Instructional Technology and Lifelong Learning Vol.5, Issue 2, 257-277 (2024)

ITALL

Research Article

ISSN: 2717-8307

The Process of Teaching and Learning from the Views of Prospective Mathematics Teachers: The Case of Algebraic Inequalities

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Abstract

This study explores the views of prospective mathematics teachers on the process of teaching and learning about algebraic inequalities. Phenomenology, one of the qualitative research methods, was used in the study. The participants of the study consisted of 6 prospective middle school mathematics teachers studying in the last year of the Elementary Mathematics Teacher Education Program of a university in the Central Anatolia Region of Türkiye in the spring semester of the 2021-2022 academic year. Data were collected through semi-structured interviews. The obtained data were analyzed using content analysis, one of the qualitative analysis methods. As a result of the analysis of the interviews with the prospective middle school mathematics teachers, three main themes were obtained. Teaching inequalities, the difficulties that students may experience in inequalities, and the methods that can be used to overcome difficulties were the main categories that emerged. Direct quotations from the prospective teachers' views were also included in the presentation of the findings. As a result of the study, it is recommended that more practice should be included in teacher education programs to improve prospective mathematics teachers' knowledge and experience of meaningful learning and overcoming difficulties.

Keywords: Mathematics education, Algebra, Inequalities, Prospective teachers, Views

Citation: Topbaş Tat, E., & Gümüş, H. (2024). The process of teaching and learning from the views of prospective mathematics teachers: the case of algebraic inequalities. *Instructional Technology and Lifelong Learning, 5*(1), 257-277. https://doi.org/10.52911/itall.1552703

Matematik Öğretmen Adaylarının Gözünden Öğretme ve Öğrenme Süreci: Cebirsel Eşitsizlikler Örneği

Özet

Bu çalışma, matematik öğretmeni adaylarının eşitsizlikleri öğrenme ve öğretme sürecine ilişkin görüşlerini araştırmaktadır. Çalışmada nitel araştırma yöntemlerinden fenomenoloji kullanılmıştır. Çalışmanın katılımcılarını 2021-2022 eğitim-öğretim yılı bahar döneminde Türkiye'nin İç Anadolu Bölgesi'ndeki bir üniversitenin İlköğretim Matematik Öğretmenliği Programı son sınıfında öğrenim gören 6 ortaokul matematik öğretmeni adayı oluşturmaktadır. Veriler yarı yapılandırılmış görüşmeler yoluyla toplanmıştır. Elde edilen veriler nitel analiz yöntemlerinden biri olan içerik analizi kullanılarak analiz edilmiştir. Ortaokul matematik öğretmeni adayları ile yapılan görüşmelerin analizi sonucunda üç ana tema elde edilmiştir. Eşitsizliklerin öğretimi, öğrencilerin eşitsizlikler konusunda yaşayabilecekleri zorluklar ve zorlukların üstesinden gelmek için kullanılabilecek yöntemler ortaya çıkan ana kategoriler olmuştur. Bulguların sunumunda öğretmen adaylarının görüşlerinden doğrudan alıntılara da yer verilmiştir. Çalışma sonucunda, matematik öğretmeni adaylarının anlamlı öğrenme ve zorlukların üstesinden gelme konusundaki bilgi ve deneyimlerini geliştirmek için öğretmen eğitimi programlarında daha fazla uygulamaya yer verilmesi önerilmektedir.

Anahtar Kelimeler: Matematik Eğitimi, Cebir, Eşitsizlikler, Öğretmen adayları, Görüşler

Date of Submission	19.09.2024
Date of Acceptance	12.11.2024
Date of Publication	31.12.2024
Peer-Review	Double anonymized - Two External
Ethical Statement	It is declared that scientific and ethical principles have been followed while carrying out and writing this study and that all the sources used have been properly cited.
Acknowledgments	-
Author(s) Contribution	**Author1:** Conceptualization, Methodology, Writing- Original draft preparation, Writing- Reviewing and Editing **Author2:** Data curation, Writing- Original draft preparation
Plagiarism Checks	Yes - Turnitin
Conflicts of Interest	The author(s) has no conflict of interest to declare.
Complaints	itall.journal@gmail.com
Grant Support	The author(s) acknowledge that they received no external funding supporting this research.
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1. Introduction

Algebra is used in almost every aspect of our lives and is defined in many different ways. Algebra can be defined as "the branch of mathematics that deals with symbolizing general numerical relationships and mathematical structures and with operating on those structures" (Kieran, 1992, p.391). The word algebra comes from the name of Harizmi's book. "The term Al Cabr, which is mentioned in the title of Harizmi's book, was used as algebra in English and French and was used as cebir in Turkish" (Baki, 2014, p.81). Algebra acts as a conceptual bridge between both sub-fields of mathematics and elements of other disciplines (Erbaş et al., 2009). It is also a field that opens the doors of abstract thinking to students.

Algebra is one of the five learning areas of the Turkish Middle School Mathematics Curriculum (Ministry of National Education [MoNE], 2018). The five basic sub-learning areas of this learning area in the curriculum are algebraic expressions, equality and equations, linear equations, algebraic expressions and identities, and inequalities. Among these sub-learning areas, algebraic expressions are included in 6th and 7th grade, equality and equations in 7th grade, linear equations, algebraic expressions, and identities and inequalities in 8th grade (MoNE, 2018). The last sub-learning area taught in middle school algebra is inequalities. Although inequalities are included in the curriculum in grade 8, the topics of equality and equations that can be the basis of this subject are included in grades 6 and 7, and the use of inequality symbols at a basic level has been taught since primary school. Inequalities are open propositions like equations and differ in that they include one of the ordering relations such as "≤", "<", ">" and "≥" instead of "=" (Argün et al., 2014). Since inequalities are also complementary to the understanding of the concepts of equivalence and equality, it is stated that the knowledge possessed is critical in realizing the conceptual understanding of equations and equation solution (Tsamir & Almog, 2001).

Algebra teaching in schools aims to provide students with competencies such as being aware of the meanings of symbolic and graphical representations, expressing mathematical results and relationships through symbols, identifying variables in problem situations by making sense of the concept of variable, and determining solution sets of equations and inequalities (Baki, 2008). However, studies conducted in this area have revealed that students at different grade levels generally have difficulties in the subjects of the algebra learning area (Abu Mokh

et al., 2019; Dede & Peker, 2007; Knuth et al., 2005; MacGregor & Stacey, 1997.). Inequalities are one of the topics that students have difficulty in algebra (Blanco & Garrote, 2007; Çoban & Yenilmez, 2020; Siagian et al., 2022; Tsamir & Almog, 2001). When we look at the studies on inequalities, it is seen that students generally have difficulties in the process of solving inequalities, in expressing and interpreting the solutions obtained (Blanco & Garrote, 2007). For example, students have been found to multiply a negative number to an inequality without reversing the direction of the inequality sign (Kroll, 1986). Similarly, Çoban and Yenilmez (2020) found that the most common difficulties with inequalities were determining the direction of inequalities in reverse, not realizing the situation of equality, writing the algebraic expression appropriate to the given expression incorrectly, showing the inequality incompletely on the number line, showing the inequality in the opposite direction on the number line, making an operation error in solving the inequality, and ignoring the negative sign.

Obstacles in the understanding of algebra can be analyzed under three headings: the structure of algebra (epistemological obstacle), students' mental development and readiness level (psycho-genetic obstacle), and deficiencies in algebra teaching (didactical obstacle) (Reconceptualising School Algebra, 1997, as cited in Dede & Argün, 2003). The obstacles related to deficiencies in algebra teaching emphasize the importance of the teacher's role in understanding algebra. The selected teaching models, the application of these models, the metaphors, examples, materials and activities used by the teacher may cause students to experience difficulties in algebra teaching. For example, misconceptions may arise due to students' incomplete understanding of the new knowledge or incorrectly associating it with the previous knowledge (Rowell et al., 1990). Similarly, concretizing mathematical concepts at the primary and secondary school levels will facilitate students' meaningful learning (Clements & McMillen, 1996). For this reasons, teachers' having sufficient knowledge in their field, keeping up with the new approaches, and being able to analyze the mistakes and misconceptions that their students may make will make algebra teaching much more understandable and effective. For this purpose, it is important to examine prospective middle school mathematics teachers' views about teaching and learning process of algebraic inequalities. Despite calls from the mathematics education community for more research in this area, there has been relatively little research into inequalities (Moon, 2019). When the

literature on inequalities is examined, it is seen that most of the studies are about students' difficulties with inequalities, besides, there are a few studies investigating prospective teachers' awareness of these difficulties, but no study has been found on their views on teaching and learning of inequalities. Therefore, the research problem of this study was determined as: What are the views of prospective middle school mathematics teachers on the process of teaching and learning inequalities? And the sub-problems were identified as: (1) What are the views of prospective middle school mathematics teachers on how the teaching of inequalities should be? (2) What are the views of prospective middle school mathematics teachers on the difficulties that students may experience with inequalities? (3) What are the views of prospective middle school mathematics teachers on the methods that can be used to overcome difficulties that students may experience with inequalities? It is thought that the answers to these questions will contribute to eliminating the deficiency in the literature on this subject by revealing the views and awareness of prospective teachers about teaching inequalities.

2. Method

In this study, the views of prospective teachers on the subject of inequalities were examined. Phenomenology, a qualitative research method, was used in the study. A phenomenological study reveals the meaning of experiences related to a particular phenomenon for individuals (Rose et al., 1995). Similarly, Creswell (2013) defines phenomenology as the meaning of people's experiences about a phenomenon or concept. The purpose of phenomenological research is to reveal in detail the thoughts of individuals about a phenomenon based on their experiences, feelings, and thoughts. In the present study, the concept of teaching inequalities is considered as a phenomenon. How do the prospective teachers handle teaching the concept of inequalities? Are the prospective teachers aware of students' difficulties with the concept of inequalities? How do they plan to solve these difficulties? So, this research aims to bring an understanding and explanation to such questions.

2.1. Participants

The convenience sampling method was used to determine the participants of the study. Convenience sampling is a non-probability sampling method in which participants are selected based on their ease of access, availability, and willingness to participate in a study

(Etikan et al., 2016; Yıldırım & Şimşek, 2006). The participants of the study consisted of 6 prospective middle school mathematics teachers studying in the last year of the Elementary Mathematics Teacher Education Program of a university in the Central Anatolia Region of Türkiye in the spring semester of the 2021-2022 academic year. Four of the prospective teachers participating in the study are female and two are male. Participants were identified on a voluntary basis.

2.2. Data Collection

In the study, a semi-structured interview form was used to determine the views of prospective mathematics teachers on the teaching of inequalities. Although a semi-structured interview begins with a predetermined set of questions, additional questions, called probes, can be generated during the interview based on the interviewee's responses (Tyson, 1991). A semi-structured interview form was prepared by the researchers, which included questions about how the teaching process should be regarding the three objectives of inequalities at the 8th-grade level, which concrete materials and technologies can be used, what kind of activities can be planned, what difficulties students may experience in this subject, how they plan to overcome these difficulties, and to what extent they consider themselves competent to overcome these difficulties. For example, the objectives of the inequalities sub-learning area of the 8th grade algebra learning area were specified and they were asked how these objectives should be taught and what attention should be paid to teaching them. The prepared interview form was presented to the expert opinion, and the interview form, which was finalized with an expert suggestion, was made ready for application. The data were collected at the end of the spring semester of the 2021-2022 academic year. Face-to-face interviews were conducted with the prospective teachers, and the interviews were recorded with the permission of the participants. Each interview lasted between 15-25 minutes.

2.3. Data analysis

In order to analyze the data obtained, the audio-recorded interviews were first transcribed and recorded as separate written documents for each prospective teacher. The names of the prospective teachers were not used when saving the files, but each prospective teacher was numbered and coded as PT1, PT2, PT3, PT4, PT5 and PT6. The same codes were also used in the presentation of the findings to indicate the prospective teachers. The data obtained were

analyzed using content analysis, one of the methods of qualitative analysis. The written data were first classified through coding and the codes were handled within the framework of sub-themes. Themes were obtained by rearranging the sub-themes. The presentation of the findings on the themes included direct quotations from the views of the prospective teachers.

3. Result

As a result of the analysis of the interviews with prospective teachers, three main themes were obtained: Views on teaching inequalities, views on the difficulties that students may experience in inequalities, and views on the methods that can be used to overcome difficulties. In this section, the themes obtained will be examined in subtitles.

3.1. Views on Teaching Inequalities

This main theme consists of the sub-themes of prerequisite knowledge, issues to be considered in teaching, and materials and activities that can be used in teaching.

3.1.1. Prerequisite Knowledge

Under this sub-theme, prospective teachers' views on the concepts and topics that are prerequisite for the inequalities sub-learning area of 8th grade algebra learning area were determined. As prerequisite concepts, prospective teachers stated the meanings of concepts such as less, more, least, greatest, greater than, less than, greater than or equal to, and less than or equal to; equality, variable and unknown, algebraic expressions, equations and linear equations. The views of the participants about the prerequisite concepts are presented in Table 1.

Table 1.

Views of Prospective Teachers for Prerequisite Concepts

	Participant					
Code	PT1	PT2	PT3	PT4	PT5	PT6
The meanings of concepts such as less, more, least, greatest, greater than, less than, greater than or equal to, and less than or equal to	X		X		X	
Equality		X	X	X	X	X
Variable and unknown		X				X
Algebraic expressions	X	X		X		X
Equations	X	X	X			X
Linear Equations				X		

As can be seen from Table 1, all prospective teachers except the PT1 have stated the concept of equality as a prerequisite for the teaching of inequalities. While the linear equations are presented as a prerequisite for inequalities by only one prospective teacher, four prospective teacher specified algebraic expressions (except for PT3 and PT5) and equations (except PT4 and PT5) as prerequisite concepts. Some examples including the opinions of prospective teachers about prerequisite concepts are presented below.

> PT2: *For example, first of all, they need to learn the concept of variable or unknown concept. Then the equation; the algebraic expression, equation they had seen before. Because, for example, in the solution of inequalities, it looks like the solutions of equations, they must first know them. … With equations, you can first know the meaning of an equality and then move on to the meaning of an inequality…*
>
> PT3: *First of all, equality. Equality is very important to learn inequality. The balance situation is very important, the students need to know it. In order to pass on to inequality, they need to know very well what the equation is. Or, the student should know the expressions such as the least and the greatest, in advance. For this, because in order to make sense of those symbols in inequality…*
>
> PT4:*… Algebraic expressions that we saw first in the sixth grade, we can say this as a prerequisite. Then they see again in the 8th grade, they see the thing this time, they see linear equations. These two issues are actually very related to this, so it would be better for me to learn equality first and then learn about the inequality.*

3.1.2. Issues to be Considered in Teaching

Under this sub-theme, it was tried to determine the views of the prospective teachers on the important issues that should be considered in the teaching of inequalities sub-learning area of the 8th grade algebra learning area. In this direction, the objectives of the inequalities sub-learning area of 8th grade algebra learning area were specified and it was asked how the

teaching of these objectives should be and what should be paid attention in teaching. Many codes have been collected under this sub-theme. For example, prospective teachers emphasized the importance and necessity of using information technologies as well as concrete materials in the teaching of inequalities. Therefore, two of these codes were determined as "use of concrete materials" and "use of information technologies". The views of the prospective teachers on the issues to be considered in the teaching of inequalities are presented in Table 2.

Table 2.

The Views of Prospective Teachers about the Issues to be Considered in Teaching

	Participant					
Code	PT1	PT2	PT3	PT4	PT5	PT6
Concretization	X	X	X	X	X	X
Use of concrete materials	X	X				X
Use of information technologies	X		X			X
Use examples from daily life or the environment	X	X	X	X	X	X
Utilizing discussions					X	
Transition between verbal and algebraic representations		X				X
Work on the number line	X	X	X	X	X	X
Benefit from activities			X			
Paying attention to readiness	X	X	X	X		
Explaining concepts and rules with reasons		X		X	X	X

According to Table 2, all of the prospective teachers mentioned the importance of using examples and problems from daily life and the environment, as well as concretization in the teaching of inequalities. They also emphasized the necessity of giving special attention to the representations and studies on the number line. Some of the prospective teachers' opinions gathered under this sub-theme are presented below:

> PT1: … *For example, let me give an example from myself. This second thing, the explanation was a stage that I never understood, you know, you change direction, but how does it change direction? It's negative, you know, it's multiplied by a negative, okay, but I never understood why. Therefore, when multiplying or dividing is with a negative number, first of all, the students should definitely find out why. If we start from there, everything will be solved in my opinion…*
>
> PT2: … *Because they can learn in a more meaningful way by associating it with daily life. If we give it directly with memorisation, the student may not know, for example, in which case to use greater than, less than or less than or equal to, greater than or equal to. But I think it may make more sense to students when they think about daily life…*

> *PT3 : ... we can use virtual manipulatives and dynamic software for better comprehension. For example, GeoGebra can be very suitable for this. Because while a variable is changing, both its increase and decrease, why it is more, why it cannot exceed that limit can be shown on GeoGebra. In this way, it makes more sense in the child's mind. ...*

When the prospective teachers were asked what kind of suggestions they had to increase the effectiveness of teaching inequalities, they made statements similar to their views in Table 2. In addition, some prospective teachers mentioned some codes under the title of suggestions to increase the effectiveness of teaching inequalities, which they did not mention under the sub-theme of issues to be considered in teaching. For example, although PT2 did not mention the use of activities in the sub-theme of issues to be considered in teaching, she suggested the use of activities in the suggestions to increase the effectiveness of teaching. In addition, while in parallel with Table 2 she talked about the use of technology, the use of concrete materials, attention to readiness and concretization, she suggested taking into account misconceptions from the codes not in the Table 2. Therefore, in addition to their views in Table 2, the prospective teachers suggested using the history of mathematics, paying attention to misconceptions, ensuring active participation of students, ensuring cooperative learning and making preparation before the lesson in order to increase the effectiveness of teaching inequalities.

3.1.3. Materials and Activities that can be Used in Teaching

Prospective teachers mentioned that there are various concrete materials that can be used in teaching inequalities and identified them as algebra tiles, number line, models of scale or seesaw and various mathematical tools. Websites, dynamic geometry software (GeoGebra, GSP, Cabri etc.), virtual manipulatives (such as NLVM), excell and smart board applications were mentioned as information technologies that can be used in teaching inequalities. The findings regarding whether the prospective teachers think of using technology in the teaching inequalities are presented in Table 3.

Table 3.

Prospective Teachers' Views on Using Concrete Materials/Information Technologies in Teaching

	Participant					
Code	PT1	PT2	PT3	PT4	PT5	PT6
I think to use concrete materials	X	X	X	X	X	X
I plan to make use of information technologies	X	X	X		X	X

According to Table 3, PT4 stated that he did not think of utilizing information technologies in teaching inequalities. Some of the other prospective teachers mentioned the importance of using appropriate concrete materials or information technology in teaching, but stated that they could not remember concrete materials or information technology that could be used in teaching inequalities and that they planned to prepare and research on this subject before the lesson. Some of the views of prospective teachers on materials that can be used in teaching inequalities are presented below:

> PT5: … *Of course I think, I think everyone should do this. Because I think the material has a great advantage especially in mathematics. … I will use materials because I believe they will be useful. … Preliminary preparation, well, I will look at articles written about inequalities. ... Inspired by them, I will either create my own material in a similar way or I will use it and present it to the students...*

As a result of the analysis of prospective teachers' views on what kind of activities they can plan in teaching inequalities, the codes of "activities related to daily life", "activities for cooperative learning", "activities involving the use of technology" and "activities involving the use of concrete materials" were obtained. In addition, one prospective teacher stated that she could not specify what kind of activity she could design, but she thought that she could design an appropriate activity with pre-lesson preparation. The findings regarding the activities that prospective teachers plan to design in teaching inequalities are presented in Table 4.

Table 4.

Prospective Teachers' Views on Activities that can be Designed in Teaching

	Participant					
Code	PT1	PT2	PT3	PT4	PT5	PT6
Activities related to daily life			X		X	
Activities for cooperative learning			X			
Activities involving the use of concrete materials	X		X	X		X
Activities involving the use of technology	X		X			

According to Table 4, while the teachers stated that they would mostly design activities involving the use of concrete materials in teaching inequalities, PT1 stated that he planned to design activities involving the use of both concrete materials and technology. In addition to these, PT3 stated that she planned to prepare activities by considering different situations.

Some of the views of prospective teachers about the activities that can be designed in teaching are presented below:

> PT3: *... For example, as I said, in GeoGebra, there are graphs, table representations that show the change of variables with the introduction of an inequality. From there, I can show the situation where the equilibrium situation in inequality is actually disrupted. ... We can also design new models for inequality through cooperative learning. For example, as I said, someone can be a grocer, like a shopkeeper, can be designed in the classroom in this way. The children go and shop, the amount of money, the equality and inequality here. … These kinds of activities can be designed in general.*
>
> *PT4: I can do something like this for inequality: I can use the tiles of the clasroom as something, I can use them as a number line. We choose a point, that point is the zero point. According to that, let's say, for example, the numbers less than 12 or let's show the set of numbers on the number line, let's think of it as an inequality. So when we show this to the students, when we fill in each tile, the students will know that they are included in that tile. But for example, we can say like this; when we say the set of numbers less than 9, we know that 9 is not included, and then the ninth tile is left blank. I can design an activity like this and teach them to understand inclusion or exclusion…*

3.2. Views on the Difficulties that Students may Experience in Inequalities

In order to determine prospective teachers' awareness of the difficulties students may experience with inequalities, they were asked what kind of difficulties the students may experience with inequalities. The prospective teachers stated that there may be difficulties in teaching inequalities, such as difficulties with $<$, $>$, $\leq$, $\geq$ symbols and their meanings, in the case of changing the direction of inequality, in showing inequality on the number line, in solving inequalities, in the use of variables, in conceptualizing inequality because of not being able to make sense of equality (prerequisite), and in moving from the concept of equality to inequality. The findings regarding the views of the prospective teachers about the difficulties that students may experience in inequalities are presented in Table 5.

Table 5.

Prospective Teachers' Views on the Difficulties that Students may Experience in Inequalities

	Participant					
Code	PT1	PT2	PT3	PT4	PT5	PT6
Difficulties with symbols <, >, ≤ , ≥ and their meanings	X	X		X	X	X
In the event of a change in the direction of inequality	X	X	X			
Showing inequality on the number line		X	X	X		
In solving inequalities		X				
With variables		X			X	
In conceptualizing inequality because of not being able to make sense of equality				X		
Moving from the concept of equality to inequality						X

According to Table 5, the prospective teachers stated that they thought that the students would have the most difficulties with the symbols for inequality and their meanings and this was followed by the statements that there would be difficulties in the case of changing the direction of the inequality and showing the inequality on the number line. An examples of prospective teachers' views on the difficulties that students may experience with inequalities is presented below:

> *PT6: ... What kind of difficulties, they may not be able to move from the concept of equality to inequality. As I have said before, they cannot understand the direction of inequality because of the equality of x+3 and 3+x. In other words, if 6>a, they can understand it as a>6. They may encounter difficulties in this way. ... If there is no equality in "less than or equal to" or "greater than or equal to" signs if it is inequality, why is it included. There were other misconceptions, but these are the ones I can think of right now.*

3.3. Views on the Methods that can be Used to Overcome Difficulties

The prospective teachers stated that students may experience certain difficulties while learning inequalities and made some suggestions about the ways to overcome these difficulties. The suggestions of the prospective teachers on this issue were coded as "getting students active", "coding for symbols", "identifying misconceptions and planning lessons accordingly", "using concrete materials", "using technology", "using daily life situations", "checking prior knowledge" and "designing activities based on constructivism". The findings obtained in this regard are presented in Table 6.

Table 6.

Prospective Teachers' Views on the Methods that can be Used to Overcome Difficulties

	Participant					
Code	PT1	PT2	PT3	PT4	PT5	PT6
Getting students active	X			X		
Coding for symbols	X					
Identifying misconceptions and planning lessons accordingly		X				X
Using concrete material			X		X	
Using technology					X	
Using daily life situations			X			
Checking prior knowledge		X	X			
Designing activities based on constructivism				X		

According to Table 6, prospective teachers mostly suggested checking prior knowledge, using concrete material, getting students active, identifying misconceptions and planning lessons accordingly to solve the problems that students may experience. Other methods suggested by the teachers included coding for symbols, utilizing technology, using daily life situations and designing activities based on constructivism. Examples of prospective teachers' views on suggestions that can be used to overcome the difficulties that students may experience in inequalities are presented below:

> PT2: … *Before, for example, while checking their readiness, if there are any misconceptions about equality, they can be eliminated first. Later on, when the subject is being taught, I can do activities in which I can recognize them, or we can guide the student with questions. if there are misconceptions, like why do you think like this questions can be asked, you know, why exactly we can find out and follow a path accordingly...*
>
> PT5: … *Afterwards, I need to have appropriate materials so that I can explain them to the student. Or I need to have appropriate technology-supported activities. … I try to explain the situation to the student in the best way possible by using materials or technology-supported activities.*

The prospective teachers were also asked to what extent they felt competent in overcoming the difficulties they might encounter. While three of the prospective teachers stated that they did not feel very competent, one prospective teacher stated that she did not think that she would have much difficulty, one prospective teacher stated that he would rate her competence as good, and one prospective teacher stated that she would rate her competence it as 7 out of 10. However, an important point that draws attention is that the prospective

teachers stated that when evaluating their level of competence, they were based on their current situation and that the parts they saw as lacking in competence could only be completed with experience. The views of the prospective teachers on how competent they feel in overcoming the difficulties that may be experienced in inequalities are given below.

> PT1: *Now, I think the place where students have difficulty is important here. I mean, based on my knowledge within the scope of the course we took before, I don't think it will be very difficult. For example, I would give myself three and a half to four out of five.*
>
> *PT2: I may not be that competent right now because as I gain more experience, I think I will be able to see where students make mistakes, what they think, why they do that, I think it would be a little more, I think I can see better by being involved in the work. Right now, not that much, right now we can know as much as we see from the lessons. Since we have never met students, we have not seen them that much in the internship, and even if we did, we did not come across those subjects, so I don't feel very adequate right now.*
>
> PT3: *Actually, I don't feel very competent at the moment because we also go to the internship and we see it there. Students are really indifferent to mathematics now, there is an incredible negative prejudice against mathematics. So I think I need to experience for a while. I need to be able to convince students that math is a good subject, that inequality, for example, is actually something that will be useful for them. For this, I think I need a process first. I don't feel competent at the moment.*
>
> PT4: *… If the top level is very good right now, I would consider myself as good…*
>
> PT5: *I don't think I am sufficient right now because I don't know much in terms of materials or technology-supported things. But it is not very difficult by searching on the internet, we have already received their teaching in our faculty. We have a certain knowledge of how we can do it, what we can do. There is no question mark in our minds. With just a few researches, we can collect them ourselves and transfer them to the students.*
>
> *PT6: …Let me say out of 10. I think that I can handle these difficulties that I will face at a level of 7 out of 10. The three points I did not give may be due to student differences, as I said, because I have not yet gained experience…. You know, I have deducted three points because of the situations that I cannot affect, that I have not yet encountered. I think that if I go to the school now, I can eliminate the difficulties, I can deal with them at the rate of 7 out of 10....I can complete the rest when I participate in working life….*

4. Discussion and Conclusion

In this study, the views of prospective middle school mathematics teachers on algebraic inequalities were determined. As a result of the research, prospective teachers' approaches to teaching inequalities, information technologies and concrete materials they are aware of for

teaching inequalities, activities they plan to do, their awareness of common difficulties encountered in the literature on inequalities, their suggestions and plans to overcome difficulties, and their views on how competent they feel in overcoming difficulties were determined. The findings were analyzed under the themes of views on teaching inequalities, views on the difficulties that students may experience in inequalities, and views on the methods that can be used to overcome the difficulties.

The theme of views on teaching inequalities was examined through the sub-themes of prerequisite knowledge, issues to be considered in teaching, and materials and activities that can be used. As prerequisite concepts, the prospective teachers stated the meanings of concepts such as less, more, least, greatest, greater than, less than, greater than or equal to, and less than or equal to; equality, variable and unknown, algebraic expressions, equations and linear equations. In teaching inequalities, they drew attention to many important points such as using concrete materials, benefiting from information technologies, using examples from daily life or the environment, ensuring the transition between verbal and algebraic representations, paying attention to readiness, and explaining concepts and rules with their reasons. As a matter of fact, Çoban and Yenilmez (2020) stated that most of the students could not understand the subject of inequalities conceptually and had difficulty in writing the inequality given verbally as a mathematical sentence. As a result of the findings, it can be said that the views of prospective teachers on the teaching of inequalities are supportive of learning the subject of inequalities by conceptualising and making sense of it. It is thought that this situation can be explained by the fact that prospective teachers acquired knowledge and experience about teaching inequalities in the course Teaching Algebra, which they took in the third year of the elementary mathematics teacher education program. In the study, it was also tried to determine the opinions of prospective teachers about various concrete materials and information technologies that can be used in teaching inequalities. Virtual manipulatives belonging to NLVM and NCTM, GeoGebra, one of the dynamic mathematics software, videos on the subject in Education Information Network (EBA), two-pan balance and seesaw models, and four-pan algebra balance are among the materials and technological tools recommended to be used in teaching inequality (Yazlık, 2019). In this direction, as a result of the findings, it can be stated that prospective teachers are aware of concrete materials and information technologies that can be used in teaching algebra, but it is necessary to improve their

knowledge on materials and information technologies specific to teaching inequality. The prospective teachers explained this situation by not being able to remember and stated that they planned to prepare the appropriate materials for the lesson with pre-lesson preparation. However, similar to some studies in the literature (e.g., McCarty, 1998; Öksüz & Ak, 2009; Ünlü, 2017), it can be said that prospective teachers agree that it is beneficial to use information technologies and concrete materials in teaching. Students, particularly younger students, learn more meaningfully in learning environments where information is represented by concrete manipulatives (Clements, 1999; Kelly, 2006). Similarly, research shows that the use of information technologies in teaching has many benefits (Seferoğlu, 2001). Therefore, the fact that prospective teachers have positive views on the use of technology and concrete materials can be considered as a positive prediction that they will include them in their future classroom practices.

It is a necessity for teachers to be aware of students' difficulties and mistakes in the learning process in order to ensure meaningful learning (Yetkin, 2003). In this direction, it has been investigated whether prospective teachers are aware of student difficulties related to inequalities. According to the findings, prospective teachers stated that they thought that students might have difficulty with symbols for inequalities and their meanings, in the case of changing the direction of an inequality, showing inequalities on the number line, solving inequalities, the use of variables and moving from the concept of equality to inequality. In parallel with the findings of this study, Çoban and Yenilmez (2020) found that although most of the students were aware of the symbols in the subject of inequalities, they did not know what they meant conceptually and had difficulties in representing the inequality on the number line. Accordingly, within the framework of the findings obtained in this study, it can be said that prospective teachers are aware of the difficulties that may be experienced regarding inequalities. It is thought that this situation can be explained by the information they gained from the courses they took in the elementary mathematics teacher education program and their own student or teaching experiences (within the scope of the teaching practice courses or tutoring, etc.) as stated by the prospective teachers in the interviews.

The suggestions of the prospective teachers about the ways to overcome the difficulties that students may experience while learning inequalities can be listed as making the students active, coding for symbols, identifying misconceptions and planning lessons accordingly,

using concrete materials, benefiting from technology, benefiting from daily life situations, checking prior knowledge and designing activities based on constructivism. Another result obtained from the findings is that prospective teachers consider themselves competent at different levels in overcoming the difficulties they may encounter in the process of teaching inequalities. However, an important point that draws attention is that the prospective teachers stated that when evaluating their level of competence, they were based on their current situation and that the parts they saw as lacking in competence could only be completed with experience. This result can be explained by the fact that, as stated by the prospective teachers in the interviews, for some prospective teachers, addressing the difficulties that can be encountered in inequalities in undergraduate courses is sufficient to see themselves as competent in this subject, while some prospective teachers need also experience factor to feel competent in this subject. In mathematics teaching, it is possible to mention the main principles that should be followed in achieving the goal and overcoming the difficulties. For example, Altun (2001) lists the basic principles necessary for effective mathematics teaching as follows: Establishing conceptual foundations, giving importance to the preconditionality relationship, giving importance to key concepts, determining the roles of the teacher and the student well in teaching, benefiting from the environment in teaching, including research studies and developing a positive attitude towards mathematics (pp. 8-15). During the teacher preparation program, prospective teachers acquire knowledge and develop beliefs about the teaching and learning process (Van Zoest & Bohl, 2005). It is recommended that the knowledge, beliefs and experiences aimed to be gained during teacher education should be at a level that increases the competences of prospective teachers towards the teaching process and improves their self-efficacy in this regard. It can also be suggested that more practices be included in teacher education programs to improve prospective mathematics teachers' knowledge and experiences about meaningful learning and overcoming difficulties. For future research, increasing the sample size to include more diversity or exploring similar perspectives in other mathematical subjects might be suggested.

5. References

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