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A Comparative Analysis of Various Optimization Methods for Solving Fully Fuzzy Transportation Problems

Tam Bulanık Ulaştırma Problemlerinin Çözümünde Çeşitli Optimizasyon Yöntemlerinin Karşılaştırmalı Analizi

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Abstract

This study investigates the solution of a Trapezoidal Fuzzy Transportation Problem (FTP) using Basirzadeh's Fuzzy Ranking Approach through various optimization methods. The FTP was drawn from Pandian and Natarajan's 2010 article in Applied Mathematical Sciences. They used the Fuzzy Zero Point Method to solve the FTP. However, the fact that this method has eleven steps, each requiring fuzzy operations, makes it a compelling process. To point out this complexity and to illustrate how easy to get the result, Basirzadeh's method has been adopted in this study. In this method, the fuzzy data is converted into crip data, and a transportation matrix is created. Then it can be solved by any transportation method and the results may remain crisp or they might be fuzzified. In this study, firstly, the FTP has been solved using the Fuzzy Zero Point Method. Not only the initial cost table and final allocation table have been shared with readers, but also all intermediate steps of the fully fuzzy solution have been demonstrated. Subsequently, some common optimization methods used in transportation problems such as Northwest Corner, Least Cost, Russel, and Vogel's methods, have been applied to Basirzadeh's method and the results have been compared. Russel's Method seems to be the best among these methods because it yielded the lowest transportation cost.

Keywords: *Fuzzy Transportation Problem, Ranking of Fuzzy Numbers, Fuzzy Zero-Point Method, Northwest Corner Method, Least Cost Method, Russel's Method, Vogel's Method.*

Jel Codes: *C60, C44.*

Öz

Bu çalışma tümüyle yamuk bulanık sayılardan oluşan bir ulaştırma probleminin Basirzadeh'in bulanık sayıların sıralanmasına dayanan yöntemi benimsenerek, çeşitli optimizasyon yöntemleri ile çözülmesini konu almaktadır. Çözümlemelerin yapıldığı sayısal örnek, Pandian ve Natajaran'ın 2010'da Applied Mathematical Sicences dergsindeki makalelerinden alınmıştır. Pandian ve Natajaran, bulanık ulaştırma probleminin çözümünde bulanık sıfır noktası yöntemini kullanmışlardır. Fakat onbir adımdan oluşan ve her adımında bulanık sayılarla işlemler yapmayı gerektiren bu yöntemin uygulanması zorlu bir süreçtir. Bu zorluğu gözler önüne sermek ve sonucu elde etmenin ne kadar kolay olduğunu göstermek için, bu çalışmada Basirzadeh'in yöntemi benimsenmiştir. Bu yöntemde, bulanık veri keskin veriye dönüştürülmekte ve bir taşıma matrisi oluşturulmaktadır. Daha sonra herhangi bir optimizasyon yöntemi ile çözülebilmekte ve sonuçlar keskin kalabilmekte ya da bulandırılabilmektedir. Bu çalışmada, ilk olarak FTP, Bulanık Sıfır Noktası Yöntemi kullanılarak çözülmüştür. Sadece başlangıç maliyet tablosu ve son atama tablosu okuyucularla paylaşılmamış, aynı zamanda tam bulanık çözümün tüm ara adımları da gösterilmiştir. Ardından, aynı bulanık ulaştırma problemi bulanık sıralama yaklaşımı benimserek, Kuzey Batı Köşesi, En Düşük Maliyet, Russel ve Vogel'in optimizasyon yöntemleri ile çözülmüş ve sonuçları kıyaslanmıştır. Bu yöntemler arasında en iyi yöntemin, en düşük ulaşım maliyetini sağlayan Russel Metodu olduğu söylenebilir.

Anahtar Kelimeler: *Bulanık Ulaştırma Problemi, Bulanık Sayıların Sıralanması, Bulanık Sıfır Noktası Yöntemi, Kuzey Batı Köşe Yöntemi, En Düşük Maliyet Yöntemi, Russel Yöntemi, Vogel Yöntemi.*

Jel Kodları: *C60, C44.*

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1. INTRODUCTION

Transportation Problems (TP) aim to find the ways of transporting the commodities from specific sources to the requested points with minimum cost. When the supply, demand, and costs are crisp values, the TP can easily be solved by using various optimization methods. But in daily life scenarios, parameters may not be crisp and certain; in fact, they are often fuzzy. What is usually done is to put them into binary logic, although they are fuzzy, whereas fuzzy logic can skillfully take into account real-life problems without imposing rigid assumptions. Because fuzzy logic enables computing with words, thinking like the human brain, using infinite-valued logic in the range of [0,1] (Zadeh, 1996).

In this study, firstly, Pandian and Natajaran's numeric example was solved using the Fuzzy Zero Point Method. Subsequently, it was solved again using Basirzadeh's fuzzy ranking approach with the Northwest Corner Method, the Least Cost Method, Russel Method, and Vogel's Method. We pursued two main objectives for doing this. The first was to share all intermediate steps of the Fuzzy Zero Point solution. The second was to demonstrate how easily one can get the solution of the Fuzzy Transportation Problem (FTP) when using a fuzzy ranking approach instead of fully fuzzy methods. Because the ranking approach does not impose any preconditions on the fuzzy numbers, they may be triangular, trapezoidal, LR number, normal or abnormal, etc. This approach facilitates the solution of problems involving fuzzy numbers by transforming them into their crisp equivalents, which can then be processed using standard optimization algorithms. The resulting solution can be either retained in its crisp form or transformed back into a fuzzy representation (fuzzification). Thus, the difficulty of working with fuzzy numbers is overcome.

This study consists of six parts; introduction, literature review, mathematical background, optimization methods, numeric example, and conclusion.

2. LITERATURE REVIEW

Transportation problems concern almost every sector, particularly in the logistics industry. In today's increasingly competitive environment, minimizing transportation costs is crucial. A management approach based on intuition and sensations, can not compete with the companies grounded in scientific decisions. Thus, the effort to solve the transportation problems with novel methods seems to never lose its popularity. A selection of studies conducted on FTP in the last four decades is seen in Table 1.

Table 1. Literature Review for Fuzzy Transportation Problem

Author(s)	Year	Method used for solving the fuzzy transportation problem
Chanas, S. Kolodziejczyk, W. Machaj, A.	1984	Fuzzy parametric programming
Chanas, S., Delgado, M., Verdegay, J. L. Vila, M. A.	1993	This paper explores these three different models; classical, interval, and fuzzy approaches with a focus on transportation problems. The results of the three FTPs were compared.

Tada, M. Ishii, H.	1996	They have developed the IFTP (integer fuzzy transportation problem), where every supply and demand value is an integer, and the values of all commodities to be transported are also integers.
Liu, S. T. Kao, C.	2004	Fuzzy Extension principle
Pandian, P. Natarajan, G.	2010	Fuzzy zero-point method with trapezoidal fuzzy numbers
Basirzadeh, H.	2011	Ranking of fuzzy numbers
Kumar, A. Kaur, A.	2011	Based on the notion that normal fuzzy numbers are insufficient for solving the transportation problem, they have employed generalized fuzzy numbers.
Kumar, A. Kaur, A.	2012	JMD representation of fuzzy trapezoidal numbers instead of existing representation of trapezoidal fuzzy numbers
Samuel, A. E.	2012	Improved zero-point method (IZPM)
Khalaf, W. S.	2014	Fuzzy ranking with triangular fuzzy numbers
Ebrahimnejad, A.	2014	Generalized trapezoidal fuzzy numbers
Khoshnavaa, A. Mozaffari, M. R.	2015	Triangular fuzzy numbers and linear multiobjective programming
Mathur, N. Srivastava, P. K. Paul, A.	2016	Fuzzy zero-point method with trapezoidal fuzzy numbers
Chandran, S. Kandaswamy, G.	2016	They used the Sudhagar score method to rank fuzzy numbers for solving FTP without converting a fuzzy problem to a crisp problem.
Hunwisai, D. Kumam, P.	2017	They have developed a method to solve the fuzzy transportation problem (FTP) using the robust ranking technique. Additionally, they have employed the allocation table method to find an initial solution.
Maheswari, P. U. Ganesan, K.	2018	Pentagonal fuzzy numbers
Balasubramanian, K. Subramanian, S.	2018	Triangular fuzzy numbers, ranking technique
Ebrahimnejad, A. Verdegay, J. L.	2018	They have proposed a new method that provides an optimal solution and optimal cost using positive intuitionistic fuzzy numbers. They used IFTP (intuitionistic fuzzy transportation problem) for solving a classical LP (linear programming problem).

Roy, H. Pathak, G. Kumar, R. Malik, Z. A.	2020	Fuzzy zero-point method with trapezoidal fuzzy numbers
Nishad, A. K. Abhishekh.	2020	A novel ranking method grounded in the ranking of intuitionistic fuzzy numbers was proposed. This method was applied to solve fully intuitionistic fuzzy transportation problems, wherein costs, supplies, and demands are all represented by intuitionistic fuzzy numbers.
Srinivasan, R. Karthikeyan, N. Renganathan, K. Vijayan, D. V.	2021	5 Step new method with triangular fuzzy numbers
Pratihar, J. Kumar, R. Edalatpanah, S. A. Dey, A.	2021	Interval type 2 fuzzy set and modified Vogel's approximation method
Arockiasironmani, A. Santhi, S.	2022	Triangular fuzzy number, trapezoidal fuzzy number, magnitude ranking function.
Fegade, M. Muley, A.	2024	Hexagonal fuzzy numbers
Agrawal, A. Singhal, N.	2024	Trapezoidal fuzzy number

3. MATHEMATICAL BACKGROUND

Definition 1: $\tilde{A}$ is convex if $\mu_{\tilde{A}}(\lambda X_1 + (1-\lambda)X_2) \geq \min(\mu_{\tilde{A}}(X_1), \mu_{\tilde{A}}(X_2))$ X_1 ve $X_2 \in X$, $\lambda \in [0, 1]$ (Kaufmann & Gupta, 1991: 21-22)

Definition 2: $\tilde{A}$ is a normal iff $\mu_{\tilde{A}}(X)=1$.

Definition 3: A triangular fuzzy number is denoted by three real numbers as follows

$\tilde{A} = (a, b, c)$. $\mu_{\tilde{A}}$ is the membership function of $\tilde{A}$:

$$\mu_{\tilde{A}} = \begin{cases} 0 & x < a \\ \dfrac{x-a}{b-a} & a \leq x \leq b \\ \dfrac{c-x}{c-b} & b \leq x \leq c \\ 0 & x > c \end{cases} \tag{1}$$

Definition 4: A trapezoidal fuzzy number is denoted by four real numbers as follows; $\tilde{A} = (a, b, c, d)$. Here, a≤b≤c≤d. $\mu_{\tilde{A}}$ is the membership function of $\tilde{A}$ (Pandian & Natarajan, 2010: 81):

$$\mu_{\breve{A}} = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \frac{d-x}{d-c}, & c \le x \le d \\ 0, & x > d \end{cases} \quad (2)$$

Definition 5: Algebraic operations on trapezoidal fuzzy numbers (Mathur et al, 2016: 3):

Let $\breve{A} = (a, b, c, d)$ and $\breve{B} = (e, f, g, h)$

(i) $\breve{A} + \breve{B} = (a + e, b + f, c + g, d + h)$

(ii) $\breve{A} - \breve{B} = (a - h, b - g, c - f, d - e)$

(iii)$\breve{A}x\breve{B} = (\min(ae, ah, de, dh), \min(bf, cf, cg, bg), \max(bf, cf, cg, bg), \max(ae, ah, de, dh))$

Definition 6: The magnitude of the trapezoidal fuzzy number (Pandian & Natarajan, 2010: 82):

$$Mag(\tilde{A}) = \frac{a+5b+5c+d}{12} \quad (3)$$

Another magnitude formula is as follows (Arockiasironmani &Santhi, 2022: 2218):

$$Mag(\tilde{A}) = \frac{a + 2b + 2c + d}{6} \quad (4)$$

Definition 7: Ranking of fuzzy numbers (Kaur & Kumar, 2012; Ebrahimnejad, 2014: 175):

$\tilde{A} = (a, b, c, d; w_1)$

$\tilde{B} = (e, f, g, h; w_2)$

$\tilde{A}$ and $\tilde{B}$ Are trapezoidal fuzzy numbers. w=min(w_1, w_2)

$$\tilde{A} \preccurlyeq \tilde{B} \quad iff \quad \tilde{A} = \frac{w(a+b+c+d)}{4} \le \tilde{B} = \frac{w(e+f+g+h)}{4} \quad (5)$$

$$\tilde{A} \succcurlyeq \tilde{B} \quad iff \quad \tilde{A} = \frac{w(a+b+c+d)}{4} \ge \tilde{B} = \frac{w(e+f+g+h)}{4} \quad (6)$$

$$\tilde{A} \cong \tilde{B} \quad iff \quad \tilde{A} = \frac{w(a+b+c+d)}{4} = \tilde{B} = \frac{w(e+f+g+h)}{4} \quad (7)$$

Definition 8: Fuzzy transportation problem (Chanas et al., 1984: 213):

$$Minimize\ \tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij} \quad (8)$$

s.t.

$$\sum_{j=1}^{n} x_{ij} \cong \tilde{S}_i \quad (i = 1,2, \dots m) \quad (9)$$

$$\sum_{i=1}^{m} x_{ij} \cong \widetilde{D}_J \quad (\mathrm{j} = 1,2, \dots \mathrm{n}) \tag{10}$$

$$x_{ij} \geq 0 \quad (\mathrm{i} = 1,2, \dots \mathrm{m};\ \mathrm{j} = 1,2, \dots \mathrm{n}) \tag{11}$$

Equation 8 is the objective function of the FTP and it is the total fuzzy cost of FTP. The goal is to minimize the total cost of transport. Equation 9 and Equation 10 are constraints on supply and demand. It's feasible iff (if and only if) $\sum \widetilde{S}_\iota = \sum \widetilde{D}_J$. In FTP, transportation costs, supply, and demands are fully trapezoidal fuzzy numbers.

Definition 9: Row totals in the transportation costs table; (S_i) indicate the quantities supplied, (D_j) column totals indicate the quantities requested c_{ij} indicate the cost of transportation between the supply and demand points.

Table 2. The Table of Transportation Costs

	1	**2**	…	**n**	**Supply**
1	C_{11}	C_{12}	…	C_{1n}	S_1
2	C_{21}	C_{22}	…	C_{2n}	S_2
…	…	…	…	…	…
n	C_{n1}	C_{n2}	…	C_{nn}	S_n
Demand	D_1	D_2	…	D_n	

4. OPTIMIZATION METHODS

In this section, the optimization methods commonly used in transportation problems have been outlined. It must be noted that the Fuzzy Zero Point Method has been used for fully fuzzy transportation problems. The other methods have been applied after ranking the fuzzy values, namely, to the crisp transportation problem. Finally, the results have been fuzzified and compared.

4.1. Fuzzy Zero Point Method

Step 1: The minimum value of each row in the transportation costs table is determined. Then, these values are subtracted from the costs. This operation must be performed for all the rows in the table.

Step 2: The same operations are applied to all columns. When the costs, supply, and demand are crisp numbers, it is easy to rank them but if they are fuzzy numbers, they must be ranked considering their magnitudes as shown in Equation 5 and Equation 6. After the subtracting process is completed, zero values will appear in the rows.

Step 3: Horizontal and vertical lines are drawn to cover all zero values. However, these lines should not be drawn arbitrarily, the number of lines should be as few as possible.

Step 4: After the lines are drawn, some cells remain under the lines, and some cells remain outside the lines. The smallest cost is determined among the costs not covered by the lines. This smallest value is subtracted from all values under the line and added to the values at the intersection of the lines.

Step 5: Supply and demand balance is checked. Column totals must be greater than or equal to the sum of the supply quantities corresponding to the zero values in the columns.

Step 6: When the supply and demand balance is achieved for all the columns, the allocation process starts (Pandian & Natarajan, 2010: 84).

Step 7: In the allocation process, the sum of rows and the sum of columns (supply and demand) must be written as fuzzy numbers.

Step 8: If the transportation cost matrix is balanced, allocation starts with the cells containing zero values.

Step 9: After the allocation process is completed, the transportation cost is obtained by multiplying the fuzzy numbers with the fuzzy costs. If desired, this fuzzy number can be defuzzify by calculating its' magnitude (See Equations 3 and 4).

4.2. The Northwest Corner Method

The method was first proposed by George B. Dantzig in 1951. In 1954; Charnes and Cooper also applied this method (Charnes&Cooper, 1954: 52). This method is carried out through the following steps:

Step 1: Supply and demand must be balanced. If, $\sum S_i = \sum D_j$ is not achieved, a dummy variable must be added.

Step 2: The starting point of the allocation is the A_{11} cell which is in the far North West. The quantity of the allocation must be equal to min(S_1, D_1). If min(S_1, D_1) equals S_1, then the first row is closed for a new allocation. The total demand for the first row is now D_1-S_1.

Step 3: All subsequent allocations will adhere to the same pattern, always to the far North West cell.

Step 4: Total transportation cost is calculated by summing the product of quantities and the transportation cost per unit.

4.3. The Least Cost Method

The least-cost method was first introduced by Hitchcock. The method focuses on delivering products from production facilities to customer locations with the minimum cost (Hitchcock, 1941: 224). This method is carried out through the following steps:

Step 1: Supply and demand must be balanced. If this is not achieved, a dummy variable must be added. The starting point of the allocation is the cell which has the least transportation cost.

Step 2: Subsequent allocations will adhere to the same pattern, always to the cell that has the least transportation cost.

Step 3: Total transportation cost is calculated by summing the product of quantities and the transportation cost per unit.

4.4. Russel's Method

Step 1: Subtraction operations are performed for each cell in the transportation costs matrix. The sum of the row and the sum of the column are subtracted from each transportation cost where it is located. Equation 12 demonstrates the subtraction operations:

$$C_{11}-S_1-D_1=a, \qquad C_{12}-S_1-D_2,=b, \qquad \ldots, \qquad C_{nn}-S_n-D_n=g \qquad (12)$$

The starting point of the allocation is the cell which has the minimum difference value. It must be noted that the differences; { a, b, …, g}, may also be negative values.

Step 2: These processes go on until all resources have been allocated.

Step 3: Total transportation cost is calculated by summing the product of quantities and the transportation cost per unit.

4.5. Vogel's Method

Linear programming problems can be solved by the simplex method basically, but this is a compelling process. Instead, Vogel's method which is far more practical can be used. However, it must be noted that Vogel's Method does not guarantee the optimal solution (Shore, H. H., 1970: 441).

Step 1: A new row and a new column are added to the far right side and the bottom of the transportation cost matrix. These new columns are the difference vectors. The values that take place in difference vectors are the differences between the least two transportation costs. For example, if there are 3 supply and 4 demand points in a transportation problem, the creation of the difference vectors is shown in Table 3.

Table 3. Calculating the Differences between Transportation Problem Matrix

	1	**2**	**3**	**4**	**Supply**	**Difference**
1	4	2	8	3	100	3-2=1
2	9	7	7	5	100	7-5=2
3	5	5	2	2	100	2-1=1
Demand	75	75	75	75		
Difference	5-4=1	5-2=3	7-2=5	3-2=1		

Step 2: The allocation starts at the point where the biggest difference takes place. In Table 3, it is seen that the starting point is the third column because its difference value is 5 and this is bigger than all of the other differences. In the third column, the minimum cost cell is A_{33}. That is the first cell to be allocated. The allocated quantity is min(S_3, D_3) and it is 75. Thus the third column is closed for any other allocations and 25 remains at the third row for allocation.

Step 2: These processes go on until all resources have been allocated.

Step 3: Total transportation cost is calculated by summing the product of quantities and the transportation cost per unit.

5. NUMERICAL EXAMPLE

A numerical example is drawn from Pandian ve Natarajan's article (2010). It was solved initially by Fuzzy Zero Point Method and all intermediate steps of the solution have been provided. Then, by adopting Basirzadeh's Fuzzy Ranking Approximation Method, the same numeric example was solved with the Northwest Corner Method, Least Cost Method, Vogel's Method, and Russel's Method. Finally, the results were compared.

The numeric example which is completely composed of trapezoidal fuzzy numbers is seen in Table 4.

Table 4. Fuzzy Transportation Costs Matrix

	1	2	3	4	Supply
1	(1,2,3,4)	(1,3,4,6)	(9,11,12,14)	(5,7,8,11)	(1,6,7,12)
2	(0,1,2,4)	(-1,0,1,2)	(5,6,7,8)	(0,1,2,3)	(0,1,2,3)
3	(3,5,6,8)	(5,8,9,12)	(12,15,16,19)	(7,9,10,12)	(5,10,12,17)
Demand	(5,7,8,10)	(1,5,6,10)	(1,3,4,6)	(1,2,3,4)	

Source: Pandian and Natarajan, 2010: 84

This fuzzy transportation problem (FTP) is balanced, because $\sum \tilde{S} = \sum \widetilde{D}$. But it can not be seen at first glance easily, because fuzzy numbers should be ordered by magnitude.

The total supply is; $\sum \tilde{S} = (6,17,21,32)$ and the total demand is; $\sum \widetilde{D} = (8,17,21,30)$, their magnitudes are; $Mag(\tilde{S}) = \frac{6+(5x17)+(5x21)+32}{12}$=19 and $Mag(\widetilde{D}) = \frac{8+(5x17)+(5x21)+30}{12}$=19. As their magnitudes are equal, this FTP is called a balanced transportation problem.

5.1. Fuzzy Zero Point Method

Step 1: The magnitudes of the trapezoidal fuzzy numbers are determined. Because the ordering of the fuzzy numbers is based on their magnitudes (Pandian, 2010: 82).

See Table 4, the $\tilde{A}_{11}$ is (1,2,3,4), $Mag(\tilde{A}_{11}) = \frac{1+(5x2)+(5x3)+4}{12} = 2{,}5\ldots.,\ Mag(\tilde{A}_{34}) = \frac{7+(5x9)+(5x10)+12}{12} = 9{,}5$.

It must be noted that the magnitudes of trapezoidal fuzzy numbers were calculated to rank the fuzzy values. We will keep using fuzzy forms of them for the solution of the Fuzzy Zero Point Method. The magnitudes of the fuzzy values are seen in Table 5.

Table 5. Magnitudes of Fuzzy Costs

	1	2	3	4
1	2,5	3,5	11,5	7,5
2	1,5	0,5	6,5	1,5
3	5,5	8,5	15,5	9,5

Step 2: The minimum value of the first row is "2,5". We subtract "2,5" from all the costs in the first row and do the same operations for all the rows. Please note that the operations are performed using trapezoidal fuzzy numbers in Table 4. Subtracting trapezoidal fuzzy costs for the rows are seen in Table 6.

Table 6. Subtracting Minimum Costs for the Rows

$\tilde{A}_{11} = 0$
$\tilde{A}_{12} = (1,3,4,6) - (1,2,3,4) = (-3,0,2,5)$
$\tilde{A}_{13} = (9,11,12,14) - (1,2,3,4) = (5,8,10,13)$
$\tilde{A}_{14} = (5,7,8,11) - (1,2,3,4) = (1,4,6,10)$
$\tilde{A}_{21} = (0,1,2,4) - (-1,0,1,2) = (-2,0,0,5)$
$\tilde{A}_{22} = 0$
$\tilde{A}_{23} = (5,6,7,8) - (-1,0,1,2) = (3,5,7,9)$
$\tilde{A}_{24} = (0,1,2,3) - (-1,0,1,2) = (-2,0,2,4)$
$\tilde{A}_{31} = 0$
$\tilde{A}_{32} = (5,8,9,12) - (3,5,6,8) = (-3,2,4,9)$
$\tilde{A}_{33} = (12,15,16,19) - (3,5,6,8) = (4,9,11,16)$
$\tilde{A}_{34} = (7,9,10,12) - (3,5,6,8) = (-1,3,5,9)$

The results of the fuzzy subtraction of minimum costs for rows are seen in Table 7.

Table 7. The Results of the Subtracting Minimum Costs for the Rows

	1	**2**	**3**	**4**	**Supply**
1	0	(-3,0,2,5)	(5,8,10,13)	(1,4,6,10)	6,5
2	(-2,0,2,5)	0	(3,5,7,9)	(-2,0,2,4)	1,5
3	0	(-3,2,4,9)	(4,9,11,16)	(-1,3,5,9)	11
Demand	7,5	5,5	3,5	2,5	

Step 3: The same operations were repeated for all the columns. The minimum value of the first column is "0". There is no need for subtraction. The same condition holds for the second column because the minimum value in the second column is "0". When we look at the third column, we can not see the minimum fuzzy value, so we look at Table 5. Subtracting trapezoidal fuzzy costs for the columns are seen in Table 8. The minimum fuzzy value takes place in A_{23}. In the fourth column, the minimum fuzzy value takes place in A_{24}.

Table 8. Subtracting Minimum Costs for the Columns

$\tilde{A}_{13} = (5,8,10,13) - (3,5,7,9) = (-4,1,5,10)$
$\tilde{A}_{23} = 0$
$\tilde{A}_{33} = (4,9,11,16) - (3,5,7,9) = (-5,2,6,13)$
$\tilde{A}_{14} = (1,4,6,10) - (-2,0,2,4) = (-3,2,6,12)$
$\tilde{A}_{24} = 0$
$\tilde{A}_{34} = (-1,3,5,9) - (-2,0,2,4) = (-5,1,5,11)$

The results of the fuzzy subtraction of minimum costs for rows are seen in Table 9.

Table 9. The Results of the Subtracting Minimum Costs for the Columns

	1	**2**	**3**	**4**	**Supply**
1	0	(-3,0,2,5)	(-4,1,5,10)	(-3,2,6,12)	6,5
2	(-2,0,2,5)	0	0	0	1,5
3	0	(-3,2,4,9)	(-5,2,6,13)	(-5,1,5,11)	11
Demand	7,5	5,5	3,5	2,5	

Step 4: Total demand for each column must be less than or equal to the total of the supplies that take place at the same row with zero values. In Table 10, the total demand is 7,5 for the first column. As 7,5<(6,5+11)=17.5, this column is balanced. The total demand for each row must be more than or equal to the total of demands that take place in the same column with the zero values. The "X" marks in Table 10 demonstrate the unbalanced columns and the unbalanced rows. It can be seen in Table 10 that only the first row and the first column are balanced.

Table 10. Balancing the Demands and the Supplies

	1	**2**	**3**	**4**	**Supply**	
1	0	(-3,0,2,5)	(-4,1,5,10)	(-3,2,6,12)	6,5	
2	(-2,0,2,5)	0	0	0	1,5	X
3	0	(-3,2,4,9)	(-5,2,6,13)	(-5,1,5,11)	11	X
Demand	7,5	5,5	3,5	2,5		
		X	X	X		

Step 5: Horizontal and vertical lines have been drawn to cover all zero values. Two lines were drawn, the first line covers the second row, and the second line covers the first column. The cells which are covered by the lines are A_{11}, A_{21}, A_{31}, A_{22}, A_{23} and A_{24}. The intersection cell is A_{21} (the intersection point of the two lines). The value of the minimum cost, which is uncovered by the lines, which is A_{12} (See Table 5), is subtracted from the covered cells (A_{11}, A_{21}, A_{31}, A_{22}, A_{23}, and A_{24}) and added to the intersection cell (A_{21}). Please note that the values in Table 5 are used for determining the minimum value. But we still subtract the trapezoidal fuzzy costs. The results after the first line drawing are seen in Table 11.

Table 11. The Results after the First Line Drawing

	1	**2**	**3**	**4**	**Supply**	
1	0	0	(-9,-1,5,13)	(-8,0,6,15)	6,5	
2	(-5,0,4,10)	0	0	0	1,5	
3	0	(-8,0,4,12)	(-10,0,6,16)	(-10,-1,5,14)	11	X
Demand	7,5	5,5	3,5	2,5		
			X	X		

As can be seen in Table 11, the third and the fourth column and the third row are not balanced. So, we have to draw lines again.

Step 6: Three lines have been drawn. The first line covers the second row, the second line covers the first column, and the third line covers the second column. The cells covered by the lines are; A_{11}, A_{21}, A_{31}, A_{12}, A_{22}, A_{32}, A_{23} and A_{24}. This time there are two intersection points. These cells are A_{21} and A_{22}. The value of the minimum cost, which is uncovered by the lines, which is A_{13} (See Table 5), is subtracted from the covered cells (A_{11}, A_{21}, A_{31}, A_{12}, A_{22}, A_{32}, A_{23}, and A_{24}) and added to intersection cells A_{21} and A_{22}.

Table 12. The Results after the Second Line Drawing As Described in Step 6

	1	2	3	4	Supply	
1	0	0	0	(-21,-5,7,24)	6,5	
2	(-14,-1,9,23)	(-9,-1,5,13)	0	0	1,5	
3	0	(-8,0,4,12)	(-23,-5,7,25)	(-23,-6,6,23)	11	X
Demand	7,5	5,5	3,5	2,5		
				X		

As can be seen in Table 12, the fourth column and the third row are not balanced. So, we have to draw lines again.

Step 7: Three lines have been drawn again. The first line covers the first row, the second line covers the second row, and the third line covers the first column. The cells covered by the lines are; A_{11}, A_{21}, A_{31}, A_{12}, A_{13}, A_{14}, A_{22}, A_{23} and A_{24} there are two intersection points; A_{11} ve A_{21}. The value of the minimum cost, among uncovered values by the lines, which is A_{34} (See Table 5), is subtracted from the covered cells (A_{11}, A_{21}, A_{31}, A_{12}, A_{13}, A_{14}, A_{22}, A_{23}, and A_{24}) and added to intersection cells A_{11} and A_{21}.

Table 13. The Results after the Third Line Drawing As Described in Step 6

	1	2	3	4	Supply	
1	(-23,-6,6,23)	0	0	(-21,-5,7,24)	6,5	
2	(-37,-7,15,46)	(-9,-1,5,13)	0	0	1,5	
3	0	(-31,-4,10,35)	(-46,-9,11,48)	0	11	X
Demand	7,5	5,5	3,5	2,5		

As can be seen in Table 13, the third row is not balanced yet. So, we have to draw lines again.

Step 8: This time, four lines have been drawn. The first line covers the first row, the second line covers the second row, the third line covers the first column, and the fourth line covers the fourth column. The cells which are covered by the lines are A_{11}, A_{21}, A_{31}, A_{12}, A_{22}, A_{13}, A_{23}, A_{14}, A_{24} and A_{23}. There are four intersection points. These cells are A_{11}, A_{21}, A_{14} and A_{24}. The value of the minimum cost, which is uncovered by the lines, is A_{34} (See Table 5), it is subtracted from the covered cells (A_{11}, A_{21}, A_{31}, A_{12}, A_{22}, A_{13}, A_{23}, A_{14}, A_{24}, and A_{23}) and added to intersection cells A_{11}, A_{21}, A_{14}, and A_{24}.

As it is seen in Table 14, the FTP is balanced.

Table 14. Balanced Quantities of FTP

	1	**2**	**3**	**4**	**Supply**
1	X	(1,5,6,10)	(-9,0,2,11)	X	(1,6,7,12)
2	X	X	(0,1,2,3)	X	(0,1,2,3)
3	(5,7,8,10)	X	(-9,-1,3,11)	(1,2,3,4)	(5,10,12,17)
Demand	(5,7,8,10)	(1,5,6,10)	(1,3,4,6)	(1,2,3,4)	

Step 9: Total transportation cost is calculated by summing the product of the fuzzy costs (See Table 4) and allocated fuzzy quantities (See Table 14)

$\tilde{z}$ =[(1,3,4,6)x(1,5,6,10)]+[(9,11,12,14)x(9,0,2,11)]+[(5,6,7,8)x(0,1,2,3)]+[(3,5,6,8)x(5,7,8,10)]+[(12,15,16,19)x(-9,1,3,11)]+[(7,9,10,12)x(1,2,3,4)]=(-274,58,188,575)

When getting the fuzzy result, it is easy to convert it to a certain value that is the magnitude of the fuzzy number (See Equation 3 and Equation 4).

$\mathrm{Mag}(\tilde{z}) = 132{,}17$

Solving an FTP using the Fuzzy Zero Point Method is a compelling process. So, in this study, we will continue our solutions by adopting Basirzadeh's method. The method is based on ranking the fuzzy numbers. The steps of the method are as follows (Basirzadeh, 2011: 1559-1560);

Step1: Convert the fuzzy data into crisp data using the magnitudes formula (See Equation 3 and Equation 4)

Step 2: Create a transportation matrix with crisp data.

Step 3: Solve it by any optimization method. The result may remain as a crisp value, or it can be fuzzified.

Solutions in the following subsections are based on Basirzadeh's Method. The FTP is the same as the previous one. It has been solved using the West Corner Method, Least Cost Method, Russel's Method, and Vogel's Method. The fuzzy results also have been calculated for all these mentioned optimization methods.

5.2. The Northwest Corner Method

The results of the allocation are seen in Table 15.

Table 15. Allocated Transportation Problem According to the Northwest Corner Method

	1	**2**	**3**	**4**	**Supply**
1	6.5	X	X	X	6,5
2	1	0.5	X	X	1,5
3	X	5	3.5	2.5	11
Demand	7,5	5,5	3,5	2,5	

Z=(2.5x6.5)+(1.75x1)+(0.5x0.5)+(8.5x5)+(15.5x3.5)+(9.5x2.5)=138.75

Fuzzy solution of the northwest corner method:

$\tilde{z} = (90.5,129,148,187.5)$

$$\text{Mag}(\tilde{z}) = \frac{90.5 + (5x129) + (5x148) + 187.5}{12} = 138.67$$

5.3. The Least Cost Method

The results of the allocation are seen in Table 16.

Table 16. Allocated Transportation Problem According to the Least Cost Method

	1	**2**	**3**	**4**	**Supply**
1	2,5 6,5	X	X	X	X
2	X	0,5 1,5	X	X	X
3	5,5 1	8,5 4	15,5 3,5	9,5 2,5	X
Demand	X	X	X	X	

Z=(2,5x6,5)+(5,5x1)+(0,5x1,5)+(8,5x4)+(15,5x3,5)+(9,5x2,5)=134,5

Fuzzy solution of the least cost method:

$\tilde{z} = (88,125,144,181.5)$

$$\text{Mag}(\tilde{z}) = \frac{88 + (5x125) + (5x144) + 181,5}{12} = 134,58$$

5.4. Russel's Method

The results of the allocation are seen in Table 17.

Table 17. Allocated Transportation Problem According to Russel's Method

	1	**2**	**3**	**4**	**Supply**
1	X	5.5	1	X	6,5
2	X	X	1.5	X	1,5
3	7,5	X	1	2,5	11
Demand	7,5	5,5	3,5	2,5	

Z = (5,5x7,5)+(3,5x5,5)+(11,5x1)+(6,5x1,5)+(15,5x1)+(9,5x1)=121

Fuzzy solution of Russel's method:

$\tilde{z}$=(74,111.5,130.5,168)

$$\text{Mag}(\tilde{z}) = \frac{74 + (5x111{,}5) + (5x130{,}5) + 168}{12} = 121$$

5.5. Vogel's Method

Step 1: The starting point of the allocation is the cell A_{24}. Because the biggest difference is in the fourth column and the least cost cell in the fourth column is A_{24} (See Table 18)

Table 18. The Results of the Differences in the First Step

	1	**2**	**3**	**4**	**Supply**	**Difference**
1	2,5	3,5	11,5	7,75	6,5	1
2	1,75	0,5	6,5	**1,5**	1,5	1
3	5,5	8,5	15,5	9,5	11	3
Demand	7,5	5,5	3,5	2,5		
Difference	0,75	3	5	**6,25**		

Step 2: The biggest difference is in the second column so the allocation has been done to the least costed cell A_{12} (See Table 19)

Tablo 19. The Results of the Differences in the Second Step

	1	2	3	4	Supply	Difference	Difference
1	2,5 1	3,5 5,5	X	X	6,5	1	1
2	X	X	X	1,5 1,5	1,5	1	X
3	5,5 6,5	X	15,5 3,5	9,5 1	11	3	3
Demand	7,5	5,5	3,5	2,5			
Difference	0,75	3	5	6,25			
Difference	3	5	4	175			

Step 3: Allocation has not been completed yet, so we keep calculating the differences. The biggest difference is in the third column, so the allocation has been done to the A_{33} which is the only empty cell (See Table 20).

Tablo 20. The Results of the Differences in the Third Step

	1	2	3	4	Supply	Difference	Difference	Difference
1	2,5 1	3,5 5,5	X	X	6,5	1	1	1
2	X	X	X	1,5 1,5	1,5	1	X	X
3	5,5 6,5	X	**15,5** 3,5	9,5 1	11	3	3	3
Demand	7,5	5,5	3,5	2,5				
Difference	0,75	3	5	6,25				
Difference	3	5	4	1,75				
Difference	3	X	**4**	1,75				

z=(2,5x1)+(5,5x6,5)+(3,5x5,5)+(15,5x3,5)+(9,5x1)+(1,5x1,5)=123,5

Fuzzy solution of Vogel's method:

$\tilde{z} = (75,114,133,172)$

$$\text{Mag}(\tilde{z}) = \frac{75 + (5x114) + (5x133) + 172}{12} = 123{,}5$$

The results of the optimization methods are seen in Table 21.

Table 21. Comparison of the Optimization Methods

Method	**z**	$\tilde{z}$	$\mathbf{Mag}(\tilde{z})$
Fuzzy Zero Point Method	X	(-274,58,188,575)	132,17
North West Corner Method (by fuzzy ranking)	138,75	(90.5,129,148,187.5)	138,67
Least Cost Method (by fuzzy ranking)	134,5	(88,125,144,181.5)	134,58
Russel's Method (by fuzzy ranking)	121	(74,111.5,130.5,168)	121
Vogel's Method (by fuzzy ranking)	123,5	(75,114,133,172)	123,5

6. CONCLUSION

In this paper, a trapezoidal fuzzy transportation problem has been solved using various optimization methods. Firstly, the Fuzzy Zero Point Method has been applied to Pandian and Natarajan's numeric Fuzzy Transportation Problem (FTP) which is a fully trapezoidal fuzzy transportation problem. Subsequently, Northwest Corner, Least Cost, Russel, and Vogel's methods have been applied to the same FTP by adopting Basirzadeh's Fuzzy Ranking Method; namely to the crisp version of the problem. Although fully fuzzy methods minimize loss of information; operating with fuzzy values is a compelling process. However, using Basirzadeh's method is far more practical, and fuzzy results can easily be obtained once the allocation has been completed. According to the findings of this study, Russel's method appears to be the best method among the methods evaluated.

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