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Forecasting the Population of Türkiye Using Grey Models

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Abstract

Population forecasting plays a significant role in determining demography, economics, and agriculture policies for developing countries. In this study, we employ the five different grey prediction models to estimate the population of Türkiye up to the year 2050. These models are given as the grey standard (GM (1,1)), grey time-varying dynamic (GM $(1,1)_t$), grey Gompertz (GGM), grey Verhulst (GVM), and grey exponential EXGM (1,1). The comparison of grey models is evaluated by mean absolute percentage error (MAPE), regression coefficient (R2), variance ratio (C), and probability of error (P). As a first application, we use a split for training (2007-2017) and testing (2018-2022) periods using from 2007 to 2022 address-based data. The EXGM (1,1) model demonstrates superiority in the analysis of training dataset. The GGM and GM $(1,1)_t$ models provide the most suitable results in the analysis of testing dataset. As a second application, we use the dataset for the period between 2007 and 2022. The GGM and GM $(1,1)_t$ were identified as the most appropriate models for predicting the 2007-2022 period. For the future population forecasts from 2023 to 2050, the results of the five models are compared with the projection values of the Turkish Statistical Institute published in 2018. The GGM is determined to be the most compatible based on the MAPE value of 0.68116 and the C value of 0.05218, and the Grey Verhulst model is provided with the most compatible R^2 value of 0.99818. According to the GGM, the population of Türkiye is projected to reach 105,948,975 up to the year 2050, 106,877,632 based on the GM (1,1) t, and 102,591,471 based on the GVM.

Keywords Population forecasting, Grey systems, Grey prediction, GM (1,1), Time series

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1. Introduction

The term population can be defined as the total number of people residing in a specific area (continent, country, city, etc.) during a given period (Aksu, 1998; Güzel, 2018). Population is also a central theme across various scientific disciplines that inherently involve the human element, such as demography, sociology, psychology, medicine, geography, and economics. Furthermore, the population plays a crucial role in the existence and sustainability of nations or communities. The existence of a nation or community is directly related to its population (Çoban & İlyas, 2017). As societies evolve, both the quantity and quality of the population have become increasingly important considerations. A population that is self-improving, well-educated, young, and possesses a high level of welfare represents one of the most fundamental and crucial resources necessary for the successful completion of a country's economic development process (Yılmaz & Karadeniz, 2021).

Population projections are a method of estimating the number of people who will live in a specific area at a future point in time (Stoto, 1983). These projections play a pivotal role in economics, demography, agriculture, sociology, and sustainability policies. In this context, population forecasting plays a critical role in several fields, including demography, sociology, economics, public policy, and sustainability. A variety of methods are employed in population forecasting, including statistical models and machine learning. Some examples of population forecasting methods are extrapolation techniques which are commonly used by researchers (Ayhan Selçuk, 2014; Rayer et al., 2009; Smith & Sincich, 1992). Artificial intelligence methods are used to predict the population of countries (Fendoğlu, 2021; Riiman et al., 2019).

Grey prediction models have been applied in various fields such as energy, economics, environment, agriculture and population. Some examples of studies related to grey prediction models are given in Table 1. Accordingly, Hsu & Wen (1998) propose a new GM (1,1) model by modifying the standard grey model (GM (1,1)) model and applying the trans-Pacific air passenger market. The improved model outperforms the original GM (1,1), the multiple regression and ARIMA models. Wen & Huang (2004) adopted the Grey Verhulst model to predict the saturation number of the population in Taiwan. The model's ability to make predictions with limited data sets represents a significant advantage. Akay & Atak (2007) propose the grey Prediction with Rolling Mechanism (GPRM) model, utilizing it to predict data from Türkiye's industrial sector and total electricity consumption and the proposed model yields more accurate results than the model of analysis of the energy demand (MAED). Wang et al. (2007) propose a new grey Verhulst model by utilizing the trapezoidal formula in the whitening equation and using data on traffic volumes and the new grey Verhulst model possesses distinct advantages over the traditional grey Verhulst model (GVM). Mostafaei & Kordnoori (2012) propose a hybrid model, the Markov residual modified grey model (MRGMM), which integrates residual modification with Markov chains to minimize errors in the grey prediction model and the MRGMM have more accurate results than the GM (1,1) model. Evans (2014) proposes the generalized grey Verhulst model which does not exhibit the symmetrical characteristic of the traditional Verhulst model for analyzing the data of the steal intensity, and the generalized grey Verhulst model provides greater flexibility for data fitting. Ding et al. (2015) propose a new grey Verhulst model that has been modified and optimized for modelling the proportion of the non-agriculture population in Shanghai. As a result, the optimized Verhulst model contributes better prediction and simulation values than other Verhulst models. Yin et al. (2018) propose two variations of the GM (1,1) model: the EOGM (1,1) and the LGM (1,1) and conducted case studies along with the traditional GM (1,1). The EOGM (1,1) provided more successful outcomes than the other models for analyzing the GDP data of China's tertiary industry. Başakın et al. (2019) apply the GM (1,1) and rolling grey prediction models using water consumption data and these models give satisfactory results when the data is small. Bilgil (2021) propose a new exponential grey exponential model (EXGM (1,1)) aiming to improve the GM (1,1) model. The EXGM (1,1) model can be used with better accuracy for predicting COVID-19 or data inferring an ongoing trend of increasing case numbers. Şimşek & Ömürbek (2021) use the GM (1,1) and linear trend analysis to forecast tourism revenue and expenses for Türkiye. The GM (1,1) model provides more reliable results compared to those obtained from linear trend analysis. Akyüz & Bilgil (2022) apply the EXGM (1,1) and GM (1,1) to predict the research and development expenditures of Türkiye. The EXGM (1,1) model provides better results than the GM (1,1), which leads to the conclusion that the EXGM (1,1) model could provide higher prediction accuracy. Cai & Wu (2022) propose a new grey Gompertz model with new information priority accumulation, NIP-GGM (1,1), to analyze the current situation of carbon emissions. Liu et al. (2022) employ the Fractional GM (1,1), FGM (1,1) model to predict the influencing factors of carbon emissions. The fitting results of the FGM (1,1) model have a better fitting effect than the GM (1,1) model. Öztürk et al. (2022) propose the optimized continuous fractional grey model (OCCFGM (1,1)) for predicting domestic energy consumption, CO2 emissions, and water consumption. Zhang et al. (2023) propose the fractional-order discrete grey Gompertz model (FDGGM), and this model is suitable with good performance for predicting the ageing population. Ding & Dang (2023) consider novel flexible nonlinear multivariable discrete grey prediction model (FNDGM) for renewable energy generation prediction. Erdinc et al. (2024) propose a new exponential fractional grey model (ECFGM (1,1)) for China's wind energy consumption. Rathnayaka & Seneviratna (2024) use a hybrid grey exponential smoothing model (HGESM) for predicting aging population density. The findings of the literature review indicate that grey prediction models are a viable approach for addressing diverse real-world scenarios with limited data sets. Therefore, the models can also enhance the precision of population forecasting.

Table 1. Grey prediction models

Authors	Content	Methods
Hsu & Wen (1998)	Trans-pacific air passenger market	GM (1,1), modified standard GM (1,1), multiple regression, ARIMA
Wen & Huang (2004)	Population saturation numbers	Grey Verhulst
Akay & Atak (2007)	Electricity consumption	GPRM, MAED
Wang et al. (2007)	Traffic volumes	Grey Verhulst, a new grey Verhulst
Mostafaei & Kordnoori (2012)	Iran's total energy consumption	MRGMM, GM (1,1)
Evans (2014)	Steal intensity of use	Generalized grey Verhulst, grey Verhulst
Ding et al. (2015)	Non-agriculture population	Modified grey Verhulst
Yin et al. (2018)	Domestic tertiary industry GDP index	GM (1,1), EOGM (1,1), LGM (1,1)
Başakın et al. (2019)	Water consumption	GM (1,1), rolling GM (1,1)
Bilgil (2021)	Covid-19 deaths of Türkiye	EXGM (1,1), GM (1,1)
Şimşek & Ömürbek (2021)	Tourism revenue and expenses	GM (1,1), linear trend analysis
Akyüz & Bilgil (2022)	Research and development expenditures	EXGM (1,1), GM (1,1)

Authors	Content	Methods
Cai & Wu (2022)	Carbon emission	NIP-GGM (1,1)
Liu et al. (2022)	Carbon peak	FGM (1,1), GM (1,1)
Öztürk et al. (2022)	Domestic energy consumption, CO2 emissions, water consumption	OCCFGM
Zhang et al. (2023)	Population aging	FDGGM
Ding & Dang (2023)	Renewable energy generation prediction	FNDGM
Erdinc et al. (2024)	China's consumption of wind energy	ECFGM (1,1)
Rathnayaka & Seneviratna (2024)	Aging population density	HGESM

The contribution of this study examines the population forecasting of Türkiye using the grey prediction models and provides an opportunity to evaluate the results with multiple options. For this aim, we use the data portal of the Turkish Statistical Institute (TurkStat). The dataset is collected from the TurkStat based on the address-based population registration system. This dataset covers the period between 2007 and 2022. The analysis is conducted using the five different grey prediction models, which are known for their effectiveness in forecasting with limited data and two applications using data partitions.

The rest of this paper is organized as follows. In Section 2, the standard GM (1,1), grey time-varying dynamic model (GM $(1,1)_t$), GGM, GVM, and EXGM (1,1) are explained, and the comparison criteria are given. Section 3 provides two applications using data partitions for the grey models and presents the model comparison. Section 4 includes some concluding remarks.

2. Grey System Theory

Grey system theory (GST) was initially proposed by Prof. Dr. Deng Ju Long in 1982 (Deng, 1982; 1989). GST has widespread applications including healthcare service quality (Aydemir & Sahin, 2019), electricity consumption (Liu et al., 2020), and sustainable development (Javanmardi et al., 2023) for analysis and forecasting. In the system, information that is fully known is referred to as "White," information about which nothing is known is called "Black," and information that is partially known is termed "Grey" (Aydemir et al., 2013; Liu & Lin, 2006; Xiao et al., 2008). The name derives from the characterization of data in these colors. The fundamental principle of the grey model is based on the greying of black data with the help of white data (Başakın et al., 2019). The main objective of grey theory is to reduce randomness and bring characteristic analysis to systems that appear complex and random.

2.1. Grey Standard Model (GM (1,1))

GM (1,1) is a widely used model among grey models (Zhang et al., 2009). GM (1,1) is particularly suitable for time series that show specific exponential growth trends (Liu et al., 2017). The model is effective in cases where we can obtain data in time series and create restricted datasets. It predicts future values based on past or present data. Some applications of different areas of the GM (1,1) are given as Wen & Huang (2004), Eren & Kaçtıoğlu (2017), Küçükerdem Öztürk & Saplıoğlu (2023) and Zhang et al. (2023).

The steps of the GM (1,1) model are as follows (Liu & Lin, 2006):

Step 1. Identify the data set (raw data set) to be used by the prediction model

$$X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(n)\right), \quad n \geq 4. \tag{1}$$

Step 2. Apply the accumulation generation operation (AGO) on the $X^{(0)}$ data set to obtain the $X^{(1)}$ data set.

$$\begin{gathered} X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), ..., x^{(1)}(n)\right) \text{ and} \\ x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \quad k = 1, 2, ..., n. \end{gathered} \tag{2}$$

Step 3. Generate the $Z^{(1)}$ data set

$$Z^{(1)} = \left(z^{(1)}(1), z^{(1)}(2), ..., z^{(1)}(n)\right) \tag{3}$$

using the following quantity

$z^{(1)}(k) = \frac{1}{2}(x^{(1)}(k) + x^{(1)}(k-1)$

$z^{(1)}(k) = x^{(1)}(k)$

Step 4. Create the Y and B matrices.

$$Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \tag{4}$$

Step 5. Determine the values of a and b,

$$[a, b]^T = \left(B^T B\right)^{-1} B^T Y \tag{5}$$

Step 6. The GM (1,1) whitening equation can be constructed using a first-order linear differential equation, which is provided below

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b \tag{6}$$

Eq. 6 is solved to obtain the values of $x^{(1)}(k)$.

Step 7. Obtain the cumulative forecast values.

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-ak} + \frac{b}{a} \tag{7}$$

Step 8. Obtain the predicted values

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k)$$
$$\hat{x}^{(0)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-ak}(1-e^{a}) \quad (8)$$
$$\hat{x}^{(0)}(1) = x^{(0)}(1)$$

2.2. Grey Time-Varying Dynamic Model (GM $(1,1)_t$)

The GM $(1,1)_t$ was designed by Wang Z.L. as a type of time-varying model, proposed in 2002 and previously discussed in his 1998 thesis, (cited by Wang et al. (2008), (Wang, 1998; 2002)).

The steps of the model are as follows (Wang et al., 2008):

Step 1. Identify the raw data set, which must consist of at least four data points.

$$X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\right), \quad n \geq 4 \quad (9)$$

Step 2. Apply the AGO to the raw data set $X^{(0)}$,

$$X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\right) \quad (10)$$

Step 3. Generate the $Z^{(1)}$ data set,

$$Z^{(1)} = \left(z^{(1)}(1), z^{(1)}(2), \ldots, z^{(1)}(n)\right) \quad (11)$$

Here:

$z^{(1)}(k) = \frac{1}{2}\left(x^{(1)}(k) + x^{(1)}(k-1)\right)$

$z^{(1)}(k) = x^{(1)}(k)$

Step 4. Create the y_n and B matrices

$$y_n = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, B = \begin{bmatrix} -z^{(1)}(2) & 2-0.5 & 1 \\ -z^{(1)}(3) & 3-0.5 & 1 \\ \vdots & \vdots & \vdots \\ -z^{(1)}(n) & n-0.5 & 1 \end{bmatrix} \quad (12)$$

Step 5. Determine the values of a, b and c,

$$[a, b, c]^T = (B^T B)^{-1} B^T y_n \quad (13)$$

Step 6. The whitening equation for the GM $(1,1)_t$ model is given as follows

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = bt + c \quad (14)$$

Step 7. Obtain the cumulative predicted values by the whitening equation:

$$\hat{x}^{(1)}(k+1) = \frac{b}{a}(k+1) - \frac{b-ac}{a^2} + \left(x^{(0)}(1) - \frac{b}{a} + \frac{b-ac}{a^2}\right)e^{-ak},\ k = 1, 2, ..., n-1$$
$$\hat{x}^{(1)}(1) = x^{(0)}(1) \tag{15}$$

Step 8. Obtain the prediction values:

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k)$$
$$\hat{x}^{(0)}(k+1) = \frac{b}{a} + e^{-ak}(1-e^{a})\left(x^{(0)}(1) - \frac{b}{a} + \frac{b-ac}{a^2}\right),\ k = 1, 2, ..., n-1 \tag{16}$$
$$\hat{x}^{(0)}(1) = x^{(0)}(1)$$

2.3. Grey Gompertz Model (GGM)

The GGM is developed based on Gompertz's law. The Gompertz function was first introduced by Benjamin Gompertz (Gompertz, 1825). The GGM is highly compatible with the GM $(1,1)_t$ model. The GGM can be developed using the GM $(1,1)_t$.

The steps of the GGM are provided below (Wang et al., 2008).

Step 1. Identify a raw data set $X^{(0)}$ consisting of at least four data points.

$$X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(n)\right),\ n \geq 4, \text{and}$$
$$X^{(0)}(k) \geq 0,\ k = 1, 2, ..., n. \tag{17}$$

Step 2. Apply a logarithmic operation on the data set $X^{(0)}$ to obtain the data set $y^{(0)}$,

$$y^{(0)} = \left(y^{(0)}(1), y^{(0)}(2), ..., y^{(0)}(n)\right) \tag{18}$$

Here:

$$y^{(0)}(i) = \ln\left(x^{(0)}(i)\right), \quad i = 1, 2, ..., n \tag{19}$$

Step 3. Apply the AGO on the data set $y^{(0)}(i)$ to obtain the data set $X^{(1)}$

$$X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), ..., x^{(1)}(n)\right) \tag{20}$$

Here:

$x^{(1)}(k) = \sum_{i=1}^{k} y^{(0)}(i),\ k = 1, 2, ..., n$

Step 4. Utilize the data set $X^{(1)}$ to derive the data set $Z^{(1)}$

$$Z^{(1)} = \left(z^{(1)}(1), z^{(1)}(2), ..., z^{(1)}(n)\right) \tag{21}$$

Here:

$z^{(1)}(k) = \frac{1}{2}\left(x^{(1)}(k) + x^{(1)}(k-1)\right)$

$z^{(1)}(k) = x^{(1)}(k)$

Step 5. Create the y_n and B matrices,

$$y_n = \begin{bmatrix} y^{(0)}(2) \\ y^{(0)}(3) \\ \vdots \\ y^{(0)}(n) \end{bmatrix}, B = \begin{bmatrix} -z^{(1)}(2) & 2-0.5 & 1 \\ -z^{(1)}(3) & 3-0.5 & 1 \\ \vdots & \vdots & \vdots \\ -z^{(1)}(n) & n-0.5 & 1 \end{bmatrix} \tag{22}$$

Step 6. Determine the values of a, b and c

$$[a,b,c]^T = \left(B^T B\right)^{-1} B^T y_n \tag{23}$$

Step 7. Obtain the prediction values

$$\begin{aligned} &\hat{x}^{(0)}(k+1) = \exp\left(\frac{b}{a} + \left(y^{(0)}(1) - \frac{b}{a} + \frac{b-ac}{a^2}\right)(1-e^a)e^{-ak}\right),\ k = 1,2,...,n-1 \\ &\hat{x}^{(0)}(1) = x^{(0)}(1) \end{aligned} \tag{24}$$

2.4. Grey Verhulst Model (GVM)

The Verhulst equation was proposed by the Belgian mathematician Pierre-François Verhulst through three articles published in French (Verhulst, 1838; 1845; 1847). In his 1845 article, Verhulst named the curve described by the function as “Logistic” (Verhulst, 1845). Verhulst countered the Malthusian view by suggesting that population growth would not continue exponentially; rather, the rate of increase would eventually slow down and approach zero, by introducing a limiting factor to the exponential growth model (İskender, 2018). The primary goal of the Verhulst model is to cap all development for the system. It is an effective model for describing processes that have a saturation zone, characterized by a sigmoid “S” curve (Kayacan et al., 2010). The Verhulst model is useful for processes that start with slow growth, accelerate, and then slow down to a saturated state where growth stops (Wang et al., 2007).

The steps of the GVM are given as follows (Ding et al., 2015; Duan & Luo, 2020; Liu et al., 2017).

Step 1. Identify the raw data set $X^{(0)}$ consisting of at least four data points.

$$\begin{aligned} &X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(n)\right), \quad n \geq 4 \text{ and} \\ &X^{(0)}(k) \geq 0,\ k = 1,2,...,n \end{aligned} \tag{25}$$

Step 2. Apply the Inverse Accumulated Generating Operation (IAGO) on the $X^{(0)}$ data set to obtain the $\tilde{X}^{(0)}$ sequence,

$$\tilde{X}^{(0)} = \left(\tilde{x}^{(0)}(1), \tilde{x}^{(0)}(2), ..., \tilde{x}^{(0)}(n)\right) \tag{26}$$

Here:

$\tilde{x}^{(0)}(k) = x^{(0)}(k) - x^{(0)}(k-1),\ k = 1,2,...,n-1$

$\tilde{x}^{(0)}(1) = x^{(0)}(1)$

Step 3. Apply the AGO on the $\tilde{X}^{(0)}$ data set to obtain the $\tilde{X}^{(1)}$ data set,

$$\tilde{X}^{(1)} = \left(\tilde{x}^{(1)}(1), \tilde{x}^{(1)}(2), ..., \tilde{x}^{(1)}(n)\right) \tag{27}$$

Here:

$\tilde{x}^{(1)}(k) = \sum_{i=1}^{k} \tilde{x}^{(0)}(i),, \; k = 1, 2, ..., n$

Step 4. Obtain the $Z^{(1)}$ data set,

$$Z^{(1)} = \left(z^{(1)}(1), z^{(1)}(2), ..., z^{(1)}(n)\right) \tag{28}$$

Here:

$z^{(1)}(k) = \frac{1}{2}\left(\tilde{x}^{(1)}(k) + \tilde{x}^{(1)}(k-1)\right)$

$z^{(1)}(1) = \tilde{x}^{(0)}(1)$

Step 5. Create the B and Y matrices,

$$B = \begin{bmatrix} -z^{(1)}(2) & \left(-z^{(1)}(2)\right)^2 \\ -z^{(1)}(3) & \left(-z^{(1)}(3)\right)^2 \\ \vdots & \vdots \\ -z^{(1)}(n) & \left(-z^{(1)}(n)\right)^2 \end{bmatrix}, Y = \begin{bmatrix} \tilde{x}^{(0)}(2) \\ \tilde{x}^{(0)}(3) \\ \vdots \\ \tilde{x}^{(0)}(n) \end{bmatrix} \tag{29}$$

Step 6. Determine the values of a and b,

$$[a, b]^T = \left(B^T B\right)^{-1} B^T Y \tag{30}$$

Step 7. The linear differential equation for the GVM is provided below,

$$\frac{d\tilde{x}^{(1)}}{dt} + a\tilde{x}^{(1)} = b\left(\tilde{x}^{(1)}\right)^2 \tag{31}$$

Step 8. Obtain the prediction values.

$$\begin{aligned} \tilde{x}^{(1)}(k+1) &= \frac{a\tilde{x}^{(1)}(1)}{b\tilde{x}^{(1)}(1) + (a - b\tilde{x}^{(1)}(1))e^{ak}}, \quad k = 1, 2, ..., n-1 \\ \tilde{x}^{(1)}(1) &= x^{(0)}(1) \end{aligned} \tag{32}$$

2.5. Grey Exponential Model (EXGM (1,1))

The EXGM (1,1) model was proposed by Bilgil (2021) as a new model based on exponential characteristics to extend the GM (1,1) and EOGM (1,1) models previously suggested by Yin et al. (2018). In the standard GM (1,1) model, the "b" parameter is typically considered a constant, which results in low sensitivity and an increasing error margin over time (Yin et al., 2018). Additionally, the GM (1,1) is suitable for systems exhibiting semi-exponential growth (Ding et al., 2015). In the EXGM (1,1) model, a decaying term (*e*-*t*) is added to the differential equation to slow down the growth trajectory (Akyuz, 2022; Bilgil, 2021). The grey effect of the EXGM (1,1) is considered both as a constant and as an exponential function of time (Bilgil, 2021).

The steps of EXGM (1,1) are as follows (Bilgil, 2021):

Step 1. Identify the raw data set $X^{(0)}$

$$X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(n)\right), \quad n \geq 4 \ \text{ and} \\ X^{(0)}(k) \geq 0, \quad k = 1, 2, ..., n \tag{33}$$

Step 2. Apply the AGO on the $X^{(0)}$ data set to obtain the $X^{(1)}$ data set,

$$X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), ..., x^{(1)}(k)\right) \tag{34}$$

Here:

$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \ k = 1, 2, ..., n$

Step 3. Obtain the $Z^{(1)}$ dataset,

$$Z^{(1)} = \left(z^{(1)}(1), z^{(1)}(2), ..., z^{(1)}(n)\right) \tag{35}$$

Here:

$z^{(1)}(k) = \frac{1}{2}\left(x^{(1)}(k) + x^{(1)}(k-1)\right)$

$z^{(1)}(k) = x^{(1)}(k))$

Step 4. Create the B and Y matrices,

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 & (e-1)e^{-2} \\ \vdots & \vdots & \vdots \\ -z^{(1)}(3) & 1 & (e-1)e^{-3} \\ -z^{(1)}(n) & 1 & (e-1)e^{-n} \end{bmatrix}, Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \tag{36}$$

Step 5. Determine the values of a, b and c,

$$[a, b, c]^T = \left(B^T B\right)^{-1} B^T Y \tag{37}$$

Step 6. The linear differential equation for the EXGM (1,1) whitening equation is presented below

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b + ce^{-t} \tag{38}$$

Step 7. Obtain the cumulative prediction values, the series $\hat{x}^{(1)}(k)$:

$$\hat{X}^{(1)} = \left(\hat{x}^{(1)}(1), \hat{x}^{(1)}(2), ..., \hat{x}^{(1)}(k)\right) \tag{39}$$

Here:

$\hat{x}^{(1)}(k) = \left(x^{(0)}(1) - \frac{b}{a} - \frac{c}{a-1}e^{-1}\right)e^{a(1-k)} + \frac{b}{a} + \frac{c}{a-1}e^{-k}$

Step 8. Obtain the prediction values

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1),\ \ k = 2, 3, \ldots, n \\ \hat{x}^{(0)}(1) = x^{(0)}(1) \tag{40}$$

2.6. Evaluation criteria

The comparison of the grey prediction models is performed using absolute percentage error (APE), mean absolute percentage error (MAPE), R^2, variance ratio (C) and probability of error (P) criteria. The formulas of APE and MAPE criteria are as follows:

$$\mathrm{APE}(k) = 100 \cdot \left| \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \right|,\ \ k = 1, 2, \ldots, n, \tag{41}$$

$$\begin{aligned} \mathrm{MAPE} &= \frac{1}{n} \sum_{k=1}^{n} \mathrm{APE}(k) \\ &= \% \frac{100}{n} \sum_{k=1}^{n} \left| \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \right| \end{aligned} \tag{42}$$

The accuracy assessment based on the obtained the MAPE values is provided in Table 2.

Table 2. Accuracy Assessment of MAPE Values

Accuracy Assessment	MAPE Value	
	Minimum	Maximum
High	0%	10%
Good	10%	20%
Acceptable	20%	50%
Unacceptable	> 50%	

Source: Lewis (1982)

R^2 formula used in the study is provided below

$$R^2 = \frac{\left(\sum_{k=1}^{n} (x(k) - \overline{x}(k)) \cdot \left(\hat{x}(k) - \overline{\hat{x}}(k)\right)\right)^2}{\sum_{k=1}^{n} (x(k) - \overline{x}(k))^2 \cdot \sum_{k=1}^{n} \left(\hat{x}(k) - \overline{\hat{x}}(k)\right)^2} \tag{43}$$

where $\overline{x}$ represents the average of the original values (raw values), and $\overline{\hat{x}}$ denotes the average of the estimated values. R^2 is used to evaluate the effect size based on Hopkins (2002) in Table 3.

Table 3. Accuracy Assessment of R^2 Values

Accuracy Assessment	R^2 Value	
	Minimum	Maximum
Perfect	1	
Nearly Perfect	0.9	1
Very Large	0.7	0.9
Large	0.5	0.7
Moderate	0.3	0.5
Small	0.1	0.3
Trivial	0.0	0.1

To assess the error rates of the models, another criterion is considered as C and P. The steps for these values are provided below (Wang et al., 2020):

$$C = \frac{S_2}{S_1} \tag{44}$$

$$P = \left\{ \left| \Delta^{(0)}(k) - \overline{\Delta}^{(0)} \right| < 0.6745 S_1 \right\} \tag{45}$$

where S_1 represents the standard deviation of the raw data (original series), S_2 represents the standard deviation of the absolute error series $\left(|x^0(k) - \hat{x}^{(0)}(k)| \right), \Delta^{(0)}(k)$ represents the absolute error series and $\overline{\Delta}^{(0)}$ represents the mean value of the absolute error series (Wang et al., 2020). The accuracy assessment for the C and P values is presented in Table 4.

Table 4. Accuracy Assessment of C and P Values

Accuracy Assessment	C Value		P Value	
	Minimum	Maximum	Minimum	Maximum
Good	0.00	0.35	0.95	1.00
Pass	0.35	0.50	0.80	0.95
Unconvincing Pass	0.50	0.65	0.70	0.80
Fail	> 0.65		0.00	0.70

Source: Wang et al. (2020)

3. Applications

In this study, the data portal for statistics obtained from TurkStat has been used to estimate the population of Türkiye up to the year 2050. To forecast the population of Türkiye, the five different grey prediction models are selected, and the methodologies employed for each grey model have been detailed.

The first general population census utilizing an address-based population registration system was conducted in 2007. From this year onwards, the “de jure” method was employed in censuses. This method entails the enumeration of the population based on the residential addresses of citizens. An examination of Türkiye’s fertility and mortality rates reveals a continuous downward trend which has resulted in the country entering a new demographic regime since the 2000s (Köse & Sertkaya

Doğan, 2022). Currently, Türkiye is in the fourth phase, which started post-2000 and is characterized by low birth and death rates (Yüceşahin, 2009). Therefore, this study does not examine population forecasting using data from the 1935-1940 period. Similarly, Wen & Huang (2004) utilized data spanning from 1946 to 2002 in their study to determine Taiwan's saturation population. However, they only utilized data from 1996 onwards because the population had reached saturation by that year. Therefore, we use the data set covering the years 2007-2022, obtained from the address-based population registration system, due to the natural flow of life, fluctuations are likely to occur within these intervals. The absence of these details is believed to result in a lack of information and an increase in the margin of error in any research conducted using this data. To develop a future projection using grey prediction models, the raw data groups $\left(X^{(0)}\right)$ that have been collected at fixed periodic intervals, typically from time series, are given in Table 5.

Table 5. The official general population census results obtained from TurkStat

Year	Population
2007	70,586,256
2008	71,517,100
2009	72,561,312
2010	73,722,988
2011	74,724,269
2012	75,627,384
2013	76,667,864
2014	77,695,904
2015	78,741,053
2016	79,814,871
2017	80,810,525
2018	82,003,882
2019	83,154,997
2020	83,614,362
2021	84,680,273
2022	85,279,553

Source: Population Statistics Portal, 2023

We consider two applications for analyzing the population dataset. In the first application, we divide the dataset into two parts. The first part is the training dataset (2007-2017) for model identification, and the second part is the testing dataset (2018-2022) for the accuracy of the prediction results. In the second application, we use the original dataset (2007-2022) and then predict the period between 2023 and 2050.

3.1. First Application

The first application is based on training (2007-2017) and testing (2018-2022) datasets. The comparison results of the five models based on MAPE, R^2, C, and P values for training and testing data sets are presented in Table 6.

Table 6. Evaluation of the results from five grey prediction models based on training and testing data sets and a comparison with TurkStat projection

	Year	Population	GM (1,1)	GM $(1,1)_t$	GGM	GVM	EXGM(1,1)
Training	2007	70,586,256	70,586,256	70,586,256	70,586,256	70,586,256	70,586,256
	2008	71,517,100	71,650,775.32	71,558,138.22	71,557,485.07	71,595,732.58	71,497,031.38
	2009	72,561,312	72,622,847.08	72,590,939.83	72,590,895.69	72,608,874.98	72,634,903.53
	2010	73,722,988	73,608,106.75	73,622,296.47	73,623,159.35	73,625,394.37	73,668,559.34
	2011	74,724,269	74,606,733.24	74,652,210.17	74,654,054.00	74,644,997.95	74,672,040.83
	2012	75,627,384	75,618,907.89	75,680,682.94	75,683,361.89	75,667,389.26	75,672,694.95
	2013	76,667,864	76,644,814.52	76,707,716.81	76,710,869.60	76,692,268.52	76,680,691.88
	2014	77,695,904	77,684,639.41	77,733,313.78	77,736,368.14	77,719,332.97	77,699,884.32
	2015	78,741,053	78,738,571.41	78,757,475.86	78,759,652.95	78,748,277.25	78,731,802.35
	2016	79,814,871	79,806,801.88	79,780,205.07	79,780,524.01	79,778,793.72	79,777,123.12
	2017	80,810,525	80,889,524.83	80,801,503.41	80,798,785.84	80,810,572.86	80,836,211.50
	Evaluation Criteria						
	MAPE		0.06833	0.052481	0.053635	0.053026	0.040527
	R2		0.999518	0.999781	0.999778	0.999759	0.999862
	C		0.015342	0.008394	0.00819	0.009848	0.007024
	P		1	1	1	1	1
	Rank		5	2	3	4	1
Testing	Year	TurkStat	GM(1,1)	GM $(1,1)_t$	GGM	GVM	EXGM(1,1)
	2018	81,867,223	81,986,936.86	81,821,372.88	81,814,247.51	81,843,303.61	81,909,319.03
	2019	82,886,421	83,099,237.27	82,839,815.47	82,826,722.76	82,876,673.78	82,996,657.12
	2020	83,900,373	84,226,628.03	83,856,833.19	83,836,029.91	83,910,370.37	84,098,423.99
	2021	84,908,658	85,369,313.87	84,872,428.03	84,841,991.95	84,944,080.02	85,214,814.59
	2022	85,911,035	86,527,502.31	85,886,601.98	85,844,436.56	85,977,489.34	86,346,024.32
	Evaluation Criteria						
	MAPE	0.368376	0.619406	0.366066	0.3489	0.402896	0.553292
	R2	0.984767	0.98357	0.984531	0.984642	0.984442	0.98359
	C	0.14657	0.395311	0.136213	0.127769	0.161044	0.300461
	P	1	1	1	1	1	1
	Rank	3	6	2	1	4	5

Accordingly, the EXGM (1,1) model consistently gives the best accurate results based on all evaluation criteria for the years between 2007 and 2017. For testing performance, grey prediction models are compared with the forecast values published by TurkStat on February 21, 2018 (TUIK, 2018). The GGM model offers the best results for MAPE with a value of 0.348900 and for C with a value of 0.127769 for the years between 2018 and 2022. The second-best result yields GM $(1,1)_t$ model. The results of the GGM and GM $(1,1)_t$ have more accurate than from the projection of TurkStat based on the MAPE, C, and P values. The TurkStat forecasts provide the best result based on the R^2 criterion. We also note that the GM (1,1) model indicates a pass value, and other grey models indicate good values based on the C criterion. Finally, these findings show that the GGM and GM $(1,1)_t$ models can provide more accurate forecast values in future simulations.

3.2. Second Application

The second application is based on the original dataset (2007-2022), and the parameters of the grey model are obtained from the period between 2007 and 2022. Then, the results of forecasting from the period between 2023 and 2050 are compared with the projection of TurkStat. Table 7 provides the prediction results obtained from the years 2007-2022 using the five grey models, and the values of the APE, MAPE, R^2, C, and P. Accordingly, the GGM yields the best results with a MAPE value of 0.12788 and an R^2 value of 0.99898. The GM (1,1) $_t$ model produces the best outcome based on the C value with 0.02286. The GVM ranks third best based on the MAPE, R^2, and C values. Regarding the P value, all models are equal to 1. Based on these results, the GGM, GM (1,1) $_t$, and the GVM are more successful than other models, respectively. These findings denote an unexpected declining trend in Türkiye's population growth and suggest that more precise forecasts for the distant future can be achieved using the GVM model, which exhibits characteristics of a logistic curve. We also note that the performance superiority of the models may also change in response to alterations in the data set.

Table 7. Evaluation of forecast results and criteria for five grey models using 2007-2022 data

Year	GM(1,1)		GM $(1,1)_t$		GGM		GVM		EXGM(1,1)	
	$\hat{X}^{(0)}$	APE	$\hat{X}^{(0)}$	APE	$\hat{X}^{(0)}$	APE	$\hat{X}^{(0)}$	APE	$\hat{X}^{(0)}$	APE
2007	70,586,256	0	70,586,256	0	70,586,256	0	70,586,256	0	70,586,256	0
2008	71,872,313	0.49668	71,455,449.9	0.0862	71,474,847	0.05908	71,663,908.38	0.20528	71,424,054	0.1301
2009	72,795,330	0.32251	72,552,900.5	0.01159	72,561,178	0.00019	72,730,105.57	0.23262	72,752,320	0.26324
2010	73,730,201	0.00978	73,635,639.4	0.11848	73,636,013	0.11798	73,784,164.69	0.08298	73,823,457	0.13628
2011	74,677,077	0.06316	74,703,864.1	0.0273	74,699,075	0.03372	74,825,438.12	0.13539	74,807,303	0.11112
2012	75,636,114	0.01154	75,757,769	0.1724	75,750,109	0.16228	75,853,314.66	0.29874	75,766,428	0.18385
2013	76,607,467	0.07878	76,797,546.1	0.16915	76,788,881	0.15785	76,867,220.5	0.26003	76,723,944	0.07315
2014	77,591,295	0.13464	77,823,384.7	0.16408	77,815,175	0.15351	77,866,620.05	0.21972	77,688,445	0.0096
2015	78,587,757	0.19468	78,835,471.8	0.11991	78,828,799	0.11144	78,851,016.5	0.13965	78,663,190	0.09889
2016	79,597,017	0.27295	79,833,991.7	0.02396	79,829,577	0.01843	79,819,952.16	0.00637	79,649,473	0.20723
2017	80,619,238	0.23671	80,819,126.2	0.01064	80,817,355	0.00845	80,773,008.71	0.04643	80,647,868	0.20128
2018	81654586	0.42595	81,791,054.8	0.25953	81,791,996	0.25839	81,709,807.15	0.35861	81658683	0.42095
2019	82,703,232	0.54328	82,749,954.5	0.48709	82,753,379	0.48298	82,630,007.64	0.63134	82,682,133	0.56865
2020	83,765,344	0.18057	83,696,000	0.09764	83,701,403	0.1041	83,533,309.12	0.09694	83,718,398	0.12442
2021	84,841,096	0.18992	84,629,363.5	0.06012	84,635,980	0.05231	84,419,448.85	0.30801	84,767,646	0.10318
2022	85,930,664	0.7635	85,550,215.2	0.31738	85,557,041.03	0.32539	85,288,201.70	0.01014	85,830,042	0.64551
Evaluation Criteria										
MAPE		0.24529		0.13284		0.12788		0.18952		0.20484
R2		0.99681		0.99895		0.99898		0.99884		0.99766
C		0.0375		0.02286		0.0231		0.02822		0.0329
P		1		1		1		1		1
Rank		5		2		1		3		4

The forecasting values of the five grey models are compared to the report values of population projection published by TurkStat on 21 February 2018 (TUIK, 2018). The forecast values for the period up to the year 2050 based on TurkStat's main scenario are illustrated in Table 8. Accordingly, Türkiye's total population is projected to be 104,749,423 by 2050 based on TurkStat reports. The forecast results of the five different grey models up to the year 2050 are given as 105,948,975 for GGM, 106,877,632 for GM $(1,1)_t$, 102,591,571 for GVM, 121,645,734 for EXGM (1,1) and 122,834,753 for GM(1,1). Figure 1 shows the forecast values of all models and the GGM, GM (1,1) t, GVM and the projection values of TurkStat have similar results.

Table 8. Forecast values of the five Grey Models and the TurkStat Main Scenario up to the year 2050

Year	TurkStat	GGM	GM (1,1) $_t$	GVM	EXGM (1,1)	GM (1,1)
2023	86,907,367	86,464,530	86,458,723	86,139379	86,905,753	87,034,224
2024	87,885,571	87,358,407	87,355,051	86,972830	87,994,945	88,151,957
2025	88,844,934	88,238,643	88,239,365	87,788435	89,097,788	89,284,045
2026	89,784,584	89,105,227	89,111,823	88,586111	90,214,453	90,430,671
2027	90,703,600	89,958,156	89,972,587	89,365808	91,345,114	91,592,022
2028	91,601,117	90,797,441	90,821,811	90,127505	92,489,944	92,768,288
2029	92,476,323	91,623,105	91,659,652	90,871211	93,649,124	93,959,661
2030	93,328,574	92,435,181	92,486,261	91,596967	94,822,831	95,166,333
2031	94,153,776	93,233,714	93,301,790	92,304836	96,011,248	96,388,502
2032	94,951,512	94,018,755	94,106,386	92,994911	97,214,559	97,626,367
2033	95,721,347	94,790,369	94,900,196	93,667306	98,432,952	98,880,129
2034	96,463,090	95,548,626	95,683,365	94,322159	99,666,615	10,014,9992
2035	97,176,768	96,293,607	96,456,036	94,959628	100,915,740	101,436,164
2036	97,862,549	97,025,397	97,218,348	95,579893	102,180,519	102,738,853
2037	98,520,720	97,744,093	97,970,442	96,183149	103,461,151	104,058,272
2038	99,151,467	98,449,795	98,712,454	96,769610	104,757,832	105,394,635
2039	99,754,923	99,142,611	99,444,519	97,339504	106,070,765	106,748,161
2040	100,331,233	99,822,655	100,166,771	97,893,073	107,400,153	108,119,069
2041	100,882,665	100,490,045	100,879,340	98,430,571	108,746,202	109,507,583
2042	101,409,507	101,144,905	101,582,358	98,952,265	110,109,121	110,913,929
2043	101,911,980	101,787,365	102,275,951	99,458,430	111,489,122	112,338,335
2044	102,390,159	102,417,557	102,960,247	99,949,351	112,886,418	113,781,035
2045	102,843,989	103,035,618	103,635,369	100,425,319	114,301,227	115,242,263
2046	103,273,571	103,641,688	104,301,442	100,886,632	115,733,767	116,722,256
2047	103,679,038	104,235,911	104,958,585	101,333,594	117,184,262	118,221,256
2048	104,060,257	104,818,433	105,606,920	101,766,514	118,652,936	119,739,507
2049	104,417,089	105,389,404	106,246,563	102,185,701	120,140,016	121,277,256
2050	104,749,423	105,948,975	106,877,632	102,591,471	121,645,734	122,834,753

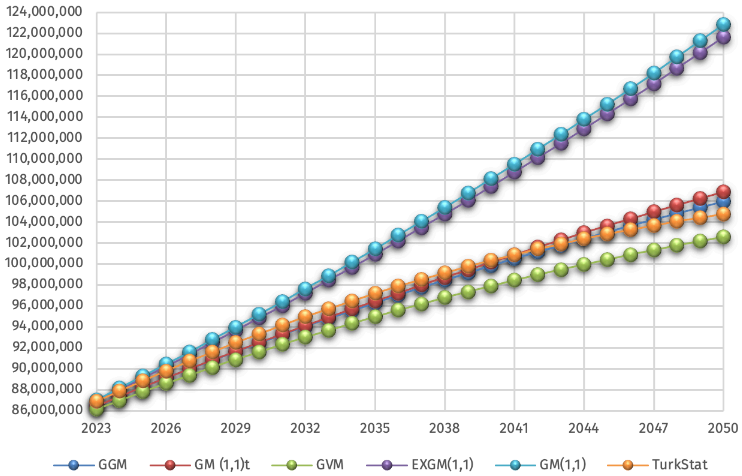

Figure 1. The plot of all forecast values of the models and TurkStat main scenario

In Table 9, the most compatible APE values are provided by the EXGM (1,1) model for the years 2023-2027, the GM (1,1) $_t$ model for the years 2028-2042, and the Grey Gompertz model for the years 2043-2050. In the evaluation of the forecast values for the years 2023-2050 based on the TurkStat main scenario, the GGM provides the most accurate results based MAPE and C values. The GM (1,1) $_t$ model provides the second best according to these criteria. GVM shows the highest compatibility, with the GGM following closely based on R^2. The EXGM (1,1) and the GM (1,1) models yield the C values of 0.96091 and 1.01902 respectively. These values significantly exceed the upper acceptable limit of 0.65. The EXGM (1,1) and the GM (1,1) models report 0.39286 and 0.35714 of the P values which are below the lower limit of 0.7. The other three models have C values below 0.1 and P values are equal to 1. From these results, the GGM is found to be more consistent with the main scenario forecast values published by TurkStat in 2018.

Table 9. Evaluation Criteria for Grey with Projection Values of TurkStat

Year	GGM	GM (1,1) $_t$	GVM	EXGM (1,1)	GM (1,1)
	APE	APE	APE	APE	APE
2023	0.50955	0.51623	0.88369	0.00186	0.14597
2024	0.59983	0.60365	1.03856	0.12445	0.30311
2025	0.68241	0.6816	1.18915	0.2846	0.49424
2026	0.75665	0.74931	1.33483	0.47878	0.7196
2027	0.82185	0.80594	1.47491	0.70726	0.97948
2028	0.87736	0.85076	1.60873	0.97032	1.27419
2029	0.92263	0.88311	1.7357	1.26822	1.60402
2030	0.95726	0.90252	1.85539	1.60107	1.96913
2031	0.97719	0.90489	1.96374	1.97281	2.37349
2032	0.98235	0.89006	2.06063	2.38337	2.81707
2033	0.97259	0.85786	2.14585	2.83281	3.29998
2034	0.94799	0.80831	2.21943	3.32099	3.82209
2035	0.90882	0.74167	2.28155	3.8476	4.38314
2036	0.85544	0.65827	2.33251	4.41228	4.98281
2037	0.78829	0.55854	2.37267	5.01461	5.6207

Year	GGM	GM (1,1) $_t$	GVM	EXGM (1,1)	GM (1,1)
	APE	APE	APE	APE	APE
2038	0.70768	0.44277	2.40224	5.65434	6.2966
2039	0.61382	0.31117	2.42135	6.33136	7.01042
2040	0.50690	0.16392	2.43011	7.04558	7.76213
2041	0.38918	0.0033	2.43064	7.79474	8.54945
2042	0.26092	0.17045	2.42309	8.5787	9.37232
2043	0.12228	0.35714	2.40752	9.39746	10.23075
2044	0.02676	0.55678	2.38383	10.25124	11.12497
2045	0.18633	0.7695	2.35179	11.14041	12.05542
2046	0.35645	0.99529	2.31128	12.06523	13.02239
2047	0.53711	1.23414	2.26222	13.02599	14.02619
2048	0.72859	1.48631	2.20424	14.0233	15.06747
2049	0.93118	1.75208	2.137	15.05781	16.14694
2050	1.14516	2.03171	2.06011	16.13022	17.26533
Evaluation Criteria					
MAPE	0.68116	0.77455	2.02581	5.91848	6.52569
R2	0.99415	0.99143	0.99818	0.96577	0.96537
C	0.05218	0.08596	0.09529	0.96091	1.01902
P	1	1	1	0.39286	0.35714
Rank	1	2	3	4	5

Figure 2 shows the box plot of the error sequence $\left(x^0(k) - \hat{x}^{(0)}(k)\right)$ based on the projection values of the main TurkStat scenario for the years 2023-2050. It is observed that the GGM model presents better conformity in the evaluation of residuals compared to other grey models.

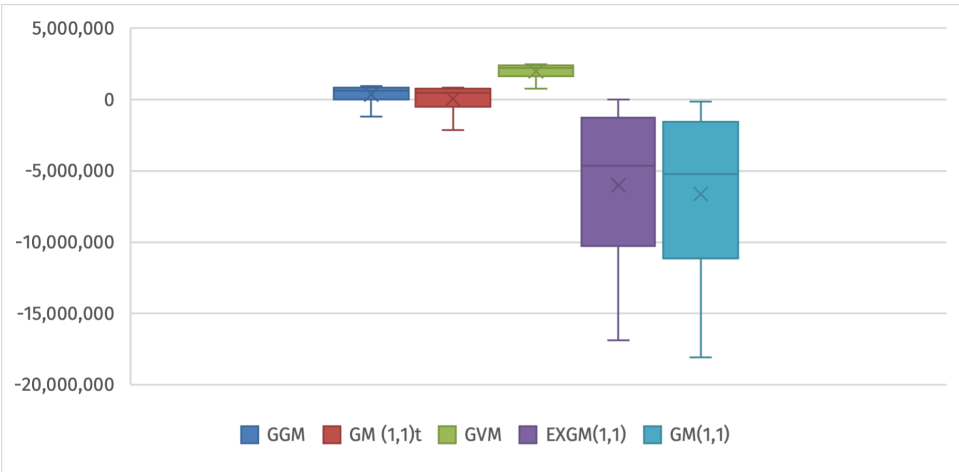

Figure 2. Box Plot of Error Series for Grey Models based on TurkStat Main Scenario

4. Conclusion

The employment of models for population forecasting provides the creation of population projections, which are critical for countries in formulating demographic, economic, and investment policies. The use of population projections allows for the informed decision-making process regarding public expenditure in areas such as education, health, infrastructure, and transportation, as it provides insight into the demographic characteristics of future societies. Consequently, countries can implement planned programs based on demographic characteristics, appropriately allocate resources to enhance the welfare of their societies and adopt measures in advance to address potential demographic challenges. This study presents alternative models for forecasting the population of Türkiye up to the year 2050. The GGM and GM $(1,1)_t$ demonstrate superior performance compared to the primary scenario forecasts of TurkStat for the testing data set period from 2018 to 2022 as the first application. The GGM, GM $(1,1)_t$ and GVM demonstrate superior performance to the other models by the evaluation criteria as the second application. The results indicate that Türkiye's population growth has unexpectedly declined, suggesting that the GVM model may exhibit superior performance compared to other models in the long term. Furthermore, this situation is consistent with the fourth and fifth stages of the demographic transition process. However, this study has several limitations. Primarily, it focuses only on the total population of Türkiye, without conducting more specific analyses such as gender, age, regions, or provinces. Therefore, grey prediction models with different variables can contribute to future research.

Declarations

Conflict of interest The authors have no competing interests to declare that are relevant to the content of this article.

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