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Deflections of Cantilever Beams Subjected to A Point Load at the Free End

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Abstract

In this study, the displacements of cantilever beams for various slenderness ratios under point load are analyzed using Timoshenko and Bernoulli-Euler beam theories. The variation of the slenderness ratio is achieved only by changing the beam length. The results from these theories are compared with those from SolidWorks, which is considered a reliable simulation software. With this comparison, the % difference rates between the simulation and theoretical results are determined. This study explains under which conditions the Timoshenko and Bernoulli-Euler beam theories should be applied and evaluates the accuracy of the simulation software. Detailed research is carried out to examine its compatibility with these two theories. Some numerical results are presented to demonstrate their validity and sensitivity.

Keywords: Beam bending, Timoshenko beam theory, Bernoulli-Euler beam theory, SolidWorks.

1. Introduction

Beams are one of the bar elements frequently used in civil engineering and building design. Beams, which are one-dimensional structural elements, are generally in vertical and horizontal positions and are exposed to loads in the direction of the bar axis and perpendicular to the bar axis. To give examples of places where they are used, we can give some examples such as buildings and bridges. However, it is possible to see objects modeled as beams in many parts of life. For example, the working principle of the atomic force microscope (AFM) can be given as an example of a cantilever beam (see Fig. 1).

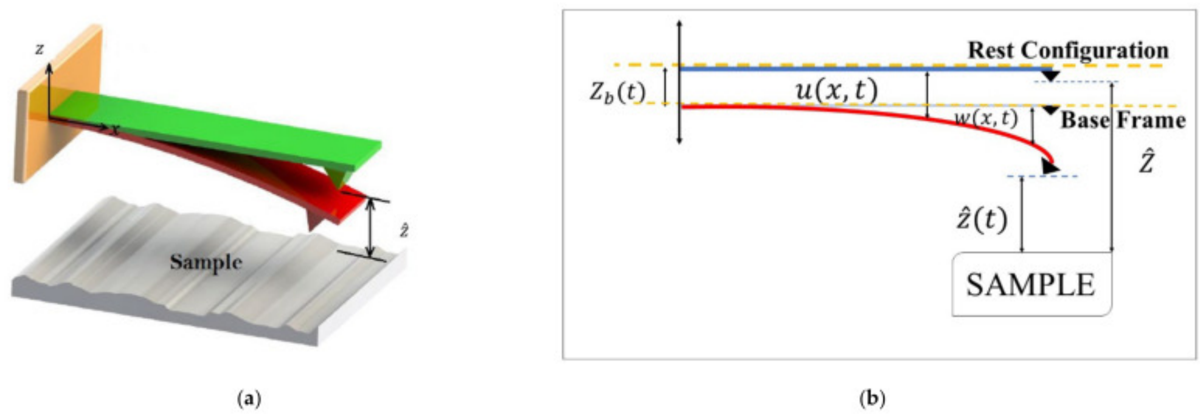

Fig. 1. The schematic view of an AFM's cantilever scanning a sample. a) 3D view of the AFM. b) 2D view of the cantilever [1].

The behavior of these elements under loads is of great importance for the safety, durability, and design of structures. Being able to analyze these behaviors accurately is essential for beams to be designed both safely and efficiently. Significant studies have been carried out on this subject from the past to present. The oldest known study among these studies was carried out by Leonhard Euler in 1744 [2]. In this study, Euler examined the displacements and buckling of beams and used algebra of variation to obtain these equations. Over the following years, this study became the pioneer of many studies to follow. Detailed information about these and the historical development of elastic stability can be found in the book "Theory of Elastic Stability" by Timoshenko and Gere [3]. There is also a study on the mechanics of materials by Gere and Timoshenko, which includes the theory of bending and displacement of beams [4]. Apart from these studies, quite extensive studies have been carried out. For example, Wang et al. [5] provide definitive solutions for buckling analysis of structural elements and aims to solve common problems in engineering structures by analytical methods. Zienkiewicz et al. [6] made a comprehensive resource on the finite element method (FEM), and this resource discusses the basic principles and applications of FEM in detail and explains how this method can be used to solve engineering and scientific problems. Bazant [7] addressed a comprehensive book that covers the basic principles of stability analysis, explaining in detail the various theories used to study the stability of structural members. Ghali et al. [8] is a comprehensive resource that combines classical and matrix methods used in structural analysis. Cook et al. [9] comprehensively explained the basic principles, mathematical foundations, and use of FEM in engineering practices and show how to use the finite element method in solving engineering problems [9]. While Akgöz et al. [10, 11] explained how to use Bernoulli-Euler beam model in the bending analysis of single-walled carbon nanotubes, in another study they presented a discrete singular convolution method for calculating the deflection analysis of beams resting on elastic foundations. Van Vinh et al. [12] investigated the static bending and buckling behaviors of bi-directional functionally graded (BFG) plates with porosity. Yaylacı et al. [13] carried out a numerical investigation on the vibration and buckling of functionally graded material (FGM) beam containing edge crack using FEM and multilayer perceptron (MLP) [13]. Azizi et al. [14] conducted a study to find deflection of a beam using finite element method based on Bernoulli-Euler and Timoshenko beam theories. Oladejo et al. [15] compared the results of deflection analysis of cantilever beam with COMSOL program to show its accuracy. Chaphalkar et al. [16] described the experimental apparatus and the associated theory which allows to obtain the natural frequencies and modes of vibration of a cantilever beam. Also, all the frequency values are analyzed with the numerical approach method by using ANSYS finite element package has been used. Hodzic [17] described the bending of cantilever beam and its analysis using finite element method. He compares the results obtained for boundary conditions with the results obtained from ANSYS simulation program. Raj et al. [18] conducted a study on modelling, simulation and analysis of cantilever beams by using ANSYS & MATLAB and theoretically by FEM for the evaluation of natural frequency and mode shape. Quang et al. [19] analyzed a three-span continuous beam using FEM and ANSYS via GUI method and APDL parameters. Ho et al. [20] compared the experimental results on deflection values of aluminum beam with the results calculated by FEM. SolidWorks 3D CAD software is used to build the beam model and perform finite element analysis. Samal et al. [21] investigated the deflection and stress distribution in a long, slender cantilever beam of uniform rectangular cross section made of linear elastic material properties that are homogeneous and isotropic. Finite element analysis of the beam was done considering various types of elements under different loading conditions in ANSYS 14.5. Balart Gimeno et al. [22-24] conducted some studies using the finite element method and the SolidWorks program. Onimowo [25] investigated the deflection and bending stress in a cantilever beam having a uniform rectangular cross section with a point load using a 3D Finite Element (FE) model. The results are validated using Bernoulli-Euler's elastic curve theory equations. Ya et al. [26] calculated the deflection of a cantilever beam was simulated under the action of uniformly distributed load. Then, compared the results obtained from the simulation with the results of Artificial Neural Networks (ANN). Talebi Rostami et al. [27] analyzed a Timoshenko beam with snowflake cross-section for different boundary conditions and variable properties. The equation of motion was solved with FEM and compared with the SolidWorks simulation. Gao [28] carried out deflection analysis of beams with the help of COMSOL program according to the finite element method and compared it with the results obtained from Timoshenko and Bernoulli-Euler beam theory. Onwubolu [29] presented to be an important resource for today's studies with his book on the use of SolidWorks.

As mentioned above, many studies have been carried out on this subject to date, since the concept of displacement has a decisive role in the design of structural elements exposed to loads. In this study, Bernoulli-Euler and Timoshenko beam theories will be compared for beams with different slenderness ratios that change as the cross-section changes along the beam length. Then, these results will be compared with SolidWorks, one of the simulation programs widely used in engineering design.

Bernoulli-Euler beam theory is one of the best-known and most used theories in engineering. This theory was developed by Jakob Bernoulli and Leonhard Euler in the 18th century. This beam theory is also known as thin beam theory or engineering beam theory. In Bernoulli-Euler beam theory, collapse and load-carrying behaviors are calculated. There are some assumptions in Bernoulli-Euler beam theory. In the approach of the Bernoulli-Euler beam theory, each beam section is perpendicular to the beam axis in case of bending. Bernoulli-Euler beam theory is independent of y-axis [30]. Therefore, the stresses in y-axis are neglected and the beam is assumed to have a straight axis. Due to these assumptions, Bernoulli-Euler beam theory is a simple beam theory to solve and use. Due to these negligible values, applying Bernoulli-Euler to beams with a high slenderness ratio will give us more accurate options. Because shear deformation and moment of inertia effects are less effective in beams with a high slenderness ratio.

Timoshenko beam theory was presented by Stephen Timoshenko in the early 20th century [31]. This beam theory has been one of the most widely used beam theories since it was presented. This is because, in addition to Bernoulli-Euler beam theory, it also uses values of shear deformation and moment of inertia due to shear force. Therefore, Timoshenko beam theory can be considered as an advanced version of the Bernoulli-Euler beam theory. Due to the shear deformation and moment of inertia values taken into calculation in the Timoshenko beam theory, it is expected that the Timoshenko beam theory will always give more accurate results than the Bernoulli-Euler beam theory. The deformed shapes of these two beam models are depicted in Fig. 2.

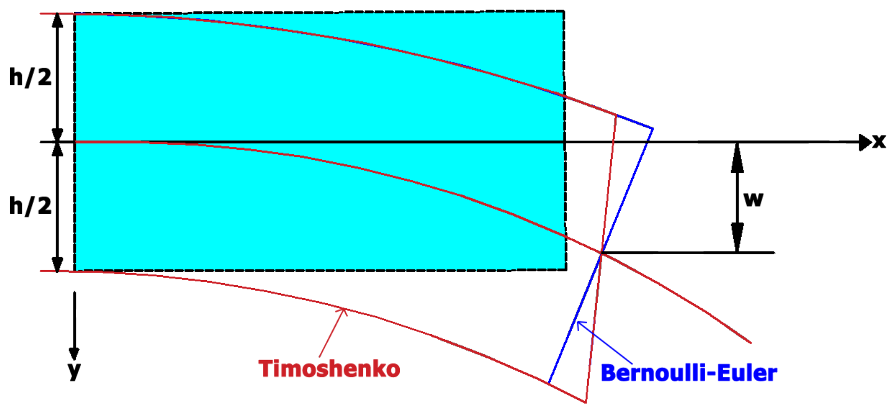

Fig. 2. Comparison of Timoshenko and Bernoulli-Euler beam deformations

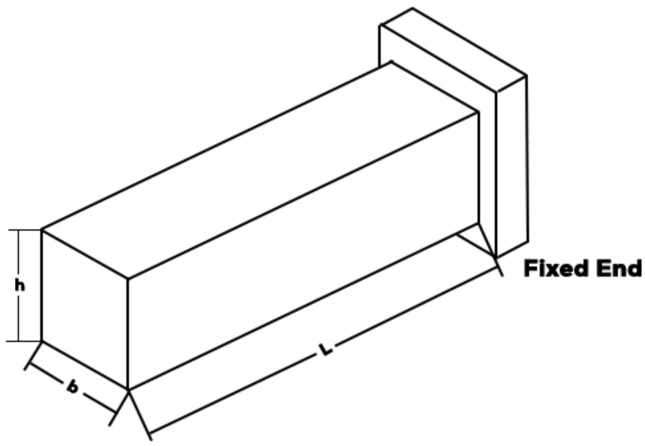

Fig. 3. Cantilever beam with rectangular cross-section

A schematic configuration of a cantilever beam with length L, height h, and width b is shown in Fig. 3. Timoshenko beam theory is also known as first-order shear deformation beam theory because it gives better results in such thick beams compared to Bernoulli-Euler beam theory. So, Bernoulli-Euler and Timoshenko beam theories are the two main approaches used in the deformation analysis of beams and have their advantages and disadvantages. Bernoulli-Euler beam theory is widely preferred because of its simplicity of solution and its realistic results on long beams. However, Bernoulli-Euler beam theory cannot provide sufficient accuracy for short and thick beams because it neglects shear deformation and moment of inertia. Although the Timoshenko beam theory involves a more complex mathematical solution, it gives more accurate results with the values of shear deformation and moment of inertia included in the calculation. Comparing the performances of these two theories on beams with different slenderness ratios is important to determine which theory is more suitable for engineering and structural design. Comparison of these two beam theories with the analysis and simulation software widely used in today's engineering applications will help us to see the compatibility between the theories and the software packages.

In this study, SolidWorks is chosen as the software to compare the theories. SolidWorks is a widely used software in engineering design. SolidWorks can perform detailed analysis of beams with complex geometries and load conditions using FEM. The simulation tools offered by SolidWorks provide the opportunity to analyze the behavior of beams under static and dynamic loads in detail. This study aims to examine how compatible SolidWorks simulations are with the theoretical results obtained with Bernoulli-Euler and Timoshenko beam theories. Particularly, which theory is more appropriate to use in displacement analyses of beams with different slenderness ratios, by comparing these theories with SolidWorks results, the validity and accuracy of the theories in practical applications will be tested. The results of the study will be a guide to providing more effective and reliable solutions in beam design.

2. Theory and Formulation

As shown in Fig.4, when a point load (P) of 1000 N is applied to a homogeneous cross-section cantilever beam with constant material properties from its non-fixed end, the displacement value according to Bernoulli-Euler beam theory can be expressed as follows:
From the moment-curvature relationship, we know that:

$$-EI\frac{d^2w}{dx^2} = M(x) \tag{1}$$

where E is the modulus of elasticity, I is the moment of inertia, and $M(x)$ is the bending moment at a distance x, and $\frac{d^2w}{dx^2}$ is the curvature of the beam. The bending moment at a distance x from the free end is given by, $M(x) = -Px$.

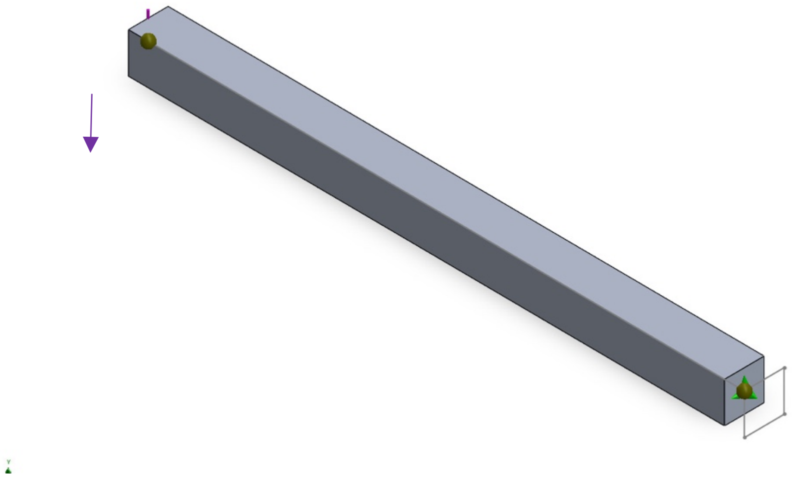

Fig. 4. Homogeneous, cantilever beam modeled in SolidWorks.

$$EI\frac{d^2w}{dx^2} = -M(x) = Px \tag{2}$$

$$\frac{d^2w}{dx^2} = \frac{P}{EI}x \tag{3}$$

Now, double integrating the above equation yields,

$$\frac{dw}{dx} = \frac{P}{EI}\int x\, dx \tag{4}$$

$$\frac{dw}{dx} = \frac{P}{2EI}\, x^2 + C_1 \tag{5}$$

$$w = \int\left(\frac{P}{2EI}x^2 + C_1\right)dx \tag{6}$$

$$w = \frac{Px^3}{6EI} + C_1 x + C_2 \tag{7}$$

Putting the boundary condition as $w = 0$ and $\frac{dw}{dx} = 0$ at $x = L$, the integral constants are determined as

$$C_1 = -\frac{PL^2}{2EI}, C_2 = \frac{PL^3}{3EI} \tag{8}$$

So, deflection at any point at distance x from the free end is given by,

$$w(x) = \frac{Px^3}{6EI} - \frac{PL^2}{2EI}x + \frac{PL^3}{3EI} \tag{9}$$

The deflection at the free end of the rectangular cantilever beam is

$$w(0) = \frac{PL^3}{3EI} \tag{10}$$

and the moment of inertia is

$$I = \frac{bh^3}{12} \tag{11}$$

It can be defined as: Again, for a homogeneous cross-section cantilever beam with constant material properties, the displacement value according to Timoshenko beam theory can be expressed as follows:

Let us assume that the clamped end is at $x = L$ and the free end is at $x = 0$. If a point load P is applied to the free end in the positive *y* direction, a free-body diagram of the beam gives us

$$-Px - M(x) \ = 0 \ \rightarrow \ M(x) \ = -Px \tag{12}$$

$$P + Q(x) = 0 \ \rightarrow \ Q(x) = -P \tag{13}$$

Therefore, from the expressions for the bending moment and shear force, we have

$$Px = EI\frac{d\varphi}{dx} \ and, -P = kAG\left(-\varphi + \frac{dw}{dx}\right). \tag{14}$$

where k is the shear correction factor, *G* is the shear modulus, *A* is the cross-sectional area, and φ is the angle of rotation of the cross-section. Integration of the first equation, and application of the boundary condition $\varphi = 0$ at $x = L$, leads to

$$\varphi(x) = -\frac{P}{2EI}\left(L^2 - x^2\right). \tag{15}$$

The second equation can then be written as

$$\frac{dw}{dx} = -\frac{P}{kAG} - \frac{P}{2EI}\left(L^2 - x^2\right). \tag{16}$$

Integration and application of the boundary condition $w = 0$ at $x = L$ gives

$$w(x) = \frac{P\,(L-x)}{kAG} - \frac{Px}{2EI}\left(L^2 - \frac{x^2}{3}\right) + \frac{PL^3}{3EI} \tag{17}$$

The deflection at the free end of the cantilever beam is

$$w(0) = \frac{PL}{kAG} + \frac{PL^3}{3EI} \tag{18}$$

Since all these variables will change with the material we choose and the dimensions of the beam, we need to give the properties and dimensions of the material we choose:

Table 1. The values used in this study for an aluminum (5052-O) homogeneous cantilever beam

Variable	Symbol	Value	Unit
Singular Load	P	1000	N
Length	L	250,500,…,5000	cm
Height	h	50	cm
Width	b	50	cm
Cross-Section Area	A	bh	cm^2
Moment of Inertia	I	$bh^3/12$	cm^4
Young's Modulus	E	7000000	N/cm^2
Poisson's Ratio	v	0.33	-
Shear Correction Factor	k	5/6	-
Shear Modulus	G	E/(2+2v)	N/cm^2

As can be seen in Table 1, these values are repeated each time by increasing the length by 250 cm to provide different slenderness ratios.

3. Results and Discussion

In this section, the displacement behavior of homogeneous cross-section beams with variable slenderness ratios under load is examined within the framework of slenderness ratios. Results are given for the cases where one end of the beam is fixed and the other end is free. Firstly, a comparison was made with the SolidWorks simulation program to prove the validity and sensitivity of the solution methods used. The Solidworks simulation program separated this cantilever beam into a certain number of finite elements as seen in Fig. 5.

In Table 2, the maximum displacement values of homogeneous cantilever beams are compared with the SolidWorks simulation of Timoshenko and Bernoulli-Euler beam theories for various slenderness ratios. When the results are examined (examining the analysis result screen in Fig. 6. may also give an idea.), it can be said that the results obtained in this study are in excellent agreement with the other results.

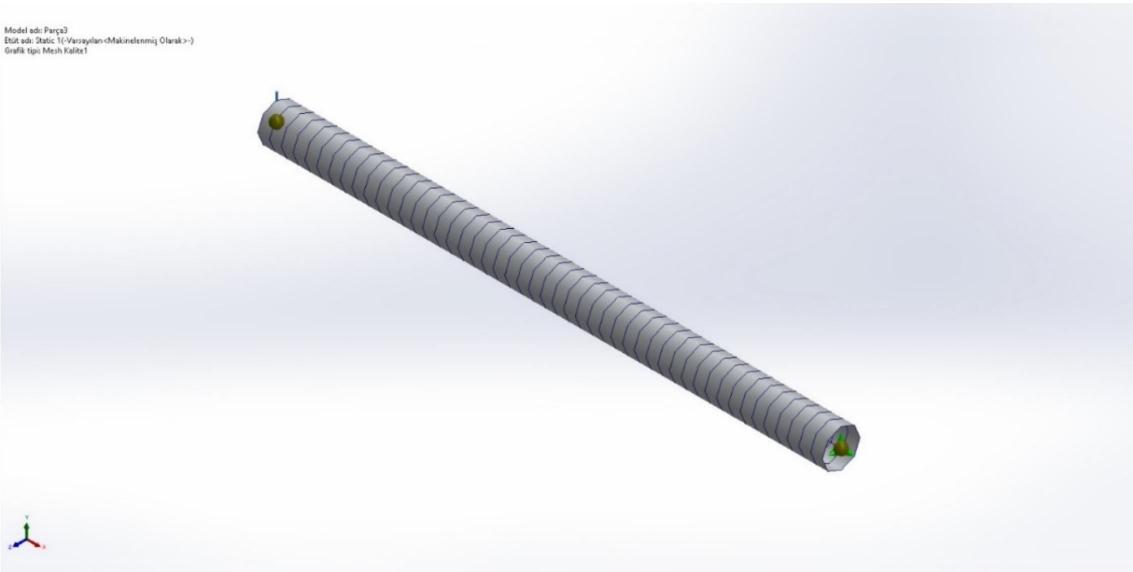

Fig. 5. Separation of the beam into finite elements.

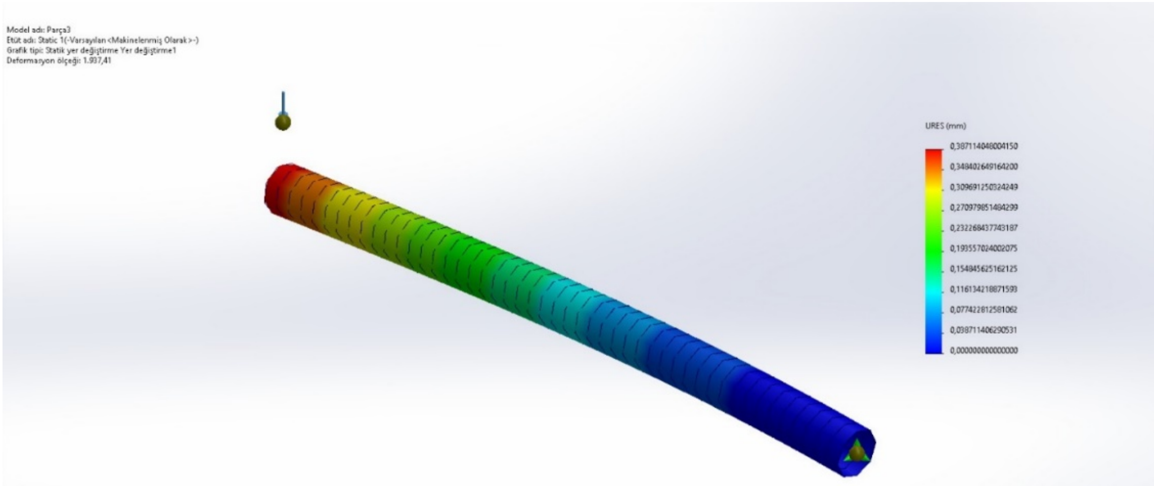

Fig. 6. Analysis result screen

Table 2. Comparison of beam theories with SolidWorks on the maximum displacements (*cm*) with respect to various slenderness ratios

Slenderness Ratio (*L/h*)	Bernoulli-Euler	Timoshenko	SolidWorks
5	0,001428571	0,001474171	0,001475231
10	0,011428571	0,011519771	0,011521892
15	0,038571429	0,038708229	0,038711405
20	0,091428571	0,091610971	0,091615212
30	0,308571429	0,308845029	0,308851331
40	0,731428571	0,731793371	0,731801891
50	1,428571429	1,429027429	1,429038048
60	2,468571429	2,469118629	2,469130993
70	3,920000000	3,920638400	3,920653152
80	5,851428571	5,852158171	5,852175236
90	8,331428571	8,332249371	8,332266808
100	11,428571430	11,42948343	11,42950439

Table 3. Comparison of % differences in maximum displacements with respect to various slenderness ratios

Slenderness Ratio (*L/h*)	Bernoulli-Euler –Timoshenko % Difference	Timoshenko – SolidWorks % Difference
5	3,093263044	1,234213101
10	0,791682375	0,523117832
15	0,353413228	0,465995258
20	0,199102790	0,399235343
30	0,088588119	0,297550460
40	0,049850137	0,243612856
50	0,031909814	0,266442410
60	0,022161754	0,181457275
70	0,016283062	0,145875785
80	0,012467195	0,138543153
90	0,009850881	0,127191497
100	0,007979363	0,108483805

Table 4. Number of Nodes - Number of Elements Comparison of SolidWorks Analyses

Slenderness Ratio (L/h)	Number of Nodes	Number of Elements
5	25	23
10	39	37
15	50	48
20	60	58
30	79	77
40	95	93
50	110	108
60	124	122
70	137	135
80	149	147
90	161	159
100	173	171

The % differences between the displacement values of a homogeneous cantilever beam for different slenderness ratios are tabulated in Table 3 and plotted in Fig. 7. It can be observed that the % difference of Bernoulli-Euler beam theory gradually decreases as the slenderness ratio value increases. Timoshenko beam theory, on the other hand, despite its low % difference, has a % difference value that decreases even more as the slenderness ratio increases.

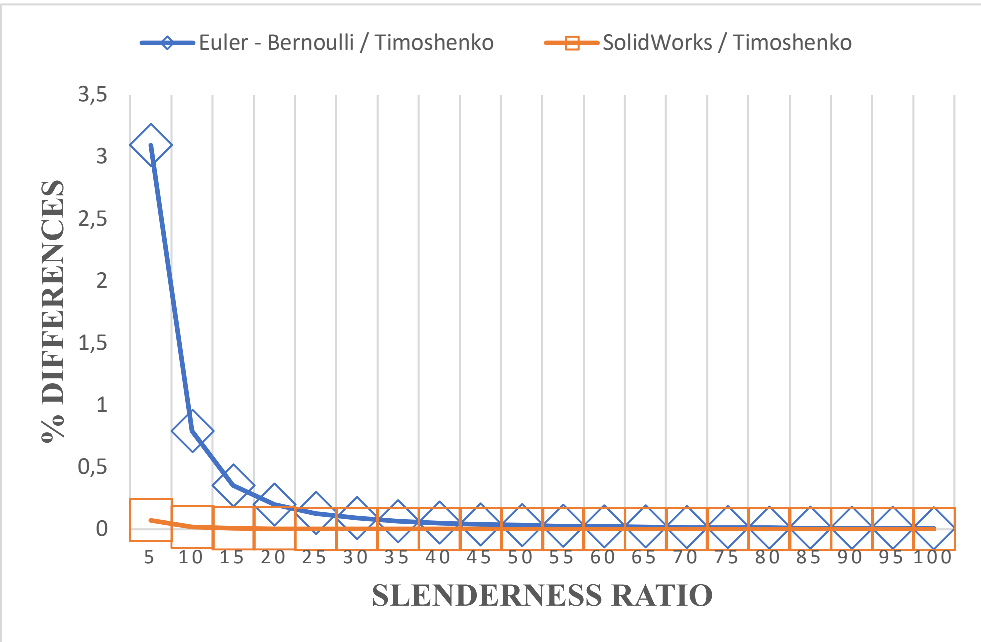

Fig. 7. Comparison of % differences

Fig. 8. and Table 4 show the Number of Nodes - Number of Elements. As can be seen here, the relationship between Number of Nodes - Number of Elements has a constant increase.

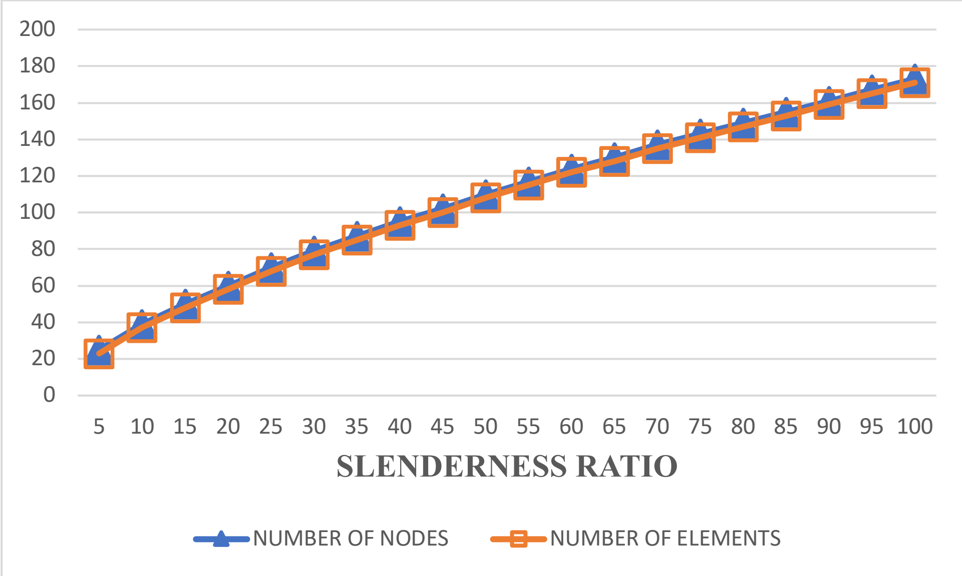

Fig. 8. Comparison of Number of Nodes - Number of Elements

4. Conclusions

In this study, the displacement behavior of cantilever beams with homogeneous cross-sections and various slenderness ratios subjected to point load was examined. The results obtained using Bernoulli-Euler and Timoshenko beam theories are compared with the results obtained from the simulation program SolidWorks. While it has been observed that the Timoshenko beam theory produces results very close to the simulation program values and the % difference decreases as the slenderness ratio increases, it has also been observed that the Bernoulli-Euler beam theory obtains results very close to reality as the slenderness ratio increases. With this inference, Timoshenko beam theory produces results very close to reality in beams with low slenderness ratio, while Bernoulli-Euler produces results very close to reality and very close to the results obtained from Timoshenko beam theory in beams with high slenderness ratio. In other words, it has been concluded that the effect of shear deformation and moment of inertia values, which are effective in Timoshenko beam theory, on the displacement value decreases as the slenderness ratio increases. Moreover, if we compare all the data we obtained, we see that SolidWorks gives us results very close to the theoretical solution. For this reason, when it is not possible to make a theoretical solution, when larger and more complex problems need to be solved in a shorter time, very good results can be obtained with SolidWorks. Since SolidWorks makes a solution based on FEM, no matter how large or complex the systems are, good results can be obtained as long as proper modeling is done. In addition, when the results obtained in this article are interpreted properly, correct inferences can be made at the nanomechanics and can be used to give ideas in studies to be carried out at the nanomechanics.

Author Contributions

Bekir Akgöz: Designed and directed the paper, Verified the theories and methods, Conceived and designed the analysis, Contributed data and analysis tools, Prepared the paper for publication.

Alper Oğulcan Söylemez: Conducted a literature review, Collected the data, Performed the analysis, Wrote the paper, Prepared the paper for publication.

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