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Investigation of the Concept Images of Mathematics Educators and Preservice Mathematics Teachers Regarding Slope, Rate of Change, and Derivative*

Matematik Eğitimcileri ve Matematik Öğretmen Adaylarının Eğim, Değişim Oranı ve Türeve İlişkin Kavram İmajlarının İncelenmesi

Ramazan EROL[**] **Elif SAYGI**[***]

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ABSTRACT: The aim of this study is to examine the concept images of mathematics educators and pre-service elementary mathematics teachers regarding the relationships between the concepts of slope, rate of change and derivative. Four mathematics educators working in different state universities and responsible for Analysis courses, three pre-service elementary mathematics teachers who successfully completed the Calculus 1 course at a state university, and four pre-service elementary mathematics teachers who successfully completed the Calculus 1 and Calculus 2 courses participated in the study. In order to determine the concept images of the participants about the concepts of slope, rate of change and derivative, the participants were asked semi-structured interview questions prepared by the researcher and application questions including the second stage questions. The audio recordings obtained from the interviews were transcribed and then the second stage questions were applied to the participants. The second stage questions were received in writing from the participants and analysed by the researcher through descriptive content analysis. As a result of this study, mathematics educators were able to use the concept images they had in the first stage questions in the second stage questions. On the other hand, pre-service elementary mathematics teachers were able to use the concept images they had in the first stage questions in the second stage questions.

Keywords: Concept image, slope, rate of change, derivative, mathematics educator.

ÖZ: Bu araştırmanın amacı matematik eğitimcilerinin ve ilköğretim matematik öğretmen adaylarının eğim, değişim oranı ve türev kavramları arasındaki ilişkilerine yönelik kavram imajlarını incelemektir. Araştırmaya farklı devlet üniversitelerinde görev yapan ve Analiz derslerinden sorumlu dört matematik eğitimcisi ve bir devlet üniversitesinde Analiz 1 dersini başarıyla tamamlamış üç ilköğretim matematik öğretmen adayı, Analiz 1 ve Analiz 2 dersini başarıyla tamamlamış dört ilköğretim matematik öğretmen adayı katılmıştır. Eğim, değişim oranı ve türev kavramları hakkında katılımcıların kavram imajlarını belirlemek üzere, katılımcılara araştırmacı tarafından hazırlanan yarı yapılandırılmış görüşme soruları ve ikinci aşama sorularının yer aldığı uygulama soruları yöneltilmiştir. Görüşme sorularından elde edilen ses kayıtları transkript edilmiş ve daha sonra katılımcılara ikinci aşama sorular uygulanmıştır. İkinci aşama sorular katılımcılardan yazılı olarak teslim alınarak araştırmacı tarafından betimsel içerik analizi ile incelenmiştir. Bu araştırmanın sonucunda matematik eğitimcileri birinci aşama sorularda sahip oldukları kavram imajlarını ikinci aşama sorularda da kullanabilmişlerdir. Bununla beraber ilköğretim matematik öğretmen adayları ise birinci aşama sorularda sahip oldukları kavram imajlarını ikinci aşama sorularda da kullanabilmiştir.

Anahtar kelimeler: Kavram imajı, eğim, değişim oranı, türev, matematik eğitimcisi.

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** *Corresponding Author:* Asst. Prof. Dr. Afyon Kocatepe University, Afyonkarahisar, Türkiye rerol@aku.edu.tr, https://orcid.org/0000-0002-2619-9084
*** Assoc. Prof. Dr., Hacettepe University, Ankara, Türkiye, esaygi@hacettepe.edu.tr, https://orcid.org/0000-0001-8811-4747

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The influential social scientist Kurt Lewin once stated, “There is nothing so practical as a good theory” (Lewin, 1951, p.169). According to Poincaré (1908), the practical value of a good theory stems from understanding mathematics learners. Therefore, studying learners through a solid theoretical framework can provide valuable insights into the research topic.

Despite its significant role, mathematics is widely acknowledged as a complex discipline to learn, often associated with fear and anxiety (Ashcraft & Ridley, 2005). According to the literature, educators and learners must have a clear and meaningful understanding of mathematical concepts (Bingölbali et al., 2016; Dreyfus & Eisenberg, 1982; Habre & Abboud, 2006; Heinze et al., 2009; Sfard, 2000, 2005). Vinner (1983) notes that incorrect examples during instruction can lead to misconceptions and incorrect learning, emphasizing the importance of accurate conceptual understanding.

Meaningful relationships among mathematical concepts are interconnected (Ministry of National Education [MoNE], 2018, 2024). Understanding these concepts’ accurate representation is crucial for meaningful learning outcomes in teaching (Bartell et al., 2013; Çakıcı et al., 2006; Goldsmith et al., 2014; Tzur & Simon, 2004). Given the spiral nature of mathematics, education progresses from patterns in preschool to advanced topics like functions, limits, and derivatives in high school, continuing into higher education.

Calculus is one of the most crucial subjects in higher education mathematics, focusing on understanding changes and predicting the future (Dreyfus, 2022; Turner & Álvarez, 2021; Vinner, 2002). The calculus course, essential from secondary school to postgraduate studies, requires high-level mathematical thinking and reasoning (Kuzu, 2021). It covers concepts such as limits, derivatives, and integrals, promoting inquiry, reasoning, and mathematical thinking (Artigue, 2002; Cohen, 2012; Ergene, 2019; Konyalıoğlu et al., 2011; Kuzu, 2021; Lithner, 2004; Mueller, 2004; Strasser, 2010; Thomas & Finney, 2001). Initially presented as groundbreaking work by Leibniz and Newton, calculus represents one of the most vital discoveries in mathematics (Bingölbali, 2010). But according to Tall (1993), the calculus course can have different meanings and interpretations, varying from country to country.

Advanced topics requiring high-level knowledge and skills are often challenging for learners to understand and internalize (Bukova, 2006; Cornu, 1981; Doruk et al., 2018; Dreyfus & Eisenberg, 1982; Grover, 2015; Tall, 1993; Tall & Vinner, 1981; Thomas & Finney, 2001; White & Mitchelmore, 1996). Studies abroad have shown that reports published by various mathematics groups in the United States during the 1980s reflect this challenge regarding the calculus course (National Council of Teachers of Mathematics [NCTM], 1987). MAA and NCTM concur that the teaching of calculus could have been more effective (Grant et al., 2016). To address these challenges, the Harvard Core Calculus Consortium Project emphasized teaching calculus concepts numerically, algebraically, and graphically (Moore & Smith, 1987). Gleason and Hallett (1992) highlighted the importance of teaching calculus through “The Rule of Three”, involving graphical, numerical, and algebraic instruction. Thus, calculus courses hold a significant place in mathematics. Understanding the foundational concepts of calculus requires a relationship with other mathematical concepts such as algebra, geometry, and trigonometry, making it the first step for subsequent mathematical topics (Sağlam, 2011). From this perspective, calculus is a fundamental course in mathematics education.

Changes in mathematics instruction have been acknowledged significantly, with many efforts to modify the curriculum and instruction. Notably, the main change in the calculus course is the increased focus on conceptualization and visualization (Habre & Abboud, 2006). Zimmermann (1991) considers visualization the core of the most substantial fundamental change in mathematics education: conceptual learning. He explains, “Conceptually, the role of visual thinking is so pivotal in understanding calculus that it would be challenging to conduct a successful calculus course without emphasising the visual aspects of any topic” (Zimmermann, 1991, p.136).

In mathematical activities, concepts are used not only in formal definitions but also in individual mental representations. These models, which exist before learning the mathematical concept, arise from experiences and are further reinforced by the learned concept (Cornu, 1981). However, even a correctly defined concept can sometimes lead to cognitive conflicts in students (Tall, 1988). Reported mental conflicts include difficulties with the representation and verbal expression of decimal numbers (Tall, 1977), the definition and expression of functions (Vinner, 1983), challenges with limits and continuity (Tall & Vinner, 1981), and the limit of functions (Ervynck, 1983).

In Turkey, middle school mathematics teacher candidates first encounter the concept of derivatives in Calculus 1 courses in the high school curriculum (MoNE, 2018, 2024). Considering the examination system, teacher candidates solve the fundamental concepts of limits, continuity, derivatives, derivative applications, and reading graphs at an operational level quickly and using test techniques (Dündar, 2015; Erdoğan, 2017).

Aim

The purpose of this study is to investigate the conceptual understanding of preservice elementary mathematics teachers and mathematics educators teaching calculus courses (Calculus 1, Calculus 2, and Calculus 3) about the concepts of slope, rate of change, and derivative, as well as the interconnections among these concepts. Students often rely on their mental images rather than formal definitions when learning mathematics. Therefore, research on concept images holds substantial significance in this context. The concept of derivative is one of the most important and fundamental concepts of calculus, both in terms of preparation for subsequent courses (e.g. differential equations) and in terms of its relationship with previous concepts (e.g. function, limit, continuity). Effective understanding of the concept of derivative requires knowledge of the slope, rate of change/instantaneous rate of change relationship. Therefore, it is important to examine the concept images of mathematics educators, who are one of the important factors in students’ learning of these concepts. A qualified mathematics teacher is expected not to have false concept images that may cause mislearning and misconceptions. In mathematics educators, concept definition and concept images about the concept of slope, rate of change and derivative are expected to be a cognitive activity between the definition cell and the image cell as in Tall and Vinner’s (1981) conceptual framework. According to some mathematics educators, regardless of the context in which the concepts of slope, rate of change and derivative are presented, it may be possible to present a coherent, meaningful interpretation of these concepts. In line with these aims, the research pursued answers to the following questions:

1. What are the concept images of pre-service elementary mathematics teachers about the relations of slope and instantaneous rate of change with derivative?
2. What are the concept images of the Mathematics Educators about the relations of slope and instantaneous rate of change with derivative?

Method

The study was based on a qualitative research approach. In the qualitative research, human behaviour is more meaningful if it is handled flexibly and holistically in research. In this context, the opinions of individuals, their behaviours, and their experiences in the natural environment have an important place in the research. In this context, a case study, one of the qualitative research designs, was used. In the case study, the boundaries of the situation are clear. It is a design that allows in-depth examination and investigation of a phenomenon, individual, group, or system (Bassey, 1999; Punch, 2005). For this reason, holistic multiple case design, one of the case study designs, would be used in the planned study. In the holistic multiple-case design, more than one situation can be perceived as holistic. Therefore, each situation is considered holistically in itself and compared with each other (Yıldırım & Şimşek, 2016). The situations in the study are analysed separately and then compared. The cases examined in this study are the concept images of each pre-service elementary mathematics teacher and mathematics educator and the concepts of slope, rate of change/instantaneous rate of change, and derivative. Concept images about each concept and concept images about the relationship of these concepts with other concepts constitute the cases of this study. Although concept images about these concepts are discussed separately, their relations with each other constitute a unity. To make sense of the study in all aspects, as Creswell et al. (2007) mentioned, many data were utilised during the study. In addition to the audio recordings of semi-structured interviews, the participants' written records were also used as data collection sources. The present study was conducted with mathematics educators working at different state universities who taught derivatives in their calculus courses (Calculus 1, Calculus 2, and Calculus 3) and prospective elementary mathematics teachers studying at a state university.

Due to the pandemic period, the study group research consisted of 7 pre-service teachers and four mathematics educators who were studying in a state university's elementary mathematics teaching programme and received a grade of BB or above in the Calculus 1 course. According to the relevant state university's undergraduate education and training examination regulations, the BB success grade, the success coefficient is 3.0, and the percentage equivalent is determined as 75-84. Because of students with a BB grade or higher have a certain level of academic success, making them suitable for analysing their concept images. In this respect, pre-service teacher participants were selected by purposive sampling. The sample consisted of n=3 pre-service teachers in the first year and n=4 in the second year. In addition, four mathematics educators working in different universities and responsible for the Calculus 1 course took part in the study.

Table 1

Demographic Information of the Pre-Service Elementary Mathematics Teachers Who Participated in the Study

Prospective Teacher	The Type of High School That Pre-Service Teachers Graduated From	Grade Level	Calculus1 Grade
Ö1	Science High School	2	BB
Ö2	Anatolian High School	2	BA
Ö3	Science High School	2	BB
Ö4	Science High School	1	AA
Ö5	Science High School	1	BA
Ö6	Anatolian High School	1	AA
Ö7	Anatolian High School	2	BB

When Table 1 is analysed, it is seen that the high schools the participants graduated from are predominantly Science High School and Anatolian High School. Considering the knowledge of prospective elementary mathematics teachers, it was thought that their concept images about slope, rate of change and derivative could be determined. Prospective elementary mathematics teachers need to be successful in the Calculus 1 course to determine their concept images about the concepts of analysis course (slope, rate of change and derivative), which require high-level knowledge (Orton, 1983).

Table 2

Demographic Information of the Mathematics Educators Who Participated in the Study

Mathematics Educator	Undergraduate Graduation	Postgraduate Graduation	Academic Title	Seniority
ME1	Elementary Mathematics Teacher Education	Elementary Mathematics Education	Asst. Prof.	5
ME2	Elementary Mathematics Teacher Education	Mathematics	Asst. Prof.	7
ME3	Secondary Mathematics Teacher Education	Mathematics	Assoc. Prof.	6
ME4	Elementary Mathematics Teacher Education	Elementary Mathematics Education	Asst. Prof.	3

Mathematics educators are responsible for Calculus 1, Calculus 2 and Calculus 3 courses. In this context, the mathematics educators participating in the study were selected by criterion sampling.

The pre-service elementary mathematics teachers who participated in the study were coded as Ö1, Ö2, Ö3, Ö4, Ö5, Ö6, and Ö7. Data were collected from the participants in order to examine the concept images of pre-service elementary mathematics teachers about the concepts of slope, rate of change and derivative. Firstly, semi-structured interviews were conducted. Then, the first stage of the second stage questions, including the application questions (questions 1, 2, and 3), were carried out. Finally, the answers of the prospective elementary mathematics teachers to the scenario questions in the second stage questions (Questions 4, 5, 6, and 7) were collected. Due to the pandemic, the data were collected face-to-face with the participants in the same settlement by taking necessary precautions. Unlike the pilot data, semi-structured interviews were conducted face-to-face with some pre-service elementary mathematics teachers (Ö6, Ö7, and Ö1). The interviews were video and audio recorded. The other participants (Ö2, Ö3, Ö4, Ö5) participated in semi-structured interviews online as in the pilot study. The data collection process started with semi-structured interview questions at the end of the autumn term 2020-2021. In the second stage (practice questions), answers were collected after the semi-structured interview questions were analysed.

The mathematics educators participating in the study were coded as ME1, ME2, ME3 and ME4. The data collection process with the mathematics educators was similar to that of the pre-service elementary mathematics teachers. The study's data were collected by determining the appropriate times for mathematics educators to participate. Semi-structured interviews were conducted face-to-face with ME1 and ME4. The second stage questions were received in written form. Semi-structured interviews with ME2 and ME3 were conducted online. The questions of the second stage were obtained from ME2 and ME3 in the digital environment.

The Data Collection Tools

The data collection tools of the study consisted of questions prepared by making use of the related literature and composed of two stages examining the relationship between slope, rate of change and derivative. The first stage questions consisted of seven semi-structured interview questions, and the second stage questions (application questions) consisted of seven open-ended questions, including one scenario about the relationship between derivative, slope and rate of change/instantaneous rate of change. Sub-dimensional questions in the second stage belong to the 1st, 2nd, and 3rd questions. Together with these sub-dimensional questions, the second stage consists of 12 questions in total. Three mathematics educators who are experts in the field examined the prepared data collection tools. In line with the expert opinions, the final version of the data collection tools was created, and the study was started.

Data Analysis

In the application dimension of the study, semi-structured interviews consisting of first-stage questions were conducted with each pre-service teacher for an average of 55 minutes. Considering the pandemic conditions, similar to the pilot application, the interviews were recorded online with video and audio recordings. In addition, semi-structured interviews with three pre-service teachers were conducted face-to-face. Face-to-face interviews were audio and video recorded in the same way. Two stages of the second stage questions were collected from pre-service elementary mathematics teachers. The answers to the first three questions were asked in the first stage. After a brief break, the answers to the last four questions containing scenarios were asked. The researcher stayed in the same environment as the participants while answering the questions. Audio and video recordings were made online with the other four pre-service teachers. The answers to the second stage questions were similarly asked online from 4 pre-service elementary mathematics teachers by taking video and audio recordings. For the second stage questions given online, pre-service teachers were asked to have their cameras turned on and to give their answers in front of the camera. The answers to the second stage questions were received digitally from the elementary mathematics teachers for the online interviews. For the second stage questions, the pre-service elementary mathematics teachers were asked to answer the first three questions at the first stage, and then a break was given. Care was taken to keep the camera on even during the break. Then, they were asked to answer the last four questions containing scenarios. Semi-structured interviews were conducted face-to-face with two mathematics educators by audio and video recording. Interviews with the other two mathematics educators were audio and video recorded online, considering the pandemic conditions. The second stage questions were received digitally from all math educators.

The researcher transcribed and analysed the first stage questions after the semi-structured interviews with pre-service elementary mathematics teachers were audio and video recorded. In the analyses, an analysis framework was used for each concept to determine the concept images about the ideas of slope, rate of change/instantaneous rate of change and derivative. Similarly, semi-structured interviews with mathematics educators, which were audio and video recorded, were transcribed and analysed. Similarly, the data of mathematics educators were analysed by using the analysis framework determined for each concept to reveal their concept images.

The second stage questions were analysed by examining the written answers given by primary school mathematics teachers. The answers given to the second stage questions were carried out online. Concept images about the concepts of slope, rate of change and derivative were revealed using the analysis frameworks by examining the semi-structured interview questions including the first stage questions and the answers given in the second stage questions. Similarly, the answers of mathematics educators to the first and second stage questions were determined by using analysis frameworks and their concept images were tried to be determined according to Tall and Vinner (1981).

The concept images of the concepts of slope, rate of change/instantaneous rate of change and derivative in the data obtained from semi-structured interview questions and open-ended questions were analysed in the context of different analysis frameworks and the concept images of mathematics educators and prospective elementary mathematics teachers were determined. As the theoretical framework, conceptual frameworks of the concepts in the study were used in the light of Tall and Vinner (1981) concept image and concept definition theoretical framework. The conceptual frameworks in the theoretical framework dimension of the study are explained as analysis frameworks in the method section.

Validity and Reliability

One of the ways to increase credibility in qualitative research is that it is carried out under expert control (Merriam & Tisdell, 2015; Yıldırım & Şimşek, 2016). In this context, all stages of the study were checked and evaluated by more than one expert. Within the scope of the research, the concept image codes determined within the scope of the data collection tool for the concepts of slope, rate of change/instantaneous rate of change and derivative were coded independently by different experts and the inter-coder consensus coefficient was calculated to determine whether there were differences between the coders. The consensus coefficient was obtained by considering the whole codes in the analysis frameworks used in the research. Reliability=Number of agreements/(Number of agreements+Number of disagreements), Reliability=19/(19+4), Reliability=.82. Inter-coder agreement is .82 and according to Fahy (2001), values above .70 are acceptable.

Slope Concept Analysis Framework

The studies and analysis frameworks related to the concept of slope are mentioned in the conceptual framework and related research section of the study. The concept images of mathematics educators and elementary mathematics teachers for the concept of slope were analysed using the analysis framework by Moore-Russo et al. (2011) in their study. Since the codes in Moore-Russo et al. (2011) analysis framework are more descriptive and detailed, this framework was used.

Table 3

Analysis Framework for the Slope Concept

Category	Slope	Code
Geometric Ratio (G)	"Rise over run" division of changes in y by changes in x	G1
	Vertical displacement (distance, change) over horizontal displacement (distance, change)	G2
Algebraic Ratio (C)	y divided by x change	C1
	Representation of the ratio with algebraic expressions, (y2-y1)/(x2-x1)	C2
Physical Property (F)	The property of the line often described using terms like "steepness" ("slope", "step", etc.); "How high it goes" or "goes up"	F
Functional Property (f)	Constant rate of change between variables	f
Parametric Coefficient (PK)	The coefficient m in the equation y = mx + b	PK
Trigonometric Understanding (T)	Tangent of the slope angle of a line	T1
	Direction component of a vector	T2
Calculus Understanding (K)	Limit	K1
	Derivative	K2
	Tangent line to a curve at a point	K3
Real World Situation (GD)	Static, physical situation (e.g., wheelchair ramp)	GD1
	Dynamic, functional situation (e.g., distance vs. time)	GD2
Determining Feature (B)	Feature determining parallel, perpendicular lines	B1
	Feature that can determine a line given a point	B2
Behavior Indicator (D)	Real number indicating the increasing, decreasing, horizontal tendencies of the line	D1
	Real number indicating the magnitude of the increase/decrease of the line	D2
	Real number indicating if the line is positive or negative, must intersect the x-axis	D3
Linear Constant (S)	Feature unaffected by translation	S1
	Constant feature unique to "straight" numbers	S2
	Constant feature independent of representation	S3

Note (Moore-Russo et al., 2011).

Rate of Change/Instantaneous Rate of Change Concept Analysis Framework

The studies and analysis frameworks related to the concept of rate of change/instantaneous rate of change are mentioned in the study's conceptual framework and related research section. The concept images of mathematics educators and elementary mathematics teachers for the rate of change/instantaneous rate of change concept were analysed using the analysis framework put forward by Hauger (1995).

Table 4

Rate of Change / Instantaneous Rate of Change Concept Analysis Framework

Category	Rate of Change	Code
Universal (Global)	Evaluates the general characteristics of the graph: increasing, decreasing, and changing rates.	D1
Interval	Addresses the directions of the average rate of change.	D2
Point-wise	Involves participation in the instantaneous rate of change.	D3

Note (Hauger, 1995).

Analysis Framework for Derivative Concept

The studies and analysis frameworks related to the concept of derivative were also mentioned in the study's conceptual framework and related research section. The concept images of mathematics educators and pre-service elementary mathematics teachers for the derivative concept were determined using the analysis framework put forward by Zandieh (2000).

Table 5

Analysis Framework for Derivative Concept

	Contexts				
	Graphical	Verbal	Physical	Symbolic	Other
Process-object pairs	Slope	Ratio	Speed	Division of Differences	
Ratio					
Limit					
Function					

Note (Zandieh, 2000).

Zandieh (2000) stated that he developed his framework by examining how the concept of derivative is defined in textbooks and by observing mathematics educators, mathematicians, calculus students in the classroom and their understanding of the concept of derivative. Each cell in Table represents one aspect of the concept of derivative. In the conceptual framework created by Zandieh (2000), the concept of derivative can be understood in the following ways:

a- graphically, the slope of a line tangent to a curve at a point, b-the instantaneous rate of change verbally,

c-physical speed or velocity,

d-can be symbolically presented as the limit of differences

Ethical Procedures

The ethics committee permission for the study, which was created from the relevant doctoral thesis, was obtained from Hacettepe University on 15. 03. 2021 with the number E-35853172-300-00001497785.

Results

Elementary Mathematics Teacher Candidates' Concept Image about the Relations of Slope and Instantaneous Rate of Change with Derivative

In the sub-problem of the research, the concept images of prospective elementary mathematics teachers about the relationship between the concepts of slope and instantaneous rate of change with the concept of derivative were analysed. The way followed here, the concept image of slope and the analysis framework of rate of change/instantaneous rate of change were analysed. In the first stage questions, "Is there a relationship between the concepts of derivative, instantaneous rate of change/rate of change and slope? If so, what kind of a relationship is there? Explain." This situation was analysed with the question in Table 6, which is one of the second stage questions.

Table 6

The Question About the Relationship Between Derivative, Slope and Instantaneous Rate of Change in the Second Stage Questions

Zeynep and Yeliz discuss the meaning of the derivative concept. Zeynep stated that the derivative at a point is equal to the rate of change at that point and the slope of the tangent drawn to the function at that point. Zeynep told Yeliz that $f(x) = x^2$ *the instantaneous rate of change, derivative and slope of the function at the point x=2 are equal to 4. Yeliz claimed that the slope at x=2 could be equal to the derivative. If you were Zeynep, how would you help Yeliz and how would you explain the relationship between these concepts?*

Table 7

Concept Images of Prospective Mathematics Teachers in the First Stage Questions

Participants		Ö1	Ö2	Ö3	Ö4	Ö5	Ö6	Ö7
		Images						
The relationship between derivative-instantaneous change and slope	Instantaneous change	Point	Point	Point	Point	X	X	X
	Slope	K2	T1	G1	K2	X	X	X
	Derivative	V-R-R/L/F	G-S/R	P-S-R	G-V-F	X	X	X

V: Verbal, R: Ratio, G: Graphical, S: Slope, P: Physical, Sp: Speed, F: Function, V: Variation, L: Limit, K2: Calculus, K3: Calculus T1: Trigonometry, G1: Real World. X: No concept images

Table 8

Concept Images of Prospective Mathematics Teachers in the Second Stage Questions

Participants		Ö1	Ö2	Ö3	Ö4	Ö5	Ö6	Ö7
		Images						
The relationship between derivative-instantaneous change and slope	Instantaneous change	Universal	Point	Universal	Interval	Point	Point	X
	Slope	G1	G1?	K2	K3	T1/K3/K2	K2	X
	Derivative	V-R/R/L	G-S/R	G-V-L	G-V-R/L/F	G-V-L	G-V-F	X

V: Verbal, R: Ratio, G: Graphical, S: Slope, P: Physical, Sp: Speed, F: Function, V: Variation, L: Limit, K2: Calculus, K3: Calculus, T1: Trigonometry, G1: Real World. X: No concept images

The concept images of prospective primary school mathematics teachers in the first and second stage questions are given in Table 7 and Table 8. When the tables are analysed:

The concept images of pre-service elementary mathematics teachers about the relationship between these three concepts were determined. One of the noteworthy points about the findings of the analyses is that when the answers of the pre-service elementary mathematics teachers to the first stage questions (semi-structured interview questions) were examined, it was seen that the concept images they had were slightly less than the concept images they had in the second stage questions (application questions). In addition, in general, prospective elementary mathematics teachers were able to use the concept images they had in the first stage questions in the second stage questions. This situation was observed in participants Ö4 and Ö6 in the first grades and participants Ö1 and Ö3 in the second grades.

When the participants were asked about all concepts one by one, it was seen that they had concept images; however, they needed help to use their concept image cells effectively in explaining the relationship between concepts. This situation drew attention, especially in the first stage of the questions. On the other hand, prospective elementary mathematics teachers tried to use their concept images to show the relationship between concepts in general, especially in the second-stage questions. However, it was determined that they needed help using their concept images about the relationship between the concepts of derivative, slope, and instantaneous rate of change in the last question in the first and second stage questions. These difficulties were their expressions that these three concepts were the same. However, it was determined that they could not explain a relationship with the concept of rate of change, although they expressed that the concept of derivative was a slope calculation.

Mathematics Educators' Concept Images of the Relationship between Slope and Instantaneous Rate of Change with the Derivative

Concept images of mathematics educators are given in Table 9 and Table 10.

Table 9

Concept Images of Mathematics Educators in the First Stage Questions

Participants		ME1	ME2	ME3	ME4
The relationship between derivative-instantaneous change and slope	Instantaneous change	Global	Point	Point	Point
	Slope	G1/G2/C1/C2/D/G2	K1/K2	K1/K2	K3/K2
	Derivative	O	V-R-L	G-V-R/L	G-I-L

V: Verbal, R: Ratio, G: Graphical, S: Slope, P: Physical, O: Other, F: Function, V: Variation, L: Limit, D: Derivative, I: Intercept, K2: Calculus, K3: Calculus, T1: Trigonometry, G1: Real World, G2: Real World, C1: Algebra, C2: Algebra

Tablo 10

Concept Images of Mathematics Educators in the Second Stage Questions

Participants		ME1	ME2	ME3	ME4
The relationship between derivative-instantaneous change and slope	Instantaneous change	Global	Point	Point	Point
	Slope	G1/G2/C1/C2/D/G2	K1/K2	K1/K2	K3/K2
	Derivative	O	V-R-L	G-V-R/L	G-I-L

V: Verbal, R: Ratio, G: Graphical, S: Slope, P: Physical, O: Other, F: Function, V: Variation, L: Limit, D: Derivative, I: Intercept, K2: Calculus, K3: Calculus, T1: Trigonometry, G1: Real World, G2: Real World, C1: Algebra, C2: Algebra

In the interviews with mathematics educators, their concept images of slope, rate of change/instantaneous rate of change and derivative concepts were examined. It was seen that the fact that mathematics educators had different experiences caused a difference in their concept images. The shaping of Piaget's schemes on how concept images are differentiated based on experience supports this situation. It can be said that mathematics educators used the concept images they had in the interview questions in the first stage questions in the application questions in the second stage questions. In addition, ME2 is the lecturer responsible for Calculus 1, Calculus 2, and Calculus 3 courses of the pre-service elementary mathematics teachers who participated in the study. In this context, it can be said that the concept images of the pre-service elementary mathematics teachers in the study and the concept images of the participant ME2 are similar. Namely, pre-service elementary mathematics teachers and ME2 frequently used the statement "*when we think of derivative, we should think of slope.*" It was seen that they used the expression frequently.

Discussion and Conclusion

Mathematics educators' and pre-service elementary mathematics teachers' concept images about slope, rate of change/instantaneous rate of change and derivative concepts were determined by using Zandieh's (2000) concept images about derivative concept, Hauger's (1995) concept images about rate of change concept and Moore-Russo et al. (2011) analysis frameworks. This situation is reported in the findings by analysing each sub-problem of the study separately. A total of 7 pre-service elementary mathematics teachers, 3 of whom were studying in the first year and 4 of whom were studying in the second year, participated in the study. In addition, 4 lecturers working at different universities responsible for Calculus1, Calculus2 and Calculus3 courses took part in the study. ME2, one of the lecturers participating in the study, is responsible for the Calculus1, Calculus2 courses of the pre-service elementary mathematics teachers in the study. Considering the situations in the study, firstly, the results about the concept images of prospective elementary mathematics teachers were given. Then, the results of the concept images of mathematics educators, which is another case of the study, are given by supporting the studies in the literature.

When the results related to the concept images of pre-service elementary mathematics teachers about the relationship between the concepts of slope, rate of change and derivative were analysed, it was seen that the concept images of pre-service elementary mathematics teachers about these three concepts were generally predisposed to explain with the concept of derivative. The results of the interviews and application questions were that we are actually doing derivative calculations. However, when they were asked about the relationship between these three concepts, it was seen that they tended to use the statements that the geometric interpretation of the derivative is the slope and the physical interpretation of the derivative is the rate of change/instantaneous rate of change. Although they were more likely to use the concept images of the concept of derivative, it is known that when the concept of derivative is defined, the slope of the beam (secant) lines is approximated to the tangent and from here, the derivative is defined by the slope of the tangent with the help of the limit. However, it was observed that the students did not have any understanding and concept image about beam (secant) lines. These results are similar to the studies in the literature (Açıkyıldız & Gökçek, 2015; Amit & Vinner, 1990; Bingölbali & Monaghan, 2008; Erdoğan, 2017; Hart, 1991; Likwambe & Christiansen, 2008; Nayir, 2013; Orton, 1983; Ubuz, 1996, 2001, 2007). It can be said that the teaching of the derivative concept may be related to the classical calculus teaching (Breen et al., 1992; Doruk et al., 2018; Duru, 2006; Grover, 2015; Sağırlı et al., 2010) and especially the reduction of the Analyses 1 course hours. It is also obvious that this situation may be different in studies with different methods and techniques (see Çekmez & Baki, 2019; Ndlovu et al., 2011). Considering the studies conducted in different countries, the results of the study confirm that the concept of derivative is an epistemologically difficult concept as mentioned by Cornu (1991).

When the related literature is examined, it can be said that giving the definitions of concepts that require high-level mathematical knowledge (limit, continuity, derivative, etc.) in only two hours of theoretical lessons will not be sufficient for conceptual learning (Aksu, 2016; Bingölbali & Monaghan, 2008; Engin, 2016; Tall & Vinner, 1981; Vinner, 1983). In particular, it is thought that it will be important to include a concept such as the concept of derivative, which is considered as the building block of the analysis course, in the minds of pre-service teachers in a more meaningful way (Engin, 2016; Gözen, 2001; Herman, 2002; Sağlam, 2011; Yeşildere, 2007). In addition, it is thought that it is important to clearly mention to the students that the concept of derivative has different representations and the relationships between these representations.

When the results related to the concept images of mathematics educators about the relationship between the concepts of slope, rate of change and derivative were analysed, it was seen that they tended to explain with the concept of derivative as stated in the results of prospective elementary mathematics teachers. When the statements and answers of the mathematics educators were examined, it was stated that "we actually examine the concept of derivative in these three concepts, we prepare the readiness of the concept of derivative". In addition, it was observed that the participants used dynamic geometry software while conveying the relationship between the concepts of slope, rate of change and derivative of ME1 and ME4 to the students. Similar to these results (Çekmez, 2013; Çekmez & Baki, 2019; Hart, 1991; Ndlovu et al., 2011; Öztoprakçı, 2014), it can be said that teaching the concept of derivative to students with dynamic geometry software helps students use concept images more effectively and internalise these concepts more. When the concept images of participant ME2 regarding the relationship between these three concepts are analysed, it can be said that he preferred to teach the concept of derivative to the students for the definition in calculus books. In addition, participant ME2 stated that it would not be beneficial for elementary mathematics teachers to use the field courses (Calculus1, Calculus2, Calculus3, Algebra, Abstract Mathematics etc.) that require high level knowledge in mathematics education in their future professional lives. This situation is similar to the findings of Kaymakçı et al. (2018). However, this situation is similar to the findings of basically you learn how to calculate with numbers, how to simplify algebraic expressions and calculate variables, how to reason about points, lines and shapes in the plane. The analysis includes these techniques and skills, but it also develops others at a finer and deeper level. Calculus, in fact, defines so many new concepts and numerical operations that you will not be able to learn everything you need in the classroom (Thomas & Finney, 2001).

When the results of the study are taken into consideration, the importance of the calculus course that requires high-level knowledge (Erol, 2013; Thomas & Finney, 2001) was in parallel with the study. From this point of view, it can be said that the concept images of prospective elementary mathematics teachers about the concept of slope, rate of change and derivative will not be clear.

Implications

In this study, the concept images of the participants about the concepts of slope, rate of change and derivative were determined. Prospective primary school mathematics teachers and mathematics educators were included in the study. In future studies, it can be considered important to include mathematics teachers in the participants as they use these three concepts in their professional lives. In this study, the concept of derivative and its sub-dimensions, which is the subject of Calculus 1 course, were examined. In the future, concept images of concepts in different content knowledge courses such as algebra, linear algebra, analytical geometry can be studied with different study groups.

Statement of Responsibility

The responsible author wrote the introduction, methods, findings and discussion, and finally the necessary conditions. The second author revised the introduction, discussion, conclusion and findings. Both authors contributed equally to the study.

Conflicts of Interest

There is no conflict of interest between the authors. The study, which was produced from the doctoral thesis of the responsible author, has the doctoral thesis supervisor as the second author.

Author Bios:

Asst. Prof. Dr. Ramazan Erol graduated from Hacettepe University, Department of Mathematics Education, under the supervision of Assoc. Prof. Dr. Elif Saygı. The corresponding author currently works as a faculty member at Afyon Kocatepe University, and the second author works as a faculty member at Hacettepe University. The two authors have previously carried out scientific studies together.

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