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ERROR CORRECTING SOFT CODES FOR ODD NUMBERS WHICH ARE EQUAL OR LESS THEN (n/2-1)

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Abstract – The main purpose of this study is to determine the new encoding and decoding method. The encoding and decoding are an important tool for Coding Theory. İn this paper, we define soft codes by using definition soft sets. Also, we explain some algebraic properties of soft codes.

Keywords – *Soft Sets , Coding Theory, Soft Codes.*

1 Introduction

Soft set theory [1] was firstly introduced by Molodtsov in 1999 as general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, etc. The soft set theory has been applied to many different fields with great success. Maji *et al.* [2] worked on theoretical study of soft sets in detail and [3] presented an application of soft set in the decision making problem using the reduction of rough sets [4]. Chen *et al.*[5] proposed parameterization reduction of soft sets, and then Kong *et al.* [6] presented the normal parameterization reduction of soft sets. We can say that The soft set has the similar applications with fuzzy sets and rough sets. H. Aktas and N. Cagman [7] has shown that every fuzzy set and every rough set can be considered as a soft set. In that sense we can say that this theory is much more general than its predecessors.

With the increasing importance of digital communications and data storage, there is a in the area of coding theory and channel modelling to design codes need for research for channels that are power limited or bandwidth limited. The purpose of a communication system is, in

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the broadest sense, the transmission of information from one point in space and time to another. We shall briefly explore the basic ideas of what information is and how it can be measured, and how these ideas relate to band width, capacity, signal-to-noise ratio, bit error rate and so on.

İn this paper, the coding theory which based digital communication is studied over soft set. Also this structure is used for single error- correcting. These codes have different applications from the other codes.

Through our study of error-control codes, we will model our data as strings of discrete symbols, often binary symbols {0,1}. When working with binary symbols, addition is done modulo 2. For example, $1 + 1 = 0\ (mod\ 2)$. We will study channels that are corrected by additive white Gaussian noise, which we can model as a string of discrete symbols that get added symbol-wise to the code word. For example, if we wish to send the code word $c = 11111$, noise may corrupt the codeword so that the $r = 01101$ is received. In this case, we would say that the error vector is $e = 10010$, since the codeword was corrupted in the first and fourth positions. Notice that $c + e = r$, where the addition is done component-wise and modulo 2. The steps of encoding and decoding that concern us are as follows:

$$m \rightarrow Encode \rightarrow c \rightarrow Noise \rightarrow c + e = r \rightarrow Decode \rightarrow \tilde{m}$$

where m is the message, c is the code word, e is the error vector due to noise, r is the received word or vector, and $\tilde{m}$ is the decoded word or vector. The hope is that $\tilde{m} = m$.

2 Preliminaries and Notation

In this section, we present the basic definitions of soft set theory [8] and coding theory [9]. We consider a binary channel which can transmit either of two symbols 0 or 1. However, due to presence of noise a transmitted zero may sometimes be received as 1, and transmitted 1 may sometimes be received as 0. When this happens we say that there is an error in transmitting the symbol. The symbols successively presented to the channel for transmission constitute the input and the she symbols received constitute the output. Error control coding is a method to detect and possibly correct errors by introducing redundancy to the stream of bits to be sent to the channel. The Channel Encoder will add bits to the message bits to be transmitted systematically. After passing through the channel, the Channel decoder will detect and correct the errors. These definition sand more detailed explanations related to the soft sets and coding theory can be found in [10,11,12] and [13] respectively.

Throughout this work, U denotes to an set of vectors, E denotes the set of code words's weight, $A \subseteq E$ and n is the code's length, $P(U)$ is the power set of U, and $A \subseteq E$. Also, 1_n and 0_n denote that every position equals to 1 and 0, respectively.

Definition 2.1. [3] A pair (F, A) is called a soft set over U where F is a mapping given by

$$F: A \rightarrow P(U)$$

In other words, a soft set over U is parametriterized family of subsets of the universe U. For $\varepsilon \in A, F(\varepsilon)$ may be considered as the set of ε- elements of the soft set (F, A).

Definition 2.2. [3] For two soft sets (F, A) and (G, B)over U, (F, A) is called a soft subset of (G, B) if

(1) $A \subset B$ and
(2) $\forall \varepsilon \in A$ $F(\varepsilon)$ and $G(\varepsilon)$ are identical approximations

This relationship is denoted by $(F, A) \tilde{\subset} (G, B)$.

Similarly, (F, A) is called a soft superset of (G, B), if (G, B) is a soft subset of (F, A). This relationship is denoted by $(F, A) \tilde{\supset} (G, B)$.

Definition 2.3. [3] Two soft sets $(F, A)$and (G, B) over U are called soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 2.4. [7] The intersection of two soft sets (F, A) and (G, B) over U is the soft set (H, C), where $C = A \cap B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon)$ or $G(\varepsilon)$ (as both are same set). This is denoted by $(F, A) \tilde{\cap} (G, B) = (H, C)$.

Definition 2.5. [7] If (F, A) and (G, B) are two soft sets, then (F, A) and (G, B) is denoted $(F, A) \wedge (G, B)$. $(F, A) \wedge (G, B)$ is defined as $(H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$ $, \forall (\alpha, \beta) \in A \times B$.

Definition 2.6. [7] The union of two soft sets (F, A) and (G, B) over U is the soft set (H, C), where $C = A \cup B$ and $\forall$ $\varepsilon \in$ C

$$H(e) = \begin{cases} F(\varepsilon) & if \quad \varepsilon \in A - B \\ G(\varepsilon) & if \ \varepsilon \ \in B - A \\ F(\varepsilon) U \ G(\varepsilon) & if \quad \varepsilon \ \in \ A \cap B \end{cases}$$

This relationship is denoted by $(F, A) \tilde{\tilde{U}} (G, B) = (H, C)$.

Definition 2.7.[14] The minimum distance of a code C is the minimum distance between any two code words in C. We can indicate as follows.

$$d(C) = min\{d(x, y) : x \neq y, x, \ y \in C\}.$$

Definition 2.8.[15] Weight $w(c)$ of a code word c is the number of nonzero components in the code words.

3 Soft Codes

Definition 3.1. Let U denotes set of vectors, $P(U)$ be the power set of U, E be the set of code words's weight, $A \subseteq E$ and n is the code's length. A soft set $(f_A, (E, n))$ on the universe U which defined by the set of ordered triads is called soft code.

$$(f_A, (E, n)) = \{(e, f_A(e), n) : e \in E, f_A(e) \in P(U), n \in N\}$$

where $f_A : E \to P(U)$.

Example 3.2. Let $(F_A, (E, n))$ be a soft code over
$U = \{0, 1, 00,10,01,11,000,100,010,001,110,101,011,111 \ldots\}$. We define $P(U)$ as the following for $n = 3$, $A = \{1,2\}$, $E = \{0,1,2,3\}$
$P(U) = \{000,100,010,001,110,101,011,111\}$. So that, we denote to soft code as follows
$(f_A, (E, n)) = \{(1, \{100,010,001\}, 3), (2, \{110,101,011\}, 3)\}$
$= \{\{(1, \{100\}, 3), (1, \{010\}, 3), (1, \{001\}, 3)\}, \{(2, \{110\}, 3), (2, \{101\}, 3), (2, \{101\}, 3)\}\}$

Definition 3.3. For a soft code $(F_A, (E, n))$ over U,

(a) $(f_A, (E, n))$ is said to be a zero soft code, if $A = \{0\}$. It is denoted $(e, f_A(e), n) = 0_n$.

(b) $(f_A, (E, n))$ is said to be a universal soft code, if $A = \{1\}$. It is denoted $(e, f_A(e), n) = 1_n$.

Definition 3.4. For three soft codes $(F_A, (E, n))$, $(G_B, (E, n))$, $(H_A, (E, n))$ over U,

(a) We define soft sub code as follows. $(G_B, (E, n))$ is soft sub code of $(F_A, (E, n))$, if $B \subseteq A$. It is denoted by $(G_B, (E, n)) \subseteq (F_A, (E, n))$.

(b) We define soft equal code as follows. $(F_A, (E, n))$ *and* $(G_B, (E, n))$ are equal soft codes, if $A = B$. It is denoted by $(G_B, (E, n)) = (F_A, (E, n))$.

Definition 3.5. Let $(F_A, (E, n))$, $(G_B, (E, n))$ and $(H_C, (E, n)) \in (P(U), (E, n))$;

(a) We define soft union code as follows. Union of $(F_A, (E, n))$ and $(G_B, (E, n))$ over U is soft code $(H_C, (E, n))$ where $C = A \cup B$, denoted by $(F_A, (E, n)) \cup (G_B, (E, n))$.

(b) We define soft intersection code as follows. İntersection of $(F_A, (E, n)$ and $(G_B, (E, n))$over U is soft code $(H_C, (E, n))$ where $C = A \cap B$, denoted by $(F_A, (E, n)) \cap (G_B, (E, n))$.

(c) Complement of $(F_A, (E, n))$over U, denoted by $(F_A, (E, n))^{\circ}$, $(G_B, (E, n)) = 1_n$ - $(F_A, (E, n))$.

(d) $(F_A, (E, n))$ and $(G_B, (E, n))$ are disjoint if $(F_A, (E, n)) \cap (G_B, (E, n)) = \emptyset$.

Proposition 3.6. Let $(F_A, (E, n)) \in (P(U), (E, n))$. Then

(a) $((F_A,(E,n))^\circ)^\circ = (F_A,(E,n))$,
(b) $0_n{}^\circ = 1_n$.

Proof. İt is clear from Definition 3.5.

Proposition 3.7. Let $(F_A,(E,n)), (G_B,(E,n), (H_C,(E,n)) \in (P(U),(E,n))$. Then

(a) $(F_A,(E,n)) \cup (F_A,(E,n)) = (F_A,(E,n))$,
(b) $(F_A,(E,n)) \cap (F_A,(E,n))^\circ = \emptyset$,
(c) $(F_A,(E,n)) \cup (G_B,(E,n) = (G_B,(E,n) \cup (F_A,(E,n))$,
(d) $((F_A,(E,n)) \cup (G_B,(E,n)) \cup (H_C,(E,n)) = (F_A,(E,n)) \cup ((G_B,(E,n))$
$\cup (H_C,(E,n))$.

Proof. İt is straightforward.

Proposition 3.4. Let$(F_A,(E,n)), (G_B,(E,n), (H_C,(E,n)) \in (P(U),(E,n))$. Then

(a) $(F_A,(E,n)) \cap (F_A,(E,n)) = (F_A,(E,n))$,
(b) $(F_A,(E,n)) \cap (G_B,(E,n) = (G_B,(E,n) \cap (F_A,(E,n))$,
(c)$((F_A,(E,n)) \cap (G_B,(E,n)) \cap (H_C,(E,n)) = (F_A,(E,n)) \cap ((G_B,(E,n)) \cap (H_C,(E,n))$

Proof. It is proved by using Definition 3.5.

Proposition 3.5. $(F_A,(E,n)), (G_B,(E,n) \in (P(U),(E,n))$. Then De Morgan's laws are valid

(a) $((F_A,(E,n)) \cap (G_B,(E,n))^\circ = ((F_A,(E,n)))^\circ \cap ((G_B,(E,n)))^\circ$,
(b) $(((F_A,(E,n)) \cup (G_B,(E,n)))^\circ = ((F_A,(E,n)))^\circ \cup ((G_B,(E,n)))^\circ$.

Proof.

(a) $$(((F_A,(E,n)) \cap (G_B,(E,n)))^\circ = 1_n - (((F_A,(E,n)) \cap (G_B,(E,n)))$$
$$= (1_n - (((F_A,(E,n)) \cap (1_n - (G_B,(E,n))))$$
$$= (F_A,(E,n)))^\circ \cap (G_B,(E,n)))^\circ$$

(b) İt can be proved similarity.

Proposition 3.6. Let $(F_A,(E,n)), (G_B,(E,n), (H_C,(E,n)) \in (P(U),(E,n))$. Then

(a) $(((F_A,(E,n)) \cup (G_B,(E,n))) \cap (H_C,(E,n)) =$
$(((F_A,(E,n) \cup (H_C,(E,n))) \cap ((G_B,(E,n)) \cup (H_C,(E,n)))$,

(b) $(((F_A,(E,n)) \cap (G_B,(E,n))) \cup (H_C,(E,n)) =$
$((F_A,(E,n) \cap (H_C,(E,n))) \cup ((G_B,(E,n))) \cap (H_C,(E,n)))$.

Proof. It is clear from Definition 3.1. and Definition 3.5.

3.1. Products of Soft Codes

In this part, we define three new definitions for soft encoding and decoding.

Definition 3.7. Let $(F_A, (E,n))$,$(G_B,(E,n)) \in (P(U),(E,n))$. We define vectorel multiplication as following.

$(F_A,(E,n)) = \{(e,\{a\},n): e \in E, \{a\} \in P(U), n \in N\}$
$(G_B,(E,n)) = \{(f,\{d\},n): f \in E, \{d\} \in P(U), n \in N\}$

Let's accept $a = (a_1 a_2 \dots a_j)$, $d = (d_1 d_2 \dots d_k)$, define vectorel multiplication as following. Also we show symbol of vectorel multiplication with "Λ".

$(F_A,(E,n)) \Lambda (G_B,(E,n)) = \{(max\ \{a_1,d_1\} max\ \{a_1,d_2\} \dots max\ \{a_1,d_k\}), (max\ \{a_2,d_1\}$
$max\{a_2,d_2\} \dots \{max\{a_2,d_k\}) \dots (max\ \{a_j,d_1\}, max\ \{a_j,d_2\} \dots max\ \{a_j,d_k\})$.

This multiplication is called as vectorel multiplication. Also this multiplication will create a basic for soft encoding and decoding. The soft encoding that set of a message which is showed by M is encoded by a soft code indicated by Ĕ. Also we make by using inverse operation decoding.

Definition 3.8. Let C be a soft code. The soft code has multiple of vectors. Each one of the vector has k information digits showed as follows.

$$(a_0, a_1, a_2, \dots, a_{k-1})$$

The each one of the soft code's elements is encoded by using Definition3.7. There are two multipliers of this product are called as message set and encoding set. The message set and encoding set is indicated M and Ĕ, respectively. $1_n$is not used for soft encoding and decoding.

Example 3.9. Let define the message set and the encoding set which are indicated $M$and Ĕ, respectively.

$M = (f_A,(E,4)) = \{(2,\{1100,1010,1001,0110,0101,0011\},4)\}$
Ĕ $= (f_A,(E,3)) = (0,\{000\},3)$

If we multiply sets of two codes,

C
$= \{(6,\{111111000000, 111000111000, 000111000111, 000111111000,000000111111\},12)\}$
.

Definition 3.10. The inverse operation of vectorel multiplication provides to find k information digit. This method is called soft decoding.

Example 3.11. Let's think Example 3.9. and try to solve the message which is called M. In this statement, we must note the following, while we multiply one digit with other digit the result code word consists from the large digits. If the digits equal one another, we write the common digit.
$m\Lambda$Ĕ $= \{(6,\{111111000000\},12)\}$

$\{xyzt\}\Lambda\{000\} = \{111111000000\}$
$x\,\Lambda\{000\} = 111 \rightarrow x = 1$
$y\,\Lambda\{000\} = 111 \rightarrow y = 1$
$z\,\Lambda\{111\} = 000 \rightarrow z = 0$
$t\,\Lambda\{111\} = 000 \rightarrow t = 0$

other elements of the message are found with similarity method.

4 $(\frac{n}{2} - 1)$ Error Correcting Soft Codes

Firstly, we proof a theorem for error correct. This theorem will generate a structure to correct.

Theorem 3.12. Distance of all of the codes which have same length and weight are always 2.

Proof.
Let x and y be same length and weight. We will examine two statements which $d(x,y)$ is even and odd.

(1) Let $d(x,y)$ be odd. In this statement, $d(x,y) = 2n + 1$ but this means $w(x) \neq w(y)$. This statement is contradiction with our acceptation.

(2) Let $d(x,y)$ be even. In this statement, let be $d(x,y) = 2n$. This sort codes are cyclic but not linear so ıf 10… is an element in code, 01… is an element in code from cyclic definition so distance is always 2.

Theorem 3.13. This collection can be 0_n or 1_n if all of the codes which have same length and weight are collected.

Proof. It is necessary to calculate the state of being one of each digit for this proof, examining all of the code words in code. Let's imagine a code which is w weight and n length.

a) We calculate the first position is 1 which are number of the code words that

$$\frac{(n-1)!}{(n-1-w+1)!(w-1)!}$$

b) Now, we calculate the second position is 1 which are number of the code words. There are two statements that $01\ldots$, $11\ldots$.

1) $01\ldots$
$\downarrow$

$$\frac{(n-2)!}{(n-2-w+1)!(w-1)!}$$

2) 11…

$$\downarrow$$

$$\frac{(n-2)!}{(n-2-w+2)!(w-2)!}$$

If we collect two statements, it will be like first statement.

$$\frac{(n-2)!}{(n-2-w+1)!(w-1)!}+\frac{(n-2)!}{(n-2-w+2)!(w-2)!}=\frac{(n-1)!}{(n-1-w+1)!(w-1)!}$$

…………………

n) …1

$$\downarrow$$

$$\frac{(n-1)!}{(n-1-w+1)!(w-1)!}$$

For end digit, we invent the same result.

Example 3.14. Let C be as following.

$$C=\{110,011,101\}.$$

In this statement, as can be seen in the code, the number of code words in which the first position 1 is 2. Number of Second and third positions respectively are repetition 2.

Theorem 3.15. The collection of elements of soft codes is 0_n or 1_n.

Proof. Let's choose two sets which are named with sets of message and encoding and show with M and E. We define as follows these sets, accept these sets have two elements. $M=\{x_1 \dots x_n, y_1 \dots y_n\}$ and $E=\{a_1 \dots a_n, b_1 \dots b_n\}$. Let's multiply by using definition 3.7..

$$M\Lambda E=$$
$$\{(x_1 \dots x_n)\Lambda(a_1 \dots a_n),(x_1 \dots x_n)\Lambda(b_1 \dots b_n),(y_1 \dots y_n)\Lambda(a_1 \dots a_n),(y_1 \dots y_n)\Lambda(b_1 \dots b_n)\}$$

If we collect these elements,

$$=$$
$$\{(x_1 \dots x_n)\Lambda(a_1 \dots a_n)+(x_1 \dots x_n)\Lambda(b_1 \dots b_n)+(y_1 \dots y_n)\Lambda(a_1 \dots a_n)+$$
$$(y_1 \dots y_n)\Lambda(b_1 \dots b_n)\}$$

$$=\{\big((x_1 \dots x_n)+(y_1 \dots y_n)\big)\Lambda(a_1 \dots a_n)+\big((x_1 \dots x_n)+(y_1 \dots y_n)\big)\Lambda(b_1 \dots b_n)\}.$$

We know that collection of soft codes can be 1_n or 0_n from Teorem 3.13. . Such as, $\big((x_1 \dots x_n)+(y_1 \dots y_n)\big)=1_n$ or 0_n. Let's accept this collection is 1_n. In this statement, $\{1_n\Lambda(a_1 \dots a_n)+1_n\Lambda(b_1 \dots b_n)\}=0_n$, ıf n is even. ıf n is odd, we can create as follows

$\{0_n \Lambda(a_1 \ldots a_n) + 0_n \Lambda(b_1 \ldots b_n)\} = (a_1 \ldots a_n) + (b_1 \ldots b_n)$,. Since E is a soft code, this result is either 1_n or 0_n.

Example 3.16. Let's think multiplication of codes $\{100,010,001\}\Lambda\{110,011,101\}$.

$100\Lambda\{110,101,011\} = \{111110110,111101101,111011011\}$ the code words' s collection is {111 000 000}= $1_3 0_3 0_3$

$010\Lambda\{110,101,011\} = \{110111110, 101111101, 011111011\}$ the total of these code words is {000 111 000}= $0_3 1_3 0_3$

$001\Lambda\{110,101,011\} = \{110110111, 101101111, 011011111\}$ the sum of these code words is {000 000 111}= $0_3 0_3 1_3$

If we write these sums in a set, it would be as follows

$$\{1_3 0_3 0_3,\ 0_3 1_3 0_3,\ 0_3 0_3 1_3\}$$

According to above theorem, the sum of the code words is either 0_9 or 1_9.

Error Correcting Soft Codes 4.1. To correct the error in the soft code the following algorithm is used.

Algorithm: These steps are followed for single error correcting soft codes.

(1) Elements of code are collected.
(2) If this collection has a mistake; this collection will be different from 0_n or 1_n.
(3) We analyses minimum distance of this collection by comparing with 0_n and 1_n.
(4) We know that minimum distance of this collection is close to 0_n or 1_n .
(5) The elements of code are compared with 0_n or 1_n.
(6) Such as we find an element which has a different distance, because this element is incorrect.
(7) All of the elements are collected but error element is not collected.
(8) This collection is collected with 0_n or 1_n, such as we find to correct element.

Example 3.17. Let's think a soft code as follows. Also we generate a mistake code word $C = \{111110110, 111011011, 111101101, 110111110, \mathbf{010001011}, 101111101, 110110111, 011011111, 101101111\}$

(1) If we collect elements of code, it is $\{110001111\}$

(2) $d(110001111, 111111111) = 3,\ \ d(110001111,000000000) = 6$

(3) this collection has to be 111111111

(4) Let find by using definition 2.7., $d(111111111, 111110110) = 2$
$d(111111111, 011111011) = 2$

………………………………….

$d(111111111, \mathbf{010001011}) = 4$ (Incorrect code word)

(5) We collect to code words but the incorrect code word is not collected. This collection is 100000100

(6) 111111111+100000100 = 011111011, it is a correct code word.

5 Conclusions

In this essay, we define a new method for low complication encoding and decoding for nonlinear binary product codes has been recommended. This technique provides an important error-correcting algorithm by using soft sets. Thus, we divide according to the weights of the linear code sets and these sets create elements of soft set. Also, a low complexity decoding algorithm was proposed for the developed nonlinear binary product codes. Finally, we provided an example illustrating the successfully application of this method.

References

[1] D. A. Molodtsov, *Soft set theory-first results*, Comput. Math. Appl., 37 (1999), 19-31.

[2] P. K. Maji, A. R. Roy and R. Biswas, *An application of soft sets in a decision making problem*, Comput. Math. Appl., 44 (2002), 1077-1083.

[3] P. K. Maji, R. Biswas and A. R. Roy, *Soft set theory*, Comput. Math. Appl., 45 (2003), 555-562.

[4] Z. Pawlak, *Rough sets*, Int. J. Comput. Inform. Sci., 11 (1982), 341-356.

[5] D. Chen, E. C. C. Tsang, D. S. Yeung and X. Wang, *The parameterization reduction of soft sets and its applications*, Comput. Math. Appl., 49 (2005), 757-763.

[6] Z. Kong, L. Gao, L. Wang and S. Li, *The normal parameter reduction of soft sets and its algorithm*, Comput. Math. Appl., 56 (2008), 3029-3037.

[7] H. Aktas. and N. Çagman, Soft sets and soft groups, Information Sciences, 177 (2007), 2726-2735.

[8] D. A. Molodtsov, *The theory of soft sets* (in Russian), URSS Publishers, Moscow, 2004.

[9] F. J. MacWilliams and N. J. A. Sloane, *The Theory of Error- Correcting Codes*, North-Holland, Amsterdam, 1977.

[10] I. S. Reed and G. Solomon , *Polynomial codes over certain finite fields*, J. Soc. Indust. Appl. Math., 8(1960), pp. 303-304.

[11] Vermanı, Lekh R., *Elements of Algebraic Coding Theory*, Chapman&Hall, 1996.

[12] Macwilliams, F.J. and Sloane, N.J.A., *The Theory of Error-Correcting Codes*, North-Holand, 1977.

[13] J. E. Roos, An algebraic study of group and nongroup error-correcting codes, Info. and Control, 8 (1965) 195-214 [6).

[14] Jan C.A. van der Lubbe., Information Theory. Cambridge Press, 1997.

[15] Adams S. S., Introduction to Algebraic Coding Theory , January 11, 2008.