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# Advanced Leader-Follower Control Strategies: Integrating Adaptation Laws with Model Predictive Control

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## Highlights

• This paper focuses on leader-follower control strategies.
• A model predictive control was applied to solve leader-follower problem.
• A special adaptation law was used to improve the stability of the controller.

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*Adaptation law*
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**Abstract**

This article investigates the intricate dynamics of the leader-follower problem within the framework of model predictive control (MPC). The study focuses on a scenario where a leader, characterized by a differential dynamic model, is diligently followed by a follower vehicle with a distinct differential dynamic model. The follower has full access to the leader's state information, facilitating real-time informed decision-making. A novel adaptation law is introduced to adjust the weighting matrix of the MPC controller, ensuring the follower approaches the leader in the tangent plane manifold by prioritizing the heading angle error. The control strategy is designed to synchronize the follower's trajectory with that of the leader, which performs various maneuvers such as lane changes, abrupt heading angle alterations, and sudden shifts in linear velocity. The leader-follower formation control problem is thoroughly investigated across diverse scenarios, including straight-line movements, circular trajectories, and intricate S-shaped paths. Comprehensive analysis demonstrates the effectiveness of MPC and the proposed adaptation law in achieving precise and adaptable formation control, significantly enhancing the understanding of leader-follower dynamics under varying conditions. This research contributes to the field by offering a robust solution for precise and reliable formation control in dynamic environments, showcasing the potential of MPC in autonomous systems.



## 1. INTRODUCTION

The burgeoning field of autonomous systems has witnessed a growing interest in the intricate dynamics of leader-follower formations, which play a pivotal role in the development of efficient and adaptive multi-vehicle systems. Effective leader-follower formations are crucial for applications ranging from unmanned aerial vehicles (UAVs) and autonomous underwater vehicles (AUVs) to spacecraft and ground robots. This article delves into the nuanced complexities of the leader-follower problem, focusing on the integration of model predictive control (MPC) and a proposed adaptation law to enhance the precision and adaptability of formation control strategies.

In the envisioned scenario, a dynamic leader, characterized by a differential model, sets the trajectory for a follower vehicle with a distinct differential dynamic model. The follower has full access to the leader's state information, enabling informed decision-making in real-time. A special adaptation law is proposed to adjust the weighting matrix of the MPC controller, ensuring the follower approaches the leader in the tangent plane manifold by prioritizing the heading angle error over other states.

The control strategy is designed to align the follower's trajectory with that of the leader. The leader executes a series of maneuvers, such as lane changes, abrupt alterations in heading angle, and sudden shifts in linear

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velocity. The leader-follower formation control problem is systematically investigated across diverse scenarios, including straight-line movements, circular trajectories, and intricate S-shaped paths. Through comprehensive analysis, this research enhances the understanding of leader-follower dynamics under varying conditions, demonstrating the efficacy of MPC and the proposed adaptation law in achieving precise and adaptable formation control.

A comprehensive literature survey reveals significant advancements and ongoing challenges in the domain of leader-follower formations. Researchers have extensively explored obstacle avoidance, energy efficiency, composite control strategies, and distributed MPC approaches. Obstacle avoidance has been a focal point in several studies, with various methodologies proposed to navigate static and dynamic environments. Energy efficiency and fuel optimization have also been critical, with efforts aimed at extending operational range and improving fuel economy. Composite control strategies integrating MPC with other control techniques, such as adaptive terminal sliding mode control and consensus control, have shown promise in enhancing formation control performance. Distributed MPC approaches have been widely investigated for their robustness and flexibility in multi-agent systems.

Despite these advancements, several gaps remain. There is a need for unified frameworks that seamlessly integrate obstacle avoidance, energy efficiency, and robust control strategies. Additionally, the complexity of composite control strategies and the scalability of distributed MPC systems require further investigation.

In summary, this article extends the current knowledge frontier in autonomous systems by unraveling the intricacies of leader-follower formations. Through rigorous analysis and diverse scenario exploration, the study provides valuable insights that pave the way for enhanced autonomy and adaptability in multi-vehicle systems. The proposed adaptation law for the MPC controller, which dynamically adjusts the weighting matrix, represents a novel contribution to the field, offering a new avenue for achieving precise and reliable formation control in dynamic environments.

The paper is structured as follows: In section 2, we summarize the key contributions of the paper. In section 3, we review the related works in this field, covering previous studies on model predictive control (MPC) in general, as well as its applications in tracking and particularly in leader-follower problems. We discuss various approaches and solution attempts proposed in the literature to address leader-follower problems. Section 4 outlines the problem statement, presents the research question and proposes a roadmap for the solution. Section 5 delves into the fundamentals of model predictive control, addressing the model-following problem which sets the stage for addressing the leader-follower formation control problem. In Section 6, we focus specifically on the leader-follower formation control using the bicycle model and MPC. Here, we conduct a detailed investigation into the leader-follower problem through numerical simulation. We analyze the effects of control horizon, adaptation law parameters, and initial errors. Section 7 provides a detailed discussion on the results and the theoretical approach proposed in this paper. Finally, section 8 concludes the paper, summarizing the findings and presenting concluding statements based on the insights gained from the study.

## 2. CONTRIBUTIONS

The proposed method in this study introduces a novel adaptation law within the framework of Model Predictive Control (MPC) to enhance the precision and adaptability of leader-follower formation control. A key contribution is the dynamic adjustment of the MPC weighting matrix, which prioritizes the heading angle error over other state variables. This adjustment ensures that the follower vehicle aligns closely with the leader's trajectory in the tangent plane manifold, facilitating stable formation control even during complex maneuvers.

The study also leverages the Kinematic Bicycle Model to capture essential vehicle dynamics, offering a balance between accuracy and computational efficiency. This model allows for precise control of both translational and rotational motions, ensuring the follower can accurately replicate the leader's path under a variety of conditions, including straight-line movements, circular trajectories, and S-shaped paths.

Another significant contribution lies in the method's ability to handle sudden changes in the leader's behavior—such as abrupt heading adjustments or velocity shifts—demonstrating the robustness of the control strategy. Through extensive simulations, the research shows that the dynamic adaptation of the control parameters improves the stability of the system by minimizing the effects of nonlinearity and ensuring reliable tracking. Furthermore, the study offers valuable insights into the trade-off between control horizon length and performance, identifying optimal parameter settings to maintain responsiveness without sacrificing stability.

This method addresses several existing challenges in autonomous vehicle coordination, such as the need for precise synchronization and adaptability under dynamic conditions. By seamlessly integrating adaptation laws into the MPC framework, the proposed approach provides a scalable solution applicable to various multi-vehicle scenarios, from ground vehicle convoys to collaborative UAV missions, thereby advancing the field of autonomous systems and multi-agent control.

## 3. RELATED WORKS

The study of leader-follower dynamics in the context of Model Predictive Control (MPC) has garnered significant attention, with various approaches explored to enhance formation control and trajectory tracking. This literature survey categorizes the reviewed works based on their specific focus areas, such as obstacle avoidance, energy efficiency, composite control strategies, and consensus control. This classification facilitates a structured discussion and critical review of the existing research.

The field of model predictive control (MPC) has been extensively explored across diverse applications, including renewable energy systems and chemical process control. Macit and Vural [1] presented a comparative study of sliding mode control, model predictive control, and PI control for a 1 MW grid-connected photovoltaic (PV) system. Their work demonstrated the superior performance of MPC in handling grid-connected PV systems, emphasizing the advantages of predictive capabilities in optimizing energy output and ensuring system stability under varying conditions. Similarly, Naregalkar and Subbulekshmi [2] focused on designing a nonlinear model predictive controller for a pH neutralization process. They introduced a Laguerre Hammerstein model identification method using least square support vector machines, highlighting the effectiveness of MPC in managing the nonlinearity and complexity inherent in chemical processes. This study underscored the potential of MPC in improving process control precision and adaptability, particularly in dynamic and sensitive applications. These contributions illustrate the versatility and robustness of MPC in handling diverse control problems, ranging from energy systems to chemical processes, further underscoring its potential in leader-follower formation control applications.

Several studies have concentrated on incorporating obstacle avoidance in leader-follower formations. Franze [3] addressed the obstacle avoidance motion planning problem for leader-follower vehicle configurations operating in static environments. Building on this, Franze et al. proposed a receding horizon control scheme based on set-theoretic ideas, tuned for agents described by linear time-invariant (LTI) systems subject to input and state constraints [4]. Xiao et al. [5] introduced a neural-dynamic optimization-based nonlinear MPC (NMPC) for leader-follower mobile robots, incorporating a separation-bearing-orientation scheme (SBOS) for regular formations and a separation-distance scheme (SDS) for obstacle avoidance. Zhao and Go [6] presented a hierarchical control strategy for leader-follower quadcopter formations to maintain specified formation shapes and avoid obstacles during flight, utilizing an MPC in the upper layer and a robust feedback linearization controller in the lower layer. Kuriki and Namerikawa [7] combined decentralized MPC with consensus-based control for collision avoidance in a multi-UAV system.

Energy efficiency in leader-follower formations is another key area of research. Yan et al. [8] aimed to extend the operation time and range of an electric-powered multi-agent system (MAS) by integrating battery-based energy awareness with distributed tracking control synthesis. He et al. [9] focused on fuel efficiency in vehicle platooning by developing a distributed economic MPC approach that improves fuel economy by using a fuel consumption criterion to design the control strategy.

The integration of multiple control strategies to enhance leader-follower formation control has been explored extensively. Lin et al. [10] proposed a composite control strategy combining MPC with adaptive terminal sliding mode control (ATSMC) for wheeled mobile robots (WMRs). Ma et al. [11] developed a leader-follower asymptotic consensus control strategy for linear MASs with unknown external disturbances and measurement noises, discussing preconditions like minimum phase condition (MPC) and observer matching condition (OMC). Qian et al. [12] addressed the formation control of underactuated autonomous underwater vehicles (AUVs) by proposing a consensus-based MPC with optimized line-of-sight guidance to handle input constraints and partial communication blockages.

Distributed MPC approaches have been widely investigated for their flexibility and robustness in leader-follower formations. Xu et al. [13] presented a distributed MPC-based formation control method for multi-UAVs in a leader-follower structure. Franze et al. [14] proposed a distributed MPC strategy for vehicles moving in an unknown obstacle scenario, incorporating receding horizon control arguments and leader-follower formations to design a flexible architecture. Pereira et al. [15] introduced a distributed MPC algorithm for spacecraft formation flying, addressing the cooperative nature of the system and considering a relative translational model for connecting different agents. Sun et al. [16] focused on formation collision avoidance for unmanned surface vehicles (USVs), highlighting the need for better responsiveness and stability compared to general ship formations.

Numerous studies have explored leader-follower formation control using MPC, addressing the challenges of trajectory tracking, collision avoidance, and communication constraints. Below are key contributions from recent works. Li et al. [17] focused on vision-based tube model predictive control for multi-robot formations: The proposed controller uses image information from an uncalibrated perspective camera mounted on the follower robot, eliminating the need for position measurement or communication among robots. This ensures robustness against model uncertainties. The system maintains visual alignment with the leader through predictive control, which allows the formation to remain stable during abrupt maneuvers and environmental disturbances. This research highlights the importance of vision-based control without communication, which aligns with our goal to develop communication-free control strategies. Ferraz and Hespanha [18] introduced distributed leader-follower control using iterative algorithms: Two iterative algorithms are presented for distributed leader-follower MPC, each using 1-hop or 2-hop neighbor information. The 2-hop approach guarantees optimal convergence to the solution by sharing data among further nodes, while the 1-hop approach balances between performance and communication cost. This framework demonstrates resilience to initial conditions and parameter deviations, making it suitable for large-scale networked robotic systems. Lim et al. [19] applied nonlinear MPC to unmanned ground vehicles (UGVs) to ensure precise control during complex maneuvers: This paper investigates the use of nonlinear MPC for controlling a fleet of unmanned ground vehicles (UGVs) in leader-follower formations. The strategy ensures that the follower UGV maintains trajectory alignment while the leader executes complex maneuvers. By continuously solving the optimization problem online, the framework achieves robust collision avoidance and smooth trajectory tracking under dynamic conditions.

The reviewed literature reveals significant advancements in leader-follower formation control across various domains. The integration of obstacle avoidance mechanisms, energy efficiency considerations, and composite control strategies demonstrates the versatility and robustness of MPC in handling complex scenarios. However, several gaps and challenges remain.

While numerous studies have proposed effective obstacle avoidance strategies, the integration of these strategies into a unified framework that can handle dynamic and unpredictable environments remains an area for further research. Research on energy efficiency and fuel optimization has shown promising results, but the trade-offs between energy consumption and control performance need more comprehensive analysis. The combination of different control strategies, such as MPC with ATSMC or consensus control, offers enhanced performance. However, the complexity of these integrated approaches necessitates further investigation into their scalability and real-time applicability. Distributed MPC has proven effective in various applications, from multi-UAV systems to spacecraft formations. Future research should focus on improving the robustness and fault tolerance of these systems, especially in highly dynamic and uncertain environments.

This paper contributes to the field by addressing the leader-follower problem within the MPC framework, introducing a novel adaptation law to adjust the weighting matrix of the MPC controller. This approach ensures that the follower approaches the leader in the tangent plane manifold by prioritizing the heading angle error. The proposed control strategy is validated across diverse scenarios, such as straight-line movements, circular trajectories, and S-shaped paths, demonstrating its effectiveness in achieving precise and adaptable formation control.

The novelty of this paper lies in the adaptation law that dynamically adjusts the MPC weighting matrix, which is a unique contribution not extensively covered in the existing literature. This method enhances the flexibility and precision of leader-follower formation control, making it a significant addition to the current body of research.

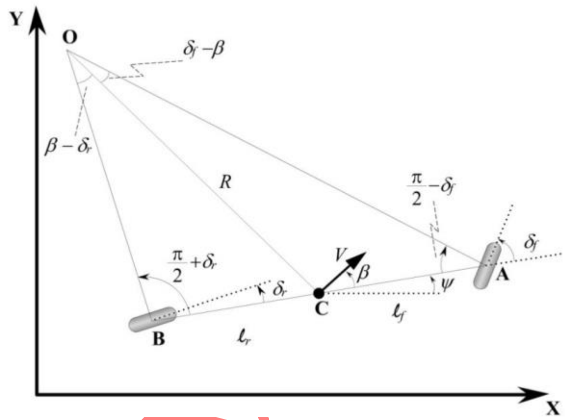


***Figure 1.** Kinematics of lateral vehicle motion*

## 4. PROBLEM STATEMENT

The advent of autonomous vehicles and multi-agent systems has catalyzed growing interest in the leader-follower problem, a fundamental challenge in the coordination and control of dynamic systems. This study elucidates the intricate dynamics inherent in the interaction between a leader and a follower vehicle. The scenario considered involves a leader vehicle with a distinctive differential dynamic model, leading a follower vehicle with a different but well-defined differential dynamic model. The follower has full access to the leader's state information, enabling precise and informed control.

The leader and follower vehicles have distinct differential dynamic models, requiring a tailored control strategy to maintain formation. Adjusting the MPC weighting matrix in real-time to prioritize the heading angle error while responding to the leader's maneuvers. The leader's execution of complex maneuvers, such as lane changes, abrupt heading angle alterations, and sudden shifts in linear velocity, poses challenges for the follower in maintaining formation. Investigating the leader-follower formation control across various scenarios, including straight-line movements, circular trajectories, and intricate S-shaped paths.

This study addresses the multifaceted challenges posed by the leader-follower problem in dynamic scenarios. Specifically, it aims to develop a control strategy that ensures the follower vehicle can accurately and adaptively follow the leader vehicle through a variety of maneuvers, maintaining a stable and coherent formation. Proposed Roadmap for Solution:

- Propose a novel adaptation law to adjust the weighting matrix of the MPC controller, ensuring that the heading angle error is weighted more than other states, enabling precise alignment with the leader's trajectory.

- Integrate the adaptation law into the MPC framework, allowing the follower to respond dynamically to the leader's maneuvers.
- Systematically investigate the control strategy across diverse scenarios, including straight-line movements, circular trajectories, and intricate S-shaped paths, to evaluate its effectiveness.
- Conduct comprehensive analyses to assess the control strategy's performance in maintaining formation accuracy and stability under varying conditions.
- Provide valuable insights into the leader-follower dynamics and the effectiveness of the proposed control strategy, contributing to the broader understanding of autonomous systems and their potential applications.

## 5. EXPLICIT MODEL FOLLOWING WITH MPC

Model Predictive Control (MPC) is a sophisticated control framework that utilizes a dynamic optimization process to predict and control system behavior over a future time horizon. It has gained prominence due to its ability to handle multi-input, multi-output (MIMO) systems while adhering to real-world constraints on control inputs and states. MPC operates by iteratively solving an optimization problem at each control step, making it suitable for complex, time-varying environments such as vehicle formation control, robotics, and process industries. This section provides a comprehensive review of MPC to build a foundation for understanding the adaptations proposed in this paper.

At each control step, MPC uses a mathematical model of the system to forecast future system states based on the current state, control inputs, and disturbances. The controller selects control actions that minimize a cost function while satisfying system constraints. The main idea is to optimize future control inputs across a finite prediction horizon but only apply the first action. This process repeats at every time step with the prediction horizon shifting forward, leading to the name receding horizon control.

A state-space model is generally used to represent the system dynamics:

$$\dot{x} = f(x,u), \quad y = h(x,u) \tag{1}$$

where $x$ is the state vector, $u$ is the control input, and $y$ represents the output. In most applications, linear approximations of the dynamics are used to simplify the optimization problem:

$$\dot{x} = Ax + Bu\,. \tag{2}$$

The state-space matrices $A$ and $B$ describe how the system evolves with respect to the current states and inputs.

MPC aims to minimize a quadratic cost function that penalizes deviations from desired states (tracking error) and excessive control effort. A typical form of the cost function over a prediction horizon $N$ is:

$$J = \sum_{k=0}^{N} \left( \| y_k - y_{\text{ref}} \|_Q^2 + \| u_k \|_R^2 \right) \tag{3}$$

where $y_k$ is the predicted state, $y_{\text{ref}}$ is the desired reference trajectory, $Q$ and $R$ are positive semi-definite weighting matrices penalizing state and input deviations, respectively

- $Q$ prioritizes state tracking, with higher weights ensuring that certain states (e.g., heading angle or position) are more strictly followed.
- $R$ penalizes aggressive or frequent changes in control inputs to avoid instability or wear on actuators.

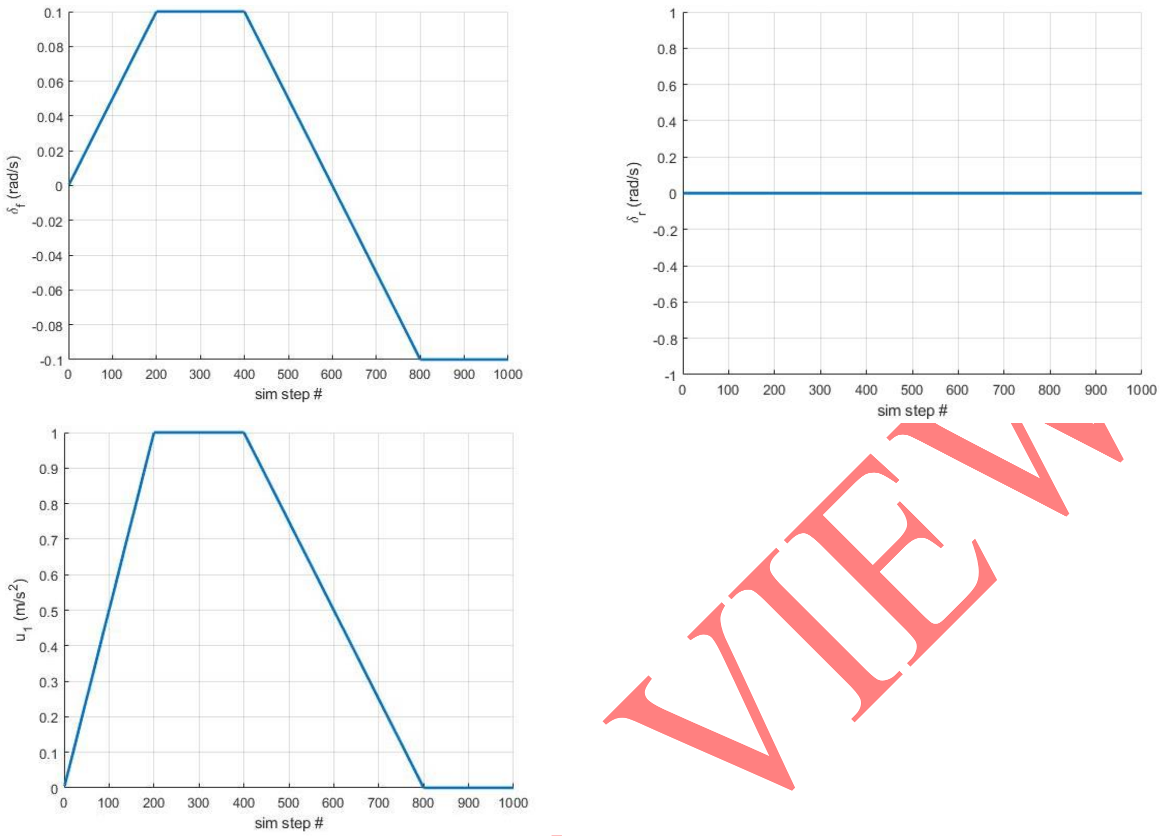


***Figure 2.*** *Leader-follower problem. Control signal* $\delta_f, \delta_r, u_1$ *respectively*

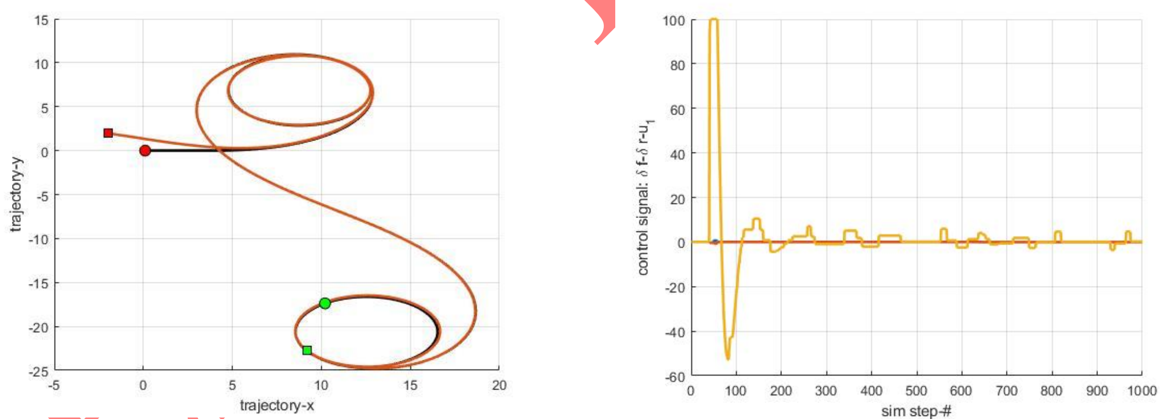


***Figure 3.*** *Leader-follower problem. Trajectory graph (Left Figure), follower control signal (Right Figure)*

This optimization balances two competing objectives: accurate tracking of reference trajectories and minimizing control effort. The weighting matrices $Q$ and $R$ play a critical role in shaping system behavior, which is a key aspect exploited by the adaptive mechanism introduced in this study.

One of MPC's strengths is its ability to explicitly handle constraints on states and control inputs. These constraints reflect real-world limits such as actuator saturation, speed limits, or safety margins:

$$u_{\min} \le u_k \le u_{\max}, \quad x_{\min} \le x_k \le x_{\max}. \tag{4}$$

Constraints ensure that the system operates safely and within feasible limits. MPC incorporates these constraints directly into the optimization problem, making it well-suited for autonomous systems like the leader-follower formation control problem discussed in this paper.

In scenarios involving rapid or large changes (e.g., abrupt speed or direction changes), handling constraints efficiently is crucial. This article extends standard MPC by dynamically adapting weighting matrices, ensuring that the controller remains effective even when the system approaches constraints.

MPC's core mechanism revolves around receding horizon optimization. At each control step, the controller predicts the system's behavior over a future horizon $N$. It solves an optimization problem that provides an optimal sequence of control inputs. However, only the first control action in the optimized sequence is applied, and the process is repeated at the next time step. This allows MPC to react to changing conditions by continuously updating predictions.

This feedback mechanism makes MPC robust to disturbances and model inaccuracies. The prediction horizon $N$ and the control horizon (the number of future inputs to optimize) are critical parameters that affect performance. If the prediction horizon is too short, the controller may fail to anticipate long-term effects. On the other hand, a long horizon increases computational complexity.

While traditional MPC provides a powerful framework for control, its performance can degrade if system conditions change rapidly or unpredictably. Adaptive MPC addresses this by adjusting key parameters-particularly the weighting matrices $Q$ and $R$-in real-time. This allows the controller to prioritize different objectives dynamically depending on the scenario.

For example, when a follower vehicle in a leader-follower formation encounters large tracking errors, the adaptation mechanism proposed in this article increases the weight on the state error (through an increase in $Q$). Conversely, when the follower is closely aligned with the leader's trajectory, the weight on control effort (through $R$) is increased to conserve energy and avoid unnecessary corrections.

The dynamic adaptation law introduced in this study refines the standard MPC framework by:

1. Scaling the weighting matrix $Q$ based on real-time tracking error thresholds.
2. Modulating control effort to maintain stability without excessive input changes.
3. Ensuring smooth trajectory tracking by emphasizing heading angle alignment, which is critical for the nonlinear vehicle dynamics involved in leader-follower scenarios.

MPC involves solving an optimization problem at every time step, which can be computationally intensive, especially for systems with long horizons or complex constraints. For real-time applications like vehicle formation control, ensuring fast optimization is essential. Methods like quadratic programming (QP) and active set algorithms are commonly employed to solve the constrained optimization problems efficiently.

This article leverages the kinematic bicycle model to simplify vehicle dynamics, making the optimization problem more tractable for real-time implementation. The adaptation law proposed ensures that the optimization remains computationally feasible by dynamically adjusting only key parameters, rather than re-computing the entire model at each step.

MPC provides a versatile and powerful framework for real-time control of dynamic systems, particularly in multi-vehicle coordination. Its ability to predict future states, handle constraints, and optimize control inputs makes it ideal for leader-follower formation problems. However, conventional MPC can struggle with rapidly changing conditions, such as abrupt maneuvers or shifting goals.

The contribution of this article lies in the adaptive adjustment of the weighting matrix $Q$, which allows the MPC controller to respond effectively to variations in the leader's behavior. By emphasizing heading angle alignment and dynamically scaling control priorities, the proposed method enhances trajectory tracking and

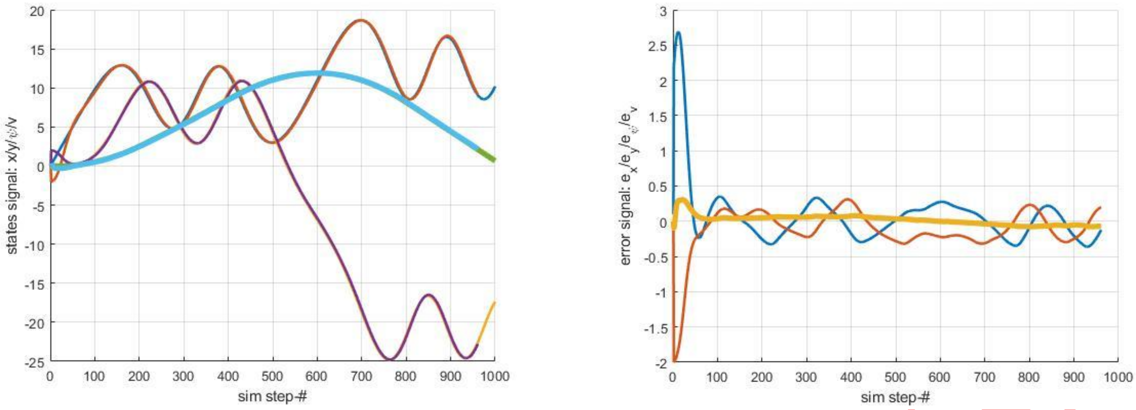


***Figure 4.*** *Leader-follower problem. Comparison of leader and follower states (Left Figure). Error signal for leader and follower states. (Right Figure)*

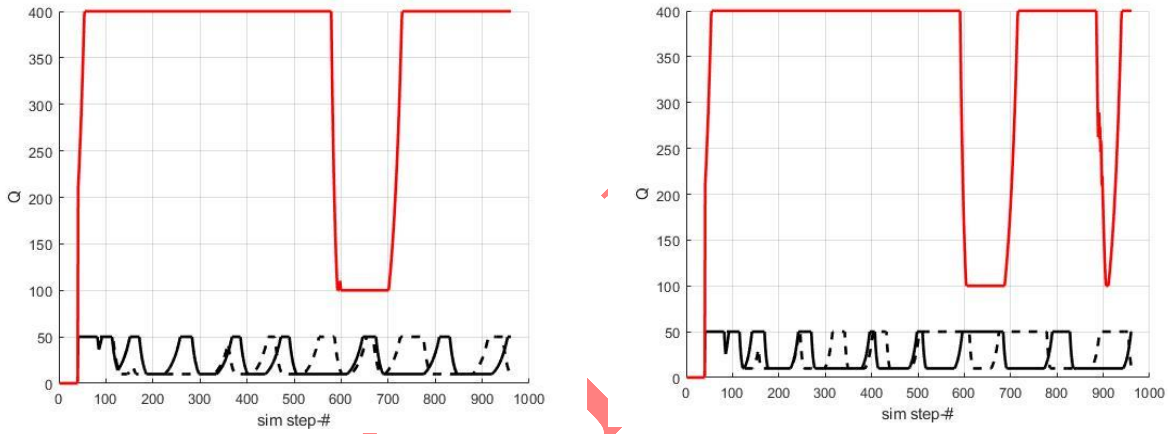


***Figure 5.*** *Leader-follower problem: Adaptive-Q:* $\epsilon_{x/y} = 0.1$ ; $\epsilon_{\psi} = 0.01$ ; $\alpha_{\psi} = 1.05/0.90$ ; $\alpha_{x/y} = 1.05/0.90$ *(Left Figure) and* $\epsilon_{x/y} = 0.1$ ; $\epsilon_{\psi} = 0.01$ ; $\alpha_{\psi} = 1.10/0.80$ ; $\alpha_{x/y} = 1.05/0.90$ *(Right Figure)*

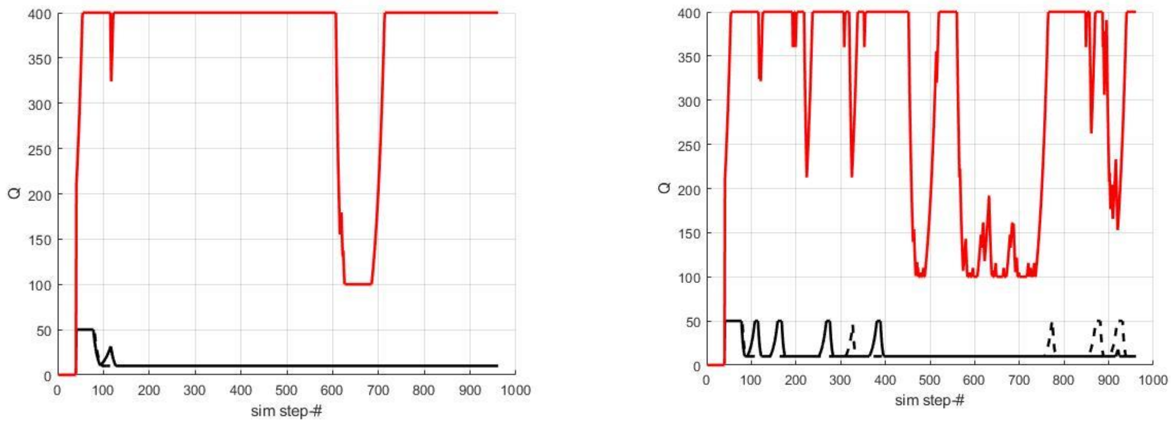


***Figure 6.*** *Leader-follower problem: Adaptive-Q* $\epsilon_{x/y} = 0.4$ ; $\epsilon_{\psi} = 0.01$ ; $\alpha_{\psi} = 1.10/0.80$ ; $\alpha_{x/y} = 1.05/0.90$ *(Left Figure) and* $\epsilon_{x/y} = 0.4$ ; $\epsilon_{\psi} = 0.01$ ; $\alpha_{\psi} = 1.05/0.90$ ; $\alpha_{x/y} = 1.05/0.90$ *(Right Figure)*

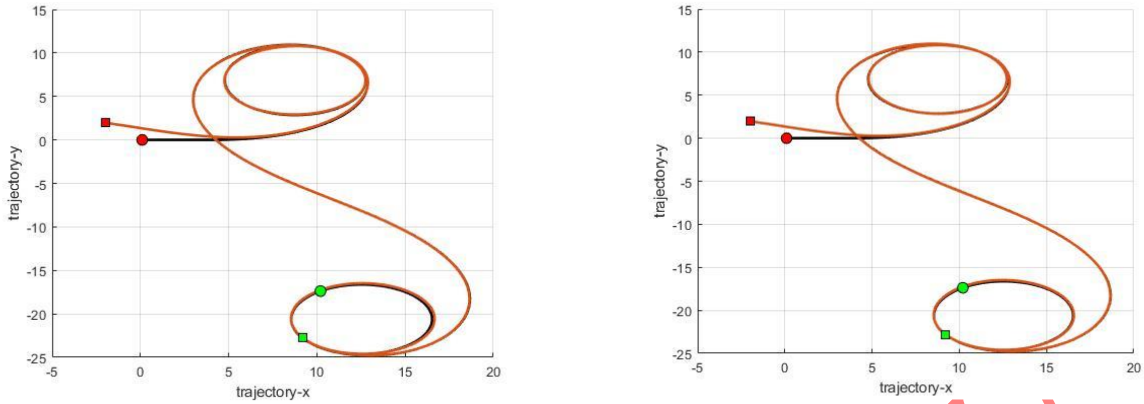


***Figure 7.*** *Leader-follower problem: Leader-Follower Trajectory* $\epsilon_{x/y} = 0.1$ *;* $\epsilon_{\psi} = 0.01$*;* $\alpha_{\psi} = 1.05/0.90$ *;* $\alpha_{x/y} = 1.05/0.90$ *(Left Figure) and* $\epsilon_{x/y} = 0.1$ *;* $\epsilon_{\psi} = 0.01$ *;* $\alpha_{\psi} = 1.10/0.80$ *;* $\alpha_{x/y} = 1.05/0.90$ *(Right Figure)*

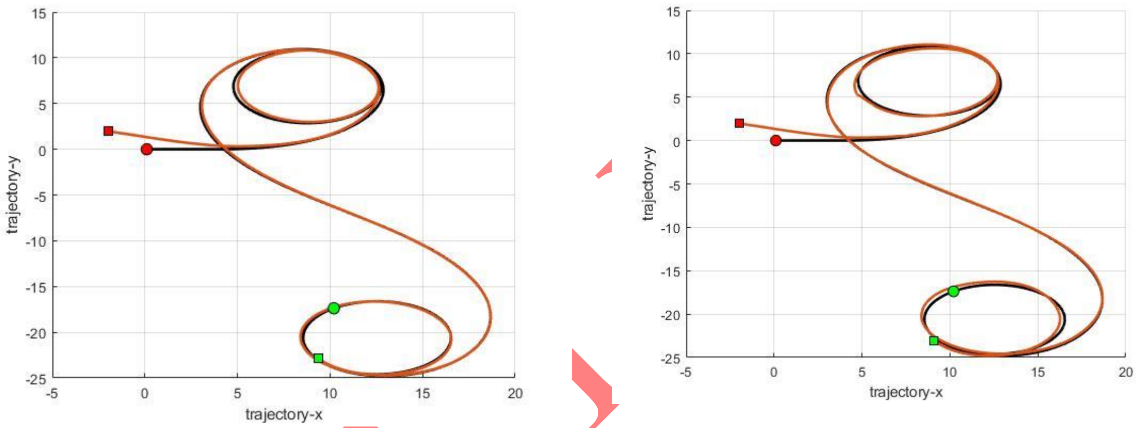


***Figure 8.*** *Leader-follower problem: Leader-Follower Trajectory* $\epsilon_{x/y} = 0.4$ *;* $\epsilon_{\psi} = 0.01$ *;* $\alpha_{\psi} = 1.05/0.90$ *;* $\alpha_{x/y} = 1.05/0.90$ *(Left Figure) and* $\epsilon_{x/y} = 0.4$ *;* $\epsilon_{\psi} = 0.01$ *;* $\alpha_{\psi} = 1.10/0.80$ *;* $\alpha_{x/y} = 1.05/0.90$ *(Right Figure)*

stability. This adaptive MPC strategy offers a novel and effective solution for maintaining precise formation control under real-world conditions, such as lane changes, circular paths, or sudden speed shifts.

Consider the nonlinear dynamic equations

$$\dot{x}(t) = f\left(x,u,t\right) \tag{5}$$

$$y(t) = h(x,u,t)\,. \tag{6}$$

Small perturbations from a nominal trajectory can be modeled by linear approximations. To do this both sides are expanded in Taylor series and terms beyond the first degree are neglected. The zeroth-degree term generates the nominal solution and the first-degree terms govern the perturbation solution. The nominal, nonlinear equations are

$$\dot{x}_o(t) = f\left(x_o(t),u_o(t),t\right) \tag{7}$$

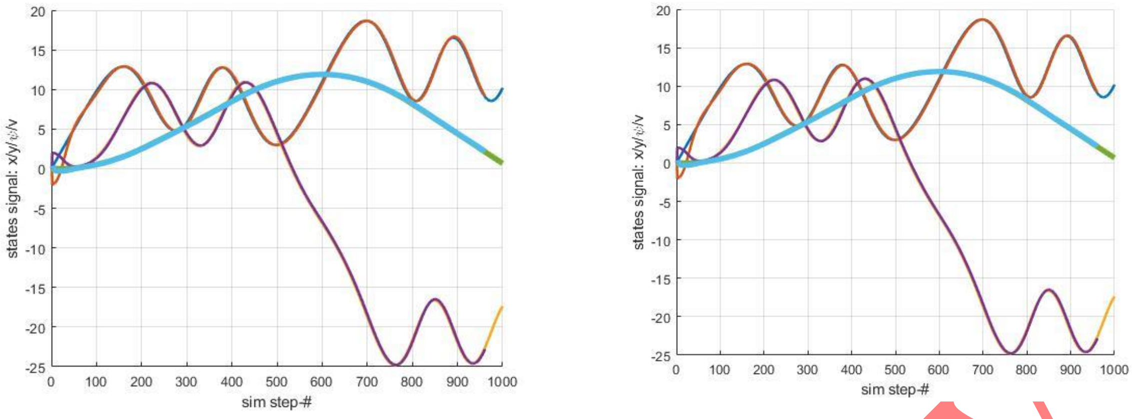


***Figure 9.*** *Leader-follower problem: Leader-Follower States* $\epsilon_{x/y}=0.1$ ; $\epsilon_{\psi}=0.01$; $\alpha_{\psi}=1.05/0.90$ ; $\alpha_{x/y}=1.05/0.90$ *(Left Figure) and* $\epsilon_{x/y}=0.1$ ; $\epsilon_{\psi}=0.01$ ; $\alpha_{\psi}=1.10/0.80$ ; $\alpha_{x/y}=1.05/0.90$ *(Right Figure)*

$$y_o(t)=h\left(x_o(t),u_o(t),t\right) \tag{8}$$

The associated linear time-varying equations that describe the perturbations from the nominal solution are

$$\Delta\dot{x}(t)=F_x(t)\Delta x(t)+F_u(t)\Delta u(t) \tag{9}$$

$$\Delta y(t)=H_x(t)\Delta x(t)+H_u(t)\Delta u(t) \tag{10}$$

where the Jacobian matrices are

$$\left\{F_x(t)=\frac{\partial f}{\partial x}\Rightarrow\left[F_x(t)\right]_{ij}=\frac{\partial f_i}{\partial x_j}\right\}_o \tag{11}$$

$$\left\{F_u(t)=\frac{\partial f}{\partial x}\Rightarrow\left[F_u(t)\right]_{ij}=\frac{\partial f_i}{\partial x_j}\right\}_o \tag{12}$$

$$\left\{H_x(t)=\frac{\partial h}{\partial x}\Rightarrow\left[H_x(t)\right]_{ij}=\frac{\partial h_i}{\partial x_j}\right\}_o \tag{13}$$

$$\left\{H_u(t)=\frac{\partial h}{\partial u}\Rightarrow\left[H_u(t)\right]_{ij}=\frac{\partial h_i}{\partial u_j}\right\}_o \tag{14}$$

$$\Delta x(t)=x(t)-x_o(t) \tag{15}$$

$$\Delta u(t)=u(t)-u_o(t) \tag{16}$$

$$\Delta y(t)=y(t)-y_o(t)\,. \tag{17}$$

The nonlinear model can be updated with the following first-order Euler integration. To update the constrained model predictive control law at each time step, it is necessary to minimize a quadratic cost function subject to a set of equality and inequality constraints. Typically, equality constraints depict the predicted state vector, which is a function of the known current state vector and the unknown control signal. These equality constraints can be eliminated by substituting them into the quadratic cost function. Consequently, it becomes evident that the quadratic cost function is contingent upon the unknown future control signal, necessitating optimization with respect to this unknown variable. Conversely, inequality constraints can be incorporated into the cost function through the use of Lagrange multipliers. Subsequently, a constrained optimization method must be applied to minimize the quadratic cost function.

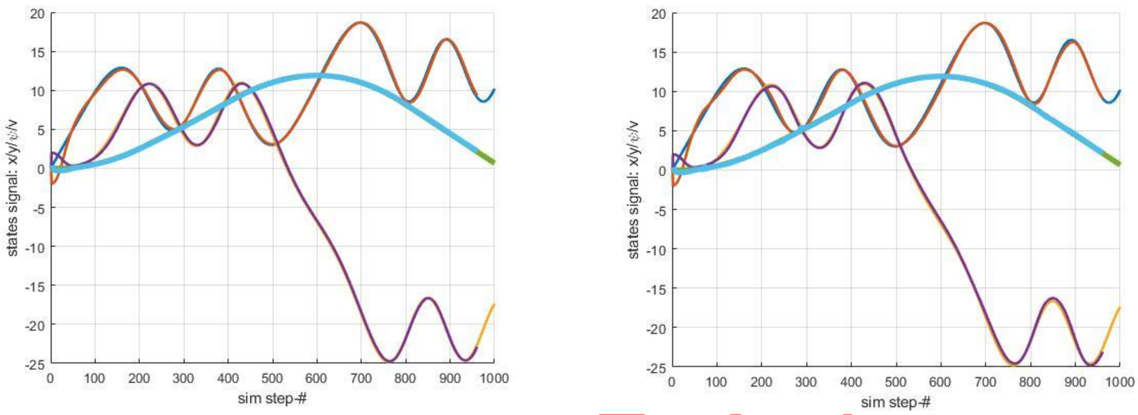


***Figure 10.*** *Leader-follower problem: Leader-Follower States* $\epsilon_{x/y} = 0.4$ ; $\epsilon_{\psi} = 0.01$ ; $\alpha_{\psi} = 1.05/0.90$ ; $\alpha_{x/y} = 1.05/0.90$ *(Left Figure) and* $\epsilon_{x/y} = 0.4$ ; $\epsilon_{\psi} = 0.01$ ; $\alpha_{\psi} = 1.10/0.80$ ; $\alpha_{x/y} = 1.05/0.90$ *(Right Figure)*

Explicit Model Following with Model Predictive Control (MPC) offers a powerful solution for the leader-follower problem in multi-agent systems. By utilizing explicit models that capture the interdependencies between agents and their environment, MPC enables each follower to predict and adapt its actions based on the anticipated behavior of the leader. This approach ensures coordinated motion among agents while respecting constraints and optimizing performance criteria. With explicit model following, MPC facilitates real-time decision-making, allowing followers to dynamically adjust their trajectories to maintain desired formations and achieve collective objectives, making it a promising strategy for a wide range of applications, including autonomous vehicle convoys, robotic swarms, and collaborative UAV missions.

For the explicit model following the truth reference is the leader model, so all linearization is made with respect to the follower state, control and output signals i.e. $x_o(t);u_o(t);y_o(t)$. It is assumed that the follower pursuits the leader with a constant time-delay denoted by $-D \equiv 1:D$. The linearized state-space model is obtained by the application of Equaions (5)-(17). The behavior of the system at the sampling times $t = kh$ is described by the difference equation

$$\Delta x_{k+(j+1)} = \Phi \Delta x_{k+j} + \Gamma \Delta u_{k+j} \tag{18}$$

$$\Delta y_{k+j} = \Delta x^r_{k+j} - \Delta x_{k+j} \tag{19}$$

$$\Delta x_{k+j} = x_{k+j} - x_{o,k} \tag{20}$$

$$\Delta u_{k+j} = u_{k+j} - u_{o,k} \tag{21}$$

$$\Delta x^r_{k+j} = x_{k-D} - x_{o,k} . \tag{22}$$

$\Delta x_k^r$ is the reference tracking signal that is supposed to be followed by the system states. The equivalent discrete-time model can be developed as follows:

$$\Phi = e^{F_x h} \quad \Gamma = \left( e^{F_x h} \int_0^h e^{-F_x \tau} d\tau \right) F_u . \tag{23}$$

If matrix $F_x$ is not invertible then series approximation of $e^{F_x \tau}$ can be used

$$\Gamma = \Phi \left[ I + \ldots + \frac{(-1)^{(k+1)}}{k!} F_x^{(k-1)} h^{(k-1)} \ldots \right] F_u h . \tag{24}$$

These linearized state-space model and the quadratic cost function indicates that the follower pursuit the leader by D -state late in discrete-time domain. The associated quadratic cost function is repeated below

$$J = \frac{1}{2} \sum_{j=1}^{N} \left\{ \Delta y_{k+j}^T Q \Delta y_{k+j} + \Delta u_{k+j}^T R \Delta u_{k+j} \right\} . \tag{25}$$

Subject to equality and inequality constraints, matrices Q and R are positive definite matrices of appropriate size, serving as penalization factors for the state and control signal, respectively. Combining the inequality constraints on incremental and absolute control signals, a unified inequality constraint set is defined as follows:

$$\mathbf{C} \Delta \mathbf{U}_k < \mathbf{d} \tag{26}$$

where

$$\mathbf{C} = \begin{bmatrix} \mathbf{I} \\ -\mathbf{I} \\ \mathbf{C}_{I/\Delta} \\ -\mathbf{C}_{I/\Delta} \end{bmatrix} \quad \mathbf{d} = \begin{bmatrix} \Delta \mathbf{U}_{max} \\ -\Delta \mathbf{U}_{min} \\ \mathbf{U}_{max} - \mathbf{u}_{k-1} \\ -\mathbf{U}_{min} + \mathbf{u}_{k-1} \end{bmatrix} . \tag{27}$$

## 6. LEADER-FOLLOWER FORMATION FOR BICYCLE MODEL OF A VEHICLE

The Kinematic Bicycle Model offers a simplified yet effective representation of vehicle dynamics, which is crucial for validating the Leader-Follower Model Predictive Control (MPC) framework. This model captures the essential aspects of vehicle motion, focusing on translational and rotational movements while abstracting away more complex dynamics. By employing the Kinematic Bicycle Model, the behavior of vehicles within the Leader-Follower MPC framework can be accurately simulated. This validation process ensures that the control strategy can effectively guide multiple vehicles in dynamic environments while adhering to safety constraints and achieving desired trajectories. The state and input vectors are defined as follows:

$$x = \begin{bmatrix} x & y & \psi & V \end{bmatrix}^T \quad u = \begin{bmatrix} \delta_f & \delta_r & u_1 \end{bmatrix}^T . \tag{28}$$

The nonlinear model of the vehicle is defined as:

$$\dot{x} = f(x, u) \tag{29}$$

where the nonlinear equations $f$ are defined as:

$$f = \begin{bmatrix} V\cos(\psi+\beta) \\ V\sin(\psi+\beta) \\ \dfrac{V\cos(\beta)}{l_r+l_f}\left(\tan(\delta_f)+\tan(\delta_r)\right) \\ u_1 \end{bmatrix} \tag{30}$$

where $\beta$ is defined as:

$$\beta = \arctan\left(\frac{l_f\tan(\delta_r)+l_r\tan(\delta_f)}{l_r+l_f}\right). \tag{31}$$

The sketch of the bicycle model and the related parameters are illustrated in Figure 1. The nonlinear dynamics of the bicycle model are linearized following standard procedures. The leader-follower problem with a constant time-delay $D$ is set up using the MPC problem framework defined in previous sections. Constraints on the control signal and time-domain specifications of state trajectories are outlined in Table 1. The linearized model of the follower is obtained as:

$$\Delta\dot{x}(t) = F_x(t)\Delta x(t) + F_u(t)\Delta u(t) \tag{32}$$

$$F_x(t) = \left.\frac{\partial f}{\partial x}\right|_o, F_u(t) = \left.\frac{\partial f}{\partial u}\right|_o. \tag{33}$$

The output model is defined as:

***Table 1.*** *Effect of Control Horizon on the Success of MPC Leader-Follower Control: e =* [ −0.20; 0.20; 0.01; 0.00 ]

| | e x 1 | e x 2 | e x 5 | e x10 |
|---|---|---|---|---|
| h=05 | X | X | X | X |
| h=10 | X | X | X | X |
| h=15 | O | O | O | X |
| h=20 | O | O | O | O |
| h=25 | O | O | O | O |

$$\Delta y(t) = \Delta x^r(t_o) - \Delta x(t) \tag{34}$$

$$\Delta x(t) = x(t) - x_o(t_o) \tag{35}$$

$$\Delta x^r(t-T) = x^r(t-T) - x_o(t) \tag{36}$$

$$\Delta u(t) = u(t) - u_o(t_o) \tag{37}$$

The fixed time delay $T$ represents a constant time delay in the continuous-time domain. Upon linearization of the nonlinear state equations $\dot{x} = f(x,u)$, the corresponding discrete-time state equations were derived as detailed earlier. The simulation parameters utilized are $h = 1\times10^{-2}$ seconds, and the vehicle dimensions are specified as $l_f = 0.2$ and $l_r = 0.2$. For the initial conditions, the leader's state is initialized as:

$$x = \begin{bmatrix} 0 & 0 & 0 & 10 \end{bmatrix}^T \tag{38}$$

while the follower's state is set as:

$$x = \begin{bmatrix} -2 & 2 & 0.1 & 0 \end{bmatrix}^T. \tag{39}$$

The weighting matrices chosen for the model predictive controller are:

$$Q = \mathrm{diag}\begin{bmatrix} 50 & 50 & 200 & 1\times10^{-2} \end{bmatrix} \tag{40}$$

$$R = \mathrm{diag}\begin{bmatrix} 1\times10^{-2} & 1\times10^{-2} \end{bmatrix}. \tag{41}$$

In the discrete-time setting, the fixed time delay is denoted as $D = 40$, and the prediction horizon for the model predictive controller is $N = 20$. The command signal for the leader vehicle is interpolated among the following control signal via points:

***Table 2.*** *Effect of Adaptation Law on the Success of MPC Leader-Follower Control:* $\epsilon_{x/y} = 0.1$ *and* $\epsilon_{\psi} = 0.01$

| | $\alpha_{\psi}$ =1.05/0.90 | $\alpha_{\psi}$ =1.10/0.80 |
|---|---|---|
| $\alpha_{x/y}$ =1.05/0.90 | O | O |
| $\alpha_{x/y}$ =1.05/0.80 | O | O |
| $\alpha_{x/y}$ =1.10/0.90 | O | X |
| $\alpha_{x/y}$ =1.10/0.80 | O | O |

$$\begin{array}{cccc} \# & \delta_f & \delta_r & u_1 \\ 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 10 \\ 3 & 1 & 0 & 10 \\ 4 & -1 & 0 & 0 \\ 5 & -1 & 0 & 0 \end{array} \tag{42}$$

The command signals are shown in Figure 2. A sophisticated heuristic strategy is employed to dynamically adjust the weighting matrices associated with the states $x, y, \psi$. The rationale behind this heuristic rule is to guide the follower vehicle to align its heading with the desired trajectory set by the leader and subsequently approach this reference path in a manner that closely resembles an exponential tangent trajectory.

***Table 3.*** *Effect of Adaptation Law on the Success of MPC Leader-Follower Control:* $\epsilon_{x/y} = 0.4$ *and* $\epsilon_{\psi} = 0.01$

| | $\alpha_{\psi}$ =1.05/0.90 | $\alpha_{\psi}$ =1.10/0.80 |
|---|---|---|
| $\alpha_{x/y}$ =1.05/0.90 | O | O |
| $\alpha_{x/y}$ =1.05/0.80 | O | O |
| $\alpha_{x/y}$ =1.10/0.90 | O | X |
| $\alpha_{x/y}$ =1.10/0.80 | O | X |

The heuristic rule is intricately designed to optimize the follower's behavior. Specifically, for the state $\psi$, the weighting matrix is adapted using a strategic approach that accounts for the alignment with the leader's trajectory. This alignment is pivotal in ensuring that the follower maintains a trajectory that smoothly converges towards the leader's intended path. The algorithm of the heuristic rule for state $x$ is defined as:

1. If $\epsilon_x > \epsilon_x^n$,

(a) $Q_x = Q_x \times \alpha_x^{\max}$

(b) $Q_x = min\ \{Q_x, Q_x^{max}\}$

2. Else

(a) $Q_x = Q_x \times \alpha_x^{\min}$

(b) $Q_x = \max\ \{Q_x, Q_x^{\min}\}$ .

This algorithm describes dynamic adaptation of weighting matrices in MPC. The goal is to optimize how the follower vehicle tracks the leader's path by adjusting the control matrix $Q_x$ based on specific conditions. It uses adaptive scaling factors to either increase or decrease control efforts based on predefined thresholds. This allows for real-time recalibration of the follower's trajectory and ensures alignment with the leader vehicle, particularly in situations involving sudden speed or directional changes.

*1. Condition 1: If the tracking error* $\epsilon_x$ *exceeds a specified threshold* $\epsilon_x^n$ *.*

*This condition suggests that if the error in the follower's position/state becomes large, the control effort needs to adapt to correct it rapidly.*

*- (a)* $Q_x = Q_x \times \alpha_x^{\max}$

*- The control matrix* $Q_x$ *is scaled up by a factor* $\alpha_x^{\max}$ *(greater than 1).*

*- This increases the importance of correcting errors, prioritizing fast convergence to the leader's trajectory.*

*- (b)* $Q_x = \min\{Q_x, Q_x^{\max}\}$

*- To prevent excessive control efforts, the matrix value is capped at a maximum allowable threshold* $Q_x^{\max}$ *. This ensures stability.*

*2. Else Condition: If the tracking error is within the acceptable range (i.e.,* $\epsilon_x \le \epsilon_x^n$ *).*

*This condition indicates that the follower is adequately tracking the leader, so the control effort can be relaxed.*

- *(a)* $Q_x = Q_x \times \alpha_x^{\min}$

  - *The control matrix* $Q_x$ *is scaled down by a factor* $\alpha_x^{\min}$ *(less than 1).*

  - *This reduces control effort when the follower performs satisfactorily, saving energy and avoiding unnecessary adjustments.*

- *(b)* $Q_x = \max\{Q_x, Q_x^{\min}\}$

  - *To maintain baseline performance, the control matrix is prevented from dropping below a minimum threshold* $Q_x^{\min}$.

This adaptive control mechanism plays a crucial role in the leader-follower formation problem. The parameters $Q_x^{\max}$ and $Q_x^{\min}$ along with the scaling factors $\alpha_x^{\max}$ and $\alpha_x^{\min}$ are designed to ensure smooth, real-time control adjustments.

Heading angle alignment is prioritized using similar adaptive mechanisms. This aligns the heading of the follower with the leader's trajectory to reduce non-linear effects. The adaptation law described optimizes performance while maintaining stability, enabling the follower to adjust seamlessly across various maneuvers (e.g., lane changes, turns, speed changes).

This algorithm ensures efficient and stable operation of autonomous vehicles by balancing precision and control effort dynamically, enhancing the overall performance of autonomous multi-vehicle systems.
The heuristic rule is meticulously defined for the $y$ state using parameters $Q_y^{min/max}$ and $\alpha_y^{min/max}$, encapsulating a sophisticated approach to trajectory optimization. In the simulated environment, specific values are assigned: $Q_x^{min} = Q_y^{min} = 10$ and $Q_x^{max} = Q_y^{max} = 50$. Furthermore, the parameters $\alpha_{x/y}^{max} = 1.05$ and $\alpha_{x/y}^{min} = 0.90$ refine the heuristic to ensure optimal trajectory adherence.

Similarly, the heuristic rule extends to the ψ state, incorporating distinct parameters: $Q_\psi^{min} = 100$ and $Q_\psi^{max} = 400$ , alongside $\alpha_\psi{}^{max} = 1.05$ and $\alpha_{psi}^{min} = 0.90$ . Notably, the weighting matrix for the $\psi$ state exceeds those for $x$ and $y$, reinforcing continuous alignment of the follower's heading with the trajectory, thereby facilitating a tangential approach to the leader's reference path.

This strategic emphasis on heading angle alignment significantly enhances stability. The heading angle is a pivotal factor introducing non-linearity into the model dynamics. Maintaining the heading angle within prescribed confidence intervals justifies the linearization of the vehicle's complex dynamics, ensuring robust and predictable behavior.

The simulation outcomes, summarized in Figures 3, provide compelling visual insights. Figure 3-a illustrates the trajectories of the leader and follower, from initial positions denoted by distinct symbols in red to final positions highlighted in blue. The follower's control signals are detailed in Figure 3-b, underscoring the effectiveness of the heuristic rule in guiding trajectory adherence. Figures 4 compares the state trajectories and associated errors, showcasing satisfactory performance and trajectory fidelity.

Next, we investigate the effect of the control horizon on the success of the MPC Leader-Follower control problem. We define an initial error $e = \left[-0.20\ 0.20\ 0.01\ 0.00\ \right]^T$ . We vary the control horizon in the range of 5, 10, 15, 20, and 25. The initial error is also scaled by multipliers of 1, 2, 5, and 10. Using these two ranges of variables, we form a matrix and test the controller across different control horizons and error ranges. The results are recorded in the matrix as "success" or "fail" and are presented in Table 1.

Table 1 shows that control horizons of 5 and 10 are insufficient for the MPC Leader-Follower control problem. A control horizon of 15 is marginally adequate, while control horizons of 20 and 25 are sufficient. Therefore, a control horizon of 20 is selected as the basis for further investigation of the MPC controller. The effects of adaptation parameters $\alpha_{x/y}$ and $\alpha_{psi}$ were also investigated.

A matrix was formed using these parameters, as presented in Tables 2 and 3. Table 2 displays results with parameters $\epsilon_{x/y} = 0.1$ and $\epsilon_{\psi} = 0.01$ , while Table 3 shows results with parameters $\epsilon_{x/y} = 0.4$ and $\epsilon_{\psi} = 0.01$ . These parameters represent the thresholds of the adaptation law.

Tables 2 and 3 indicate that a slowly varying adaptation is optimal. Additionally, a reasonably small threshold yields better results. Consequently, the adaptation parameters were set at $\alpha_{x/y} = 1.05/0.90$ and $\alpha_{\psi} = 1.05/0.90$ . The results of the adaptation law, specifically the adaptive $Q$ matrix for various adaptation parameters, are illustrated in Figures 5-6. These figures demonstrate that the $Q$ matrix for the heading angle is consistently larger than the $Q$ matrix for the x and y coordinates. Consequently, the tracking problem will be embedded in the tangent plane of the leader's trajectory. Since the heading is the primary source of nonlinearity, aligning the follower's heading angle with the leader's heading angle will simplify the tracking problem and minimize any nonlinearities arising from the leader/follower dynamic model.

Figures 7-10 illustrate the leader-follower control problem and compare the states of the leader and the follower. These graphical sketches clearly demonstrate the success of the adaptation law and its strategy. The follower approaches the leader within the tangent manifold, which enhances the stability of any inherent linearization operations performed in the analysis of the MPC leader-follower control problem. The comparisons of the states between the leader and the follower show a strong correspondence, confirming the effectiveness of the control strategy.

## 7. RESULTS AND DISCUSSION

### 7.1. Discussion on the Results

The results from the study confirm the effectiveness of the proposed Model Predictive Control (MPC) framework, enhanced by a novel adaptation law, in addressing the leader-follower formation problem. The control strategy was systematically evaluated under multiple trajectory scenarios—including straight-line paths, circular movements, and complex S-shaped trajectories—demonstrating its robustness and adaptability. Below are the key findings, supported by discussions on their significance.

The follower vehicle successfully tracked the leader’s trajectory in all tested scenarios, maintaining minimal state errors. The introduction of the dynamic adaptation law, which prioritized heading angle error, significantly improved formation accuracy. This was crucial when the leader executed abrupt changes, such as lane shifts or sudden velocity alterations. The follower’s alignment with the leader in the tangent plane manifold ensured smooth transitions even under non-linear dynamics. This discrete-time equation governed the system’s stability, ensuring the follower adapted quickly to leader maneuvers without excessive deviation.

Through simulations, it was found that control horizons $h = 20$ and $h = 25$ offered the best balance between responsiveness and stability. In contrast, shorter horizons such as $h = 5$ or $h = 10$ resulted in the system becoming unstable or unresponsive to leader actions. These findings are reflected in Table 1, where longer control horizons consistently yielded “success” across diverse conditions.

The quadratic cost function $J$ optimized the trade-off between state error and control effort, with positive-definite weighting matrices $Q$ and $R$. A control horizon of 20 was chosen for further simulations, providing optimal performance across varying scenarios.

The adaptive weighting matrix, which emphasized heading angle alignment, played a key role in stabilizing the formation. Simulations showed that: $\alpha_x = 1.05, 0.90$, $\alpha_\psi = 1.10, 0.80$ yielded the most consistent tracking results. Tables 2 and 3 illustrate how smaller adaptation thresholds ensured smooth transitions without introducing oscillations or overshooting. A gradual adaptation process was essential for maintaining stability, particularly during rapid changes in the leader's trajectory.

The use of the Kinematic Bicycle Model to simulate vehicle dynamics proved effective in capturing real-world behaviors. By focusing on translational and rotational movements, the model abstracted unnecessary complexities while maintaining key nonlinearities—particularly in heading angle alignment

$$\psi(t) = \arctan\left(\frac{l_r \tan(\delta_f) + l_f \tan(\delta_r)}{l_r + l_f}\right). \tag{43}$$

This alignment of heading angles ensured smooth formation tracking. The weighting matrix for the heading angle, consistently larger than those for $x$- and $y$-coordinates, mitigated potential nonlinearities that could arise from abrupt turns or speed changes.

Figures 7 to 10 show the close correspondence between leader and follower trajectories, validating the effectiveness of the proposed strategy. In challenging scenarios, such as circular and S-shaped paths, the follower vehicle was able to maintain precise alignment with the leader. The success of the strategy underscores the robustness of the MPC framework, which dynamically adjusted control inputs in real-time to match the leader's path. The results confirm that the integration of MPC with a dynamic adaptation law provides a highly effective solution for leader-follower formation control.

### 7.2. Stability and Robustness of the Proposed Adaptive MPC Algorithm

The proposed Model Predictive Control (MPC) framework in this study leverages a dynamic adaptation law that adjusts the control weighting matrices in real-time, with a particular focus on the heading angle alignment in the first plane. This design ensures that the system remains stable and robust across various dynamic scenarios, such as lane changes, circular trajectories, and sudden shifts in speed or direction. In this section, we explore the stability and robustness characteristics of the proposed adaptive MPC algorithm and how these qualities are achieved through the combination of real-time adaptation and prioritization of heading angle error.

A key component of the proposed algorithm is the real-time adaptation of the weighting matrix $Q$. The matrix $Q$ controls the relative importance of tracking different state variables (such as position and heading angle). In traditional MPC implementations, the weights are fixed, making it challenging to respond effectively to sudden or significant deviations from the desired trajectory. In contrast, the proposed algorithm introduces adaptive scaling factors that increase or decrease the elements of $Q$ based on the instantaneous tracking error.

- When tracking error is large, particularly in the heading angle, the algorithm increases the weights for the state variables, prioritizing fast convergence to the reference trajectory.
- When the tracking error is small, the weights are reduced, which relaxes the control effort, avoids overcorrection, and promotes energy efficiency.

This dynamic adaptation of weights ensures that the system remains stable under varying conditions. By continuously balancing between aggressive corrections and smooth control, the system avoids oscillatory or unstable behavior that could arise from abrupt changes in control inputs.

Formally, stability is ensured by regulating the error dynamics through the adaptive matrix updates:

$$Q_x(t+1)=\alpha_x^{\max} Q_x(t), \quad \text{if} \quad \epsilon_x > \epsilon_x^n \tag{44}$$

$$Q_x(t+1)=\alpha_x^{\min} Q_x(t), \quad \text{otherwise .} \tag{45}$$

This ensures that the closed-loop system behaves predictably, even under conditions of large tracking errors, and avoids excessive control actions when unnecessary.

The heading angle alignment plays a critical role in the robustness of the proposed MPC algorithm. The alignment between the leader and the follower's heading angle ensures that the follower vehicle remains closely aligned with the tangent plane of the leader's trajectory. This alignment is crucial because the heading angle introduces nonlinear dynamics into the system, making control more challenging if not carefully managed.

To handle these nonlinearities, the proposed algorithm prioritizes heading angle errors over other states when adjusting the control matrix. The adaptation law ensures that:

$Q_\psi > Q_x, Q_y$ where $Q_\psi$ corresponds to the weight associated with the heading angle error, and $Q_x, Q_y$ correspond to the weights for positional errors. By assigning a higher weight to the heading angle, the system ensures that the nonlinear effects caused by misalignment are corrected quickly. This prevents large deviations from the desired path, which could otherwise destabilize the system.

This strategy enhances robustness by minimizing the impact of external disturbances or sudden maneuvers (e.g., sharp turns or velocity changes). A well-aligned heading angle reduces the need for frequent or drastic positional corrections, improving the system's ability to remain stable over time.

The leader-follower formation control problem involves nonlinear dynamics, especially when the leader executes complex maneuvers. These dynamics are addressed by the adaptive weighting strategy, which ensures that the follower's trajectory is embedded in the tangent manifold of the leader's path. Aligning the follower's heading angle with the leader's trajectory simplifies the tracking problem, reducing nonlinear effects.

The adaptive MPC algorithm handles time-varying conditions (such as abrupt changes in speed or direction) by continuously recalculating optimal control actions based on updated predictions. The receding horizon approach ensures that the control actions are always based on the latest system state, while the adaptive matrix adjustment prevents the system from becoming overly sensitive to minor disturbances.

- For example, if the leader vehicle performs a sudden lane change, the follower can quickly increase the weight on the heading angle error, ensuring that the follower reorients itself efficiently without destabilizing the formation.
- Conversely, during steady-state operations, the control effort can be reduced by decreasing the weights on both heading and positional errors, ensuring smooth and energy-efficient performance.

This ability to adapt to changing conditions enhances robustness and ensures that the control system maintains performance across a wide range of scenarios, from straight-line paths to complex S-shaped trajectories.

The proposed adaptive MPC algorithm also ensures robust performance by managing constraints effectively. Constraints on both the states (such as the follower's position) and control inputs (such as steering angle or acceleration) are embedded within the optimization problem. The adaptation law dynamically adjusts the weighting matrices to prevent the system from approaching constraint limits too aggressively.

This feature ensures that:

- The follower vehicle remains within safe operating limits at all times.
- The control actions are smooth, even in the presence of input saturation or other physical constraints.

The robustness of the algorithm is further enhanced by the ability to adjust priorities dynamically. For instance, during a sudden maneuver, the system may momentarily relax some control constraints (e.g., limit on steering angle) to prioritize alignment with the leader. Once the maneuver is complete, the control effort is scaled back to prevent unnecessary input saturation.

The proposed adaptive MPC framework ensures asymptotic stability by constructing the control problem as a constrained quadratic optimization. As the adaptive weighting law ensures that the control problem remains convex, the solution to the optimization problem guarantees that the tracking error converges to zero over time.

Mathematically, the stability condition can be expressed as:

$$\lim_{t\to\infty} \| y(t) - y_{\text{ref}}(t) \| = 0 . \tag{46}$$

This convergence is achieved by:
1. Increasing the heading angle weight during large tracking errors to guarantee rapid re-alignment.
2. Reducing control efforts when the system is stable, preventing oscillations or unstable behaviors caused by excessive corrections.

The adaptive framework maintains feasibility of the optimization problem at every step by ensuring that the constraints remain satisfied under all operating conditions. As a result, the closed-loop system is both stable and robust, even under unpredictable environmental disturbances or leader vehicle maneuvers.

The proposed adaptive MPC algorithm offers significant stability and robustness enhancements by combining dynamic adaptation of weighting matrices with heading angle prioritization. Key contributions to stability and robustness include:

- Real-time adjustment of weights to ensure fast convergence to the desired trajectory without excessive corrections.
- Prioritization of heading angle alignment, which reduces nonlinear effects and ensures that the follower tracks the leader's path smoothly.
- Continuous recalculation of control actions in a receding horizon framework, ensuring responsiveness to time-varying conditions.
- Effective management of constraints, maintaining safe operation while optimizing performance.

## 8. CONCLUSIONS

This research delves into the intricate dynamics of the leader-follower problem, focusing on formation control strategies using Model Predictive Control (MPC). The study presents a scenario where a leader vehicle with a specific dynamic model guides a follower vehicle with a distinct dynamic model, with the follower having full access to the leader's state information. The proposed adaptation law dynamically adjusts the weighting matrix of the MPC controller, ensuring that the follower vehicle aligns its trajectory with the leader's in the tangent plane manifold. This approach emphasizes the heading angle error, promoting stable and precise formation control.

The Kinematic Bicycle Model effectively captures essential vehicle dynamics, facilitating the simulation of the Leader-Follower MPC framework. This model abstracts complex dynamics, focusing on translational and rotational movements. By employing this model, the simulation accurately replicates vehicle behavior within the Leader-Follower MPC framework. Simulation parameters include a time step $h = 1\times10^{-2}$

seconds, with vehicle dimensions $l_f = 0.2$ and $l_r = 0.2$. The initial conditions for the leader and follower states, and the weighting matrices $Q$ and $R$, are carefully selected to optimize performance.

Extensive simulations explore the leader executing maneuvers such as lane changes, heading angle adjustments, and sudden shifts in linear velocity, reflecting real-world scenarios. These maneuvers present significant challenges for maintaining formation, demonstrating the robustness of the proposed MPC strategy.

The leader-follower formation control problem is systematically investigated across various scenarios, including straight-line movements, circular trajectories, and complex S-shaped paths. This comprehensive analysis underscores the effectiveness of MPC and the adaptation law in achieving precise and adaptable formation control.

The dynamic adaptation of the weighting matrix $Q$ enhances the follower's ability to track the leader's trajectory closely. Emphasizing the heading angle error stabilizes the formation control and simplifies handling nonlinearities. The results indicate that a control horizon of 20 is optimal for the MPC Leader-Follower control problem, balancing responsiveness and stability. The adaptation parameters $\alpha_{x/y}$ and $\alpha_\theta$ significantly impact controller performance. A slow adaptation with small thresholds yields the best results, ensuring the follower's trajectory remains aligned with the leader's intended path. The leader's varied maneuvers in the simulation validate the MPC framework's robustness, demonstrating its capability to handle dynamic and complex scenarios.

This study significantly advances our understanding of leader-follower dynamics, showcasing the effectiveness of MPC in achieving precise and adaptable formation control. The research demonstrates that the Kinematic Bicycle Model, combined with a dynamic adaptation law for the MPC controller, provides robust performance in trajectory tracking under diverse conditions. The findings contribute valuable insights into autonomous systems, robotics, and multi-agent coordination, paving the way for further development and refinement of autonomous vehicle strategies in real-world applications. The proposed approach ensures that the follower maintains a stable and precise formation relative to the leader, enhancing the potential for practical implementation in autonomous vehicle convoys, robotic swarms, and collaborative UAV missions.

Future works can focus on enhancing the robustness and scalability of the leader-follower control strategies proposed in this study. One potential direction is the incorporation of real-time obstacle avoidance mechanisms to improve the framework's performance in dynamic and unpredictable environments. Additionally, further research can explore the integration of energy-efficient algorithms to optimize fuel consumption, especially for larger fleets or long-duration operations. Another avenue lies in expanding the application to heterogeneous multi-agent systems, such as combining aerial and ground vehicles, which would introduce new challenges in synchronization and control. Investigating the effects of communication delays and sensor noise on the system's stability is also crucial for real-world deployment. Finally, the adaptation law could be further refined using machine learning techniques to enhance its responsiveness under varied operational scenarios, paving the way for broader use in autonomous vehicles and robotic swarms.



## CONFLICTS OF INTEREST

No conflict of interest was declared by the author.

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