	[−5,5]	-1.0316
$F_{16}(x) = [1 + (x_1\ + x_2\ + 1)^2(19 - 14x_1\ + 3x_1^2 - 14x_2\ + 6x_1x_2 + 3x_2^2)] * [30 + (2x_1\ - 3x_2\ )^2(18 - 32x_1\ + 12x_1^2 + 48x_2\ - 36x_1x_2 + 2x_2^2)]$	2	[−5,5]	-1.0316

Table 2 Using frequency 100 and number of items 5

Function	ABC	WOA	ABCWOA
F1	1.0444e-10	2.0246e-41	0
F2	9.6791e-07	1.6006e-25	0
F3	1.7878e-07	70121.1481	1.106e-272
F4	1.5448e-05	79.9442	6.8175e-28
F5	0.0040749	28.7177	0.025852
F6	1.5692e-10	1.4552	7.2226e-08
F7	0.0052746	0.0019287	0.60139
F8	-2230.9123	-8713.2418	-2094.6025
F9	6.9647	0	0
F10	5.1781e-06	4.4409e-15	4.4409e-15
F11	0.0098573	0	0
F12	3.9099e-12	0.046243	0.00024966
F13	1.9942e-11	0.83173	0.00012812
F14	0.00069064	0.00033453	0.00031245
F15	-1.0316	-1.0316	-1.0316
F16	3	3	3

Table 3 Using frequency 250 and number of items 5

Function	ABC	WOA	ABCWOA
F1	3.485e-06	1.0413e-15	0
F2	0.00023305	8.8026e-12	0
F3	0.0043285	86184.8331	5.9759e-152
F4	0.0017774	1.0603	1.5683e-09
F5	0.12367	28.7605	0.076329
F6	6.2318e-06	2.2564	3.4758e-07
F7	0.01054	0.0039246	0.15095
F8	-2094.9144	-7918.9488	-2094.6058
F9	6.9647	0	0
F10	0.00088075	1.3146e-07	7.9936e-15
F11	0.0074123	1.2879e-14	0
F12	4.5156e-09	0.25834	5.7205e-05
F13	2.2886e-07	0.76708	0.00019614
F14	0.00031911	0.011256	0.00075824
F15	-1.0316	-1.0316	-1.0316
F16	3	3.0017	3

Table 3 Using frequency 500 and number of items 5

Function	ABC	WOA	ABCWOA
F1	2.2539e-24	2.4774e-105	0
F2	1.7809e-13	4.0432e-71	0
F3	8.4283e-20	42743.963	2.4703e-323
F4	1.6095e-12	46.0478	5.3762e-71
F5	0.94103	28.7527	0.00043173
F6	2.9983e-24	1.3531	3.1469e-09
F7	0.0033792	0.00015082	0.8238
F8	-1897.5128	-12334.9275	-2094.8756
F9	6.9647	0	0
F10	1.3154e-12	4.4409e-15	8.8818e-16
F11	0.0098647	0	0
F12	3.6252e-25	0.086897	4.5053e-05
F13	2.573e-25	0.59082	7.8302e-06
F14	0.00052025	2.9821	0.00032294
F15	-1.0316	-1.0316	-1.0316
F16	3	3	3

8.Discussion of Numerical Results

The results in the previous table show the success of the hybrid algorithm (ABCWOA) in feedback on the optimal resolution for (16) of the standard test functions with a high measurement compared with the synthetic Bee settlement algorithm again and again with the Whale Optimization Algorithm WOA, and this confirms the success of the hybridization process. The proposition algorithm ABCWOA gave better results from the algorithm ABC and WOA, we note that the functions (F11, F9, F2, F1) have given an ideal solution that is better than the algorithm itself, i. The algorithm of whale optimization, but it improves in the hybrid algorithm. As for the functions (F11, F8, F7), we notice an improvement in the Whale Optimization Algorithm, and it gets worse in the synthetic bee colony algorithm. As for the functions (F15, F14, F10), they may be bad with constancy in the value of convergence when crossing.

Table 4 Using frequency 1000 and number of items 5

Function	ABC	WOA	ABCWOA
F1	9.536e-47	1.0397e-208	0
F2	1.8945e-24	3.0999e-136	0
F3	1.5979e-18	22776.1519	2.9644e-323
F4	2.1646e-23	79.0986	1.3765e-224
F5	0.11947	28.7487	0.00063479
F6	0	0.62392	1.9829e-08
F7	0.01665	1.4017e-05	0.028716
F8	-2252.8289	-12568.7956	-2094.9129
F9	1.9899	0	0
F10	4.4409e-15	4.4409e-15	4.4409e-15
F11	0.0098573	0	0
F12	9.4233e-32	0.0198082	6.5582e-08
F13	1.3498e-32	0.67866	8.543 e-15
F14	0.00071787	0.00035035	0.00032294
F15	-1.0316	-1.0316	-1.0316
F16	3	3	3

9. Conclusions and recommendations

First: Conclusions

The hybridization of the meta-heuristic evolutionary algorithm with the heuristic algorithm enhanced the performance of the combined approach by accelerating convergence and improving solution quality. This integration increased both the exploration and search capabilities of the algorithm. Numerical results demonstrated the effectiveness of the hybrid algorithm in addressing various optimization problems. Comparative analysis between the Bee Colony Optimization (BCO) and Whale Optimization Algorithm (WOA) revealed that the hybrid algorithm consistently achieved superior results. The comprehensive optimal solutions were attained for most test functions, as indicated by the results with a difference with other algorithms (F16).

Second: Recommendations

In light of the conclusions reached by this research, the following recommendations can be presented:
1. Attempt to hybridize the ant colony algorithm with other algorithms that depend on the swarm system and improve it.
2. Attempt to hybridize the bee colony algorithm with classical optimization algorithms and compare the results of this hybrid algorithm with the hybrid algorithm with the swarm.
3. Attempt to hybridize the whale optimization algorithm with classical optimization algorithms and compare the results of this hybrid algorithm with the hybrid algorithm with the swarm.
4. Apply the proposed hybrid algorithm to one of the operations research problems of the (NP-hard) type

10.Reference

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