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COMPREHENSION OF SOFT BINARY PIECEWISE GAMMA OPERATION: A NEW OPERATION FOR SOFT SETS

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ABSTRACT

Soft set theory gained popularity as a cutting-edge approach to handling uncertainty-related problems and modeling uncertainty after being introduced by Molodtsov in 1999. Numerous theoretical and practical applications have been conducted by using of the theory. This paper presents a novel soft set operation, called the "soft binary piecewise gamma operation". Its basic algebraic characteristics are thoroughly examined. Furthermore, this operation's distributions over a number of soft set operations are investigated. We prove that given certain assumptions, the soft binary piecewise addition operation determines a commutative semigroup and a right-left system, and soft binary piecewise gamma operation, along with some other types of soft sets, form many important algebraic structures, such as semirings and near-semirings in the collection of soft sets over the universe by considering the algebraic properties of the operation and its distribution rules together.

Keywords: Soft sets, soft set operations, conditional complements, soft binary piecewise gamma operation

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1 Introduction

Fuzzy set theory, interval mathematics, and probability theory are a few theories that may be used to explain uncertainty; however, each of these theories has disadvantages of its own. Soft Set Theory is a revolutionary approach to describing uncertainty and using it to solve issues involving uncertainty. In 1999, Molodtsov [1] provided the first description of it. This idea has been successfully applied in several mathematical fields since its conception. Measurement theory, game theory, probability theory, Riemann integration, and Perron integration are a few of these areas that have been researched.

Soft set operations were first studied by Maji et al. [2] and Pei and Miao [3]. Ali et al. [4] provided a number of soft set operations, including restricted and extended soft set operations. In their work on soft sets, Sezgin and Atagün [5] offered aspects of the restricted symmetric difference of soft sets. In addition, they went over the fundamentals of soft set operations and gave examples of how they connect to one another. A thorough examination of the algebraic structures of soft sets was carried out by Ali et al. [6]. A number of academics were interested in soft set operations, and they researched the subject thoroughly in [7–18].

Eren and Çalışıcı [19] introduced the idea of the soft binary piecewise difference operation in soft sets. Furthermore, Sezgin and Çalışıcı [20] conducted a thorough analysis of the soft binary piecewise difference operation. While Sezgin et al. [21] initially suggested the extended difference of soft sets, Stojanovic [22] defined and examined the characteristics of the extended symmetric difference of soft sets.

Two additional complement operations were introduced to the literature by Çağman [23], and Sezgin et al. [24] worked on them as well as numerous other unique binary set operations. Aybek [25] proposed a wide range of additional restricted and extended soft set operations using these new binary operations. While Akbulut [26], Demirci [27], and Sarıalioğlu [28] continued to work on changing the structure of extended operations in soft sets, they focused on the complementary extended soft set operations. Complementary soft binary piecewise operations were examined by [29–38] by notably altering the form of the soft binary piecewise operation in soft sets. Two seminal works on piecewise operations for soft binary systems are Yavuz [38] and Sezgin and Yavuz [39]. Studies [40–47] concerning different types of soft equity are also crucial.

Algebraic structures, sometimes referred to as mathematical systems or structures, have long piqued the curiosity of mathematicians. Sorting algebraic structures according to the properties of the operation done to a set is one of the main problems in algebraic mathematics. One of the most well-known ideas in binary algebraic structures is the extension of rings, including near-rings, semirings, and semifields. Scholars have long been interested in learning more about this topic. Vandiver provided the first definition of "semirings" [48]. Semirings have been extensively studied in more recent times, particularly in relation to their applications (see [48]). Semirings are important in geometry, pure mathematics, and the solution of many problems in the information sciences and applied mathematics [49–61]. Semirings have important applications in pure mathematics and geometry. Hoorn and Rootselaar discussed the nearsemiring [62]. More general than a near-ring or semiring, a seminearring is an algebraic structure also known as a nearsemiring in mathematics. Concepts of soft set operations are essential to soft sets, just as operations from classical algebra are to classical set theory. Thus, we may be able to better understand the algebraic structure of soft sets if we consider it in terms of.

We want to make a major contribution to the field of soft set theory by introducing the "soft binary piecewise gamma operation" and thoroughly examining the algebraic structures associated with it, as well as other soft set operations in the collection of soft sets over the universe. The structure of this study is as follows: The fundamental concepts of soft sets and other algebraic structures are reviewed in Section 2. The third section presents a thorough examination of the algebraic characteristics of the recently proposed soft set operation. These properties enable us to demonstrate that, under certain assumptions, the soft binary piecewise plus operation determines both a right-left system with the right identity empty soft set and a commutative semigroup. Section 4 looks at how the soft binary piecewise gamma operation is distributed throughout several soft set operations, such as restricted, extended, and soft binary piecewise operations. Considering the distribution laws and the algebraic features of the soft set operations, a thorough examination of the algebraic structures formed by the set of soft sets with these operations is provided. It is demonstrated that a variety of significant algebraic structures, such as semirings and nearsemirings, are formed in the collection of soft sets over the universe by using the soft binary piecewise gamma operation and various forms of soft sets. Section 5 discusses the significance of the study's results and how they could apply to the subject.

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2 Preliminaries

This section provides a number of algebraic structures as well as several basic concepts in soft set theory.

Definition 2.1. Let U be the universal set, E be the parameter set, P(U) be the power set of U, and let $K \subseteq E$. A pair (F, K) is called a soft set on U. Here, F is a function given by $F: K \rightarrow P(U)$ [1].

The set of all soft sets over U is denoted by $S_E(U)$. Let K be a fixed subset of E, then the set of all soft sets over U with the fixed parameter set K is denoted by $S_K(U)$. In other words, in the collection $S_K(U)$, only soft sets with the parameter set K are included, while in the collection $S_E(U)$, soft sets over U with any parameter set can be included. Clearly, the set $S_K(U)$ is a subset of the set $S_E(U)$.

Definition 2.2. Let (F,K) be a soft set over U. If F(e)=∅ for all e∈K, then the soft set (F,K) is called a null soft set with respect to K, denoted by $\emptyset_K$. Similarly, let (F,E) be a soft set over U. If F(e)=∅ for all e∈E, then the soft set (F,E) is called a null soft set with respect to E, denoted by $\emptyset_E$ [4].

It is known that a function F: ∅ ⟶ K, where the domain is the empty set, is referred to as the empty function. Since the soft set is also a function, it is evident that by taking the domain as ∅, a soft set can be defined as F: ∅ ⟶ P(U), where U is a universal set. Such a soft set is called an empty soft set and is denoted as $\emptyset_\emptyset$. Thus, $\emptyset_\emptyset$ is the only soft set with an empty parameter set [6].

Definition 2.3. Let (F,K) be a soft set over U. If F(e)=U for all e∈K, then the soft set (F,K) is called an absolute soft set with respect to K, denoted by U_K. Similarly, let (F,E) be a soft set over U. If F(e)=U for all e∈E, then the soft set (F,E) is called an absolute soft set with respect to E, denoted by U_E [4].

Definition 2.4. Let (F,K) and (G,Y) be soft sets over U. If K⊆Y and F(e) ⊆G(e) for all e∈K, then (F,K) is said to be a soft subset of (G,Y), denoted by $(F,K)\tilde{\subseteq}(G,Y)$. If (G,Y) is a soft subset of (F,K), then (F,K) is said to be a soft superset of (G,Y), denoted by $(F,K)\tilde{\supseteq}(G,Y)$. If $(F,K)\tilde{\subseteq}(G,Y)$ and $(G,Y)\tilde{\subseteq}(F,K)$, then (F,K) and (G,Y) are called soft equal sets [3].

Definition 2.5. Let (F,K) be a soft set over U. The soft complement of (F,K), denoted by $(F,K)^r = (F^r,K)$, is defined as follows:, $F^r(e)=U-F(e)$, for all e∈K [4].

Çağman [23], introduced two new complements as novel concepts in set theory. For ease of representation, we denote these binary operations as + and θ, respectively. For two sets T and Y, these binary operations are defined as T+Y=T'∪Y and TθY=T'∩Y'. Sezgin et al. [24] investigated the relationship between these two operations and also introduced three new binary operations, examining their relationships with each other. For two sets T and Y, these new operations are defined as T*Y=K'∪Y', TγY= T'∩Y, T**λ**Y=T∪Y' [24].

As a summary for soft set operations, we can categorize all types of soft set operations as follows: Let "⊗" be used to represent the set operations (i.e., here, ⊗ can be ∩, ∪,\, Δ, +,θ, *, λ,γ), then all type of soft set operations are defined as follows:

Definition 2.6. Let (F, K) and (G, Y) be two soft sets over U. The restricted ⊗ operation of (F, K) and (G, Y) is the soft set (H, P), denoted by (F, K) $\otimes_{\mathrm{R}}$ (G, Y)= (H, P), where P = K ∩ Y≠ ∅ and for all e ∈ P, H(e) = F(e)⊗G(e). Here, if P = K ∩ Y = ∅, then (F, K) $\otimes_{\mathrm{R}}$(G, Y)= $\emptyset_{\emptyset}$ [4, 5,6, 25].

Definition 2.7. Let (F, K) and (G, Y) be two soft sets over U. The extended ⊗ operation (F, K) and (G,Y) is the soft set (H,P), denoted by (F, K) $\otimes_{\varepsilon}$(G, Y) = (H, P), where P = K ∪ Y, and for all e ∈ P,

$$\mathrm{H(e)} = \begin{cases} \mathrm{F(e)}, & \mathrm{e} \in \mathrm{K} - \mathrm{Y} \\ \mathrm{G(e)}, & \mathrm{e} \in \mathrm{Y} - \mathrm{K} \\ \mathrm{F(e)} \otimes \mathrm{G(e)}, & \mathrm{e} \in \mathrm{K} \cap \mathrm{Y} \end{cases}$$

[2,4,21,22,25].

Definition 2.8. Let (F, K) and (G, Y) be two soft sets over U. The complementary extended ⊗ operation (F, K) and (G,Y) is the soft set (H,P), denoted by (F, K) $\overset{*}{\otimes}_{\varepsilon}$(G, Y) = (H, P), where P = K ∪ Y, and for all e ∈ P,

$$\mathrm{H(e)} = \begin{cases} \mathrm{F'(e)}, & \mathrm{e} \in \mathrm{K} - \mathrm{Y} \\ \mathrm{G'(e)}, & \mathrm{e} \in \mathrm{Y} - \mathrm{K} \\ \mathrm{F(e)} \otimes \mathrm{G(e)}, & \mathrm{e} \in \mathrm{K} \cap \mathrm{Y} \end{cases}$$

[26-28].

Definition 2.9. Let (F,K) and (G,Y) be two soft sets on U. The complementary soft binary piecewise ⊗ operation of (F,K) and (G,Y) is the soft set (H,K), denoted by $(\mathrm{F},\mathrm{K}) \underset{\otimes}{\overset{*}{\sim}} (\mathrm{G},\mathrm{Y}) = (\mathrm{H},\mathrm{K})$, where for all e ∈ K,

$$\mathrm{H(e)} = \begin{cases} \mathrm{F'(e)}, & \mathrm{e} \in \mathrm{K} - \mathrm{Y} \\ \mathrm{F(e)} \otimes \mathrm{G(e)}, & \mathrm{e} \in \mathrm{K} \cap \mathrm{Y} \end{cases}$$

[29-37].

Definition 2.10. Let (F,K) and (G,Y) be two soft sets on U. The soft binary piecewise ⊗ operation of (F,K) and (G,Y) is the soft set (H,K), denoted by $(\mathrm{F},\mathrm{K}) \overset{\sim}{\otimes} (\mathrm{G},\mathrm{Y}) = (\mathrm{H},\mathrm{K})$, where for all e ∈ K,

$$\mathrm{H(e)} = \begin{cases} \mathrm{F(e)}, & \mathrm{e} \in \mathrm{K} - \mathrm{Y} \\ \mathrm{F(e)} \otimes \mathrm{G(e)}, & \mathrm{e} \in \mathrm{K} \cap \mathrm{Y} \end{cases}$$

[19,20,38,39]. For more about soft sets, we refer to [63-76].

Definition 2.11. Let (S, ⋆) be an algebraic structure. An element s ∈S is called idempotent if s^2=s. If s^2=s for all s∈S then the algebraic structure (S,⋆) is said to be idempotent. An idempotent semigroup is called a band, an idempotent and commutative semigroup is called a semilattice, and an idempotent and commutative monoid is called a bounded semilattice [77].

In a monoid, although the identity element is unique, a semigroup/groupoid can have one or more left identities; however, if it has more than one left identity, it does not have a right identity element, thus it does not have an identity element. Similarly, a semigroup/groupoid can have one or more right identities; however, if it has more than one right identity, it does not have a left identity element, thus it does not have an identity element [78].

Similarly, in a group, although each element has a unique inverse, in a monoid, an element can have one or more left inverses; however, if an element has more than one left inverse, it does not have a right inverse, thus it does not have an inverse. Similarly, in a monoid, an element can have one or more right inverses; however, if an element has more than one right inverse, it does not have a left inverse, thus it does not have an inverse [78].

Definition 2.12. If a semigroup (S,*) has a left identity and every element has a right inverse, then the semigroup is called a left-right system, and if the semigroup has a right identity and every element has a left inverse, then the semigroup is called a right-left system. The difference between the left-right system and the group is that a group has a left (resp., right) identity, and every element has a left (resp., right) inverse [79].

Definition 2.13. Let S be a non-empty set, and let "+" and "⋆" be two binary operations defined on S. If the algebraic structure $(S, +, \star)$ satisfies the following properties, then it is called a semiring:

i. $(S, +)$ is a semigroup.
ii. $(S, \star)$ is a semigroup,
iii. For all $x, y, z \in S$, $x\star(y + z) = x\star y + x\star z$ and $(x + y)\star z = x\star z + y\star z$

If for all $x,y\in S$, $x+y=y+z$, then S is called an additive commutative semiring. If $x\star y=y\star x$ for all $x,y\in S$, then S is called a multiplicative commutative semiring. If there exists an element $1\in S$ such that $x\star 1=1\star x=x$ for all $x\in S$ (multiplicative identity), then S is called semiring with unity. If there exists $0\in S$ such that for all $x\in S$, $0\star x=x\star 0=0$ and $0+x=x+0=x$, then 0 is called the zero of S. A semiring with commutative addition and a zero element, is called a hemiring [48].

Definition 2.14. Let S be a non-empty set, and let "+" and "⋆" be two binary operations defined on S. If the algebraic structure $(S, +, \star)$ satisfies the following properties, then it is called a nearsemiring (or seminearring):

i. $(S,+)$ is a semigroup.
ii. $(S, \star)$ is a semigroup.
iii. For all $x,y,z\in S$, $(x+y)\star z = x\star z+y\ast z$ (right distributivity)

If the additive zero element 0 of S (that is, for all $x\in S$, $0+x=0+x=x$) satisfies that for all $x\in S$, $0\star x=0$ (left absorbing element), then $(S, +, \star)$ is called a (right) nearsemiring with zero. If $(S, +, \star)$ additionally satisfies $x\star 0=0$ for all $x\in S$ (right absorbing element), then it is called a zero symmetric nearsemiring [62]. We refer to [80] for possible implications of network analysis and graph applications with regard to soft sets, which are defined by the divisibility of determinants, and [81-83] for more about soft algebraic structures of soft sets.

3 Soft binary piecewise gamma operation

The soft binary piecewise gamma operation is a novel soft set operation that is presented in this section. In addition to giving an example of the operation, it looks at the distribution rules and algebraic structures that the operation forms in $S_E(U)$, as well as the general algebraic characteristics of the operation and its relationships with other soft set operations.

Definition 3.1. Let (F,K) and (G,Y) be soft sets over U. The soft binary piecewise theta of (F,K) and (G,Y) is the soft set (H,K), denoted by, $(F,K)\underset{\gamma}{\sim}(G,Y) = (H,K)$, where for all ǎ∈K,

$$H(ǎ)=\begin{cases} F(ǎ), & ǎ\in K-Y \\ F'(ǎ)\cap G(ǎ), & ǎ\in K\cap Y \end{cases}$$

Example 3.2. Let $E=\{e_1,e_2,e_3,e_4\}$ be the parameter set $K=\{e_1, e_4\}$ and $Y=\{e_2, e_3, e_4\}$ be the subsets of E and $U=\{h_1,h_2,h_3,h_4,h_5,h_6\}$ be the initial universe set. Assume that (F,K) and (G,Y) are the soft sets over U defined as following:

$$(F,K)=\{(e_1, \{h_2, h_4,h_6), (e_4,\{h_1,h_2,h_5, h_6\})\}$$

$$(G,Y)=\{(e_2,\{h_1, h_2\}), (e_3,\{h_2,h_3,h_4, h_5\}),(e_4,\{h_2, h_3,h_5\})\}$$

Let $(F,K)\underset{\gamma}{\sim}(G,Y)=(H,K)$, where for all ǎ∈K,

$$H(ǎ)=\begin{cases} F(ǎ), & ǎ\in K-Y \\ F'(ǎ)\cap G(ǎ), & ǎ\in K\cap Y \end{cases}$$

Here since $K=\{e_1,e_4\}$ and $K-Y=\{e_1\}$, for all $ǎ \in K-Y=\{e_1\}$, $H(ǎ)=F(ǎ)$ and so $H(e_1)=F(e_1)=\{h_2, h_4,h_6\}$; for all $ǎ \in K\cap Y=\{e_4\}$, $H(ǎ)=F'(ǎ)\cap G(ǎ)$ and so $H(e_4)=F'(e_4)\cap G(e_4)=\{h_3,h_4\}\cap\{h_2,h_3, h_5\}=\{h_3\}$. Thus,

$$(F,K)\underset{\gamma}{\sim}(G,Y)=\{(e_1,\{h_2, h_4,h_6\}),(e_4, \{h_3\})\}.$$

Theorem 3.3. Algebraic Properties of the Operation

1) The set $S_E(U)$ is closed under $\underset{\gamma}{\sim}$. That is, when (F,K) and (G,Y) are two soft sets over U, then so is $(F,K)\underset{\gamma}{\sim}(G,Y)$.

Proof: It is clear that $\underset{\gamma}{\sim}$ is a binary operation in $S_E(U)$. That is,

$$\underset{\gamma}{\sim} : S_E(U) \times S_E(U)\to S_E(U)$$

$$((F,K), (G,Y)) \to(F,K)\underset{\gamma}{\sim}(G,Y)= (H,K)$$

Hence, the set $S_E(U)$ is closed under $\underset{\gamma}{\sim}$. Similarly,

$$\underset{\gamma}{\sim} : S_K(U) \times S_K(U)\to S_K(U)$$

$$((F,K),(G,K)) \rightarrow (F,K)\tilde{\gamma}(G,K)=(H,K)$$

That is, let K be a fixed subset of the set E and (F,K) and (G,K) be elements of $S_K(U)$, then so is $(F,K)\tilde{\gamma}(G,K)$. Namely, $S_K(U)$ is closed under $\tilde{\gamma}$.

2) Let (F,K), (G,Y), (H,D) be soft sets over U. Then, $[(F,K)\tilde{\gamma}(G,Y)]\tilde{\gamma}(H,D)=(F,K)\tilde{\gamma}[(G,Y)\tilde{\gamma}(H,D)]$, where $K\cap Y'\cap D=K\cap Y\cap D=\emptyset$.

Proof: Let first handle the left hand side (LHS) of the equality and let $(F,K)\tilde{\gamma}(G,Y)=(T,K)$, where for all $\breve{a}\in K$,

$$T(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in K-Y \\ F'(\breve{a})\cap G(\breve{a}), & \breve{a}\in K\cap Y \end{cases}$$

Let $(T,K)\tilde{\gamma}(H,D)=(M,K)$, where for all $\breve{a}\in K$,

$$M(\breve{a})=\begin{cases} T(\breve{a}), & \breve{a}\in K-D \\ T'(\breve{a})\cap H(\breve{a}), & \breve{a}\in K\cap D \end{cases}$$

Thus,

$$M(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in (K-Y)-D=K\cap Y'\cap D' \\ F'(\breve{a})\cap G(\breve{a}), & \breve{a}\in (K\cap Y)-D=K\cap Y\cap D' \\ F'(\breve{a})\cap H(\breve{a}), & \breve{a}\in (K-Y)\cap D=K\cap Y'\cap D \\ [F(\breve{a})\cup G'(\breve{a})]\cap H(\breve{a}), & \breve{a}\in (K\cap Y)\cap D=K\cap Y\cap D \end{cases}$$

Let $(G,Y)\tilde{\gamma}(H,D)=(K,Y)$, where for all $\breve{a}\in Y$,

$$K(\breve{a})=\begin{cases} G(\breve{a}), & \breve{a}\in Y-D \\ G'(\breve{a})\cap H(\breve{a}), & \breve{a}\in Y\cap D \end{cases}$$

Let $(F,K)\tilde{\gamma}(K,Y)=(S,K)$, where for all $\breve{a}\in K$,

$$S(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in K-Y \\ F'(\breve{a})\cap K(\breve{a}), & \breve{a}\in K\cap Y \end{cases}$$

Hence,

$$S(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in K-Y \\ F'(\breve{a})\cap G(\breve{a}), & \breve{a}\in K\cap (Y-D)=K\cap Y\cap D' \\ F'(\breve{a})\cap [G'(\breve{a})\cap H(\breve{a})], & \breve{a}\in K\cap (Y\cap D)=K\cap Y\cap D \end{cases}$$

Considering K-Y in the S function, since $K-Y=K\cap Y'$, if $\breve{a}\in Y'$, then $\breve{a}\in D-Y$ or $\breve{a}\in (Y\cup D)'$. Thus, if $\breve{a}\in K-Y$, then $\breve{a}\in K\cap Y'\cap D'$ or $\breve{a}\in K\cap Y'\cap D$. Thus, M=S for $K\cap Y'\cap D= K\cap Y\cap D=\emptyset$. That is, under suitable conditions, the operation $\tilde{\gamma}$ is associative $S_E(U)$.

3) Let (F,K), (G,K), (H,K) be soft sets over U. Then, $[(F,K)\tilde{\gamma}(G,K)]\tilde{\gamma}(H,K)\neq(F,K)\tilde{\gamma}[(G,K)\tilde{\gamma}(H,K)]$

Proof: Consider first the LHS and let $(F,K)\tilde{\gamma}(G,K)=(T,K)$, where for all $\breve{a}\in K$,

$$T(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in K\text{-}K=\emptyset \\ F'(\breve{a})\cap G(\breve{a}), & \breve{a}\in K\cap K=K \end{cases}$$

Let $(T,K)\ \tilde{\gamma}\ (H,K)=(M,K)$, where for all $\breve{a}\in K$,

$$M(\breve{a})=\begin{cases} T(\breve{a}), & \breve{a}\in K\text{-}K=\emptyset \\ T'(\breve{a})\cap H(\breve{a}), & \breve{a}\in K\cap K=K \end{cases}$$

Thus,

$$M(\breve{a})=\begin{cases} T(\breve{a}), & \breve{a}\in K\text{-}K=\emptyset \\ [F(\breve{a})\cup G'(\breve{a})]\cap H(\breve{a}), & \breve{a}\in K\cap K=K \end{cases}$$

Now consider the RHS. Let $(G,K)\tilde{\gamma}(H,K)=(L,K)$, where for all $\breve{a}\in K$,

$$L(\breve{a})=\begin{cases} G(\breve{a}), & \breve{a}\in K\text{-}K=\emptyset \\ G'(\breve{a})\cup H(\breve{a}), & \breve{a}\in K\cap K=K \end{cases}$$

Let $(F,K)\ \tilde{\gamma}\ (L,K)=(N,K)$, where for all $\breve{a}\in K$,

$$N(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in K\text{-}K=\emptyset \\ F'(\breve{a})\cap L(\breve{a}), & \breve{a}\in K\cap K=K \end{cases}$$

Hence,

$$N(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in K\text{-}K=\emptyset \\ F'(\breve{a})\cap [G'(\breve{a})\cup H(\breve{a})], & \breve{a}\in K\cap K=K \end{cases}$$

It is seen that $M\neq N$. That is, for the soft sets whose parameter set is the same, the operation $\tilde{\gamma}$ is not associative.

4) Let (F,K), (G,Y) be soft sets over U. Then, $(F,K)\ \tilde{\gamma}(G,Y)\neq(G,Y)\tilde{\gamma}(F,K)$.

Proof: $(F,K)\ \tilde{\gamma}(G,Y)=(H,K)$, where for all $\breve{a}\in K$,

$$H(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in K\text{-}Y \\ F'(\breve{a})\cap G(\breve{a}), & \breve{a}\in K\cap Y \end{cases}$$

Let $(G,Y)\ \tilde{\gamma}\ (F,K)=(T,Y)$, where for all $\breve{a}\in Y$,

$$T(\breve{a})=\begin{cases} G(\breve{a}), & \breve{a}\in Y\text{-}K \\ G'(\breve{a})\cap F(\breve{a}), & \breve{a}\in Y\cap K \end{cases}$$

Here, while the parameter set of the soft set of the LHS is K, the parameter set of the soft set of the RHS is Y. Thus, by the definition of soft equality,

$$(F,K)\ \tilde{\gamma}\ (G,Y)\neq(G,Y)\ \tilde{\gamma}\ (F,K).$$

That is, $\tilde{\gamma}$ does not have the commutative property in $S_E(U)$. Moreover, $\tilde{\gamma}$ is not commutative in $S_K(U)$, where K is a fixed parameter set such that $K\subseteq E$.

Theorem 3.3.1. By Theorem 3.3 (1), (2) ve (4), $(S_E(U),\tilde{\gamma})$ is a commutative but not idempotent semigroup, under the condition K∩Y'∩D=K∩Y∩D=∅, where (F,K), (G,Y), and (H,D) are elements of $S_E(U)$.

By Theorem 3.3. (3) since $\tilde{\gamma}$ is not associative in $S_K(U)$ where K⊆ E is a fixed parameter set, $(S_K(U),\tilde{\gamma})$ is not a semigroup, however, it is obvious that it is a commutative groupoid.

5) Let (F, K) be soft set over U. Then, $(F,K)\tilde{\gamma}(F,K)=\emptyset_K$.

Proof: Let $(F,K)\tilde{\gamma}(F,K)=(H,K)$, where for all ð∈K,

$$H(ð)=\begin{cases} F(ð), & ð\in K-K=\emptyset \\ F'(ð)\cap F(ð), & ð\in K\cap K=K \end{cases}$$

Thus, for all ð∈K, H(ð)=F'(ð)∩F(ð)= ∅, thus $(H,K)=\emptyset_K$. That is, the operation $\tilde{\gamma}$ is not idempotent in $S_E(U)$.

6) Let (F, K) be soft set over U. Then, $(F,K)\tilde{\gamma}\emptyset_K=\emptyset_K$.

Proof: Let $\emptyset_K$=(S,K), where for all ð∈K, S(ð)=∅. $(F,K)\tilde{\gamma}(S,K)=(H,K)$, where for all ð∈K,

$$H(ð)=\begin{cases} F(ð), & ð\in K-K=\emptyset \\ F'(ð)\cap S(ð), & ð\in K\cap K=K \end{cases}$$

Thus, for all ð∈K, H(ð)=F'(ð)∩S(ð)=F'(ð)∩∅=∅. Hence $(H,K)=\emptyset_K$. That is, the right absorbing element of the operation $\tilde{\gamma}$ is the soft set $\emptyset_K$ in $S_K(U)$.

7) Let (F, K) be soft sets over U. Then, $\emptyset_K\tilde{\gamma}(F,K)=(F,K)$.

Proof: Let $\emptyset_K$=(S,K), where for all ð∈K, S(ð)=∅. $(S,K)\tilde{\gamma}(F,K)=(H,K)$, where for all ð∈K,

$$H(ð)=\begin{cases} S(ð), & ð\in K-K=\emptyset \\ S'(ð)\cap F(ð), & ð\in K\cap K=K \end{cases}$$

Hence, for all ð∈K, H(ð)= S'(ð)∩F(ð)= U∩F(ð)= F(ð), that is, (H,K)=(F, K). That is, the left identity element of the operation $\tilde{\gamma}$ is the soft set $\emptyset_K$ in $S_K(U)$.

8) Let (F, K) be soft set over U. Then, $(F,K)\tilde{\gamma}\emptyset_E=\emptyset_K$.

Proof: Let $\emptyset_E$=(S,E), where for all ð∈E, S(ð)=∅. $(F,K)\tilde{\gamma}(S,E)=(H,K)$. Thus, for all ð∈K,

$$H(ð)=\begin{cases} F(ð), & ð\in K-E=\emptyset \\ F'(ð)\cap S(ð), & ð\in K\cap E=K \end{cases}$$

Hence, for all ð∈K, H(ð)=F'(ð)∩S(ð)=F'(ð)∩∅= ∅. Thus, $(H,K)=\emptyset_K$.

9) Let (F,K) be soft set over U. Then, $(F,K)\,\tilde{\gamma}\,\emptyset_{\emptyset}$=(F,K).

Proof: Let $\emptyset_{\emptyset}$=(S, ∅) ve (F,K)$\tilde{\gamma}$(S, ∅)=(H,K), where for all ă∈K,

$$H(ă)=\begin{cases} F(ă), & ă\in K-\emptyset=K \\ F'(ă)\cap S(ă), & ă\in K\cap\emptyset=\emptyset \end{cases}$$

Hence, for all ă∈K, H(ă)=F(ă) olup, (H,K)=(F,K). That is, $\emptyset_{\emptyset}$ is the right identity element for the operation $\tilde{\gamma}$ in $S_E(U)$.

10) Let (F,K) be soft set over U. Then, $\emptyset_{\emptyset}\,\tilde{\gamma}\,(F,K)=\emptyset_{\emptyset}$.

Proof: Let $\emptyset_{\emptyset}$=(S, ∅) and (S,∅)$\tilde{\gamma}$(F,K)=(H,∅). Since $\emptyset_{\emptyset}$ is the only soft set whose parameter set is the empty set, (H,∅)= $\emptyset_{\emptyset}$. That is, in $S_E(U)$, for the operation $\tilde{\gamma}$, the left inverse of each element with respect to the right identity element $\emptyset_{\emptyset}$ is the soft set $\emptyset_{\emptyset}$ Moreover, in $S_E(U)$, the left absorbing element of the $\tilde{\gamma}$ operation is the soft set $\emptyset_{\emptyset}$.

Theorem 3.3.2. From the properties of (1), (2), (9), and (10), ($S_E(U)$, $\tilde{\gamma}$) is a right-left system with the right identity $\emptyset_{\emptyset}$ and the left inverses of each element is $\emptyset_{\emptyset}$ under the condition K∩Y∩D =K∩Y'∩D=∅, where (F,K), (G,Y), and (H, D) are the elements of $S_E(U)$.

11) Let (F,K) be soft set over U. Then, $U_K\,\tilde{\gamma}\,(F,K)=\emptyset_K$.

Proof: Let U_K=(T,K), where for all ă∈K, T(ă)=U. Let (T,K) $\tilde{\gamma}$ (F,K)=(H,K), where for all ă∈K,

$$H(ă)=\begin{cases} T(ă), & ă\in K-K=\emptyset \\ T'(ă)\cap F(ă), & ă\in K\cap K=K \end{cases}$$

Hence, for all ă∈K, H(ă)= T'(ă)∩F(ă)=∅∩F(ă)= ∅ so (H,K)=$\emptyset_K$.

12) $(F,K)\,\tilde{\gamma}\,U_K=(F,K)^r$.

Proof: Let U_K=(T,K), where for all ă∈K, T(ă)=U. $(F,K)\,\tilde{\gamma}\,(T,K)$=(H,K), where for all ă∈K,

$$H(ă)=\begin{cases} F(ă), & ă\in K-K=\emptyset \\ F'(ă)\cap T(ă), & ă\in K\cap K=K \end{cases}$$

Hence, for all ă∈K, H(ă)=F'(ă)∩T(ă)=F'(ă)∩U= F'(ă) so (H,K)=$(F,K)^r$.

13) Let (F,K) be soft set over U. Then, $(F,K)\,\tilde{+}\,U_E=(F,K)^r$.

Proof: Let U_E=(T,E), where for all ă∈E, T(ă)=U. $(F,K)\,\tilde{\gamma}\,(T,E)$=(H,K), where for all ă∈K,

$$H(ǎ)=\begin{cases} F(ǎ), & ǎ\in K-E=\emptyset \\ F'(ǎ)\cap T(ǎ), & ǎ\in K\cap E=K \end{cases}$$

Hence, for all ǎ∈K, H(ǎ)=F'(ǎ)∩T(ǎ)=F'(ǎ)∩U=F'(ǎ), Thus, $(H,K)=(F,K)^r$.

14) Let (F,K) be soft set over U. Then, $(F,K)\underset{\gamma}{\sim}(F,K)^r=(F,K)^r$.

Proof: Let $(F,K)^r=(H,K)$, where for all ǎ∈K, H(ǎ)=F'(ǎ). $(F,K)\underset{\gamma}{\sim}(H,K)=(T,K)$, where for all ǎ∈K,

$$T(ǎ)=\begin{cases} F(ǎ), & ǎ\in K-K=\emptyset \\ F'(ǎ)\cap H(ǎ), & ǎ\in K\cap K=K \end{cases}$$

Hence, for all ǎ∈K, T(ǎ)=F'(ǎ)∩H(ǎ)=F'(ǎ)∩F'(ǎ)= F'(ǎ). Thus $(T,K)=(F,K)^r$
That is, in the set $S_E(U)$, the relative complement of each soft set is its right absorbing element for the operation $\underset{\gamma}{\sim}$.

15) Let (F,K) be soft set over U. Then, $(F,K)^r\underset{\gamma}{\sim}(F,K)=(F,K)$.

Proof: Let $(F,K)^r=(H,K)$, where for all ǎ∈K, H(ǎ)=F'(ǎ). Let $(H,K)\underset{\gamma}{\sim}(F,K)=(T,K)$. for all ǎ∈K,

$$T(ǎ)=\begin{cases} H(ǎ), & ǎ\in K-K=\emptyset \\ H'(ǎ)\cap F(ǎ), & ǎ\in K\cap K=K \end{cases}$$

Thus, for all ǎ∈K, T(ǎ)= H'(ǎ)∩F(ǎ)= F(ǎ)∩F(ǎ)=F(ǎ). Thus (T,K)=(F,K). That is, in the set $S_E(U)$, the relative complement of each soft set is its left identity element for the operation $\underset{\gamma}{\sim}$.

16) Let (F,K), (G,Y) be soft sets over U. Then, $[(F,K)\underset{\gamma}{\sim}(G,Y)]^r=(F,K)\overset{*}{\underset{\lambda}{\sim}}(G,Y)$.

Proof: Let $(F,K)\underset{\gamma}{\sim}(G,Y)=(H,K)$, where for all ǎ∈K,

$$H(ǎ)=\begin{cases} F(ǎ), & ǎ\in K-Y \\ F'(ǎ)\cap G(ǎ), & ǎ\in K\cap Y \end{cases}$$

Let $(H,K)^r=(T,K)$, where for all ǎ∈K,

$$T(ǎ)=\begin{cases} F'(ǎ), & ǎ\in K-Y \\ F(ǎ)\cup G'(ǎ), & ǎ\in K\cap Y \end{cases}$$

Thus, $(T,K)=(F,K)\overset{*}{\underset{\backslash}{\sim}}(G,Y)$.

17) Let (F,K), (G,K) be soft sets over U. Then, $(F,K)\underset{\gamma}{\sim}(G,K)=U_K \Leftrightarrow (F,K)=\emptyset_K$ and $(G,K)=U_K$.

Proof: Let $(F,K)\underset{\gamma}{\sim}(G,K)=(T,K)$, where for all ǎ∈K,

$$T(ǎ)=\begin{cases} F(ǎ), & ǎ\in K\text{-}K=\emptyset \\ F'(ǎ)\cap G(ǎ), & ǎ\in K\cap K=K \end{cases}$$

Since $(T,K)=U_K$, for all $ǎ\in K$, $T(ǎ)=F'(ǎ)\cap G(ǎ)=U \Leftrightarrow$ for all $ǎ\in K$, $F'(ǎ)=U$ ve $G(ǎ)=U \Leftrightarrow$ for all $ǎ\in K$, $F(ǎ)=\emptyset$ and $G(ǎ)=U \Leftrightarrow (F,K)=\emptyset_K$ and $(G,K)=U_K$.

18) Let (F,K), (G,Y) be soft sets over U. Then, $\emptyset_K \mathrel{\tilde{\subseteq}} (F,K)\,\tilde{\gamma}\,(G,Y)$ and $\emptyset_Y \mathrel{\tilde{\subseteq}} (G,Y)\,\tilde{\gamma}\,(F,K)$.

19) Let (F,K), (G,Y) be soft sets over U. Then, $(F,K)\,\tilde{\gamma}\,(G,Y) \mathrel{\tilde{\subseteq}} U_K$ and $(G,Y)\,\tilde{\gamma}\,(F,K) \mathrel{\tilde{\subseteq}} U_Y$.

20) Let (F,K), (G,K) be soft sets over U. Then, $(F,K)\,\tilde{\gamma}\,(G,K) \mathrel{\tilde{\subseteq}} (F,K)^r$ and $(F,K)\,\tilde{\gamma}\,(G,K) \mathrel{\tilde{\subseteq}} (G,K)$.

Proof: Let $(F,K)\,\tilde{\gamma}\,(G,K)=(H,K)$, where for all $ǎ\in K$,

$$H(ǎ)=\begin{cases} F(ǎ), & ǎ\in K\text{-}K=\emptyset \\ F'(ǎ)\cap G(ǎ), & ǎ\in K\cap K=K \end{cases}$$

Since for all $ǎ\in K$, $H(ǎ)=F'(ǎ)\cap G(ǎ)\subseteq F'(\omega)$, $(F,K)\,\tilde{\gamma}\,(G,K) \mathrel{\tilde{\subseteq}} (F,K)^r$. $(F,K)\,\tilde{\gamma}\,(G,K) \mathrel{\tilde{\subseteq}} (G,K)$ can be shown similarly.

21) Let (F,K), (G,K), (H,Z), (H,K) be soft sets over U. Then, If $(F,K)\mathrel{\tilde{\subseteq}}(G,K)$, then $(H,Z)\,\tilde{\gamma}\,(F,K) \mathrel{\tilde{\subseteq}} (H,Z)\,\tilde{\gamma}\,(G,K)$ and $(G,K)\,\tilde{\gamma}\,(H,K) \mathrel{\tilde{\subseteq}} (F,K)\,\tilde{\gamma}\,(H,K)$.

Proof: Let $(F,K)\mathrel{\tilde{\subseteq}}(G,K)$, where for all $ǎ\in K$, $F(ǎ)\subseteq G(ǎ)$, so $G'(ǎ)\subseteq F'(ǎ)$. Let $(H,Z)\,\tilde{\gamma}\,(F,K)=(W,Z)$, where for all $ǎ\in Z$,

$$W(ǎ)=\begin{cases} H(ǎ), & ǎ\in Z\text{-}K \\ H'(ǎ)\cap F(ǎ), & ǎ\in Z\cap K \end{cases}$$

Let $(H,Z)\,\tilde{\gamma}\,(G,K)=(L,Z)$, where for all $ǎ\in Z$,

$$L(ǎ)=\begin{cases} H(ǎ), & ǎ\in Z\text{-}K \\ H'(ǎ)\cap G(ǎ), & ǎ\in Z\cap K \end{cases}$$

Thus, for all $ǎ\in Z\text{-}K$, $W(ǎ)=H(ǎ)\subseteq H(ǎ)=L(ǎ)$, for all $ǎ\in Z\cap K$, $W(ǎ)=H'(ǎ)\cap F(ǎ)\subseteq H'(ǎ)\cap G(ǎ)=L(ǎ)$, $(H,Z)\,\tilde{\gamma}\,(F,K) \mathrel{\tilde{\subseteq}} (H,Z)\,\tilde{\gamma}\,(G,K)$. Under the same conditions, for all $ǎ\in K$, $G'(ǎ)\cap H(ǎ)\subseteq F'(ǎ)\cap H(ǎ)$, $(G,K)\,\tilde{\gamma}\,(H,K) \mathrel{\tilde{\subseteq}} (F,K)\,\tilde{\gamma}\,(H,K)$.

22) Let (F,K), (G,K), (H,Z), (H,D) be soft sets over U. Then, If $(H,Z)\,\tilde{\gamma}\,(F,K) \mathrel{\tilde{\subseteq}} (H,Z)\,\tilde{\gamma}\,(G,K)$ then $(F,K)\mathrel{\tilde{\subseteq}}(G,K)$ needs not be true. Similarly, if $(G,K)\,\tilde{\gamma}\,(H,K) \mathrel{\tilde{\subseteq}} (F,K)\,\tilde{\gamma}\,(H,K)$ then $(F,K)\mathrel{\tilde{\subseteq}}(G,K)$ needs not be true. That is, the converse of Theorem 3.3. (21) is not true.

Proof: To demonstrate that the converse of Theorem 3.3. (21) is not true, let's provide an example. Let E={e_1,e_2,e_3,e_4,e_5} be the parameter set, K={e_1,e_3} and Z={e_1,e_3,e_5} be two subsets of E, U={h_1,h_2,h_3,h_4,h_5} be the universal set. Let (F,K), (G,K), and (H,Z) be soft sets over U as follows:

$$(F,K)=\{(e_1,\{h_2,h_5\}),(e_3,\{h_1,h_2,h_5\})\},(G,K)=\{(e_1,\{h_2\}),(e_3,\{h_1,h_2\})\},$$
$$(H,Z)=\{(e_1,U),(e_3,U),(e_5,\{h_2,h_5\})\}.$$

Let (H,Z)$\tilde{\gamma}$(F,K) = (L,Z), where for all ᾰ ∈ Z-K={e_5}, L(e_5)=H(e_5)={h_2,h_5}, *for all* ᾰ ∈ Z ∩K={e_1,e_3}, L(e_1)=H'(e_1)∩F(e_1)=∅, L(e_3)=H'(e_3)∩F(e_3)=∅. Thus, (H,Z)$\tilde{\gamma}$(F,K)={(e_1, ∅),(e_3, ∅),(e_5,{h_2,h_5})}.

Now let (H,Z)$\tilde{\gamma}$(G,K) = (W,Z), where W(e_5)=H(e_5)={h_2,h_5}, W(e_1)=H'(e_1) ∩G(e_1)=∅,W(e_3)=H'(e_3)∩G(e_3)=∅.

Thus, (H,Z)$\tilde{\gamma}$(G,K)={(e_1,∅),(e_3, ∅),(e_5,{h_2,h_5})}.

Thus, (H,Z)$\tilde{\gamma}$(F,K) $\tilde{\subseteq}$ (H,Z)$\tilde{\gamma}$(G,K), however (F,K) $\tilde{\subseteq}$ (G,K) is not a soft subset of. Similarly if (G,K) $\tilde{\gamma}$(H,K) $\tilde{\subseteq}$ (F,K) $\tilde{\gamma}$(H,K), then (F,K)$\tilde{\subseteq}$ (G,K) needs not to be true can be shown by taking (H,K)={(e_1,∅),(e_3,∅)} in the same example.

23) Let (F,T), (G,T), (K,T), (L,T) be soft sets over U. Then, If (F,T)$\tilde{\subseteq}$ (G,T) and (K,T)$\tilde{\subseteq}$ (L,T), then (G,T)$\tilde{\gamma}$(K,T)$\tilde{\subseteq}$(F,T)$\tilde{\gamma}$(L,T) and (L,T)$\tilde{\gamma}$(F,T) $\tilde{\subseteq}$(K,T)$\tilde{\gamma}$(G,T).

Proof: Let (F,T)$\tilde{\subseteq}$ (G,T), where for all ᾰ ∈T, F(ᾰ)⊆ G(ω). Thus, for all ᾰ ∈T, G'(ᾰ)⊆ F'(ᾰ). (G,T)$\tilde{\gamma}$(K,T)=(M,T). Thus, for all ᾰ ∈T, M(ᾰ)=G'(ᾰ)∩K(ᾰ). (F,T)$\tilde{\gamma}$(L,T)=(N,T). Thus, for all ᾰ ∈T, N(ᾰ)=F'(ᾰ)∩L(ᾰ). for all ᾰ ∈T, G'(ᾰ)⊆ F'(ᾰ) so, M(ᾰ)=G'(ᾰ)∩K(ᾰ)⊆F'(ᾰ)∩L(ᾰ)=N(ᾰ). Hence, for (G,T)$\tilde{\gamma}$(K,T) $\tilde{\subseteq}$(F,T) $\tilde{\gamma}$(L,T). Under the same conditions, it can be similarly shown that (L,T)$\tilde{\gamma}$(F,T) $\tilde{\subseteq}$(K,T)$\tilde{\gamma}$(G,T).

4 Distributions

This section provides a detailed examination of the distribution of the soft binary piecewise gamma operation over various soft set operations, leading to the discovery of several intriguing algebraic structures.

Proposition 4.1. Let (F,K), (G,Y), and (H,D) be soft sets over U. Then, the soft binary piecewise gamma operation distributes over restricted operations as follows:

LHS Distributions: The equations are satisfied with the condition K∩(YΔD)=∅.

1) (F,K) $\tilde{\gamma}$[(G,Y) $\cap_R$(H,D]=[(F,K)$\tilde{\gamma}$ (G,Y)]$\cap_R$[(F,K) $\tilde{\gamma}$ (H,D)].

Proof: Firstly, consider the LHS. Let $(G,Y)\cap_R(H,D)=(M,Y\cap D)$, where for all $\breve{a}\in Y\cap D$, $M(\breve{a})=G(\breve{a})\cap H(\breve{a})$. Let $(F,K)\,\tilde{\gamma}\,(M,Y\cap D)=(N,K)$, where for all $\breve{a}\in K$,

$$N(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in K-(Y\cap D) \\ F'(\breve{a})\cap M(\breve{a}), & \breve{a}\in K\cap(Y\cap D) \end{cases}$$

Hence,

$$N(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in K-(Y\cap D) \\ F'(\breve{a})\cap[G(\breve{a})\cap H(\breve{a})], & \breve{a}\in K\cap(Y\cap D) \end{cases}$$

Now consider the RHS, i.e., $[(F,K)\,\tilde{\gamma}\,(G,Y)]\cap_R[(F,K)\,\tilde{\gamma}\,(H,D)]$.$(F,K)\,\tilde{\gamma}\,(G,Y)=(V,K)$, where for all $\breve{a}\in K$,

$$V(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in K-Y \\ F'(\breve{a})\cap G(\breve{a}), & \breve{a}\in K\cap Y \end{cases}$$

Let $(F,K)\,\tilde{\gamma}\,(H,D)=(W,K)$, where for all $\breve{a}\in K$,

$$W(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in K-D \\ F'(\breve{a})\cap H(\breve{a}), & \breve{a}\in K\cap D \end{cases}$$

Assume that $(V,K)\cap_R(W,K)=(T,K)$, where for all $\breve{a}\in K$, $T(\breve{a})=V(\breve{a})\cap W(\breve{a})$,

$$T(\breve{a})=\begin{cases} F(\breve{a})\cap F(\breve{a}), & \breve{a}\in(K-Y)\cap(K-D) \\ F(\breve{a})\cap[F'(\breve{a})\cap H(\breve{a})], & \breve{a}\in(K-Y)\cap(K\cap D) \\ [F'(\breve{a})\cap G(\breve{a})]\cap F(\breve{a}), & \breve{a}\in(K\cap Y)\cap(K-D) \\ [F'(\breve{a})\cap G(\breve{a})]\cap[F'(\breve{a})\cap H(\breve{a})], & \breve{a}\in(K\cap Y)\cap(K\cap D) \end{cases}$$

Hence,

$$T(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in K\cap Y'\cap D' \\ \emptyset, & \breve{a}\in K\cap Y'\cap D \\ \emptyset, & \breve{a}\in K\cap Y\cap D' \\ F'(\breve{a})\cap G(\breve{a})\cap H(\breve{a}), & \breve{a}\in K\cap Y\cap D \end{cases}$$

Considering the parameter set in the first row of the N function, we can consider $K-(Y\cap D)=K\cap(Y\cap D)'$ and if $\breve{a}\in(Y\cap D)'$, then $\breve{a}\in Y-D$ or $\breve{a}\in D-Y$ or $\breve{a}\in Y\cup D$. Thus, if $\breve{a}\in K-(Y\cap D)$ then $\breve{a}\in K\cap(Y\cap D')$ or $\breve{a}\in K\cap(Y'\cap D)$ or $\breve{a}\in K\cap(Y'\cap D')$. Thus, for $K\cap Y'\cap D=K\cap Y\cap D'=\emptyset$, it is seen that N=T. It is obvious that the condition $K\cap Y'\cap D=K\cap Y\cap D'=\emptyset$ is equivalent to the condition $K\cap(Y\Delta D)=\emptyset$.

2) $(F,K)\,\tilde{\gamma}\,[(G,Y)\cup_R(H,D)]=[(F,K)\,\tilde{\gamma}\,(G,Y)]\cup_R[(F,K)\,\tilde{\gamma}\,(H,D]$

3) If $K\cap Y'\cap D'=K\cap Y\cap D'=\emptyset$, $(F,K)\,\tilde{\gamma}\,[(G,Y)\backslash_R(H,D]=[(F,K)\,\tilde{\gamma}\,(G,Y)]\backslash_R[(F,K)\,\tilde{\gamma}\,(H,D)]$.

RHS Distributions: The following hold, where $K\cap Y\cap D=\emptyset$.

1)$[(F,K)\cup_R(G,Y)]\,\tilde{\gamma}\,(H,D)=[(F,K)\,\tilde{\gamma}\,(H,D)]\cup_R[(G,Y)\,\tilde{\gamma}\,(H,D)]$.

Proof: First, consider the LHS. Let $(F,K)\cup_R(G,Y)=(M,K\cap Y)$, where for all $\breve{a}\in K\cap Y$, $M(\breve{a})=F(\breve{a})\cup G(\breve{a})$. $(M,K\cap Y)\,\tilde{\gamma}\,(H,D)=(N,K\cap Y)$, where for all $\breve{a}\in K\cap Y$,

$$N(\breve{a})=\begin{cases} M(\breve{a}), & \breve{a}\in(K\cap Y)-D \\ M'(\breve{a})\cap H(\breve{a}), & \breve{a}\in(K\cap Y)\cap D \end{cases}$$

Hence,

$$N(\breve{a})=\begin{cases} F(\breve{a})\cup G(\breve{a}), & \breve{a}\in(K\cap Y)-D=K\cap Y\cap D' \\ [F'(\breve{a})\cap G'(\breve{a})]\cap H(\breve{a}), & \breve{a}\in(K\cap Y)\cap D \end{cases}$$

Now consider the RHS, i.e., $[(F,K)\,\tilde{\gamma}\,(H,D)]\cup_R[(G,Y)\,\tilde{\gamma}\,(H,D)]$. $(F,K)\,\tilde{\gamma}\,(H,D)=(V,K)$, where for all $\breve{a}\in K$,

$$V(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in K-D \\ F'(\breve{a})\cap H(\breve{a}), & \breve{a}\in K\cap D \end{cases}$$

Let $(G,Y)\,\tilde{\gamma}\,(H,D)=(W,Y)$, where for all $\breve{a}\in Y$,

$$W(\breve{a})=\begin{cases} G(\breve{a}), & \breve{a}\in Y-D \\ G'(\breve{a})\cap H(\breve{a}), & \breve{a}\in Y\cap D \end{cases}$$

Let $(V,K)\cup_R (W,Y)=(T,K\cap Y)$, where for all $\breve{a}\in K\cap Y$, $T(\breve{a})=V(\breve{a})\cup W(\breve{a})$,

$$T(\breve{a})=\begin{cases} F(\breve{a})\cup G(\breve{a}), & \breve{a}\in(K-D)\cap(Y-D)=K\cap Y\cap D' \\ F(\breve{a})\cup[G'(\breve{a})\cap H(\breve{a})], & \breve{a}\in(K-D)\cap(Y\cap D)=\emptyset \\ [F'(\breve{a})\cap H(\breve{a})]\cup G(\breve{a}), & \breve{a}\in(K\cap D)\cap(Y-D)=\emptyset \\ [F'(\breve{a})\cap H(\breve{a})]\cup[G'(\breve{a})\cap H(\breve{a})], & \breve{a}\in(K\cap D)\cap(Y\cap D)=K\cap Y\cap D \end{cases}$$

Hence,

$$T(\breve{a})=\begin{cases} F(\breve{a})\cup G(\breve{a}), & \breve{a}\in K\cap Y\cap D' \\ [F'(\breve{a})\cup G'(\breve{a})]\cap H(\breve{a}), & \breve{a}\in K\cap Y\cap D \end{cases}$$

Thus, it can be seen that $N=T$ for $K\cap Y\cap D=\emptyset$.

2)$[(F,K)\cap_R(G,Y)]\,\tilde{\gamma}\,(H,D)=[(F,K)\,\tilde{\gamma}\,(H,D)]\cap_R[(G,Y)\,\tilde{\gamma}\,(H,D)]$.

3)$[(F,K)\setminus_R(G,Y)]\,\tilde{\gamma}\,(H,D)=[(F,K)\,\tilde{\gamma}\,(H,D)]\setminus_R[(G,Y)\,\tilde{\gamma}\,(H,D)]$.

Corollary 4.1.1. $(S_E(U),\cup_R,\tilde{\gamma})$ is an additive commutative and additive idempotent semiring without zero and unity under certain conditions.

Proof: Ali et al. [6] showed that $(S_E(U),\cup_R)$ is a commutative, idempotent monoid with identity element $\emptyset_E$, that is, a bounded semilattice (hence a semigroup). By Theorem 3.3.1, $(S_E(U),\tilde{\gamma})$ is a noncommutative and not idempotent semigroup under the condition $T\cap Z'\cap M = T\cap Z\cap M=\emptyset$, where (F,T), (G,Z) and (H,M) are soft sets over U. Besides, by Proposition 4.1. (2), $\tilde{\gamma}$ distributes over $\cup_R$ from LHS under the condition $K\cap(Y\Delta D)=\emptyset$. Besides, by Proposition 4.1. (1), $\tilde{\gamma}$ distributes over $\cup_R$ from RHS under the condition $T\cap Z\cap M=\emptyset$.

Thus, $(S_E(U),\cup_R,\tilde{\gamma})$ is an additive commutative and additive idempotent semiring without zero and unity under certain conditions.

Corollary 4.1.2. $(S_E(U),\cap_R,\tilde{\gamma})$ is an additive commutative and additive idempotent semiring without zero and unity under certain conditions.

Proof: Ali et al. [6] showed that $(S_E(U),\cap_R)$ is a commutative, idempotent monoid with identity element U_E, that is, a bounded semilattice (hence a semigroup). By Theorem 3.3.1, $(S_E(U),\tilde{\gamma})$ is a noncommutative and not idempotent semigroup under the condition $T\cap Z'\cap M = T\cap Z\cap M = \emptyset$, where (F,T), (G,Z) and (H,M) are soft sets over U. Besides, by Proposition 4.1. (1), $\tilde{\gamma}$ distributes over $\cap_R$ from LHS under the condition $K\cap(Y\Delta D)=\emptyset$ and Proposition 4.1. (2), $\tilde{\gamma}$ distributes over $\cap_R$ from RHS under the condition $T\cap Z\cap M=\emptyset$. Thus, $(S_E(U),\cap_R,\tilde{\gamma})$ is an additive commutative and additive idempotent semiring without zero and unity under certain conditions.

Here note that since the algebraic structure $(S_E(U),\backslash_R)$ is not associative, and so it cannot be a semigroup, thus it cannot be expected to form a semring with $\tilde{\gamma}$.

Proposition 4.2. Let (F,K), (G,Y), and (H,D) be soft sets over U. Then, the distributions of the soft binary piecewise gamma operation over extended soft set operations are as follows:

LHS Distributions: The following are satisfied with the condition $K\cap(Y\Delta D)=\emptyset$.

1) $(F,K)\tilde{\gamma}[(G,Y)\cup_\varepsilon(H,D)]=[(F,K)\tilde{\gamma}(G,Y)]\cup_\varepsilon[(F,K)\tilde{\gamma}(H,D)]$.

Proof: First handle the LHS. Let $(G,Y)\cup_\varepsilon(H,D)=(M,Y\cup D)$, where for all ẟ$\in Y\cup D$,

$$M(ẟ)=\begin{cases} G(ẟ), & ẟ\in Y-D \\ H(ẟ), & ẟ\in D-Y \\ G(ẟ)\cup H(ẟ), & ẟ\in Y\cap D \end{cases}$$

Let $(F,K)\tilde{\gamma}(M,Y\cup D)=(N,K)$, where for all ẟ$\in K$,

$$N(ẟ)=\begin{cases} F(ẟ), & ẟ\in K-(Y\cup D) \\ F'(ẟ)\cap M(ẟ), & ẟ\in K\cap(Y\cup D) \end{cases}$$

Hence,

$$N(ẟ)=\begin{cases} F(ẟ), & ẟ\in K-(Y\cup D)=K\cap Y'\cap D' \\ F'(ẟ)\cap G(ẟ), & ẟ\in K\cap(Y-D)=K\cap Y\cap D' \\ F'(ẟ)\cap H(ẟ), & ẟ\in K\cap(D-Y)=K\cap Y'\cap D \\ F'(ẟ)\cap[(G(ẟ)\cup H(ẟ)], & ẟ\in K\cap Y\cap D=K\cap Y\cap D \end{cases}$$

Now consider the RHS. Let $[(F,K)\tilde{\gamma}(G,Y)]\cup_\varepsilon[(F,K)\tilde{\gamma}(H,D)]$. $(F,K)\tilde{\gamma}(G,Y)=(V,K)$, where for all ẟ$\in K$,

$$V(ẟ)=\begin{cases} F(ẟ), & ẟ\in K-Y \\ F'(ẟ)\cap G(ẟ), & ẟ\in K\cap Y \end{cases}$$

Let $(F,K)\,\tilde{\gamma}\,(H,D)=(W,K)$, where for all $\breve{a}\in K$,

$$W(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in K-D \\ F'(\breve{a})\cap H(\breve{a}), & \breve{a}\in K\cap D \end{cases}$$

Let $(V,K)\cup_{\varepsilon}(W,K)=(T,K)$, where for all $\breve{a}\in K$,

$$T(\breve{a})=\begin{cases} V(\breve{a}), & \breve{a}\in K-K=\emptyset \\ W(\breve{a}), & \breve{a}\in K-K=\emptyset \\ V(\breve{a})\cap W(\breve{a}), & \breve{a}\in K\cap K=K \end{cases}$$

Hence,

$$T(\breve{a})=\begin{cases} F(\breve{a})\cup F(\breve{a}), & \breve{a}\in(K-Y)\cap(K-D)=K\cap Y'\cap D' \\ F(\breve{a})\cup[F'(\breve{a})\cap H(\breve{a})], & \breve{a}\in(K-Y)\cap(K\cap D)=K\cap Y'\cap D \\ [F'(\breve{a})\cap G(\breve{a})]\cup F(\breve{a}), & \breve{a}\in(K\cap Y)\cap(K-D)=K\cap Y\cap D' \\ [F'(\breve{a})\cap G(\breve{a})]\cup[F'(\breve{a})\cap H(\breve{a})], & \breve{a}\in(K\cap Y)\cap(K\cap D)=K\cap Y\cap D \end{cases}$$

Thus,

$$T(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in(K-Y)\cap(K-D)=K\cap Y'\cap D' \\ F(\breve{a})\cup H(\breve{a}), & \breve{a}\in(K-Y)\cap(K\cap D)=K\cap Y'\cap D \\ G(\breve{a})\cup F(\breve{a}), & \breve{a}\in(K\cap Y)\cap(K-D)=K\cap Y\cap D' \\ F'(\breve{a})\cap[G(\breve{a})\cup H(\breve{a})], & \breve{a}\in(K\cap Y)\cap(K\cap D)=K\cap Y\cap D \end{cases}$$

It can be seen that N=T for $K\cap Y'\cap D=K\cap Y\cap D'=\emptyset$. It is obvious that the condition $K\cap Y'\cap D=K\cap Y\cap D'=\emptyset$ is equivalent to the condition $K\cap(Y\Delta D)=\emptyset$.

2) $(F,K)\,\tilde{\gamma}\,[(G,Y)\cap_{\varepsilon}(H,D)]=[(F,K)\,\tilde{\gamma}\,(G,Y)]\cap_{\varepsilon}[(F,K)\,\tilde{\gamma}\,(H,D)]$.

3) If $K\cap Y'\cap D'=K\cap Y'\cap D=\emptyset$, then $(F,K)\,\tilde{\gamma}\,[(G,Y)\setminus_{\varepsilon}(H,D)]=[(F,K)\,\tilde{\gamma}\,(G,Y)]\setminus_{\varepsilon}[(F,K)\,\tilde{\gamma}\,(H,D)]$.

RHS Distributions: The followings hold, where $K\cap Y\cap D=\emptyset$.

1) $[(F,K)\cap_{\varepsilon}(G,Y)]\,\tilde{\gamma}\,(H,D)=[(F,K)\,\tilde{\gamma}\,(H,D)]\cap_{\varepsilon}[(G,Y)\,\tilde{\gamma}\,(H,D)]$.

Proof: First consider the LHS. Let $(F,K)\cap_{\varepsilon}(G,Y)=(M,K\cup Y)$, where for all $\breve{a}\in K\cup Y$,

$$M(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in K-Y \\ G(\breve{a}), & \breve{a}\in Y-K \\ F(\breve{a})\cap G(\breve{a}), & \breve{a}\in K\cap Y \end{cases}$$

Let $(M,K\cup Y)\,\tilde{\gamma}\,(H,D)=(N,K\cup Y)$, where for all $\breve{a}\in K\cup Y$,

$$N(\breve{a})=\begin{cases} M(\breve{a}), & \breve{a}\in(K\cup Y)-D \\ M'(\breve{a})\cap H(\breve{a}), & \breve{a}\in(K\cup Y)\cap D \end{cases}$$

Thus,

$$N(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in(K-Y)-D=K\cap Y'\cap D' \\ G(\breve{a}), & \breve{a}\in(Y-K)-D=K'\cap Y\cap D' \\ F(\breve{a})\cap G(\breve{a}), & \breve{a}\in(K\cap Y)-D=K\cap Y\cap D' \\ F'(\breve{a})\cap H(\breve{a}), & \breve{a}\in(K-Y)\cap D=K\cap Y'\cap D \\ G'(\breve{a})\cap H(\breve{a}), & \breve{a}\in(Y-K)\cap D=K'\cap Y\cap D \\ [F'(\breve{a})\cup G'(\breve{a})]\cap H(\breve{a}), & \breve{a}\in(K\cap Y)\cap D=K\cap Y\cap D \end{cases}$$

Now consider the RHS, i.e., $[(F,K)\widetilde{\gamma}(H,D)]\cap_{\varepsilon}[(G,Y)\widetilde{\gamma}(H,D)]$. $(F,K)\widetilde{\gamma}(H,D)=(V,K)$, where for all $\breve{a}\in K$,

$$V(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in K-D \\ F'(\breve{a})\cap H(\breve{a}), & \breve{a}\in K\cap D \end{cases}$$

Let $(G,Y)\widetilde{\gamma}(H,D)=(W,Y)$, where for all $\breve{a}\in Y$,

$$W(\breve{a})=\begin{cases} G(\breve{a}), & \breve{a}\in Y-D \\ G'(\breve{a})\cap H(\breve{a}), & \breve{a}\in Y\cap D \end{cases}$$

Assume that $(V,K)\cap_{\varepsilon}(W,Y)=(T,K\cup Y)$, where for all $\breve{a}\in K\cup Y$,

$$T(\breve{a})=\begin{cases} V(\breve{a}), & \breve{a}\in K-Y \\ W(\breve{a}), & \breve{a}\in Y-K \\ V(\breve{a})\cap W(\breve{a}), & \breve{a}\in K\cap Y \end{cases}$$

Thus,

$$T(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in(K-D)-Y=K\cap Y'\cap D' \\ F'(\breve{a})\cap H(\breve{a}), & \breve{a}\in(K\cap D)-Y=K\cap Y'\cap D \\ G(\breve{a}), & \breve{a}\in(Y-D)-K=K'\cap Y\cap D' \\ G'(\breve{a})\cap H(\breve{a}), & \breve{a}\in(Y\cap D)-K=K'\cap Y\cap D \\ F(\breve{a})\cap G(\breve{a}), & \breve{a}\in(K-D)\cap(Y-D)=K\cap Y\cap D' \\ F(\breve{a})\cap[G'(\breve{a})\cap H(\breve{a})], & \breve{a}\in(K-D)\cap(Y\cap D)=\emptyset \\ [F'(\breve{a})\cap H(\breve{a})]\cap G(\breve{a}), & \breve{a}\in(K\cap D)\cap(Y-D)=\emptyset \\ [F'(\breve{a})\cap H(\breve{a})]\cap[G'(\breve{a})\cap H(\breve{a})], & \breve{a}\in(K\cap D)\cap(Y\cap D)=K\cap Y\cap D \end{cases}$$

Hence,

$$T(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in K\cap Y'\cap D' \\ F'(\breve{a})\cap H(\breve{a}), & \breve{a}\in K\cap Y'\cap D \\ G(\breve{a}), & \breve{a}\in K'\cap Y\cap D' \\ G'(\breve{a})\cap H(\breve{a}), & \breve{a}\in K'\cap Y\cap D \\ F(\breve{a})\cap G(\breve{a}), & \breve{a}\in K\cap Y\cap D' \\ F'(\breve{a})\cap G'(\breve{a})\cap H(\breve{a}), & \breve{a}\in K\cap Y\cap D \end{cases}$$

Thus, it can be seen that N=T for $K\cap Y\cap D=\emptyset$.

2) $[(F,K)\cup_{\varepsilon}(G,Y)]\widetilde{\gamma}(H,D)=[(F,K)\widetilde{\gamma}(H,D)]\cup_{\varepsilon}[(G,Y)\widetilde{\gamma}(H,D)]$.

3) $[(F,K)\setminus_{\varepsilon}(G,Y)]\widetilde{\gamma}(H,D)=[(F,K)\widetilde{\gamma}(H,D)]\setminus_{\varepsilon}[(G,Y)\widetilde{\gamma}(H,D)]$.

4) $[(F,K)\Delta_{\varepsilon}(G,Y)]\widetilde{\gamma}(H,D)=[(F,K)\widetilde{\gamma}(H,D)]\Delta_{\varepsilon}[(G,Y)\widetilde{\gamma}(H,D)]$.

5) $[(F,K)+_{\varepsilon}(G,Y)]\widetilde{\gamma}(H,D)=[(F,K)\widetilde{\gamma}(H,D)]+_{\varepsilon}[(G,Y)\widetilde{\gamma}(H,D)]$.

6) $[(F,K)\gamma_{\varepsilon}(G,Y)]\widetilde{\gamma}(H,D)=[(F,K)\widetilde{\gamma}(H,D)]\gamma_{\varepsilon}[(G,Y)\widetilde{\gamma}(H,D)]$.

7) $[(F,K)*_{\varepsilon}(G,Y)]\widetilde{\gamma}(H,D)=[(F,K)\widetilde{\gamma}(H,D)]*_{\varepsilon}[(G,Y)\widetilde{\gamma}(H,D)]$.

8) $[(F,K)\ \theta_\varepsilon(G,Y)]\tilde{\gamma}(H,D)=[(F,K)\tilde{\gamma}\ (H,D)]\theta_\varepsilon[(G,Y)\ \tilde{\gamma}(H,D)]$.

Corollary 4.2.1. $(S_E(U),\ \cup_\varepsilon,\tilde{\gamma})$ and $(S_E(U),\cap_\varepsilon,\tilde{\gamma})$ are additive commutative and additive idempotent (right) nearsemirings with zero but without unity and zero symmetric property under certain conditions. Similarly, $(S_E(U),\backslash_\varepsilon,\tilde{\gamma})$, $(S_E(U),\Delta_\varepsilon,\tilde{\gamma})$, $(S_E(U),+_\varepsilon,\tilde{\gamma})$, $(S_E(U),\gamma_\varepsilon,\tilde{\gamma})$, $(S_E(U),\lambda_\varepsilon,\tilde{\gamma})$, $(S_E(U),*_\varepsilon,\tilde{\gamma})$, $(S_E(U),\theta_\varepsilon,\tilde{\gamma})$ are additive commutative not idempotent (right) nearsemirings with zero, but without unity and zero symmetric property under certain condition.

Proof: Ali et al. [6] showed that $(S_E(U),\ \cup_\varepsilon)$ is a commutative, idempotent monoid with identity $\emptyset_\emptyset$, that is, a bounded semilattice (hence a semigroup). By Theorem 3.3.1, $(S_E(U),\tilde{\gamma})$ is a noncommutative and not idempotent semigroup under the condition $T\cap Z'\cap M = T\cap Z\cap M =\emptyset$, where (F,T), (G,Z) and (H,M) are soft sets over U. Besides, by Theorem 3.3 (10), $\emptyset_\emptyset\tilde{\gamma}(F,T)=\emptyset_\emptyset$, that is $\emptyset_\emptyset$ is the left absorbing element for $\tilde{\gamma}$ in $S_E(U)$, furthermore by Proposition 4.2, $\tilde{\gamma}$ distributes over $\cup_\varepsilon$ from RHS under the condition $T\cap Z\cap M=\emptyset$. Thus, $(S_E(U),\ \cup_\varepsilon,\ \tilde{\gamma})$ is an additive commutative and additive idempotent (right) nearsemiring with zero, but without unity under certain conditions. Moreover, since $(F,K)\tilde{\gamma}\emptyset_\emptyset \neq \emptyset_\emptyset$, $(S_E(U),\cup_\varepsilon,\tilde{\gamma})$ is a (right) nearsemiring without zero symmetric property. Similarly, $(S_E(U),\cap_\varepsilon,\tilde{\gamma})$ is an additive commutative and additive idempotent (right) nearsemiring with zero but without unity and zero symmetric property under certain condition. Furthermore, $(S_E(U),\backslash_\varepsilon,\tilde{\gamma})$, $(S_E(U),\Delta_\varepsilon,\tilde{\gamma})$, $(S_E(U),+_\varepsilon,\tilde{\gamma})$, $(S_E(U),\gamma_\varepsilon,\tilde{\gamma})$, $(S_E(U),\lambda_\varepsilon,\tilde{\gamma})$, $(S_E(U),*_\varepsilon,\tilde{\gamma})$, $(S_E(U),\theta_\varepsilon,\tilde{\gamma})$ are all additive commutative not idempotent (right) nearsemirings with zero but without unity and zero symmetric property under certain conditions. Here note that, Aybek (2024) showed that the first operation is associative in $S_E(U)$ under the condition $T\cap Z\cap M=\emptyset$ (for Δ_ε, without any condition).

Corollary 4.2.2. $(S_E(U),\ \cup_\varepsilon,\tilde{\gamma})$ and $(S_E(U),\ \cap_\varepsilon,\tilde{\gamma})$ are noncommutative and additive idempotent semirings without zero and unity under certain conditions

Proof: Ali et al. [6] showed that $(S_E(U),\cup_\varepsilon)$ is a commutative, idempotent monoid with identity $\emptyset_\emptyset$, that is, a bounded semilattice (hence a semigroup). By Theorem 3.3.1, $(S_E(U),\tilde{\gamma})$ is a noncommutative and not idempotent semigroup under the condition $T\cap Z'\cap M = T\cap Z\cap M =\emptyset$, where (F,T), (G,Z), and (H,M) are soft sets over U. Besides, by Proposition 4.2, $\tilde{\gamma}$ distributes over $\cup_\varepsilon$ from LHS under the condition $T\cap(Z\Delta M)=\emptyset$, and $\tilde{\gamma}$ distributes over $\cup_\varepsilon$ from RHS under the condition $T\cap Z\cap M=\emptyset$. Thus, $(S_E(U),\ \cup_\varepsilon,\tilde{\gamma})$ is a noncommutative and additive idempotent semiring without zero and unity under certain conditions. One can similarly show that $(S_E(U),\ \cap_\varepsilon,\tilde{\gamma})$ is a noncommutative and additive idempotent semiring without zero and unity under certain conditions.

Corollary 4.2.3. $(S_E(U),\backslash_{\varepsilon},\tilde{\gamma})$ is a noncommutative, and not idempotent semiring without zero and unity under certain conditions.

Proof: Ali et al. [6] showed that $(S_E(U), \backslash_{\varepsilon})$ is a monoid with identity $\emptyset_{\emptyset}$, that is, a bounded semilattice (hence a semigroup). By Theorem 3.3.1, $(S_E(U),\tilde{\gamma})$ is a noncommutative and not idempotent semigroup under the condition T∩Z'∩M = T∩Z∩M =∅, where (F,T), (G,Z) and (H,M) are soft sets over U. Besides, by Proposition 4.2, $\tilde{\gamma}$ distributes over $\backslash_{\varepsilon}$ from LHS under the condition T∩Z'∩M = T∩Z'∩M'=∅, and $\tilde{\gamma}$ distributes over $\backslash_{\varepsilon}$ from RHS under the condition T∩Z∩M=∅. Thus, $(S_E(U),\backslash_{\varepsilon},\tilde{\gamma})$ is a noncommutative, and not idempotent semiring without zero and unity under certain conditions.

Proposition 4.3. Let (F,K), (G,Y), (H,D) be soft sets on U. Then, the distribution of the soft binary piecewise gamma operation over soft binary piecewise operations are as follows:

LHS Distributions: The following hold, where K∩(YΔD) =∅.

1) $(F,K)\tilde{\gamma}[(G,Y)\tilde{\cup}(H,D)]=[(F,K)\tilde{\gamma}(G,Y)]\tilde{\cup}[(F,K)\tilde{\gamma}(H,D)]$.

Proof: First consider the LHS. Let $(G,Y)\tilde{\cup}(H,D)=(M,Y)$, where for all $\breve{\alpha}\in Y$,

$$M(\breve{\alpha})=\begin{cases} G(\breve{\alpha}), & \breve{\alpha}\in Y-D \\ G(\breve{\alpha})\cup H(\breve{\alpha}), & \breve{\alpha}\in Y\cap D \end{cases}$$

Let $(F,K)\tilde{\gamma}(M,Y)=(N,K)$, where for all $\breve{\alpha}\in K$,

$$N(\breve{\alpha})=\begin{cases} F(\breve{\alpha}), & \breve{\alpha}\in K-Y \\ F'(\breve{\alpha})\cap M(\breve{\alpha}), & \breve{\alpha}\in K\cap Y \end{cases}$$

Thus,

$$N(\breve{\alpha})=\begin{cases} F(\breve{\alpha}), & \breve{\alpha}\in K-Y \\ F'(\breve{\alpha})\cap G(\breve{\alpha}), & \breve{\alpha}\in K\cap(Y-D)=K\cap Y\cap D' \\ F'(\breve{\alpha})\cap[G(\breve{\alpha})\cup H(\breve{\alpha})], & \breve{\alpha}\in K\cap Y\cap D=K\cap Y\cap D \end{cases}$$

Now consider the RHS. Let $[(F,K)\tilde{\gamma}(G,Y)]\tilde{\cup}[(F,K)\tilde{\gamma}(H,D)]$. $(F,K)\tilde{\gamma}(G,Y)=(V,K)$, where for all $\breve{\alpha}\in K$,

$$V(\breve{\alpha})=\begin{cases} F(\breve{\alpha}), & \breve{\alpha}\in K-Y \\ F'(\breve{\alpha})\cap G(\breve{\alpha}), & \breve{\alpha}\in K\cap Y \end{cases}$$

Let $(F,K)\tilde{\gamma}(H,D)=(W,K)$. Thus, for all $\breve{\alpha}\in K$,

$$W(\breve{\alpha})=\begin{cases} F(\breve{\alpha}), & \breve{\alpha}\in K-D \\ F'(\breve{\alpha})\cap H(\breve{\alpha}), & \breve{\alpha}\in K\cap D \end{cases}$$

Asume that (V,K) $\tilde{\cup}$ (W,K)=(T,K), where for all ǎ∈K,

$$T(ǎ)=\begin{cases} V(ǎ), & ǎ\in K-K=\emptyset \\ V(ǎ)\cup W(ǎ), & ǎ\in K\cap K=K \end{cases}$$

Thus,

$$T(ǎ)=\begin{cases} F(ǎ)\cup F(ǎ) & ǎ\in(K-Y)\cap(K-D)=K\cap Y'\cap D' \\ F(ǎ)\cup[F'(ǎ)\cap H(ǎ)], & ǎ\in(K-Y)\cap(K\cap D)=K\cap Y'\cap D \\ [F'(ǎ)\cap G(ǎ)]\cup F(ǎ) & ǎ\in(K\cap Y)\cap(K-D)=K\cap Y\cap D' \\ [F'(ǎ)\cap G(ǎ)]\cup[F'(ǎ)\cap H(ǎ)], & ǎ\in(K\cap Y)\cap(K\cap D)=K\cap Y\cap D \end{cases}$$

Hence,

$$T(ǎ)=\begin{cases} F(ǎ), & ǎ\in K\cap Y'\cap D' \\ F(ǎ)\cup H(ǎ), & ǎ\in K\cap Y'\cap D \\ G(ǎ)\cup F(ǎ), & ǎ\in K\cap Y\cap D' \\ F'(ǎ)\cap[G(ǎ)\cup H(ǎ)], & ǎ\in K\cap Y\cap D \end{cases}$$

We will consider the function K-Y in N, if ǎ∈K-Y, since K-Y=K∩Y', if ǎ∈Y', from ǎ∈D-Y or ǎ∈(Y∪D), from here, if ǎ∈K-Y, ǎ∈K∩ Y It becomes '∩D' or ǎ∈K∩Y'∩D. Thus, it appears that N=T for K∩Y'∩D= K∩Y∩D'=∅. It is obvious that the condition K∩Y'∩D=K∩Y∩D'=∅ is equivalent to the condition K∩(YΔD)=∅.

2) (F,K)$\tilde{\gamma}$[(G,Y)$\tilde{\cap}$(H,D)]=[(F,K) $\tilde{\gamma}$(G,Y)] $\tilde{\cap}$ [(F,K)$\tilde{\gamma}$ (H,D)].

3) If K∩Y'∩D'=∅, then (F,K)$\tilde{\gamma}$[(G,Y)$\tilde{\setminus}$ (H,D)]=[(F,K) $\tilde{\gamma}$(G,Y)] $\tilde{\setminus}$ [(F,K) $\tilde{\gamma}$ (H,D)].

RHS Distributions: The following hold, where K∩Y∩D=∅.

1) [(F, K) $\tilde{\cap}$ (G,Y)]$\tilde{\gamma}$ (H,D)=[(F,K) $\tilde{\gamma}$(H,D)] $\tilde{\cap}$ [(G,Y) $\tilde{\gamma}$ (H,D)].

Proof: First consider the LHS. Let (F, K)$\tilde{\cap}$(G,Y)=(M,K), where for all ǎ∈K,

$$M(ǎ)=\begin{cases} F(ǎ), & ǎ\in K-Y \\ F(ǎ)\cap G(ǎ), & ǎ\in K\cap Y \end{cases}$$

Let (M,K) $\tilde{\gamma}$ (H,D)=(N,K), where for all ǎ∈K,

$$N(ǎ)=\begin{cases} M(ǎ), & ǎ\in K-D \\ M'(ǎ)\cap H(ǎ), & ǎ\in K\cap D \end{cases}$$

Thus,

$$N(ǎ)=\begin{cases} F(ǎ), & ǎ\in(K-Y)-D=K\cap Y'\cap D' \\ F(ǎ)\cap G(ǎ), & ǎ\in(K\cap Y)-D=K\cap Y\cap D' \\ F'(ǎ)\cap H(ǎ), & ǎ\in(K-Y)\cap D=K\cap Y'\cap D \\ [F'(ǎ)\cup G'(ǎ)]\cap H(ǎ), & ǎ\in(K\cap Y)\cap D=K\cap Y\cap D \end{cases}$$

Now consider the RHS. Let [(F,K) $\tilde{\gamma}$(H,D)]$\tilde{\cap}$[(G, Y) $\tilde{\gamma}$ (H,D)]. (F, K) $\tilde{\gamma}$ (H,D)=(V,K), where for all ǎ∈K,

$$V(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in K-D \\ F'(\breve{a})\cap H(\breve{a}), & \breve{a}\in K\cap D \end{cases}$$

Let $(G,Y)\,\tilde{\gamma}\,(H,D)=(W,Y)$, where for all $\breve{a}\in Y$,

$$W(\breve{a})=\begin{cases} G(\breve{a}), & \breve{a}\in Y-D \\ G'(\breve{a})\cap H(\breve{a}), & \breve{a}\in Y\cap D \end{cases}$$

Assume that $(V,K)\,\tilde{\cap}\,(W,Y)=(T,K)$, where for all $\breve{a}\in K$,

$$T(\breve{a})=\begin{cases} V(\breve{a}), & \breve{a}\in K-Y \\ V(\breve{a})\cap W(\breve{a}), & \breve{a}\in K\cap Y \end{cases}$$

Hence,

$$T(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in(K-D)-Y=K\cap Y'\cap D' \\ F'(\breve{a})\cap H(\breve{a}), & \breve{a}\in(K\cap D)-Y=K\cap Y'\cap D \\ F(\breve{a})\cap G(\breve{a}), & \breve{a}\in(K-D)\cap(Y-D)=K\cap Y\cap D' \\ F(\breve{a})\cap[G'(\breve{a})\cap H(\breve{a})], & \breve{a}\in(K-D)\cap(Y\cap D)=\emptyset \\ [F'(\breve{a})\cap H(\breve{a})]\cap G(\breve{a}), & \breve{a}\in(K\cap D)\cap(Y-D)=\emptyset \\ [F'(\breve{a})\cap H(\breve{a})]\cap[G'(\breve{a})\cap H(\breve{a})], & \breve{a}\in(K\cap D)\cap(Y\cap D)=K\cap Y\cap D \end{cases}$$

Thus,

$$T(\breve{a})=\begin{cases} F(\breve{a}), & \breve{a}\in K\cap Y'\cap D' \\ F'(\breve{a})\cap H(\breve{a}), & \breve{a}\in K\cap Y'\cap D \\ F(\breve{a})\cap G(\breve{a}), & \breve{a}\in K\cap Y\cap D' \\ F'(\breve{a})\cap G'(\breve{a})\cap H(\breve{a}), & \breve{a}\in K\cap Y\cap D \end{cases}$$

Thus, it can be seen that N=T for $K\cap Y\cap D=\emptyset$.

2) $[(F,K)\,\tilde{\cup}\,(G,Y)]\,\tilde{\gamma}\,(H,D)=[(F,K)\,\tilde{\gamma}\,(H,D)]\,\tilde{\cup}\,[(G,Y)\,\tilde{\gamma}\,(H,D)]$

3) $[(F,K)\,\tilde{\backslash}\,(G,Y)]\,\tilde{\gamma}\,(H,D)=[(F,K)\,\tilde{\gamma}\,(H,D)]\,\tilde{\backslash}\,[(G,Y)\,\tilde{\gamma}\,(H,D)]$.

4) $[(F,K)\,\tilde{\Delta}\,(G,Y)]\,\tilde{\gamma}\,(H,D)=[(F,K)\,\tilde{\gamma}\,(H,D)]\,\tilde{\Delta}\,[(G,Y)\,\tilde{\gamma}\,(H,D)]$.

5) $[(F,K)\,\tilde{+}\,(G,Y)]\,\tilde{\gamma}\,(H,D)=[(F,K)\,\tilde{\gamma}\,(H,D)]\,\tilde{+}\,[(G,Y)\,\tilde{\gamma}\,(H,D)]$.

6) $[(F,K)\,\tilde{\gamma}\,(G,Y)]\,\tilde{\gamma}\,(H,D)=[(F,K)\,\tilde{\gamma}\,(H,D)]\,\tilde{\gamma}\,[(G,Y)\,\tilde{\gamma}\,(H,D)]$.

7) $[(F,K)\,\tilde{*}\,(G,Y)]\,\tilde{\gamma}\,(H,D)=[(F,K)\,\tilde{\gamma}\,(H,D)]\,\tilde{*}\,[(G,Y)\,\tilde{\gamma}\,(H,D)]$.

8) $[(F,K)\,\tilde{\theta}\,(G,Y)]\,\tilde{\gamma}\,(H,D)=[(F,K)\,\tilde{\gamma}\,(H,D)]\,\tilde{\theta}\,[(G,Y)\,\tilde{\gamma}\,(H,D)]$.

9) $[(F,K)\,\tilde{\lambda}\,(G,Y)]\,\tilde{\gamma}\,(H,D)=[(F,K)\,\tilde{\gamma}\,(H,D)]\,\tilde{\lambda}\,[(G,Y)\,\tilde{\gamma}\,(H,D)]$.

Corollary 4.3.1. $(S_E(U),\tilde{\cup},\tilde{\gamma})$ and $(S_E(U),\tilde{\cap},\tilde{\gamma})$ are noncommutative and additive idempotent semirings without zero and unity under certain conditions.

Proof: Yavuz [38] showed that $(S_E(U),\tilde{\cup})$ are idempotent, noncommutative semigroups (that is a band) under the condition $T\cap Z'\cap M=\emptyset$, where (F,T), (G,Z) and (H,M) are soft sets over U. By Theorem 3.3.1, $(S_E(U),\tilde{\gamma})$ is a noncommutative and not idempotent semigroup under the condition $T\cap Z'\cap M = T\cap Z\cap M=\emptyset$, where (F,T), (G,Z) and (H,M) are soft sets over U. Besides, by Proposition 4.3, $\tilde{\gamma}$ distributes over $\tilde{\cup}$ from LHS under the condition $T\cap(Z\Delta M)=\emptyset$, and $\tilde{\gamma}$ distributes over $\tilde{\cup}$ from RHS under the condition $T\cap Z\cap M=\emptyset$. Thus, $(S_E(U),\tilde{\cup},\tilde{\gamma})$ is a noncommutative and additive idempotent semiring without zero and unity under certain conditions. One can similarly show that $(S_E(U),\tilde{\cap},\tilde{\gamma})$ is a noncommutative and additive idempotent semiring without zero and unity under certain conditions.

Corollary 4.3.2. $(S_E(U),\tilde{\setminus},\tilde{\gamma})$ is a noncommutative, and not idempotent semiring without zero and unity under certain conditions.

Proof: Yavuz [38] showed that $(S_E(U),\tilde{\setminus})$ is a noncommutative semigroup (that is a band) under the condition $T\cap Z'\cap M = T\cap Z\cap M=\emptyset$, where (F,T), (G,Z), and (H,M) are soft sets over U. By Theorem 3.3.1, $(S_E(U),\tilde{\gamma})$ is a noncommutative and not idempotent semigroup under the condition $T\cap Z'\cap M = T\cap Z\cap M=\emptyset$. Besides, by Proposition 4.3, $\tilde{\gamma}$ distributes over $\tilde{\setminus}$ from LHS under the condition $T\cap(Z'\cap M')=\emptyset$, and $\tilde{\gamma}$ distributes over $\tilde{\setminus}$ from RHS under the condition $T\cap Z\cap M=\emptyset$. Thus, $(S_E(U),\tilde{\setminus},\tilde{\gamma})$ is a noncommutative, and not idempotent semiring without zero and unity under certain conditions.

Corollary 4.3.3. $(S_E(U),\tilde{\Delta},\tilde{\gamma})$, $(S_E(U),\tilde{+},\tilde{\gamma})$, $(S_E(U),\tilde{\gamma},\tilde{\gamma})$, $(S_E(U),\tilde{*},\tilde{\gamma})$, $(S_E(U),\tilde{\theta},\tilde{\gamma})$ are all not idempotent and noncommutative (right) nearsemirings without zero and unity under the condition $T\cap Z'\cap M = T\cap Z\cap M=\emptyset$, where (F,T), (G,Z), and (H,M) are soft sets over U. Here, note that Yavuz (2024) showed that the first operation is associative in $S_E(U)$ under the condition $T\cap Z'\cap M=T\cap Z\cap M=\emptyset$ (for $\tilde{\Delta}$, under the condition $T\cap Z'\cap M=\emptyset$).

4 Conclusion

Parametric techniques like soft sets and soft operations are useful when dealing with uncertain data. By introducing new soft operations and figuring out their algebraic features and applications, new approaches to tackling parametric data problems may be discovered. The work presents a unique type of soft set operation in this way. By putting forth a novel soft set operation that we refer to as the "soft binary piecewise plus operation" and thoroughly examining the algebraic structures that underlie it as well as other novel soft set operations in the class of soft sets, we hope to make a substantial contribution to the area of soft set theory.

Specifically, the distributions of the soft binary piecewise gamma operation over different types of soft set operations are analyzed, and the whole algebraic properties of this novel soft set operation are fully examined. A detailed examination of the algebraic structures formed by these operations is obtained in the set of soft sets with taking into account the distribution laws and the algebraic features of the soft set operations. We show that numerous significant algebraic structures, such as semirings and nearsemirings, are formed in the collection of soft sets over the universe with the soft binary piecewise gamma operation and other types of soft set operations.

- $(S_E(U),\widetilde{\gamma})$ is a noncommutative, and not idempotent semigroup under certain conditions, moreover $(S_E(U),\widetilde{\gamma})$ is a right-left system under certain conditions.
- $(S_E(U),\cup_R,\widetilde{+})$, $(S_E(U),\cap_R,\widetilde{+})$ are additive commutative and additive idempotent semirings without zero and unity under certain conditions.
- $(S_E(U),\cup_\varepsilon,\widetilde{\gamma})$ and $(S_E(U),\cap_\varepsilon,\widetilde{\gamma})$ are additive commutative and additive idempotent (right) nearsemirings with zero, but without unity and zero symmetric property under certain conditions.
- $(S_E(U),\backslash_\varepsilon,\widetilde{\gamma})$, $(S_E(U),\Delta_\varepsilon,\widetilde{\gamma})$, $(S_E(U),+_\varepsilon,\widetilde{\gamma})$, $(S_E(U),\gamma_\varepsilon,\widetilde{\gamma})$, $(S_E(U),\lambda_\varepsilon,\widetilde{\gamma})$, $(S_E(U),*_\varepsilon,\widetilde{\gamma})$, $(S_E(U),\theta_\varepsilon,\widetilde{\gamma})$ are additive commutative not idempotent (right) nearsemirings with zero, but without unity and zero symmetric property under certain condition.
- $(S_E(U),\cup_\varepsilon,\widetilde{\gamma})$ and $(S_E(U),\cap_\varepsilon,\widetilde{\gamma})$ are noncommutative and additive idempotent semirings without zero and unity under certain conditions.
- $(S_E(U),\backslash_\varepsilon,\widetilde{\gamma})$ is a noncommutative, and not idempotent semiring without zero and unity under certain conditions.
- $(S_E(U),\widetilde{\cup},\widetilde{\gamma})$ and $(S_E(U),\widetilde{\cap},\widetilde{\gamma})$ are noncommutative and additive idempotent semirings without zero and unity under certain conditions.
- $(S_E(U),\widetilde{\backslash},\widetilde{\gamma})$ is a noncommutative, and not idempotent semirings without zero and unity under certain conditions.
- $(S_E(U),\widetilde{\Delta},\widetilde{\gamma})$, $(S_E(U),\widetilde{+},\widetilde{\gamma})$, $(S_E(U),\widetilde{\gamma},\widetilde{\gamma})$, $(S_E(U),\widetilde{*},\widetilde{\gamma})$, $(S_E(U),\widetilde{\theta},\widetilde{\gamma})$ are all not idempotent and noncommutative (right) nearsemirings without zero and unity under the conditions.

We completely understand their use by looking at new soft set operations and the algebraic structures of soft sets. In addition to offering new examples of algebraic structures, this might further the fields of soft set theory and classical algebraic literature. The goal of this study is to obtain the particular algebraic structures produced in the set of soft set over a universal set by combining different kinds of soft set operations with the soft binary piecewise gamma operation. This kind of thorough investigation should improve our understanding of the use of soft sets. Further research may look at further variations of soft binary piecewise operations and the distributions and characteristics that go along with them.

5 References

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