Turk. J. Math. Comput. Sci.
16(2)(2024) 481–489
© MatDer
DOI : 10.47000/tjmcs.1555972

Spread of Crime Dynamics: A Mathematical Approach

Michael Aguadze[1], Ana Vivas[2], Sujan Pant[2], Kubilay Dagtoros[2,*]

[1]*Department of Electrical Engineering, Norfolk State University, Norfolk, VA, 23504, United States.*
[2]*Department of Mathematics, Norfolk State University, Norfolk, VA, 23504, United States.*

Received: 25-09-2024 • Accepted: 11-12-2024

Abstract. In this work, the spread of crime dynamics in the US is analyzed from a mathematical perspective. An epidemiological model is established, including five compartments: Susceptible (S), Latent 1 (E_1), Latent 2 (E_2), Incarcerated (I), and Recovered (R). A system of differential equations is used to model the spread of crime. A result demonstrating the positivity of the solutions for the system is included. The basic reproduction number and the stability of the disease-free equilibrium are calculated following epidemiological theories. Numerical simulations are performed with US-specific parameter values. Understanding the dynamics of the spread of crime helps to determine what factors may work best to reduce violent crime effectively.

2020 AMS Classification: 37N25, 91D10, 92D25

Keywords: Spread of crime, mathematical modeling, crime dynamics, stability analysis.

1. Introduction

The United States of America has one of the highest per capita incarceration rates in the world [14]. In fact, every single US state has a higher incarceration rate than most nations on Earth. The number of incarcerated individuals and all the entities involved in this process cost billions of dollars to US taxpayers. Criminal activity has been considered a contagious phenomenon by several authors [14, 22]. In this work, an epidemiological model is presented under the assumption that an individual can begin criminal activity induced by the influence of their peers or by an intrinsic desire to commit criminal activity without being induced by others.

Criminality is a social phenomenon that can be spread within social communities that share a common demographic identity that includes race, ethnicity, economic opportunity, education, and political socialization. Many authors have studied this phenomenon from different perspectives [5, 9, 13, 15, 23]. Further, relevant literature indicates that criminality and recidivism can be largely attributed to structural social disparities embedded in the legal, political, and economic institutions [2, 10]. Additionally, some studies indicate that criminal tendencies are more prominent when an individual has experienced childhood trauma. A study conducted on incarcerated women illustrates this relation in [20]. Previous work has used compartmental models to study the dynamics of crime [16, 22, 24]. This work aims to provide an understanding of this social science phenomenon through a mathematical lens. A compartmentalized modeling method is used to understand the dynamics of at-risk populations [4, 6, 17–19].

The model assumes that the total population N is divided into five compartments, which include: S (Susceptible individuals with no criminal behavior), E_1 (Latent 1 individuals with criminal behavior, who have never entered the legal system and have never been incarcerated), E_2 (Latent 2 repeat offenders, who were incarcerated, released, and

*Corresponding Author
Email addresses: maguadze@nsu.edu (M. Aguadze), alvivas@nsu.edu (A. Vivas), spant@nsu.edu (S. Pant), kdagtoros@nsu.edu (K. Dagtoros)

committed a crime again), I (Incarcerated individuals), and R (Released individuals). In the analysis, we evaluate the basic reproduction number [8, 11, 12], and calculate the disease-free equilibrium, the endemic equilibrium, and stability. Additionally, simulations are included using data for the US. Simulations are performed with data from the at-risk population, which is primarily made up of racial and ethnic minorities experiencing socio-economic disparities. The results reinforce the findings that crime and incarceration rates are associated with structural inequalities stemming from racial prejudice toward minority communities. Hence, this analysis has policy implications for understanding the spread and dynamics of violent crime.

This paper is organized as follows. The second section provides a description of the compartments and parameters used to model the dynamics of crime spread. For modeling purposes, assumptions related to the compartments are included, similar to those in [4]. We provide a brief description of certain human behaviors exhibited by individuals who have been incarcerated, as well as the reasons why two-thirds of individuals released from jail or prison reoffend [14]. The third section presents the mathematical analysis of the model, including results on the positivity of the solutions and the stability of the disease-free equilibrium. The basic reproduction number (the rate of secondary infection after exposure to a criminal individual) is evaluated using the next-generation matrix method [12]. Numerical simulations for the reproduction number are also included in this section. The fourth section presents numerical simulations for various scenarios, with variations in the most relevant parameters of the model. Identifying the parameters that can help increase the recovery flow of first-time offenders back to law-abiding citizens is a key focus of this work.

2. Modeling the Spread of Crime

The spread of crime dynamics is considered a contagious disease. Due to social interactions in the community, several factors can influence an individual to commit a crime. Assume that the total population is divided into five groups: Susceptible (S) (not infected, potentially at risk), Exposed 1 (E_1) (has the disease for the first time, in this case, committing a crime, but neither caught nor found guilty), Infected (I) (in this case, incarcerated, as they are showing as being infected, whereas others exposed may be committing crimes but did not enter the judicial system, based on crimes reported), Recovered individuals (R) (in this case released from incarceration), and Exposed 2 (E_2) (has gone back to committing crime). Exposed 2 could flow into infected (incarcerated). Along with the groups mentioned above, the model allows for population flow from Susceptible (or law-abiding) to Exposed 1 (committing crimes). Some individuals can commit a crime without having contact with other criminally active individuals. Also, someone who has recovered may transition to law-abiding and avoiding the risk of Exposed 2 (Recidivism). Recidivism refers to the behavior as a result in the rearrest or re-conviction or a return to prison within 3 years after release [3]. We observe that reducing the contact rate between criminal-active individuals and non-criminal-active individuals will reduce overall crime.

The flow chart in Figure 1 represents the dynamics of the transition of individuals among compartments. A detailed description of the parameters is provided in Table 1.

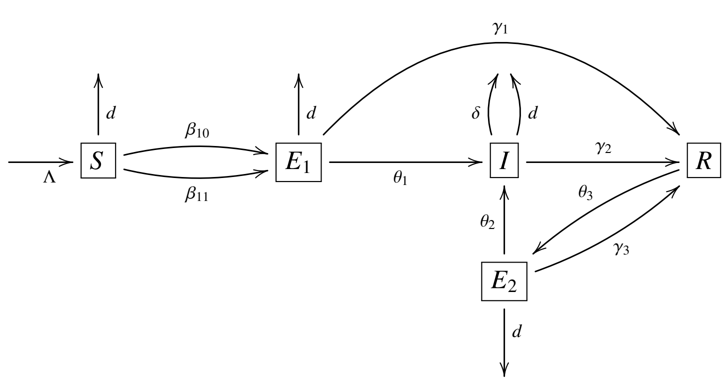

Figure 1. Flow diagram among compartments

Table 1. The description of parameters

Parameters	Description	value (default)
Λ	Population birth rate	0.012 [21]
β_{10}	Effective rate into criminal activity without having contact with other criminally active people	0.18 [7]
β_{11}	Effective contact rate into criminal activity as a result of having contact with other criminally active people	0.24 [14]
θ_1	Incarceration rate (police, courts, correctional systems)	0.0035 [21]
θ_2	Re-Incarceration rate (police, courts, correctional systems)	0.0044 [3]
θ_3	Recidivist rate (repeated offenders)	0.6666 [3]
γ_1	Recovery rate from criminal activity not related to the experience of incarceration	0.9933 [3]
γ_2	Recovery rate from criminal activity related to the experience of incarceration	0.794 [3]
γ_3	Recovery rate for recurrent criminals related with the experience incarceration	0.3334 [3]
δ	Mortality rate associated with incarceration	0.0033 [7]
d	Population death rate	0.0132 [1]

The dynamics for the spread of crime can be modeled by the following system of ordinary differential equations:

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - \beta_{11}\frac{S(E_1+E_2)}{N} - (\beta_{10}+d)S\\
\frac{dE_1}{dt} &= \beta_{11}\frac{S(E_1+E_2)}{N} + \beta_{10}S - (d+\theta_1+\gamma_1)E_1\\
\frac{dE_2}{dt} &= \theta_3 R - (\theta_2+\gamma_3+d)E_2\\
\frac{dI}{dt} &= \theta_1 E_1 + \theta_2 E_2 - (\gamma_2+\delta+d)I\\
\frac{dR}{dt} &= \gamma_1 E_1 + \gamma_2 I + \gamma_3 E_2 - (\theta_3+d)R.
\end{aligned} \tag{2.1}$$

In this setting, the total population is $N = S + E_1 + E_2 + I + R$.

3. Mathematical Analysis of SE_1IE_2R System

This section includes a result demonstrating both the positivity and long-term behavior (boundedness) of the solutions to System (2.1).

Theorem 3.1. *If each compartment is non-negative at $t = 0$, then each compartment remains non-negative for all $t > 0$. Moreover,*

$$\lim_{t\to\infty} N(t) \leq \frac{\Lambda}{d}.$$

Proof. Assume that T is the maximum time for the epidemic. That is,

$$T = \sup\{S > 0, E_1 \geq 0, E_2 \geq 0, I \geq 0, R \geq 0\} \in [0, t].$$

Consider the System (2.1) for $T > 0$. From E_1-equation,

$$\frac{dE_1}{dt} \geq -(\theta_1+\gamma_1+d)E_1,$$

which implies $E_1(T) \geq E_1(0)\exp\{-(\theta_1+\gamma_1+d)t\}$. Hence, $E_1(T) \geq 0$ for all $T > 0$. Similarly, from E_2-equation,

$$\frac{dE_2}{dt} \geq -(\theta_2+\gamma_3+d)E_2$$

which implies $E_2(T) \geq E_2(0)\exp\{-(\theta_2+\gamma_3+d)t\}$. Hence, $E_2(T) \geq 0$ for all $T > 0$. Additionally, from S-equation,

$$\frac{dS}{dt} \geq -\left\{\beta_{11}\frac{S(E_1+E_2)}{N} + \beta_{10}\right\}S$$

which implies

$$S(T) \geq S(0)\exp\left\{\int_0^t -\left\{\beta_{11}\frac{S(E_1+E_2)}{N}+\beta_{10}\right\}ds\right\}.$$

Hence, $S(T) \geq 0$ for all $T > 0$. The positivity of the remaining compartments can be demonstrated similarly.

Now, to analyze the long-term behavior of the total population N, summing the left-hand sides of System (2.1) yields

$$\frac{dN}{dt} = \Lambda - \delta I - dN.$$

This implies

$$\frac{dN}{dt} \leq \Lambda - dN,$$

or equivalently,

$$\frac{dN}{dt} + dN \leq \Lambda.$$

Then, it follows that

$$N(t) \leq \frac{\Lambda}{d} + \left(N_0 - \frac{\Lambda}{d}\right)\exp(-dt).$$

Since $N_0 - \frac{\Lambda}{d}$ is constant and $d > 0$, we have

$$\lim_{t\to\infty} N(t) \leq \frac{\Lambda}{d},$$

as desired.

The feasible region D for System (2.1) is therefore

$$D = \left\{(S, E_1, E_2, I, R) \in \mathbb{R}_+^5 : N \leq \frac{\Lambda}{d}\right\}.$$

□

3.1. **The Basic Reproduction Number.** Several factors are considered when an individual is inclined to commit a crime. Psychological and sociological factors both play an important role in modeling the spread of crime. In this study, the assumption that crime can spread through social interactions leads to the interpretation of the basic reproduction number as the number of secondary infections [8, 11], meaning the number of new criminals generated after effective contact between a susceptible individual and a criminal individual.

To evaluate $\mathscr{R}_0$, the next-generation matrix method [12] is used. System (2.1) is rearranged for simplicity, noting that the newly infectious individuals are those in the compartment E_1. The disease-free equilibrium is denoted by $x_0 = (E_1, E_2, S, I, R) = (0, 0, N, 0, 0)$. Following the notation in [12], the matrices $\mathscr{F}$ and $\mathscr{V}$ are given by

$$\mathscr{F} = \begin{bmatrix} \beta_{11}\frac{S(E_1+E_2)}{N} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \mathscr{V} = \begin{bmatrix} -\beta_{10}S + (\theta_1+\gamma_1+d)E_1 \\ -\theta_3 R + (\theta_2+\gamma_3+d)E_2 \\ -\Lambda + \beta_{11}\frac{S(E_1+E_2)}{N} + (\beta_{10}+d)S \\ -\theta_1 E_1 - \theta_2 E_2 + (\gamma_2+\delta+d)I \\ -\gamma_1 E_1 - \gamma_2 I - \gamma_3 E_2 + (\theta_3+d)R \end{bmatrix}.$$

According to the next-generation matrix method in [12], the matrices F and V, defined as $F = D\mathscr{F}(x_0)$ and $V = D\mathscr{V}(x_0)$, are given by

$$F = \begin{bmatrix} \beta_{11} & \beta_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad V = \begin{bmatrix} \theta_1+\gamma_1+d & 0 & -\beta_{10} & 0 & 0 \\ 0 & \theta_2+\gamma_3+d & 0 & 0 & 0 \\ \beta_{11} & \beta_{11} & \beta_{10}+d & 0 & 0 \\ -\theta_1 & -\theta_2 & 0 & \gamma_2+\delta+d & 0 \\ -\gamma_1 & -\gamma_3 & 0 & -\gamma_2 & \theta_3+d \end{bmatrix}.$$

The basic reproduction number is the spectral radius of FV^{-1} and depends on certain parameters of the model:

$$\mathscr{R}_0 = \frac{\beta_{11}(\beta_{10} + d)}{\beta_{10}\beta_{11} + (\theta_1 + \gamma_1 + d)(\beta_{10} + d)}.$$

The importance of calculating the basic reproduction number lies in the epidemiological principle that if $\mathscr{R}_0 < 1$, the epidemic can be controlled. Otherwise, it may develop into a pandemic. Developing measures to control the parameters that most significantly affect $\mathscr{R}_0$ is the primary goal of this work. The following graph illustrates the variation of $\mathscr{R}_0$ with respect to certain key parameters.

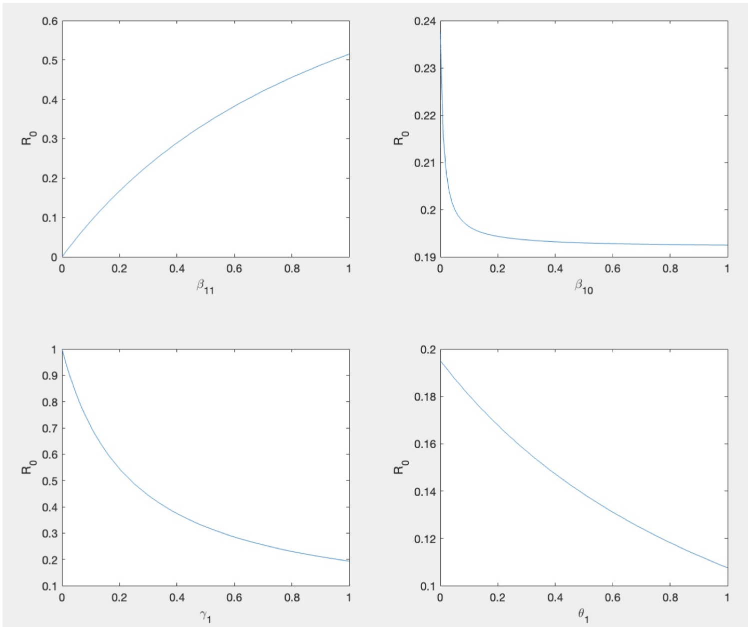

Figure 2. The basic reproduction number $\mathscr{R}_0$ is under control for the parameters: $0 \leq \beta_{11} \leq 1$, transmission rate $0 \leq \beta_{10} \leq 1$, recovery rate $0 \leq \gamma_1 \leq 1$, incarceration rate $0 \leq \theta_1 \leq 1$.

In Figure 2, note that $\mathscr{R}_0$ increases as β_{11} increases, meaning that if susceptible individuals have effective contact with active criminal individuals, the number of secondary criminal individuals increases. Additionally, as the incarceration rate θ_1 increases, $\mathscr{R}_0$ decreases. This is consistent with the fact that criminal individuals in jail cannot have contact with susceptible individuals, and therefore, the number of secondary criminals is reduced.

3.2. **Stability for the Disease-free Equilibrium.** The local stability of the disease-free equilibrium x_0 can be shown by using the Jacobian for the System (2.1) evaluated at x_0. The following result presents the eigenvalues for the Jacobian of System (2.1) and demonstrates that under particular conditions on the parameters, the real parts of the eigenvalues are negative.

Theorem 3.2. *The disease-free equilibrium x_0 for System* (2.1) *is stable.*

Proof. The Jacobian for System (2.1) at x_0, is given by

$$J(x_0) = \begin{bmatrix} \beta_{11} - (d + \theta_1 + \gamma_1) & \beta_{11} & \beta_{10} & 0 & 0 \\ 0 & -(\theta_2 + \gamma_3 + d) & 0 & 0 & 0 \\ -\beta_{11} & -\beta_{11} & -(\beta_{10} + d) & 0 & 0 \\ \theta_1 & \theta_2 & 0 & -(\gamma_2 + \delta + d) & \theta_3 \\ \gamma_1 & \gamma_3 & 0 & \gamma_2 & -(\theta_3 + d) \end{bmatrix}.$$

The real parts of the eigenvalues of $J(x_0)$ are as follows:

$$\lambda_1 = -(\theta_2 + \gamma_3 + d),$$
$$\lambda_2 = -\left\{\frac{d}{2} + \frac{\gamma_2 + \delta + d}{2} + \frac{\theta_3}{2}\right\},$$
$$\lambda_3 = -\left\{\frac{d}{2} + \frac{\gamma_2 + \delta + d}{2} + \frac{\theta_3}{2}\right\},$$
$$\lambda_4 = \frac{\beta_{11} - (\beta_{10} + \theta_1 + \gamma_1 + 2d)}{2},$$
$$\lambda_5 = \frac{\beta_{11} - (\beta_{10} + \theta_1 + \gamma_1 + 2d)}{2}.$$

All the eigenvalues have a real negative part under the condition that $\beta_{11} < \beta_{10} + \theta_1 + \gamma_1 + 2d$, so the disease-free equilibrium is asymptotically stable. This means that the solutions approach x_0 as t tends to infinity. □

The next section will include numerical simulations for System (2.1), using parameters from Table 1. Additionally, some simulations will vary certain parameter values related to the basic reproduction number.

4. Numerical Simulations

Numerical simulations were performed using the most recent parameter values for the United States. As mentioned earlier, Table 1 lists the references from which the parameters were obtained.

Figure 3 shows the dynamics of all the compartments for the model. Notice that as time increases, the populations in the latent E_1 and E_2 compartments do not vanish, meaning that there will always be a number of criminally active individuals. Furthermore, those who serve time in jail or prison and are released will return to being criminally active.

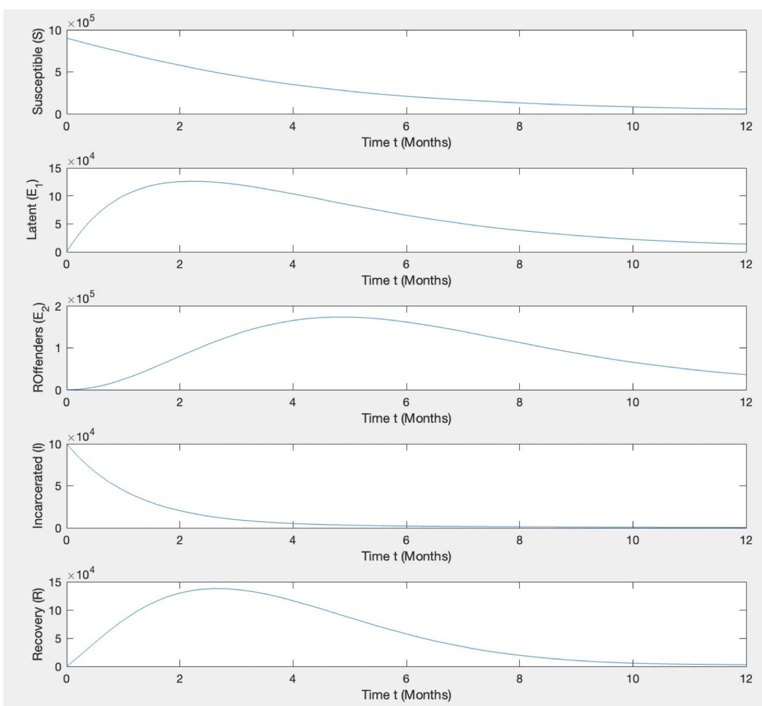

Figure 3. The latent population E_1 has a peak around the second month of the year, while the latent population E_2 peaks around the fourth and a half month of the year. The recovery population R decreases after the second and a half months as a consequence of re-incidence. Parameter values used for the simulations were taken from Table 1.

One of the motivations of this work is to analyze how the incarcerated population can be decreased. Figure 4 shows the behavior of the incarcerated population when varying θ_1 (incarceration rate) and θ_2 (re-incarceration rate). Note that the number of incarcerated individuals increases and reaches a maximum around month six at most levels when varying θ_2. This is in agreement with the fact that, without programs to support released individuals from jail or prison in reintegrating them into society, they will return to committing crimes. When varying θ_1, the maximum values for the incarcerated population occur around month three. It is important to note that if the incarceration rate θ_1 is at a very low level, the incarcerated population decreases during the time interval under consideration.

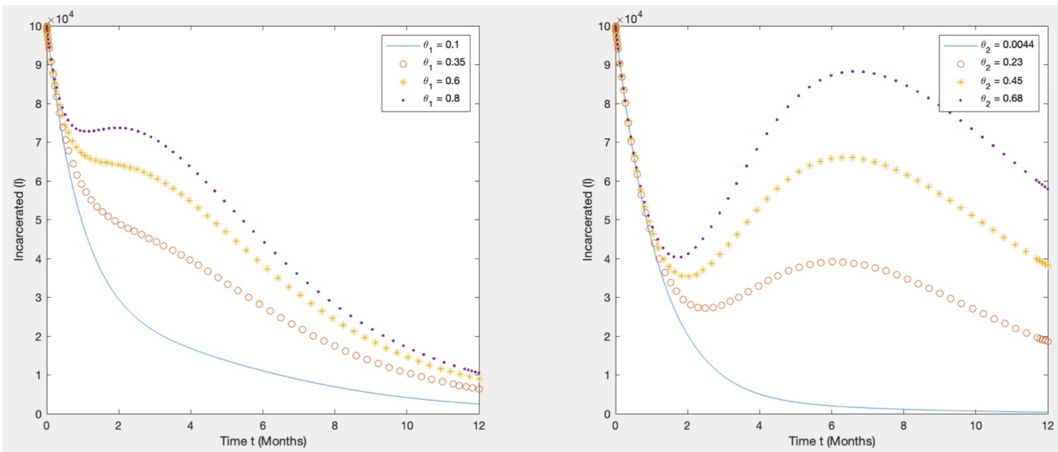

FIGURE 4. Incarcerated individuals I can be reduced and get to a minimum (blue solid graphs) when θ_1 (incarceration rate) and θ_2 (re-incarceration) are at their minimum values. Varying θ_1 = 0.1, 0.35, 0.6, 0.8, and θ_2 = 0.0044, 0.23, 0.45, 0.68, with other parameters fixed.

A topic of interest when analyzing the dynamics of the spread of crime is the influence of criminally active individuals on non-criminal individuals. The most vulnerable population is teenagers within minorities. One important parameter to analyze is the effective contact rate β_{11}. In Figure 5, it can be observed that as β_{11} increases, the number of individuals in the latent compartment E_1 also increases, with maximum values around the second and third months after the contact. Another influencing parameter for the latent population is θ_1. In Figure 5, observe that for high values of θ_1, the population in this compartment decreases significantly.

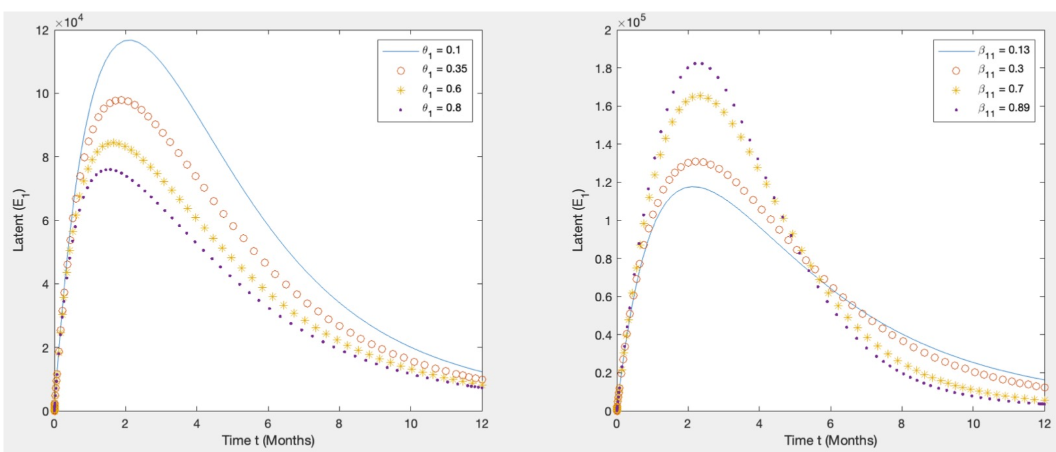

FIGURE 5. Latent 2 individuals E_2 reach their maximum when the incarceration rate θ_1 is minimum θ_1 = 0.1, 0.35, 0.6, 0.8, and reach their minimum when the effective rate contact with criminals β_{11} is maximum β_{11} = 0.13, 0.30, 0.70, 0.89, with other parameters fixed.

Another relevant topic of interest is the E_2 population, individuals who, after being released from jail/prison, re-enter criminal activity and return to jail/prison. Figure 6 shows that the higher the value of θ_2, the fewer individuals remain in this compartment, but most importantly, if the recovery rate γ_3 increases, then the population in this compartment decreases.

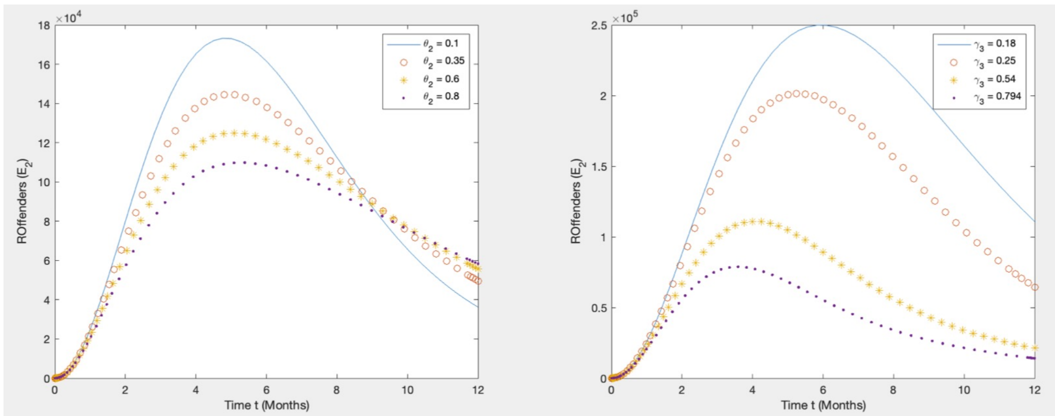

Figure 6. Latent 2 individuals E_2 are larger (blue solid graphs) when θ_1 has the lowest value, and if the recovery rate γ_1 is low, this class remains higher. Plots are obtained when varying $\theta_2 = 0.1, 0.35, 0.6, 0.8$ and $\gamma_3 = 0.18, 0.25, 0.54, 0.794$, while keeping other parameters fixed.

5. Conclusions

The spread of crime is a very complex problem that tremendously affects the United States population. To find feasible solutions to this problem, multiple entities need to be involved. Evidence shows that the most vulnerable sectors of the population have a higher risk of being involved in criminal activities. Since incarceration and recidivism rates are higher for minorities, intervention programs should be offered for those sectors of the population. Providing alternatives such as educational opportunities and extracurricular activities to individuals at high risk of committing criminal activities, along with other measures from local governments, will reduce criminal activity among teenagers. After an individual is released from jail/prison, the local government should implement programs to support the individual and their families, facilitating the reintegration of this individual into society.

The number of secondary infected individuals depends on certain parameters. These parameters can be controlled through specific measures implemented by the entities involved. For example, reducing the contact rate between criminally active individuals and non-criminally active individuals will reduce the number of secondary infected individuals.

Increasing the incarceration rate produces the effect of reducing the number of secondary infected individuals, which in epidemiology is one of the most important goals (to keep this number below one if possible). However, once an individual enters the criminal judicial system, breaking this chain requires significant effort from all the entities involved. It has been proven that recidivism is very high in communities with social and economic disadvantages. The judicial system in the US should re-evaluate the types of crimes that warrant sending a young person to jail, because this is a prime age when an individual can recover from criminal activity with the appropriate support from the government and their families.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

Authors Contribution Statement

The authors have contributed equally. All authors have read and agreed to the published version of the manuscript.

References

[1] Ahmad, F.B., Cisewski, J.A., Xu, J., Anderson, R.N., *Provisional mortality data - United States, 2022*, Morb. Mortal. Wkly. Rep., **72**(2023), 488–492.

[2] Akers, R.L., Sellers, C.S., Criminological Theories: Introduction, Evaluation, and Application, Oxford University Press, 2012.

[3] Alper, M., Durosse, M.R., Markman, J., *Update on prisoner recidivism*, NCJ 250975, (2018).

[4] Banks, H.T., Castillo-Chavez, C., *Bioterrorism: Mathematical modeling applications in homeland security*, Society for Industrial and Applied Mathematics, Philadelphia, PA, 2003.

[5] Becker, G., *Crime and punishment: An economic approach*, J. Politic. Econ. **76**(1968), 169–217.

[6] Brauer, F., Driessche, P.V.D., *Models for transmission of disease with immigration of infectives*, Math. Biosci., **171**(1995), 143–154.

[7] Carson, E.A., *Mortality in state and federal prisons 2001-2019*, U.S. Department of Justice, Office of Justice Programs Bureau of Justice Statistics, (2021).

[8] Castillo-Chavez, C., Feng, Z., Huang, W., *On the computation of R_0 and its role on global stability*, Mathematical Approaches for Emerging and Reemerging Infectious Diseases: An Introduction, Vol. 125 of IMA, Springer, New York, (2002), 229–250.

[9] Clinard, M.B., Meier, R.F., Sociology of Deviant Behavior, Wadsworth, Belmont, 2010.

[10] Crane, J., *The epidemic theory of ghettos and neighborhood effects on dropping out and teenage childbearing*, Amer. J. Sociol., **96**(1991), 1226–1259.

[11] Diekmann, O., Heesterbeek, J.A.P., Metz, J.A.J., *On the definition and the computation of the basic reproduction ratio R_0 in models for infectious diseases in heterogeneous populations*, J. Math. Biol., **28**(4)(1990), 365–382.

[12] Driessche, P.V.D., Watmough, J., *Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission*, Math. Biosci., **180**(1)(2002), 29–48.

[13] Ehrlich, I., *On the usefulness of controlling individuals: An economic analysis of rehabilitation, incapacitation and deterrence*, Amer. Econ. Rev., **71**(3)(1981), 307–322.

[14] FBI Report 2018, *Crime in the Unite States*, https://ucr.fbi.gov/crime-in-the-u.s/2018/crime-in-the-u.s.-2018/topic-pages/nations-two-crime-measures.

[15] Glueck, S., Glueck, E.T., Unravelling Juvenile Delinquency, Harvard University Press, Cambridge, 1950.

[16] Gordon, M.B., *A random walk in the literature on criminality: A partial and critical view on some statistical analyses and modeling approaches*, Eur. J. Appl. Math., **21**(2010), 283–306.

[17] Hethcote, H.W., *The mathematics of infectious diseases*, SIAM Rev., **42**(4)(2000), 599–653.

[18] Kretzschmar, M., Wallinga, J., *Mathematical models in infectious disease epidemiology*, Modern Infectious Disease Epidemiology, Springer New York, NY, (2009), 209–221.

[19] Lawson, D., Marion, G., An Introduction to Mathematical Modeling, Bioinformatics and Statistics Scotland, 2008.

[20] Lehrer, D., *Trauma-informed care: The importance of understanding the incarcerated women*, J. Correct. Health Care, **27**(2)(2021), 121–126.

[21] Martin, J.A., Hamilton, B.E., Osterman, M.J.K., *Births in the United States, 2022*, NCHS Data Brief, **477**(2023).

[22] McMillon, D., Simon, C.P., Morenoff, J., *Modeling the underlying dynamics of the spread of crime*, PLoS One, **9**(4)(2014).

[23] Santonja, F.J., Tarazona, A.C., Villanueva, R.J., *A mathematical model of the pressure of an extreme ideology on a society*, Comput. Math. Appl., **56**(2008), 836–846.

[24] Zhao, H., Feng, Z., Castillo-Chavez, C., *The dynamics of poverty and crime*, J. Shanghai Normal Univ., Nat. Sci. Math., **43**(5)(2014), 486–495.