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On the Philosophical Roots of the Naïve and Axiomatic Set Theories: *Determinatio est Negatio*

Naif ve Aksiyomatik Küme Teorilerinin Felsefi Kökenleri Üzerine: *Determinatio est Negatio*

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ABSTRACT

The principle *determinatio est negatio*—that determination is achieved through negation—has philosophical roots extending back to Plato and Aristotle, and it later influenced early modern thinkers such as Francisco Suárez and Spinoza. This paper has two aims. The first demonstrates how the principle of negation functions as a tool for conceptual determination across various philosophical frameworks, and the second demonstrates that the principle plays a key role in the analysis and resolution of the Burali-Forti paradox within the context of the naïve and axiomatic set theories. In the first section, the analysis focuses on the evolution of the principle from ancient philosophy to early modern metaphysics, examining Plato's dialectics, Aristotle's metaphysical distinctions, Suárez's scholastic theories, and Spinoza's monist metaphysics. The second section shifts to mathematics, where *determinatio est negatio* plays a key role in resolving set-theoretical paradoxes, particularly the Burali-Forti paradox. By exploring Cantor's solution and its reliance on the distinction between consistent and inconsistent sets, this study demonstrates how this principle is essential for avoiding self-referential inconsistencies. The contributions of Zermelo and von Neumann, who developed axiomatic frameworks to address these paradoxes, further elaborate the philosophical foundations of set theory. Ultimately, the study reveals a deep connection between metaphysical principles and the mathematical treatment of paradoxes, emphasizing the ongoing relevance of *determinatio est negatio* in both domains.

Keywords: *Determinatio est negatio*, The Burali-Forti paradox, Cantor, set theory, self-reference

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ÖZ

Determinatio est negatio ilkesi—yani belirlemenin olumsuzlama yoluyla gerçekleştirildiği anlayışı—Platon ve Aristoteles'e kadar uzanan felsefi köklere sahiptir ve daha sonra Francisco Suárez ve Spinoza gibi erken modern düşünürleri etkilemiştir. Söz konusu ilkeyi analiz eden bu çalışmanın iki amacı vardır. İlki, olumsuzlama ilkesinin çeşitli felsefi sistemler boyunca kavramsal belirleme aracı olarak nasıl işlediğini göstermek; ikincisi ise ilkenin, naif ve aksiyomatik kümeler teorisi bağlamında Burali-Forti paradoksunun analizinde ve çözümünde önemli bir rol oynadığını ortaya koymaktır. İlk bölümde, analiz bu ilkenin antik felsefeden erken modern metafiziğe olan evrimine odaklanmakta ve Platon'un diyalektiği, Aristoteles'in metafiziksel ayrımları, Suárez'in skolastik teorileri ve Spinoza'nın monist metafiziği incelenmektedir. İkinci bölümde ise, *determinatio est negatio* ilkesinin özellikle Burali-Forti paradoksunu çözmedeki rolü vurgulanarak matematik alanına geçiş yapılmaktadır. Cantor'un çözümü ve tutarlı ve tutarsız kümeler arasındaki ayrımı *determinatio est negatio* bağlamında incelenerek, bu ilkenin öz-referanssal paradokslardan kaçınmada oynadığı önemli rol gösterilmektedir. Zermelo ve von Neumann'ın Burali-Forti paradoksu için geliştirdikleri aksiyomatik çerçeveler, kümeler teorisinin felsefi temellerini daha da aydınlatmaktadır. Sonuç olarak, çalışma, metafizik ilkeler ile paradoksların matematiksel çözümü arasındaki derin bağlantıyı ortaya koyarak *determinatio est negatio* ilkesinin her iki alandaki devam eden önemini vurgulamaktadır.

Anahtar Kelimeler: *Determinatio est negatio*, Burali-Forti paradoksu, Cantor, kümeler teorisi, öz-referans

Introduction

The principle *determinatio est negatio*—that determination is achieved through negation—has deep roots extending back to classical philosophical traditions. While it may not have been explicitly named as such in ancient philosophy, its influence can be traced through the works of Plato and Aristotle, whose foundational ideas resonate with early modern thinkers like Francisco Suárez and Spinoza. At its core, this principle involves the conceptual distinction of objects by negating certain properties in one object that are affirmed in another, thereby enabling the differentiation of objects. The first section of this paper examines how this shared rationale appears in different philosophical frameworks from different historical periods. By tracing its evolution from ancient to early modern metaphysics, I explore how *determinatio est negatio* shapes conceptual determination. Specifically, I analyze the role of negation in Plato's dialectics, Aristotle's metaphysical distinctions, Suárez's scholastic theories, and Spinoza's monist philosophy. This analysis reveals the enduring significance of negation in shaping our understanding of concepts and existence. The second section transitions from metaphysics to mathematics, exploring how *determinatio est negatio* plays a crucial role in addressing a well-known foundational set-theoretical paradox, i.e., the Burali-Forti paradox. By examining the paradox in detail, this section demonstrates how various solutions to self-referential inconsistencies, including those by Cantor, Zermelo, and von Neumann, draw on this same principle.

To introduce the first section, I begin with Plato's application of negation as a tool for generating conceptually determinate definitions. In *Sophist*, the Eleatic Visitor critiques early philosophers' treatment of non-being, arguing that negation functions not to oppose concepts but to differentiate them. Plato's dialectical approach emphasizes that negation can be used to distinguish mutually exclusive concepts without leading to contradiction. This method of conceptual differentiation, later formalized as logical disjunction, plays a fundamental role in subsequent metaphysical and mathematical thought. Following this analysis, I turn to Aristotle, whose exploration of negation in logic and metaphysics furthers the discussion. In his *Metaphysics*, Aristotle distinguished between contradictions, which are logically unrepresentable, and contraries, which are representable and essential for understanding conceptual determination. Through examples such as the negation of qualities, Aristotle demonstrates how the absence of certain properties clarifies the nature of a being, thus reinforcing negation's role in defining objects.

This section then moves on to Suárez, who bridges Aristotelian thought with early modern philosophy. In his nuanced theory of predication, Suárez distinguishes between intrinsic and extrinsic denominations, showing how negative predicates like indivisibility or blindness establish conceptual distinctions by marking privation or absence. His analysis offers insights into conceptual distinctions achieved through these forms of predication.

The exploration culminates in Spinoza's metaphysics, in which *determinatio est negatio* takes on a central role. Spinoza's assertion that determination is synonymous with negation, particularly in his distinction between substance and modes, highlights the integral role of negation in defining the essence of things. Spinoza's concept of substance as *causa sui* and its paradoxical misinterpretations illustrate how negation not only helps in resolving philosophical paradoxes but also lays the groundwork for understanding self-referential mathematical paradoxes like the Burali-Forti paradox within his metaphysical framework.

The second section of the article delves into the intersection of philosophy and mathematics, particularly in the treatment of set-theoretical paradoxes. Cantor's approach to the Burali-Forti paradox exemplifies how *determinatio est negatio* underpins the resolution of these foundational issues. This section provides both symbolic and natural language explanations of the paradox and Cantor's solution, which is rooted in distinguishing between consistent and inconsistent multiplicities. This distinction, influenced by Spinoza's metaphysical ideas, offers a framework for understanding and mitigating self-referential paradoxes.

In addressing the Burali-Forti paradox—where contradictions arise from the assumption of a set of all ordinals—Cantor's solution relies on redefining sets to avoid such inconsistencies. His approach, grounded in *determinatio est negatio*, distinguishes well-defined sets from those that are inherently contradictory. This principle not only resolves specific paradoxes but also paves the way for the development of axiomatic set theory. The final section of the section examines the evolution of set theory in response to these challenges. The contributions of Zermelo and von Neumann, who formalized axiomatic solutions to avoid self-referential paradoxes, reflect the enduring influence of *determinatio est negatio*. Zermelo's axiom of separation and von Neumann's hierarchical approach illustrate how logical and ontological insights derived from this principle were carried forward into modern set theory.

In conclusion, this study demonstrates that *determinatio est negatio* lies at the heart of both the philosophical and mathematical solutions to the Burali-Forti paradox, revealing the deep connections between metaphysical principles and the foundations of set theory.

1. Genesis and Rationale of the Principle *Determinatio Est Negatio*

Although not explicitly formulated as *determinatio est negatio*, the rationale behind the principle can be traced back to the classical logic of the philosophical traditions of ancient philosophers such as Plato and Aristotle.[1] Plato discusses the significance of conceptual delineation to determine the content of concepts in his *Sophist*, where the rationale of the principle plays a crucial role in the dialectic concerning the concepts of non-being and being without falling into contradiction.[2] The debate begins with the Eleatic visitor's claim that earlier philosophers have avoided or misunderstood the nature of non-being or nothing, which is closely related to being (or something) rather than being its opposite.[3] The visitor argues that when "we say something is not, it seems, we're not saying that it is the opposite of what is, we're just saying it is different".[4]

The central point made by the visitor is that the function of negation is to distinguish between extensionally exclusive concepts. As can be noticed, the basic idea is later developed as a case of disjunction, where concepts without shared objects are differentiated. The visitor underscores the point when says that when a "negation is uttered we will not concede that it signals an opposite, but only this much, that 'not' and 'not-' when prefixed to the names that follow them point to something other than those names—or rather other than the things to which the names following the negation relate".[5] When these considerations are applied to the debate on nothing and being, negation of being does not refer to absolute nothing but to something that differs from a particular being or a mode, or a type of being.[6] Plato presents an early exploration of how negation can imply some kind of positive determination by distinction. Distinction can be made through predicates affirmatively, or negatively by the negation of predicates. Mesquita, providing examples of both affirmative and negative sentences, highlights the role of negation in determination as follows:

> Thus, to say, for instance, "the table is brown" or "the table is not green" is always to say "that which it is", for it always involves speaking, either affirmatively or negatively, of something that is. In light of this, we can say that the outcome of the Sophist is the discovery of negation as a fundamental determination of being itself.[7]

Aristotle also explored the function of negation as determining and distinguishing the extension of concepts in *Metaphysics*, interpreting the role of negation not merely a form of non-being but an absence of a quality that is integral to defining what a thing is or could be. Aristotle focused on the principle of *determinatio est negatio* in terms of the negation of predicates whereby the qualities of a being are determined by distinction through negation. Making a clear

[1] It is possible to list numerous philosophers from various traditions and eras; e.g., see Plotinus, *The Enneads*, eds. Lloyd P. Gerson & et al. (Cambridge: Cambridge University Press, 2018), specifically the enneads V. 3. 14, VI. 7. 17-18 and VI. 9. 3-4; and also see Proclus, *Elements of Theology*, trans. E. R. Dodds (Oxford: Oxford University Press, 1971), specifically the propositions 23, 25, 89, 91 and 92. Yet, I think Plato, Aristotle and Suárez should suffice to illustrate the principle and its function within different conceptual systems as well as providing some foreknowledge about its significance in Spinoza's system. More on the function of negation and the relations between Platon, the medieval philosophers and Spinoza, see Friedrich Heinrich Heinemann, "The Meaning of Negation," *Proceedings of the Aristotelian Society, 44* (1943): 127–152.

[2] Plato, "Sophist", In *Theaetetus and Sophist*, ed. & trans. Christopher Rowe (Cambridge: Cambridge University Press, 2015), 237b-259d. To facilitate the reader's ability to locate the quotes, I reference passages according to their original pagination in all texts when available.

[3] See Plato, "Sophist", 237b-c.

[4] Plato, "Sophist", 257-b1.

[5] Plato, "Sophist", 257b5-c5.

[6] Plato, "Sophist", 190c5-d1. The principle plays a crucial role in Plato's distinction among the terms *apeiron, mikton* and *peras*, which later helps Cantor to design the hierarchy and the ontology of his set theory. Yet, for brevity, I contend myself with referencing the relevant texts: see Plato, *Philebus*, trans. C. B. Gosling (London: Oxford University Press, 1975); Georg Cantor, *Foundations of a General Theory of Manifolds (Grundlagen einer Allgemeinen Mannigfaltigkeitslehre)*, trans. Uwe Parpart (New York: Campaigner Publications, 1976), 93; Karl Hauser, "Cantor's Concept of Set in the Light of Plato's Philebus," *The Review of Metaphysics 63* (4) (2005), 783-805.

[7] António Pedro Mesquita, "Plato's Eleaticism in the Sophist: The Doctrine of Non-Being", in *Plato's Sophist Revisited*, Eds. Beatriz Bossi & Thomas M. Robinson (Berlin: De Gruyter, 2013), 179.

distinction between contradictions and contraries,[8] Aristotle argued that negating the quality of a being "involves the addition of the differentia to the single thing and not just the negating factor, since the negation of a thing in this way marks an absence [of a quality]".[9] To elaborate, while a contradiction has the form $F(x) \wedge \neg F(x)$, contrariness is in the form $F(x) \vee \neg F(x)$. While the former is conceptually unrepresentable, the latter is representable and underscores the logical possibility of distinction by negation of a quality, thus determining $\neg F(x)$ in contrast to $F(a)$.

Aristotle provides two clear examples of this. The first is that if "to be a man just is either to be a non-man or not to be a man, then it will be something else other than what it is to be a man".[10] By negating the quality or property of being a man, one does not deny the property by something possess, which eventually determines it as a non-man thing by the principle *determinatio est negatio*. The second example involves a situation where "someone is asked whether something is white and replies that it is not, he has not denied anything other than its so being, and this not so being is a negation",[11] which eventually determines and distinguishes the thing in question by non-whiteness and in accordance with the principle. Aristotle scholars support this interpretation, contending that his arguments "support the notions that: any character and its denial divide all beings into two discrete classes, instances of only one of these classes can be predicated essentially of any individual subject".[12]

Suárez was a prominent scholastic philosopher who bridged the gap between Aristotle and early modern philosophers. Despite belonging to different philosophical traditions, the rationale behind the principle *determinatio est negatio* underpins many metaphysical debates in Suárez's and Spinoza's works. By Suárez's time, the scholastics had developed a nuanced theory of predication that distinguished between extrinsic and intrinsic denominations. Intrinsic denominations refer to cases in which an object is predicated based on its intrinsic properties, such as predicating a human being with the predicate 'thinking animal'. In contrast, extrinsic denominations are transcendental and phenomenological in essence, i.e., they pertain not to the essence of the object but to how it appears to us or how we predicate it based on properties that are not essential but accidental. In other words, intrinsic denominations refer to properties inherent to the object, such as mobility in animals, and extrinsic denominations refer to properties attributed to an object based on its relation to something external, e.g., calling a person a stranger in relation to the native inhabitants of a place.

Suárez, dividing extrinsic denominations into two types, defines one type as that which "includes those things which consist in a negation or privation, for in this way we say that a thing_{r} is indivisible, that a moral act is evil, that a human being is blind, and the like".[13] Note that Suárez uses negative predicates as examples, i.e., indivisible, evil, and blind. Indivisibility is the negation of divisibility; evil is the negation of goodness and blindness is the negation of sight. All these negative predicates create contrariety in the Aristotelian sense and determine the other or the opposites based on privation or lack. Without negation, as one might expect, we encounter oneness, or unitas ultima, which, according to Suárez "$\text{signifies}_{\text{d}}$ a negation of division in a being".[14] In such a case, there is no determination of the other through negation because there is no negation. In other words, without negation, it is impossible to determine the others and create conceptual distinctions.

Regarding Spinoza's framework, let me begin with the passage in which he explicitly states the principle. Spinoza argued that

> With regard to the statement that figure is a negation and not anything positive, it is obvious that matter in its totality, considered without limitation, can have no figure, and that figure applies only to finite and determinate bodies. For he who says that he apprehends a figure, thereby means to indicate simply this, that he apprehends a determinate thing and the manner of its determination. This determination therefore does not pertain to the thing in regard to its being; on the contrary, it is its non-being. So since figure is nothing but determination, and determination is negation, figure can be nothing other than negation, as has been said.[15] (italics mine).

As is evident from the passage, almost all properties such as 'having a figure' carry geometric connotations and function as limiting, therefore negative properties. In the case of an object with a figure, there is geometric determination that distinguishes it from *the other*—a point Spinoza makes when he asserts that determination does not pertain to the thing in relation to its being but rather to its non-being. When considered in its totality, matter without limitation—hence without negation—is indeterminate with respect to the property of having a figure. In other words, as in the frameworks of Aristotle and Suárez, determination is achieved through negation. Consequently, indeterminate objects such as

8 See Aristotle, *The Metaphysics*, trans. Hugh Lawson-Tancred (London: Penguin Books, 1998), 1055b.
9 Aristotle, *The Metaphysics*, 1004a.
10 Aristotle, *The Metaphysics*, 1007a.
11 Aristotle, *The Metaphysics*, 1012a.
12 Edward C. Halper, *One and Many in Aristotle's Metaphysics* (Las Vegas: Parmenides Publishing, 2009), xx-xxi.
13 Francisco Suárez, *Metaphysical Disputations III & IV*, trans. Shane Duarte (Washington: The Catholic University of America Press, 2023), 17.
14 Suárez, *Metaphysical Disputations III & IV*, 101.
15 Baruch Spinoza, "Letters", in *Spinoza Complete Works*, Ed. Michael L. Morgan, trans. Samuel Shirley (Indianapolis: Hackett, 2002b), 892.

substance or the totality of matter—i.e., the attribute of extension considered as the sum of all extended objects—cannot possess a figure because they lack determination through negation.

When analyzed in the context of Spinoza's metaphysics, the principle *determinatio est negatio* is integral to (1) his distinction between modes and substance, as well as in (2) his conception of substance as *causa sui*. Beginning with the first point, as is known, Spinoza's monist metaphysics posits that there is only one substance, which he identifies as God or Nature (famously *Deus sive Natura*), and everything else is a mode or expression of this single infinite substance. Spinoza's proposition that "*Two or more distinct things are distinguished from one another either by the difference of the attributes of the substances or by the difference of the affections of the substances*",[16] makes clear that he differentiates things either by their essential properties and intrinsic denominations—i.e., by the difference of the attributes of the substances—or by the accidental properties and extrinsic denominations, that is, by the difference in the affections of the substances. While the former distinction has an essentialist character, the latter takes on a transcendental and phenomenological character.

In this framework, any finite thing is not an independent entity but a determined modification of the infinite substance. Spinoza's proposition that "Absolutely infinite substance is indivisible",[17] makes it obvious that, for him, to define something is identical to conceptually limiting and determining it by negation. A finite thing can only be understood in terms of what it is not, in contrast to the infinite substance. Determination, therefore, entails negation, because to define something or give it boundaries is to distinguish it from everything else—to negate all other possibilities. Arguing that there is no vacuum in Nature, Spinoza concludes that "the parts cannot be distinct in reality; that is, corporeal substance, insofar as it is substance, cannot be divided".[18] Similar to Suárez's conception of *one*, which is implied by the negation of division in being, Spinoza negates division concerning the essence of substance. This renders concepts like *modes* in his system transcendental and phenomenological. Because substance is indivisible in essence, we conceptually rely on negation and extrinsic denomination to determine and distinguish modes from substance.

It is crucial to note that, similar to Aristotle and Suárez, the distinction between modes and substance is not based on the notion of contradiction but on the notion of contrariety. Spinoza distinguished modes from substance with reference to the concept of *cause*. He asserts that "Substance cannot be produced by anything else (Cor. Pr. 6) and is therefore self-caused [*causa sui*]; that is (Def. 1), its essence necessarily involves existence; that is, existence belongs to its nature".[19] There are two basic themes in his proposition. The first is Spinoza's distinction between causa sui and causatum ab alio, that is, between substance as own cause and the things caused by external causes. *Causa sui* denotes an intrinsic denomination, whereas *causatum ab alio* denotes an extrinsic denomination. This leads to the second theme: modes do not exist as objects distinct from the substance. Their distinction is purely conceptual and is grounded in the negation of the intrinsic property of *causa sui*. This is why Spinoza contends that "we can have true ideas of them [nonexistent modifications] since their essence is included in something else, with the result that they can be conceived through that something else, although they do not exist in actuality externally to the intellect".[20] In other words, distinguishing between modes and substance through the negation of the conception of *causa sui* leads to the concept of *causatum ab alio*, which does not correspond to any being that exists outside the intellect. It is a conceptual determination and distinction based on negation.

In light of these considerations, when one analyzes (2), namely Spinoza's conception of substance as *causa sui*, it becomes clear that his conception is not rooted in contradiction. However, in a cursory reading, the term *causa sui* may appear paradoxical in two ways. First, it may seem to imply that for something to be its own cause, it must come into existence at a particular time, say, at T. Yet, for the very same thing to be brought into existence at T, it would also have to be non-existent at that time, which seems to suggest the simultaneous existence and non-existence of the object in question at T. However, Spinoza defined *causa sui* as an intrinsic property that negates the notion of an external cause is negated. He states that "By that which is self-caused [*causa sui*] I mean that whose essence involves existence; or that whose nature can be conceived only as existing".[21] Second, the term *causa sui* may sound as though substance is a cause and it is its own cause. This interpretation leads to the following contradiction: where the substance is both the caused and the cause. Before it comes into existence as a caused being, it must exist as a cause, even if it does not yet exist as a caused being. Friedman highlights the inference gap in the misinterpretation of the term *causa sui*, noting that the conclusion that substance cannot be caused by anything other than itself does not imply that it is caused by itself

16 Baruch Spinoza, "Ethics", in *Spinoza Complete Works*, Ed. Michael L. Morgan, trans. Samuel Shirley (Indianapolis: Hackett, 2002a), 218.
17 Spinoza, "Ethics", 224.
18 Spinoza, "Ethics", 226.
19 Spinoza, "Ethics", 219.
20 Spinoza, "Ethics", 220.
21 Spinoza, "Ethics", 217. Also see Spinoza, "Letters", 766.

in a mechanistic sense. While it is not contradictory to claim that substance has no external cause and that existence is its essential property, it is contradictory to infer that substance is its own mechanistic cause from the fact that it has no external cause.[22]

Spinoza's definition of *causa sui* is a negative definition that draws not only on his notion of causality in relation to substance and modes but also on his distinction between finite and infinite beings, where finite beings correspond to modes and the infinite being corresponds to substance. Spinoza elaborates on this distinction by being finite and being infinite in his well-known *Letter on Infinity*, where he identifies three common failures in distinguishing between the finite and infinite. As two of these failures are relevant to the aims of the study, let me quote Spinoza, omitting the third:

> The question of the infinite has universally been found to be very difficult, indeed, insoluble, through failure to distinguish between that which must be infinite by its very nature or by virtue of its definition, and that which is unlimited not by virtue of its essence but by virtue of its cause. Then again, there is the failure to distinguish between that which is called infinite because it is unlimited, and that whose parts cannot be equated with or explicated by any number, although we may know its maximum or minimum.[23]

To elaborate on these failures, Spinoza provides definitions for *mode, substance, and eternity*, all of which are grounded in the principle *determinatio est negatio*. Spinoza argues that

> The affections of Substance I call Modes. The definition of Modes, insofar as it is not itself a definition of Substance, cannot involve existence. Therefore, even when they exist, we can conceive them as not existing. From this it further follows that when we have regard only to the essence of Modes and not to the order of Nature as a whole, we cannot deduce from their present existence that they will or will not exist in the future or that they did or did not exist in the past. Hence it is clear that we conceive the existence of Substance as of an entirely different kind from the existence of Modes. This is the source of the difference between Eternity and Duration. It is to the existence of Modes alone that we can apply the term Duration; the corresponding term for the existence of Substance is Eternity, that is, the infinite enjoyment of existence or—pardon the Latin—of being (*essendi*).[24]

When scrutinized, Spinoza's definition of mode distinguishes modes from substance based on the idea that while modes exist contingently, substance exists necessarily. In other words, necessary existence is an intrinsic property of substance, whereas this mode of existence is negated in the essence of modes, rendering them to contingent beings. A key point to approach with caution in Spinoza's distinction is that the definition of mode is based on extrinsic denominations, and their relationship with substance is not a part-whole relation. Those who fail to distinguish the nature of existence of modes and the nature of the existence of substance may mistakenly interpret this relationship as one of part and whole. Attributing such a relation to modes and substance occurs when one conflates the transcendental and phenomenological (the extrinsic essence of modes' existence) with the essential and intrinsic nature of substance's existence. In this context, substance should never be understood as composed of parts. Spinoza strongly criticizes this misinterpretation, calling it "bordering on madness".[25] He provides a compelling counterexample: "It is as if, by simply adding circle to circle and piling one on top of another, one were to attempt to construct a square or a triangle or any other figure of a completely different nature".[26]

2. *Determinatio Est Negatio* and The Solutions For The Burali-Forti Paradox

The failure to distinguish between modes and substance based on the nature of their distinct modes of existence plays a significantly pivotal role in Cantor's treatment of paradoxes within his system. Before delving further into this topic, it is useful to briefly highlight Spinoza's philosophical influence on Cantor. As mentioned in the introduction, Cantor was an attentive reader of Spinoza.[27] He studied *Ethics* in detail, documenting his notes in a separate notebook. Bussotti and Tapp inform us that "Cantor began to study the Ethics in detail in 1871. He copied the text of Spinoza's definitions, axioms, and propositions onto the left-hand sides of the pages of a notebook and added his own summaries and commentaries on the right-hand sides".[28] Further evidence for Spinoza's influence on Cantor is found in Cantor's reading of *Spinoza's Letter on Infinity*, which he described as "'most important' (*höchst bedeutend*) and found it 'full of content' (*inhaltsvoll*)".[29] These connections between Cantor and Spinoza suggest that Cantor's treatment of set-theoretical paradoxes, particularly the Burali-Forti paradox, is rooted in the principle *determinatio est negatio*.

[22] See Joel I. Friedman, "Was Spinoza fooled by the ontological argument?," *Philosophia* 11 (3-4) (1982), 315-316.
[23] Spinoza, "Letters", 787.
[24] Spinoza, "Letters", 788.
[25] Spinoza, "Letters", 788.
[26] Spinoza, "Letters", 788.
[27] See Christian Tapp, "On Some Philosophical Aspects of the Background to Georg Cantor's Theory of Sets," *Philosophia Scientiæ* 5 (2005), 157-173, and Lester Stauffer, "Spinoza, Cantor, and infinity," *Southern Philosophical Studies* 15 (1993), 74–81.
[28] Paolo Bussotti; Christian Tapp, "The influence of Spinoza's concept of infinity on Cantor's set theory," *Studies in History and Philosophy of Science Part A. 40* (1) (2009), 25 (fn. 3).
[29] Bussotti & Tapp, "The influence of Spinoza's concept of infinity on Cantor's set theory", 25 (fn. 5).

Cantor applies the principle *determinatio est negatio* in his set theory and distinguishes between what he calls *inconsistent multiplicities* and *consistent multiplicities*. Accordingly,

> A multiplicity can be such that the assumption that all of its elements are 'together' leads to a contradiction, so that it is impossible to understand the multiplicity as a unity, as 'a completed thing'. I call such multiplicity absolutely infinite or inconsistent multiplicity. . . . If, on the other hand, the totality of the elements of a multiplicity can be thought of as being 'together' without contradiction, so that it is possible to combine them into 'one thing', I call them a consistent multiplicity or a 'set'.[30]

The concept of indeterminate totality found in Aristotle, Suárez, and Spinoza reappears in Cantor's theory as the notion of inconsistent multiplicity. Cantor observes, "Most people even confuse the transfinite with the undifferentiated One, the so-called Absolute, i.e., the absolute maximum, which is, of course, not subject to any determination and thus not within the realm of mathematics".[31] Clearly, Cantor recognizes that due to the absence of determination through negation, such objects cannot be considered complete, as there is no limiting property by which they can be defined.[32] Unlike consistent multiplicities, inconsistent multiplicities cannot be understood as complete multiplicities, doing so leads to self-referential paradoxes. Cantor explains that the paradoxical nature of inconsistent multiplicities arises from their lack of determination, highlighting his awareness of the dangers of incorporating such multiplicities into his system:

> For the totality of all alephs is one that cannot be conceived as a determinate, well-defined, completed set. If this were the case, then this totality would be followed in size by a determinate aleph, which would therefore both belong to this totality (as an element) and not belong, which would be a contradiction.[33]

Cantor's analysis of the indeterminate, and thus inconsistent, multiplicity of the totality of alephs reveals that such multiplicities are paradoxical because they simultaneously affirm and negate the property of 'being followed in size by a determinate aleph'. The aleph numbers are arranged in a hierarchy of size, with each aleph succeeded by a larger one—of course assuming the continuum hypothesis. The aleph that would follow the totality of all alephs would, by definition, be larger than this totality. Consequently, this larger aleph cannot be contained within the totality of alephs, which creates a contradiction. According to the definition of the totality of all alephs, this totality must contain the larger aleph, yet it cannot, which, when combined with the first contradiction, eventually results in a paradox. The paradox exemplifies impredicative definitions in which negation fails to determine the object in question. For comparison, consider the two misinterpretations of Spinoza's conception of *causa sui*. In the first misinterpretation, a substance must be both existent and non-existent at a given time T. In the second misinterpretation, it must exist as a cause only when it does not exist as a caused being.[34] Both cases arise from an inappropriate application of negation, leading to contradictions similar to those found in Cantor's analysis of the inconsistent multiplicity of alephs.

To avoid paradoxes, Cantor employs the principle *determinatio est negatio* and explains that "by removing the predicate 'finite' while retaining the predicate 'definite,' one obtains what I call the actually infinite number, starting with ω, and then extending to ω+1, ω+2, and so on".[35] Note that removing the predicate 'finite' does not necessarily entail the removal of the predicate 'definite'. Cantor distinguishes his transfinite ordinal and cardinal numbers from the Absolute in three key respects, reflecting Spinoza's influence on his thinking as follows:

> First, as it pertains to the transcendent, eternal, omnipotent God, or *Natura Naturans*, where it is referred to as the Absolute; second, as it manifests in concrete reality or *Natura Naturata*, which I term the Transfinite; and third, as it can be considered in the abstract, that is, as it is conceptualized by human cognition in the form of actual infinities, or as I have referred to them, transfinite numbers, or in the even broader context of transfinite order types (e.g., cardinal or ordinal numbers).[36]

In light of the analyses above, I will now examine the Burali-Forti paradox and demonstrate that the solutions proposed by Cantor, Zermelo and von Neumann share the rationale of the principle *determinatio est negatio*. In 1897, Burali-Forti published a paper that provided a thirty-five-step proof of a self-referential paradox in Cantor's set theory.[37]

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30 Georg Cantor, *Briefe*, eds. Herbert Meschkowski & Winfried Nilson (Berlin: Springer Verlag, 1991), 407.

31 Cantor, Briefe, 181.

32 See Anne Newstead, "Cantor on Infinity in Nature, Number, and the Divine Mind," *American Catholic Philosophical Quarterly 83* (4) (2009), 533-553, and Ignacio Jané, "The Role of the Absolute Infinite in Cantor's Conception of Set," *Erkenntnis 42* (3) (1995), 375-402.

33 Cantor, *Briefe*, 388.

34 The same applies to the transfinite ordinals as well. Cantor contends that the first transfinite ordinal number ω "does not refer to the Absolute, i.e., the absolutely greatest (or God), which can only be determined by itself and not by us. Rather, each transfinite number is capable of further increase. The smallest of all transfinite whole numbers is denoted by ω, followed by ω+1, ω+2, and so on. Each of these numbers has a specific meaning as the cardinality of well-ordered sets" (Cantor, *Briefe*, 174). See also Georg Cantor, "Mitteilungen zur Lehre vom Transfiniten", in *Gesammelte Abhandlungen Mathematischen und Philosophischen Inhalts*, ed. E. Zermelo (Berlin: Julius Springer, 1932), 407.

35 Cantor, *Briefe*, 139.

36 Cantor, *Briefe*, 252. For more on Cantor and God and the inconsistent multiplicities, see Alasdair R. Thomas-Bolduc, "Cantor, God, and Inconsistent Multiplicities," *Studies in Logic, Grammar and Rhetoric. 44* (57) (2016), 133-146.

37 See Cesare Arzelà Burali-Forti, "Question on Transfinite Numbers", in *From Frege to Gödel: A Source Book in Mathematical Logic, 1879-1931, ed.* Jean van Heijenoort (Cambridge and Massachusetts: Harvard University Press, 1897), 105-117.

This proof assumes a set U, which is the set of all ordinals, and that U is a well-ordered set. According to Cantor's conception of well-orderedness, a set is well-ordered if it has a smallest element, a largest element, and intermediary element(s).[38]Cantor, *Briefe*, 407. For example, consider a set A = {a, b, c}, where a is the smallest, b is the intermediary, and c is the largest element. The set can be ordered as a < b < c. When we assume the existence of a set of all ordinals U in Cantor's theory and that U is a well-ordered set, the paradox arises. For brevity, rather than detailing Burali-Forti's thirty-five-step proof,[39] This section illustrates the structure of the paradox through a simpler version based on the ordinal structure of the von Neumann construction. The simpler version is as follows:

1. $\forall x\ (x \in \omega \leftrightarrow \mathrm{Ord}(x))$
2. $\forall x\ (\mathrm{Ord}\ (x) \rightarrow (\forall y\ (y \in x \rightarrow \mathrm{Ord}(y))))$.
3. $\forall A\ (\forall z\ (z \in A \rightarrow z \in \omega) \wedge A \neq \varnothing \rightarrow \exists x\ (x \in A \wedge \forall y\ (y \in A \rightarrow (x \in y \vee x = y))))$.
4. $\forall x\ (x \in \omega \rightarrow \mathrm{Ord}(x)) \wedge \forall x,y\ (x \in \omega \wedge y \in \omega \wedge x \in y \rightarrow x \neq y)$
5. $\omega \in \omega$
6. $\forall x\ (x \in \omega \rightarrow x \notin x)$.
7. $\perp$

The first step reads that ω is the set of all ordinals. The second step states that because of the transitivity of ordinals, every element x of ω contains their elements, that is, y, which are also in ω. In other words, every element of the elements of ω is in ω. The third step asserts that ordinal numbers are well-ordered, meaning they can be arranged in a sequence in which each has a distinct position. The fourth step reads that as ω is transitive and well-ordered, it is an ordinal itself and must behave as an ordinal, i.e., it can be ordered in a sequence as well. The fifth step is where the paradox is unfolded, i.e., since ω is an ordinal and the set of all ordinals, it must be smaller than itself in terms of its rank in the order, which is the first problem. The sixth step asserts that no set can be its own element,[40] implying that ω cannot be smaller than itself in terms of its rank in the order. Combining steps 5 and 6 leads to the paradox, where ω is smaller than itself and not smaller than itself in terms of its rank in the order. Burali-Forti's formulation of the paradox concludes that "$\Omega + 1 > \Omega$ and $\Omega + 1 \leq \Omega$".[41,42]

The crux of the Burali-Forti paradox lies in two key assertions: first, there is a set of all ordinals, and second, by definition, this set is an element of itself. To analyze the paradox further, it is useful to recall two elements from Spinoza's metaphysics: (1) the two misinterpretations of Spinoza's conception of *causa sui*, and (2) the absoluteness of substance.

Starting with (1), in the first misinterpretation, substance is interpreted as both existent and non-existent at a given time T because of the self-referential nature of substance being both a cause and a caused being. In the second misinterpretation, substance is said to exist as a cause only when it does not exist as a caused being, which similarly arises from self-reference. The Burali-Forti paradox reflects this self-referential reasoning, where ω is considered both an element of itself and the set of itself. Thus, the structure of the paradox mirrors the structure of the paradoxical misinterpretations of Spinoza's *causa sui*.

Regarding (2), the Burali-Forti paradox assumes ω as the ultimate set of all ordinals, analogous to the absolute substance in Spinoza's metaphysics. The all-comprehensive notion of substance in Spinoza's system is one cause of the misinterpretations, as it leads to a self-referential conclusion. In set theory, assuming such an all-encompassing set results in the paradox that ω is the element of itself and the set of itself, akin to how substance is misinterpreted as being its own cause in the two misinterpretations mentioned above.

In this regard, the main question is whether Cantor made such mistaken assumptions of considering ω to be an all-comprehensive set and that it contained itself as an element. In fact, he did neither of these. He never asserted that ω is the set of all ordinals nor that it is an element of itself. On the contrary, he explicitly defines ω as "not the maximum of the finite numbers (there is no such thing), but ω is the minimum of all infinite ordinal numbers".[43] Cantor was aware of the paradoxical outcomes that could arise from assuming and including an all-comprehensive set in the ontology of

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39 For more on the Burali-Forti paradox, see Irving M. Copi, "The Burali-Forti Paradox,". *Philosophy of Science 25* (4) (1958), 281–286; Barkley Rosser, "The Burali-Forti Paradox," *The Journal of Symbolic Logic 7* (1) (1942), 1–17; Christopher Menzel, "CANTOR AND THE BURALI-FORTI PARADOX," The Monist 67 (1) (1984), 92–107; and Geoffrey Hellman, "On the significance of the Burali-Forti paradox," *Analysis 71* (4) (2011), 631–637.

40 In Zermelo-Fraenkel set theory, this point is an axiom, called *the regularity axiom*, which is $\forall x\ (x \neq \varnothing \rightarrow \exists y\ (y \in x \wedge y \cap x = \varnothing))$, meaning that no element is an element of itself.

41 Burali-Forti, "Question on Transfinite Numbers", 111.

42 In the notation ω, Ω, and U denote to the same number, i.e., the first transfinite ordinal number.

43 Cantor, "Mitteilungen zur Lehre vom Transfiniten", 407.

his theory. Instead of allowing for an indefinite absolute totality, he adhered to the principle of *determinatio est negatio* by determining ω through negating the predicate 'finite' and affirming the predicate 'definite'.[44]

This principle, which Cantor used to address the paradoxes in his set theory, evolved into what is known as *the unrestricted comprehension principle*. This principle prevents the inclusion of excessively large and inconsistent multiplicities in formal systems, thereby avoiding self-referential paradoxes. In response to these concerns, Zermelo and Fraenkel axiomatized this rationale in their axiomatic set theory, encapsulating it in what is called *the axiom of separation*. This axiom restricts the formation of sets to avoid inconsistencies by allowing the creation of subsets from only existing sets based on specific criteria, thus preventing the creation of problematic self-referential sets. To illustrate how the philosophical heritage of Plato, Aristotle, Suárez, Spinoza, and other philosophers has influenced the development of axiomatic set theories, I first discuss Zermelo's solution to prevent self-referential paradoxes and then analyze von Neumann's approach.

Zermelo formulated the axiom of separation as follows: "Axiom III. (Axiom of separation.) Whenever the propositional function **F** *(x)* is definite for all elements of set *M, M* possesses a subset M_F, containing as elements precisely those elements x of M for which **F** *(x)* is true".[45] The rationale of the axiom is akin to Plato's distinction between *apeiron* and *peras*[46] as well as Spinoza's conceptual distinction between the absolute substance and its determinate modes. It also mirrors Cantor's distinction between consistent and inconsistent multiplicities. In axiomatic set theory, absolute totalities are considered to be illegitimate. In contrast, only sets that contain definite elements determined by a function are considered legitimate. A function, in this context, affirms a property for a specific set of elements that negates the same property for the others. This aligns with the principle *determinatio est negatio*, which stipulates that sets are determined by the properties they affirm and those they negate. By this principle, a set is defined by its membership criteria, which involve the negation of properties for elements that are not part of the set.

As previously mentioned, von Neumann developed a solution for self-referential paradoxes that aligns with the principle *determinatio est negatio*. Von Neumann's approach integrates this principle into his system in a manner that parallels Spinoza's ideas, demonstrating a clever adaptation of Spinoza's original strategy.

Just as Spinoza incorporates the absolute being, the all-inclusive substance, into his metaphysics, von Neumann introduces an all-inclusive object into his system, which he designates as V. In von Neumann's framework, there are two kinds of objects: sets and classes. A class is defined as a set if it is a subclass of a higher class. While all classes can be subclasses of some higher class and thus categorized as sets, there is one notable exception: the non-set class of all sets, denoted V.[47] The reason for this is that V is the ultimate or absolute class of all sets. Note that it is not the set of all sets but the non-set class of all sets. Designed in this way, V cannot result in any paradoxes because it is not a set, it cannot be a subclass of itself. This design ensures that V, being the ultimate or absolute class of all sets, avoids paradoxes because it is not a set itself and thus cannot be a subclass of itself. The Burali-Forti paradox arises because it allows ω to be its own element. Von Neumann addressed this issue by asserting, "Rather than being completely prohibited, they are only declared incapable of being arguments (they are not I-objects!). This suffices to avoid the antinomies".[48]

Von Neumann's strategy successfully eliminates the self-referential paradoxes that arise from allowing inconsistent multiplicities or an absolute totality into a system. When examined in relation to Spinoza's system, it becomes clear that the success of von Neumann's approach is due to its shared rationale with Spinoza's ideas. Although there is no direct textual evidence that von Neumann read Spinoza and explicitly adapted his system, the fundamental rationale underlying von Neumann's hierarchy mirrors that of Spinoza's system. Spinoza includes the concept of substance as *causa sui*, but he avoids paradox by ensuring that the term *causa sui* is not interpreted paradoxically. In Spinoza's system, substance is an absolute, all-inclusive entity; however, it does not lead to paradoxes because of its non-self-referential nature. Similarly, von Neumann permitted the non-set class of all sets in his system. This class, V, is defined negatively, similar to Spinoza's *causa sui*. By ensuring that V is not a set itself but a non-set class, von Neumann's approach avoids paradoxes. This agrees with the principle *determinatio est negatio*, where the definition of V excludes it from being a subclass of itself, thereby preventing paradoxical situations. In other words, both systems integrate an

44 See Cantor, *Briefe*, 139.

45 Ernst Zermelo, "Investigations in the foundations of Set Theory", in *From Frege to Gödel: A Source Book in Mathematical Logic, 1879-1931*, ed. I. Jean van Heijenoort (Cambridge and Massachusetts: Harvard University Press, 1908), 202.

46 See Cantor, *Foundations of a General Theory of Manifolds (Grundlagen einer Allgemeinen Mannigfaltigkeitslehre)*, 93, and Plato, *Philebus*.

47 John von Neumann, "An Axiomatization of Set Theory", in *From Frege to Gödel: A Source Book in Mathematical Logic, 1879-1931*, ed. I. Jean van Heijenoort (Cambridge and Massachusetts: Harvard University Press, 1925), 403.

48 Von Neumann, "An Axiomatization of Set Theory", 401.

absolute entity into their frameworks without resulting in paradoxes through a definition that negates self-referential implications.

Conclusion

In conclusion, the principle *determinatio est negatio*—the idea that determination is achieved through negation—plays a fundamental role in both philosophical and mathematical thought. This study has traced its evolution from the metaphysical inquiries of ancient philosophers like Plato and Aristotle, through the scholastic theories of Francisco Suárez, to Spinoza's monist metaphysics, illustrating how negation serves as a crucial tool for conceptual differentiation across various frameworks. In philosophy, this principle has been instrumental in shaping how thinkers across different eras approached the problem of defining and distinguishing concepts. Plato's dialectical use of negation, Aristotle's metaphysical distinctions between contradictions and contraries, and Suárez's nuanced theory of predication all underscore the centrality of negation in conceptual determination.

The second section demonstrated that *determinatio est negatio* is not limited to philosophical discourse; it also plays a pivotal role in the development of set theory, particularly in resolving self-referential paradoxes like the Burali-Forti paradox. Cantor's solution, which distinguishes between consistent and inconsistent sets, is deeply rooted in this principle. Moreover, the contributions of Zermelo and von Neumann to axiomatic set theory further exemplify how philosophical insights into negation have influenced the logical frameworks used to address foundational mathematical problems.

Ultimately, this study shows that the principle of *determinatio est negatio* creates a profound link between metaphysical inquiry and mathematical formalism, highlighting its enduring significance. Whether in the realm of philosophical metaphysics or in the formal structures of modern set theory, the principle continues to shape our understanding of how distinctions are made, paradoxes are resolved, and concepts are defined.

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