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# Domination Scattering Number in Graphs

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**Abstract** — Scattering number measures the stability of a graph by determining how well vertices are spread throughout the graph. However, it may not always be distinctive for different graphs, especially when comparing the same scattering numbers. In this study, we aim to provide a more nuanced and sensitive measure of stability for graphs by introducing domination scattering numbers, a new measure of graph stability. This parameter likely captures additional structural characteristics or dynamics within the graph that contribute to its stability or resilience. Moreover, we investigate the domination scattering numbers of the graphs $P_n$, $C_n$, $K_{1,n}$, $K_{m,n}$, and $P_n \times C_3$.

**Keywords** *Vulnerability, domination number, scattering number*

**Mathematics Subject Classification (2020)** 05C40, 05C69

## 1. Introduction

Network stability depends on nodes (processing) and links (communications or transport). Whenever a link or node is lost, the effectiveness of the network decreases. Communication networks should be stable during initial disruptions and future reconstructions. A network's stability can be measured by its cost of disruption. Analyzing the stability of a network against disruption is crucial in various fields like telecommunications, transportation, and ecology. Here are some fundamental concepts to consider [1–3]:

*i.* The number of non-functioning nodes in a network depends on several factors, such as the nature of the disruption. It is important to determine the number of these nodes.

*ii.* By analyzing how many groups still have mutual communication after a network outage, the network's topology needs to be evaluated.

*iii.* In terms of difficulty, connecting a network that has been disrupted varies widely based on factors such as the scale of the disruption, the nature of the network, available resources, and expertise.

Modeling a communication network as a graph is a common and effective approach to analyzing its stability and behavior. In this graph model, the following concepts are involved:

*i.* Vertices (Nodes): Each node in the graph represents a distinct entity within the communication network. These entities could be devices, e.g., computers, routers, or smartphones, communication endpoints, such as users or servers, or any other relevant network component.

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*ii.* Edges (Links): Each edge in the graph represents a communication link or connection between two nodes. These links could be physical connections, e.g., cables or wireless, or logical connections, such as virtual circuits or network paths.

By representing the considered network as a graph, various graph theory concepts and algorithms can be applied to analyze its properties, connectivity, and stability. We have some graph theoretical parameters to obtain the stability of communication networks, e.g., connectivity, integrity, toughness, and scattering number [1, 4–7, 11]. Edge versions of these graph parameters are also defined. The scattering number is handy for measuring the stability of a graph. However, it does not provide good results for some families of graphs, and the edge scattering number does not yield satisfactory results for certain graphs. In other words, these parameters are not distinctive between some families of graphs. This paper investigates a new parameter for stability, considering this situation.

If scattering numbers and dominance are thought together, then when a small group of decision-makers has effective communication links with each other, dominance in graphs can provide a valuable model for deciding what to do [12]. In essence, removing a minimum dominating set like $X$ can trigger a cascade of adverse effects, culminating in chaos within the network. It highlights the critical role played by centralized decision-makers and effective communication channels in maintaining organizational stability and functionality [12]. The motivation of this paper is to choose the dominating set of a graph instead of the set $X$ when calculating the scattering number. By this choice, this paper introduces a new graph parameter.

## 2. Preliminaries

Throughout this paper, we use the notation $w(G)$ to denote the order of the most significant component. We provide some basic definitions to be needed in the following sections.

**Definition 2.1.** [7] The scattering number of a noncomplete connected graph $G$ is defined by

$$\mathrm{sc}(G) = \max\{w(G-X) - |X| \ : \ X \subset V(G) \text{ and } w(G-X) \geq 2\}$$

where the notation $|X|$ represents the cardinality of $X$. Moreover, a set $X \subset V(G)$ is called a scatter-set of $G$ if $\mathrm{sc}(G) = w(G-X) - |X|$.

Some results for this parameter are provided as follows:

**Theorem 2.2.** [3] If $G$ is a noncomplete connected graph of order $n$, then

$$2\eta(G) - n \leq \mathrm{sc}(G) \leq \eta(G) - \kappa(G)$$

where $\eta(G)$ and $\kappa(G)$ are independence number and connectivity number of the graph $G$, respectively.

We then present the cartesian product of two graphs.

**Definition 2.3.** [6, 13] Let $G$ and $H$ be two graphs, $V_G$ and $V_H$ be the sets of vertices of $G$ and $H$, respectively, $V = V_G \times V_H$, and $m, n \in V$ such that $m = (m_1, m_2)$ and $n = (n_1, n_2)$. Then, the cartesian product of $G$ and $H$, denoted by $G \times H$, is defined by vertices in $V$ that $m$ and $n$ are adjacent in $G \times H$ if and only if $m_1 = n_1$ and the vertices $m_2$ and $n_2$ in $V_H$ are adjacent in $H$ or $m_2 = n_2$ and the vertices $m_1$ and $n_1$ in $V_G$ are adjacent in $G$.

**Theorem 2.4.** [14] Let $m \geq 2$ and $n \geq 2$. Then,

$$\mathrm{sc}\left(K_{1,m} \times P_n\right) = \begin{cases} m-1, & n \text{ is even} \\ m-2, & n \text{ is odd} \end{cases}$$

For more information about scattering numbers, refer to [3, 7, 11, 14–17]. The edge version of the scattering number has been defined by Aslan [18].

**Definition 2.5.** [18, 19] The edge scattering number of a noncomplete connected graph G is defined by

$$\mathrm{es}(G) = \max\{w(G - X) - |X| \ : \ X \subseteq E(G) \text{ and } w(G - X) \geq 2\}$$

where the notation $|X|$ represents the cardinality of $X$. Moreover, a set $X \subseteq E(G)$ is called an edge scatter set (es-set) of $G$ if $\mathrm{es}(G) = w(G - X) - |X|$.

Some results for the edge scattering number are provided as follows:

**Theorem 2.6.** [18] The edge-scattering number of the cycle graph $C_n$ is 0. Moreover, the edge-scattering number of the complete bipartite graph $K_{m,n}$ is $2 - m$ where $2 \leq m \leq n$.

**Theorem 2.7.** [18] If $n \geq 3$ is a positive integer, then $\mathrm{es}(K_2 \times P_n) = 0$. If $n \geq 4$ is a positive integer, then $\mathrm{es}(K_2 \times C_n) = -1$.

We mention another important concept of stability.

**Definition 2.8.** [12] A nonempty subset $X \subset V(G)$ is called a dominating set of $G$ if every vertex not in $X$ is adjacent to at least one vertex in $X$. A dominating set is called minimal if none of its proper subsets is a dominating set. The minimum cardinality of all the dominating graph sets $G$ is called the domination number of the graph and is denoted by $\gamma(G)$.

Any subset of vertices of a graph $G$ is a dominating set. In other words, the subset that gives the scattering number can be a dominating set. The motivation of this paper is to use the dominating set when investigating the stability measurement.

## 3. Domination Scattering Number of a Graph

In this section, we first define a new parameter as stability measurement.

**Definition 3.1.** The domination scattering number of a noncomplete graph $G$ is

$$\mathrm{ds}(G) = \max\{w(G - X) - |X| : w(G - X) \geq 2 \text{ and } X \text{ is a dominating set}\}$$

where the notation $|X|$ represents the cardinality of $X$. Moreover, a set $X \subset V(G)$ is called a domination scatter set (ds-set) of $G$ if $\mathrm{ds}(G) = w(G - X) - |X|$.

We provide an example showing that this parameter is more distinctive than the scattering and edge scattering numbers. In other words, the stability parameter we define offers better results than other parameters for some graph families.

Consider the graphs $G_1$, $G_2$, and $G_3$, each having the same number of vertices. A pertinent question arises: "Can the relevance of the domination scattering number as a measure of stability in graphs be evaluated by analyzing its properties and effectiveness in distinguishing graphs based on their structural flexibility and variations in dominance?" In other words, are $G_1$, $G_2$, and $G_3$ distinguished by the domination scattering number?

We can find many examples of graphs that suggest that $\mathrm{ds}(G)$ is a suitable measure of stability in that it can distinguish between graphs. Consider Figure 1 as an example.

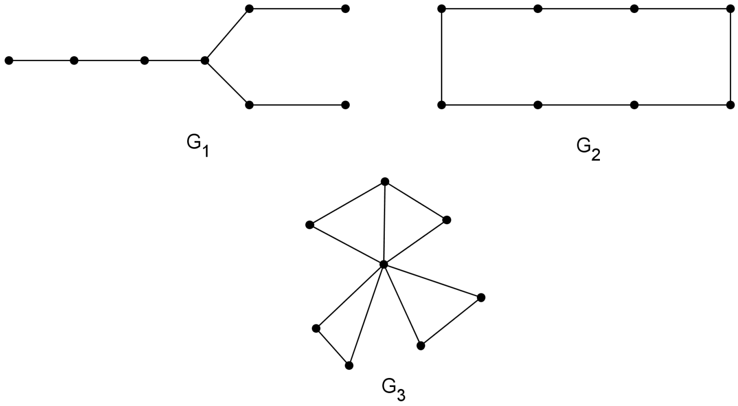


**Figure 1.** Graphs $G_1$, $G_2$, and $G_3$, each having the same number of vertices

The scattering number, edge scattering number, and domination scattering number of graphs in Figure 1 are calculated and listed in Table 1.

**Table 1.** Scattering, edge scattering, and domination scattering numbers of graphs in Figure 1

| | sc($G$) | es($G$) | ds($G$) |
|---|---|---|---|
| $G_1$ | 2 | 1 | 1 |
| $G_2$ | 0 | 0 | 0 |
| $G_3$ | 2 | 0 | 2 |

It can be observed from Table 1 that $\mathrm{sc}(G_1) = \mathrm{sc}(G_3) = 2$. Therefore, scattering numbers do not distinguish between graphs $G_1$ and $G_3$. Since $\mathrm{ds}(G_1) \neq \mathrm{ds}(G_3)$, the domination scattering number distinguishes between graphs $G_1$ and $G_3$. We can also say the same for the graphs $G_2$ and $G_3$. Table 1 shows $\mathrm{es}(G_2) = \mathrm{es}(G_3) = 0$. Therefore, edge-scattering numbers do not distinguish between graphs $G_2$ and $G_3$. However, since $\mathrm{ds}(G_2) \neq \mathrm{ds}(G_3)$, we say that the domination scattering number distinguishes between graphs $G_2$ and $G_3$.

Consequently, the new parameter defined in this study is more distinctive for these graphs than others. In other words, the graph parameter we defined is a suitable indicator of its stability. Therefore, we investigate which graphs the parameter we defined is better for. We provide the domination scattering number of several graphs.

### 3.1. Domination Scattering Number of Some Graphs

In this subsection, we provide the results obtained by the new parameter. Firstly, we start with the path graph $P_n$.

**Theorem 3.2.** Let $n \in \mathbb{Z}^+$ and $n \geq 5$. Then, $\mathrm{ds}(P_n) = 1$.

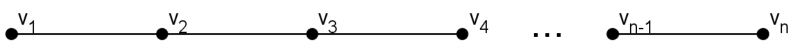


**Figure 2.** Path graph $P_n$

Proof. Let $X$ be a dominating set of $P_n$ and $V(P_n) = \{v_1, v_2, v_3, \cdots, v_n\}$ (see Figure 2). From [20], since $\gamma(P_n)=\lceil \frac{n}{3} \rceil$, we have there different cases:

**Case 1:** Let $n \equiv 0 \pmod 3$. If we remove $|X| \geq \frac{n}{3}$ vertices, then $w(P_n - X) \leq \frac{n}{3} + 1$. Therefore,

$$\mathrm{ds}(P_n) = \max\{w(P_n - X) - |X|\} \leq \max\left\{\frac{n}{3} + 1 - \frac{n}{3}\right\} \leq 1$$

If we choose $X^* = \{v_2, v_5, v_8, \cdots, v_{n-1}\}$ such that $|X^*| = \frac{n}{3}$ and $w(P_n - X) = \frac{n}{3} + 1$, then

$$\mathrm{ds}(P_n) = 1 \tag{3.1}$$

**Case 2:** Let $n \equiv 1 \pmod 3$. By removing $|X| \geq \lceil \frac{n}{3} \rceil$ vertices, we have $w(P_n - X) \leq \lceil \frac{n}{3} \rceil + 1$. Therefore,

$$\mathrm{ds}(P_n) \leq \max\left\{ \left\lceil \frac{n}{3} \right\rceil + 1 - \left\lceil \frac{n}{3} \right\rceil \right\} \leq 1$$

If we choose $X^* = \{v_2, v_5, v_8, \cdots, v_{n-5}\} \cup \{v_{n-3}, v_{n-1}\}$ such that $|X^*| = \lceil \frac{n}{3} \rceil$ and $w(P_n - X) = \lceil \frac{n}{3} \rceil + 1$, then

$$\mathrm{ds}(P_n) = 1 \tag{3.2}$$

**Case 3:** Let $n \equiv 2 \pmod 3$. If $|X| \geq \lceil \frac{n}{3} \rceil$ vertices are removed, then $w(P_n - X) \leq \lceil \frac{n}{3} \rceil + 1$. Therefore,

$$\mathrm{ds}(P_n) \leq \max\left\{ \left\lceil \frac{n}{3} \right\rceil + 1 - \left\lceil \frac{n}{3} \right\rceil \right\} \leq 1$$

Let $X^* = \{v_2, v_5, v_8, \cdots, v_{n-3}\} \cup \{v_{n-1}\}$ be a vertex cut. Then, $|X^*| = \lceil \frac{n}{3} \rceil$ and $w(P_n - X) = \lceil \frac{n}{3} \rceil + 1$. Hence,

$$\mathrm{ds}(P_n) = 1 \tag{3.3}$$

From (3.1)-(3.3), $\mathrm{ds}(P_n) = 1$. $\square$

**Theorem 3.3.** Let $n \in \mathbb{Z}^+$ and $n \geq 4$. Then, $\mathrm{ds}(C_n) = 0$.

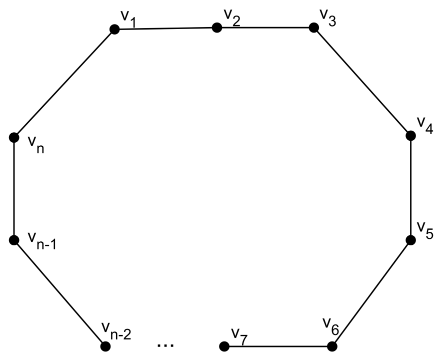

**Figure 3.** Cycle graph $C_n$

Proof. Let $X$ be a dominating set of $C_n$ and $V(C_n) = \{v_1, v_2, v_3, \cdots, v_n\}$ (see Figure 3). From [20], since $\gamma(C_n) = \lfloor \frac{n+2}{3} \rfloor$, we have three different cases:

**Case 1:** Let $n \equiv 0 \pmod 3$. If $|X| \geq \lfloor \frac{n+2}{3} \rfloor$ vertices are removed, then $w(C_n - X) = \lfloor \frac{n+2}{3} \rfloor$ and

$$\mathrm{ds}(C_n) \leq \max\left\{ \left\lfloor \frac{n+2}{3} \right\rfloor - \left\lfloor \frac{n+2}{3} \right\rfloor \right\} \leq 0$$

Hence, if we choose $X^* = \{v_2, v_5, v_8, ..., v_{n-1}\}$, then $|X^*| = \lfloor \frac{n+2}{3} \rfloor$ and $w(C_n - X) = \lfloor \frac{n+2}{3} \rfloor$. Hence,

$$\mathrm{ds}(C_n) = 0 \tag{3.4}$$

**Case 2:** Let $n \equiv 1 \pmod 3$. If $|X| \geq \frac{n+2}{3}$ vertices are removed, then $w(C_n - X) \leq \frac{n+2}{3}$. Therefore,

$$\mathrm{ds}(C_n) \leq \max\left\{ \frac{n+2}{3} - \frac{n+2}{3} \right\} \leq 0$$

Let $X^* = \{v_2, v_5, v_8, ..., v_{n-2}\} \cup \{v_n\}$ be a vertex cut. By the choice of $|X^*|$, we obtain $|X^*| = \frac{n+2}{3}$ and $w(C_n - X) = \frac{n+2}{3}$. Then,

$$\mathrm{ds}(C_n) = 0 \tag{3.5}$$

**Case 3:** Let $n \equiv 2 \pmod 3$. By removing $|X| \geq \lfloor \frac{n+2}{3} \rfloor$ vertices, we have $w(C_n - X) \leq \lfloor \frac{n+2}{3} \rfloor$. Then,

$$\mathrm{ds}(C_n) \leq \max\left\{ \left\lfloor \frac{n+2}{3} \right\rfloor - \left\lfloor \frac{n+2}{3} \right\rfloor \right\} \leq 0$$

If we choose $X^* = \{v_2, v_5, v_8, ..., v_{n-3}\} \cup \{v_{n-1}\}$ or $X^* = \{v_2, v_5, v_8, ..., v_{n-3}\} \cup \{v_n\}$ as a vertex cut, then $|X^*| = \lfloor \frac{n+2}{3} \rfloor$ and $w(C_n - X^*)$=$\lfloor \frac{n+2}{3} \rfloor$. Then,

$$\mathrm{ds}(C_n) = 0 \tag{3.6}$$

From (3.4)-(3.6), $\mathrm{ds}(C_n) = 0$. □

**Theorem 3.4.** If n$\in \mathbb{Z}^+$ and $n \geq 2$, then $\mathrm{ds}(K_{1,n}) = n - 1$.

Proof. Let $X$ be a dominating set of $K_{1,n}$ and $v$ be a vertex with maximum degree. If we remove $|X| \geq$1 vertices, then $w(K_{1,n} - X) \leq n$. Then,

$$\mathrm{ds}(K_{1,n}) \leq \max\{n-1\} \leq n - 1$$

If we choose $X^* = \{v\}$ such that $|X^*| = 1$ and $w(K_{1,n} - X^*) = n$, then $\mathrm{ds}(K_{1,n}) = n - 1$. □

**Theorem 3.5.** If $n, m \in \mathbb{Z}^+$ and $n \geq m$, then $\mathrm{ds}(K_{m,n}) = n - m$.

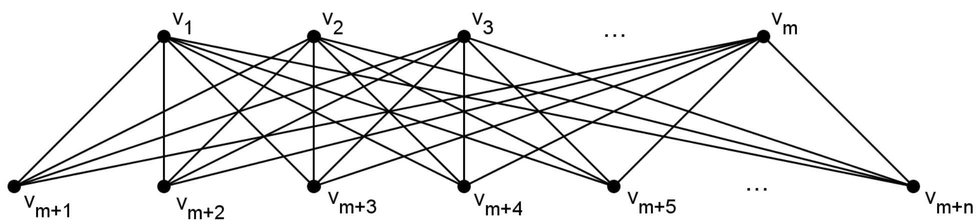


**Figure 4.** Complete bipartite graph $K_{m,n}$

Proof. Let $X$ be a dominating set of $K_{m,n}$ and $V(K_{m,n}) = \{v_1, v_2, v_3, \cdots, v_{m+n-1}, v_{m+n}\}$ (see Figure 4). From [16], since $\gamma(K_{m,n}) = 2$ and $w(K_{m,n} - X) > 1$, then $|X|$ must be at least $m$. If we remove $|X| \geq m$ vertices, then $w(K_{m,n} - X) \leq n$. Therefore,

$$\mathrm{ds}(K_{m,n}) \leq \max\{n - m\} \leq n - m$$

If we take $X^* = \{v_1, v_2, v_3, ..., v_m\}$, then $w(K_{m,n} - X^*) = n$. Hence, $\mathrm{ds}(K_{m,n}) = n - m$. □

### 3.2. Cartesian Product and Domination Scattering Number

In this subsection, we provide the $\mathrm{ds}(P_n \times C_3)$ value.

**Theorem 3.6.** If $n \in \mathbb{Z}^+$ and $n \geq 4$, then

$$\mathrm{ds}(P_n \times C_3) = \begin{cases} 1 - \frac{2n}{3}, & n \equiv 0 \pmod 3 \\ \lceil \frac{n}{3} \rceil - n, & \text{otherwise} \end{cases}$$

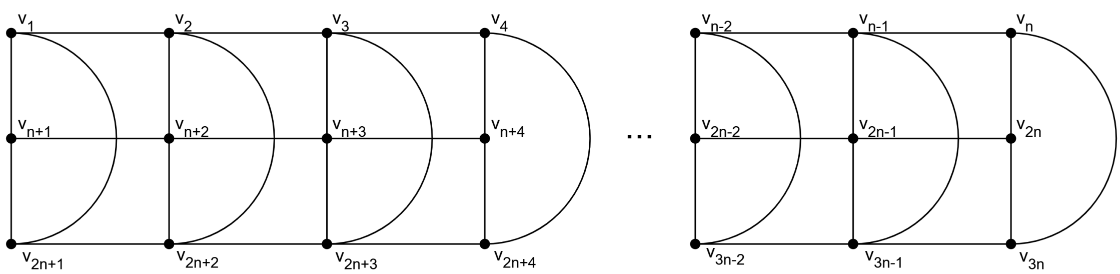


**Figure 5.** Graph $P_n \times C_3$

PROOF. Let $X$ be a dominating set and $V(P_n \times C_3) = \{v_1, v_2, v_3, ..., v_{3n-1}, v_{3n}\}$ (see Figure 5). From [21], since

$$\gamma(P_n \times C_3) = \begin{cases} \lceil \frac{3n}{4} \rceil + 1, & n \equiv 0 \pmod 4 \\ \lceil \frac{3n}{4} \rceil, & \text{otherwise} \end{cases}$$

then $|X|$ must be at least $\lceil \frac{3n}{4} \rceil$. Then, we consider two different cases:

**Case 1:** Let $n \equiv 0 \pmod 3$. If we remove $|X| = \lceil \frac{3n}{4} \rceil$ vertices, then $w((P_n \times C_3) - X) = 1$. If we remove $|X| = \lceil \frac{3n}{4} \rceil + k$ vertices such that $k \in \mathbb{Z}^+$ and $|X| < n$, then $w((P_n \times C_3) - X) \leq 1 + k$. Thus,

$$\operatorname{ds}(P_n \times C_3) \leq \max\left\{1 + k - \left(\left\lceil \frac{3n}{4} \right\rceil + k\right)\right\} \leq 1 - \left\lceil \frac{3n}{4} \right\rceil \tag{3.7}$$

If we remove $|X| \geq n$ vertices, then $w((P_n \times C_3) - X) \leq \frac{n}{3} + 1$. Thus,

$$\operatorname{ds}(P_n \times C_3) \leq \max\left\{\frac{n}{3} + 1 - n\right\} \leq 1 - \frac{2n}{3} \tag{3.8}$$

Since $1 - \left\lceil \frac{3n}{4} \right\rceil \leq 1 - \frac{2n}{3}$, for all $n \geq 4$, then $\operatorname{ds}(P_n \times C_3) \leq 1 - \frac{2n}{3}$ from (3.7) and (3.8).

Hence, if we choose

$$X^* = \{v_2, v_5, v_8, ..., v_{n-1}\} \cup \{v_{n+2}, v_{n+5}, v_{n+8}, ..., v_{2n-1}\} \cup \{v_{2n+2}, v_{2n+5}, v_{2n+8}, ..., v_{3n-1}\}$$

then $|X^*| = n$ and $w((P_n \times C_3) - X^*) = \frac{n}{3} + 1$. Then,

$$\operatorname{ds}(P_n \times C_3) = 1 - \frac{2n}{3} \tag{3.9}$$

**Case 2:** Let $n \equiv 1 \pmod 3$ or $n \equiv 2 \pmod 3$. If we remove $|X| = \lceil \frac{3n}{4} \rceil$ vertices, then $w((P_n \times C_3) - X) = 1$. If we remove $|X| = \lceil \frac{3n}{4} \rceil + k$ vertices such that $k \in \mathbb{Z}^+$ and $|X| < n$, then $w((P_n \times C_3) - X) \leq 1 + k$. Thus,

$$\operatorname{ds}(P_n \times C_3) \leq \max\left\{1 + k - \left(\left\lceil \frac{3n}{4} \right\rceil + k\right)\right\} \leq 1 - \left\lceil \frac{3n}{4} \right\rceil \tag{3.10}$$

If we remove $|X| \geq n$ vertices, then $w((P_n \times C_3) - X) \leq \lceil \frac{n}{3} \rceil$. Therefore,

$$\operatorname{ds}(P_n \times C_3) \leq \max\left\{\left\lceil \frac{n}{3} \right\rceil - n\right\} \leq \left\lceil \frac{n}{3} \right\rceil - n \tag{3.11}$$

Since $1 - \lceil \frac{3n}{4} \rceil \leq \lceil \frac{n}{3} \rceil - n$, for all $n \geq 4$, then $\operatorname{ds}(P_n \times C_3) \leq \lceil \frac{n}{3} \rceil - n$ from (3.10) and (3.11).

If we choose

$$X^* = \{v_2, v_5, v_8, ..., v_{n-2}\} \cup \{v_{n+2}, v_{n+5}, v_{n+8}, ..., v_{2n-2}\} \cup \{v_{2n+2}, v_{2n+5}, v_{2n+8}, ..., v_{3n-2}\} \cup \{v_{2n}\}$$

while $n \equiv 1 \pmod 3$ and then $|X^*| = n$ and $w((P_n \times C_3) - X^*) = \lceil \frac{n}{3} \rceil$. Therefore,

$$\operatorname{ds}(P_n \times C_3) = \left\lceil \frac{n}{3} \right\rceil - n \tag{3.12}$$

If we choose

$$X^{**} = \{v_2, v_5, v_8, ..., v_{n-3}\} \cup \{v_{n+2}, v_{n+5}, v_{n+8}, ..., v_{2n-3}\} \cup \{v_{2n+2}, v_{2n+5}, v_{2n+8}, ..., v_{3n-3}\} \cup \{v_{n-1}, v_{2n}\}$$

while $n \equiv 2 \pmod 3$, then $|X^{**}| = n$ and $w((P_n \times C_3) - X^{**}) = \lceil \frac{n}{3} \rceil$. Hence,

$$\operatorname{ds}(P_n \times C_3) = \left\lceil \frac{n}{3} \right\rceil - n \tag{3.13}$$

By (3.9), (3.12), and (3.13), the results are obtained. □

## 4. Conclusion

The vulnerability of a communication network measures the network's resistance to the disruption of its operation after the failure of specific processors or communication links. Network designers aim to design networks with less vulnerability or more reliability. In other words, network designers care about network stability. For this reason, the vulnerability values of graphs (networks) are investigated by modeling networks with graphs. In this study, first of all, it was observed that the scattering and edge scattering numbers among the vulnerability measurements in graphs were insufficient to distinguish some graph families. Afterward, a new parameter was defined to distinguish these graph families, called the domination scattering number. The vertices removed from the graph in this parameter are also components of any dominant cluster in the graph. In this article, the domination scattering number for basic graphs is calculated. The domination scattering number of the graph $P_n \times C_3$ is also provided. In future research, the primary objective can be to obtain graphs corresponding to real-life networks using graph operations, such as the graph $P_n \times C_3$. Subsequently, the aim can be to calculate the domination scattering numbers of these graphs. However, an essential question warrants investigation: Can the domination scattering number of a graph be calculated in polynomial time? Moreover, the following questions are anticipated that obtaining answers to these questions will benefit network designers:

*i.* Which graph family has the smallest or largest domination scattering number?

*ii.* What are the relationships between the domination scattering number and other graph parameters?

*iii.* What are the values of the domination scattering numbers for a graph's total, line, and middle graphs?

## Author Contributions

All the authors equally contributed to this work. This paper is derived from the first author's doctoral dissertation supervised by the second author. They all read and approved the final version of the paper.

## Conflicts of Interest

All the authors declare no conflict of interest.

## Ethical Review and Approval

No approval from the Board of Ethics is required.

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