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Implementing Mathematical Modelling with Calculus of Variations to Design a Disaster Tent

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Abstract. This manuscript shares results from a mathematical modelling project developed by a mathematics educator and a high school student to solve a real-life problem; durable disaster tents. The authors worked together to first design a tent, CaTent, by implementing biomimicry with design thinking. Through the process of mathematical modelling, the authors mathematise the problem with catenary which can be obtained by solving a calculus of variations problem. Then, reaching the equation for catenary curve modelling the poles of CaTent, the length of a pole is obtained, approximately 7.2834 meters. The total length of three poles necessary for a CaTent would be 21.8503 meters approximately, while the total amount of poles needed for a common disaster tent would be approximately 40.32 meters.

Keywords. Mathematical modelling, calculus of variations, catenary, design thinking, biomimicry.

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This manuscript describes the process of development of a mathematical modelling project with a design thinking approach. The project is completed by a team of a high school student and a mathematics education scholar. The scholar guided the team in design thinking, mathematical modelling and the underlying mathematical topic of calculus of variations. In this manuscript the design created, CaTent, will be introduced with mathematical underlying explanations to reach the CaTent Design.

As part of mentoring the high school student, the mentor scholar adopted a design thinking approach. The purpose of this collaboration was to enhance a talented high school students' proficiency in STEM (Science, Technology, Engineering and Mathematics) through completing a mathematical modelling project. The definition of STEM has been a focus of attention for years (Akaygun, Aslan-Tutak, & Özel, 2020). There may be different approaches to STEM education, yet the central idea is to integrate at least two disciplines to create a solution for a complex problem. The nature of STEM education allows for an interdisciplinary perspective for examining real-life problems and designing solutions for them. The Turkish High School mathematics curriculum has emphasised mathematical modelling but provides limited opportunities for STEM education applications. The second author of this project is a science high school student who is very interested in mathematics but has limited experience in STEM education projects. Through the mentoring of the first author, a scholar in mathematics education, the team was able to adopt a STEM education approach to design a real-life object using mathematical modelling.

Design Thinking and Mathematical Modelling

The literature on design thinking provides various approaches. The chair of the innovation company IDEO, Brown (2008) identifies three stages of design thinking: inspiration, ideation and implementation (p. 5). Brown discusses the iterative nature of this process while need for following some steps for a fruitful design thinking process. Girgin (2021) summarizes design thinking as an interdisciplinary approach of Brown's five steps of design thinking: empathize, define, ideate, prototype and test. Even though these steps are considered as part of a process to follow, at the same time they are non-linear and dynamic (Scheer, Noweski & Meinel; 2011). The designer would use the information gathered at empathize stage to define problem while the last stage (test) would inform both empathize and define stage for the designer to understand the situation further. Also, the designer may experience an iterative process for ideate stage and prototype stage. Upon producing a prototype based on an idea, a designer may create new ideas. This iterative nature of design thinking may seem challenging for many novice designers. Considering educational settings, students might find the work of design daunting task, thus having steps to define the design process helps educators to make design thinking accessible for many students. For this study, the authors implemented design thinking process for a real life problem that incorporates STEM approach. There are various educational approaches to implement design thinking with students such as Double-Diamond Model by British Design Council, Hynes, Stanford d. School.

When these models examined, the iterative nature of design process as well as the process of understanding to implementation stay same (Karatas & Aslan-Tutak, 2023). For this study, the authors used Stanford d. School Framework which provides flexibility to be used in or out of school context as well as well-defined steps to be followed. As in their own website, Stanford d. School has vision of “A place for explorers & experimenters at Stanford University.” The d. school can be considered one of the maker spaces that were spreading through many engineering universities. While working on various projects, the experts at d.school incorporated a systemic way of guiding students in design projects. For the current study, Stanford d. School approach is adapted to lead the second author, a high school student, into designing a solution for shelter problem in disasters. This framework can be considered in six steps: empathise, define, ideate, prototype, test, and assess. In this manuscript, the authors describe a project they used in a mentoring context instead of a school setting.

Mathematical models represent complex real-world systems mathematically, thus leading to solutions for them. “A mathematical model is a mathematical structure that approximates the features of a phenomenon.” (Swetz & Hartzler, 1991, p.1). Mathematical modelling has been the centre of many national curricula and the Turkish Mathematics curriculum. Teaching mathematics with mathematical modelling tasks has a promising effect on students' achievement and process skill development. While a student completing a mathematical modelling task, it can be considered to have cyclical process with four steps: description, manipulation, translation (or prediction), and verification (Lesh, & Doerr, 2003). This four step process starts with learners to describe the real-life phenomena as transformed in mathematical word problem which can be done through considering variables with their relationships. Then mathematical concepts are used to solve the problem in manipulation step. This leads to translation step in which learners make interpretations of the results for real-life phenomena. Sometimes learners may stop this stage but it’s crucial to complete the final step of verification as learners reflect on their interpretations to check for errors. This last step allows learners to have a chance to revise and repeat the modelling process, thus making the task of mathematical modelling dynamic rather than finding a single solution.

Even though the process can be considered in four steps to complete, this task is not an easy one for many learners (Schaap et al., 2011; Sol et al., 2011). When a learner lacks the necessary mathematical knowledge, conceptual or procedural, both description and manipulation steps becomes difficult for the learner. Also, managing the process of the mathematical modelling turns into a challenge for a novice leaner. Teachers need to support learners in the process of mathematical modelling from mathematical aspect as well as guiding through the steps of the modelling.

It should be noted that mathematical modelling is not limited with classroom settings. Indeed, mathematical modelling plays a central role in various STEM education tasks (Aslan-Tutak, 2020). STEM education calls for integration of two or more disciplines from science, technology, engineering and mathematics. Biomimicry is defined as “a design method that draws on the inspiration of Nature for more sustainable solutions to human challenges.” (Chen, Klotz & Ross, 2016, p. 497). The aim of adopting biomimicry approach in designs is to look for patterns in nature that can be utilized for engineering problems. Biomimicry is based on the assumption of nature presenting efficient solutions to various problems that humans face (Biomimicry Institute, 2019). Sanne, Risheim and Impelluso (2019) conducted a project with primary and lower secondary school students from Norway. Researchers aimed to incorporate sustainability in the four-module program that was prepared to use biomimicry to integrate science, engineering and mathematics to provide learning opportunities to students. The findings show that “It had positive affect on student understanding of the role of mathematics. It inspired some students to study engineering. It enabled them to see the value of sustainable engineering design.”

Within the scope of the project presented in this manuscript, a tent has been designed to meet the sheltering needs of people in natural disasters and emergencies in a fast and safe way. Mathematical modelling and biomimicry principles, which are effective methods in proposing solutions to many problems in real life, were used together to design these tents that will be fast and safe to install. In this way, a solution has been produced / provided for the design of the tent model away from aesthetic concerns and suitable for different climatic and real life conditions. This manuscript aims to share the mathematical modelling process of a scholar and a high school student with a design thinking and mathematical modelling approach.

Method

The methods section of the manuscript consists of a real-life problem that provides the basis, and mathematisation of the problem by using definitions from calculus of variations, an advanced branch of mathematics.

The Real-life Problem: Sheltering During Natural Disasters

Due to natural disasters and emergencies, many people become homeless and live in tents. Especially in earthquake-prone regions such as Türkiye, it is imperative to find fast and safe solutions in emergencies. In the earthquake on 6 February 2023, many people lost their homes, and in this process, people's urgent need for shelter arose, so many tents were needed. One of the most challenging factors in using tents is the *rapid* construction of safe and *durable* tents.

Mathematising the Problem through Biomimicry

Biomimicry briefly means finding solutions to various problems inspired by nature and imitating nature. Examples of biomimicry are swimming flippers and ducks. Ducks' feet push water more easily due to their structure. This inspired some designers to develop products by examining the feet of ducks. Designs found in nature have survived to the present day with their durability. People use these designs to solve problems in daily life.

A chain of infinitesimally small identical links, fixed at both ends, will take the shape of a certain geometrical curve under gravity. This curve is called catenary. *Catenary* is a mathematical curve. It is the natural shape of a chain in equilibrium when held at both ends. The algebraic equation of the curve was first revealed with the answers given by Leibniz, Huygens and Johann Bernoulli to the question posed by Jacob Bernoulli in 1691. Through the study of "calculus of variations", it was found that this shape is actually related to cosh, a hyperbolic trigonometric function (Çağlar, 2023).

The curve of the catenary is widely observed in nature, naturally providing an optimal balance with gravity, it is therefore used in the design of some organisms or structures. Some examples of catenary in nature are as follows:

- Spider Web: The webs spun by spiders usually follow the chain curve. This curve helps the spider web to provide the best balance and to be resistant to external factors such as wind.

- Catfish web: Underwater, catfish weave a web similar to a chain curve. This structure provides optimal stability and resistance underwater, helping them to catch their prey more effectively.
- Mushrooms with large forks: Some mushroom species mimic the chain curve with their large forked structures. This helps the cork to cover a large area and provide the best durability.
- Load Bearing Cable Systems: In the transport or telecommunications industry, the chain curve is used to balance the loads carried by cable systems and distribute the tension evenly. This principle is particularly common in suspension bridges and cable car systems.

Mankind has encountered various natural disasters since its existence and natural disasters have left deep traces on people. For this reason, various structures have been built to meet the needs for shelter and protection. Among the designs in these structures, igloos and dome-shaped structures have come to the fore with their durability (Handy, 2018). The name of the shape that forms the dome in these structures is defined as 'catenary'. The catenary shape adds strength/durability to the structure by effectively distributing the forces and providing a perfect balance. The tent design is called *CaTent*. For the real-life problem of this project, designing a durable tent with *rapid* construction, the authors first determined the tent's specifications, which were chosen based on the common disaster tents from Kızılay.

The CaTent design principles:

1- For the tent frame, 3 poles and a durable fabric would be used to cover the poles.

2- Two points at a certain distance are joined by poles to form the skeleton of the tent. These poles will be placed at equal intervals and 60 degree angles around a circular axis and all of them are designed to have a chain curve shape.

3- The roof height of the poles of this tent, which is a hemispherical shape, is 2.55 metres, the same as the existing disaster tents. The diameter of the circular axis will be 4.6 metres.

4- After the skeleton of the tent is formed, the skeleton will be covered with a durable fabric.

This design will enable the tent to be set up quickly and easily and to be more durable than other tents. The visual representation of CaTent is given in Figure 1 that is produced by using ThinkerCad.

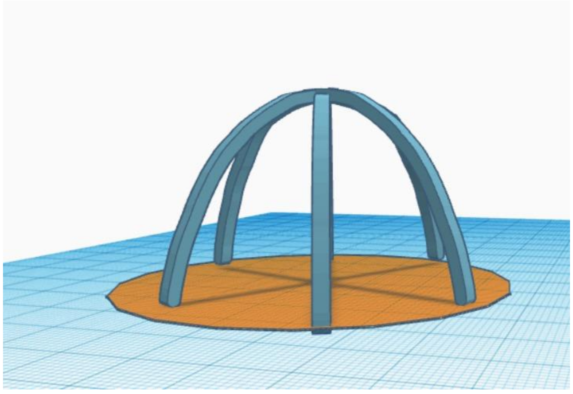

Figure 1. CaTent Design.

This design requires the production of poles with a certain length. When the length of the poles and the distance between the two ends of the poles are predetermined, the poles will take a unique shape. In our case, the aim is to determine the length of the poles in order to create the shape of the catenary. This requires solving an arc length problem for the catenary curve.

Solving Arc Length Problem with Calculus of Variations

Functionals constitute the basic building block of variation calculus. Functionals can be defined as a relation from one set of continuous and differentiable functions to another set. In applied mathematics, in the calculus of variation, functionals are used to determine the maximum/minimum functions. The simplest example of using functionals in this way is to prove that the linear function is the function that gives the shortest distance between two points in the same plane. Euler-Lagrange equations are needed to find the maximum and minimum problems where functionals are employed. This theorem originated as an equation developed by two famous mathematicians, Euler and Lagrange, while working on a problem in the 1750s. The Euler-Lagrange equation allows to find the function that gives the minimum/maximum value when the functional derivative is equal to zero (Forsyth, 1960). The Euler-Lagrange equations are expressed by the formula

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$$

For example, when finding the equation of a curve that has the shortest distance between two points, the functional is defined based on the arc length integration formula.

$$S = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$The\ Functional: F = \sqrt{1+(y')^2}$$

Solving the Euler-Lagrange equation for this functional would result into

$$m = \frac{dy}{dx}$$

This means the solution is a linear function.

In the design of the tent, which is the subject of this project, the Euler-Lagrange theorem will be used to find the equation that gives the curve equation between two points. In this project, the pillars of the tent design can be considered as ‘durability according to natural conditions’. The equation giving the surface area of the 3D object formed by rotating a curve is as follows:

$$Surface\ Area = \int 2\pi\, y\, ds$$

This equation can be written as follows

$$Surface\ Area = \int_{x_1}^{x_2} 2\pi y \sqrt{1+(y')^2}dx$$

This equation can then be used to define function, which would be examined to find the curve that minimises the surface area.

$$F = 2\pi y \sqrt{1+(y')^2}$$

Solving Euler-Lagrange equation for this functional would provide the equation for the curve which would be a durable tent design.

Results

Mathematical Model for CaTent

Since the application of the Euler-Lagrange Theorem involves differentiation with respect to x and since the functional we are studying does not include the variable x, the Euler-Lagrange equation can be solved as follows

$$f - y' \frac{\partial f}{\partial y'} = c$$

Thus in order to reach the equation for CaTent poles, the following equation needs to be solved:

$$2\pi y \sqrt{1+(y')^2} - y' \frac{\partial(2\pi y \sqrt{1+(y')^2})}{\partial y'} = c$$

Dividing both sides with 2π

$$y\sqrt{1+(y')^2} - y'\frac{\partial(y\sqrt{1+(y')^2})}{\partial y'} = K$$

Completing the partial derivative in the second expression

$$y\sqrt{1+(y')^2} - (y')^2 y\frac{1}{\sqrt{1+(y')^2}} = K$$

$$\frac{\left[y\sqrt{1+(y')^2}\right]\left[\sqrt{1+(y')^2}\right] - (y')^2 y}{\sqrt{1+(y')^2}} = K$$

$$\frac{y\,(1+(y')^2) - (y')^2 y}{\sqrt{1+(y')^2}} = K$$

Thus,

$$\frac{y}{\sqrt{1+(y')^2}} = K$$

The aim is to reach a differential equation to solve

$$\sqrt{1+(y')^2} = \frac{y}{K}$$

$$1+(y')^2 = \frac{y^2}{K^2}$$

$$y' = \sqrt{(\frac{y^2}{K^2} - 1)}$$

Thus,

$$\frac{dx}{dy} = \frac{K}{\sqrt{y^2 - K^2}}$$

Separating the variables

$$dy\frac{K}{\sqrt{y^2 - K^2}} = dx$$

Solving the equtaion

$$\int dy \frac{K}{\sqrt{y^2 - K^2}} = \int dx$$

This problem was solved by Hass (2000) as follows:

$$x = K\ cosh^{-1}\left(\frac{y}{K}\right) + \ C \ \ where\ C\ is\ integral\ constant$$

Thus,

$$y = K\ cosh\left(\frac{x - C}{K}\right)$$

$$when\ the\ integration\ constant\ C = 0$$

The result is the hyperbolic trigonometric cosine function (cosh). It was previously discussed that the catenary, with its biomimicry, is a possible design element for CaTent. With the equation obtained above, it is seen that the catenary is the solution.

This equation needs further manipulation to model CaTent poles. C in the equation comes as integration constant. It can be assumed to be zero. In order to depict the influence of this constant, graphs of different equations can be examined. The authors used Desmos to sketch two catenary equations as shown in Figure 2.

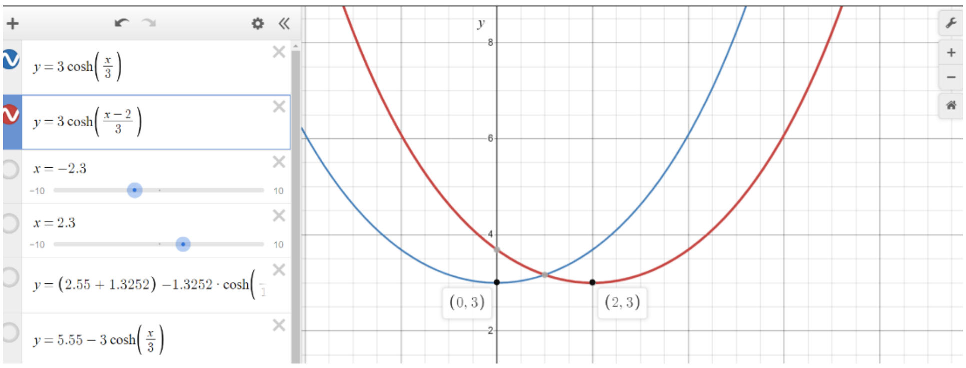

Figure 2. Graphs for c=0, c=2, and k=3.

The value of c would not influence the solving for the CaTent pole design. So, after this point, it will be assumed that c = 0. However, as shown in Figure 2, the curve has concave-up shape while the design of CaTent requires the shape to be concave-down. Also the intersection of the curve with y-axis needs to be a positive real number which will model the height of the CaTent. Thus the equation for CaTent pole design would be

$$y = a - K\ cosh\left(\frac{x}{K}\right)$$

Calculating Pole Length of CaTent

In order to calculate the lengths of the metal rods to be used for the CaTent, it is necessary to calculate the length of the chain curve equation. The values of the Kızılay Tent were used to interpret the CaTent design. Based on the Kızılay Tent, the centre height should be 2.55 m, and the diameter should be 4.6 m (the radius being 2.3 m) in order to have a similar floor area to the Kızılay Tent. In order for the graph of the catenary to fulfil these conditions, it must intercept the y-axis at (0, 2.55), and the zeros of the function must be (2.3,0) and (-2.3,0). In order to find the mathematical equation that gives the CaTent design, constants (**a** and **k**) must be determined.

- Catenary intercepting (0, 2.55) means that $2.55 = a - k \rightarrow a = k + 2.55$
- Catenary intercepting (2.3,0) means that $0 = k + 2.55 - k\cosh(2.3/k)$. Solving this equation requires advanced calculators. The authors used Wolfram Alpha to find the solution as $k \approx 1.3252$

Thus the equation becomes

$$\boldsymbol{y = 3.8752 - 1.3252\ cosh\ (\frac{x}{1.3252})}$$

This graph of this equation is produced by using desmos as shown in Figure 3

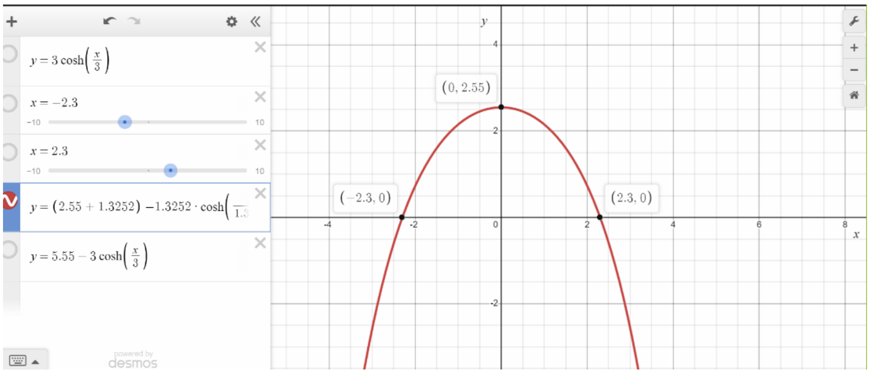

Figure 3. Graph Equation for CaTent Poles.

In order to calculate how much metal rod is needed in the CaTent design, the length of the curve between the points of the equation obtained must be calculated. Due to this catenary equation being symmetric according to y-axis, the integration will be calculated from x=0 to x=2.3.

$$\frac{S}{2} = \int_0^{2.3} \sqrt{1 + \left(\frac{d}{dx}(\mathbf{3.8752 - 1.3252\ cosh}(x/\mathbf{1.3252}))\right)^2}\ dx$$

$$= \int_0^{2.3} \sqrt{1 + (sinh(x/1.3252))^2}dx$$

$$= \int_0^{2.3} cosh\left(\frac{x}{1.3252}\right)\ dx$$

$$= 1.3252\ sinh\left(\frac{2.3}{1.3252}\right) - 1.3252\ sinh\left(\frac{0}{1.3252}\right)$$

$$= 1.3252\ sinh\left(\frac{2.3}{1.3252}\right)$$

$$= 1.3252 \cdot 2.7480$$

$$\frac{S}{2} \cong 3.6417$$

This result is for half of a pole. Since the CaTent design requires three poles, the necessary pole for this design can be obtained at approximately 21.8502 m.

Discussion and Conclusion

The authors also compared this length with the total length of poles for Kızılay tent design. It was calculated that the poles to form a tent with the given base dimensions should be 3.25 metres and 3.84 metres, respectively, and 40.32 metres of metal rods were needed for a total of 6 rods. As a result, the CaTent model requires less material for poles for the tent as well as maintaining the existing dimensions of the existing tent models. The CaTent model is faster and safer than the existing tent models, as well as durable and sustainable.

With this mathematical modelling project, authors proposed a tent design, CaTent, which then examined. The authors were able to merge concepts, calculus of variations and biomimicry through design thinking and mathematical modelling. This examination of CaTent may contribute to further designs of buildings as well as disaster tents by improving durability of structures. For example, this design may inspire more efficient energy production of solar panels and wind turbines.

Last but not least, one should not forget a key point, material selection. In order to ensure that the rods used in tent design do not break when bent and are even more durable, help from material science experts can be obtained during the design phase.

About Authors

First Author: Dr. Aslan-Tutak is an associate professor at the Mathematics and Science Education Department at Bogazici University. She has undertaken roles in commissions to develop the national mathematics curricula. Also, she has served in the Thematic Working Group TWG20 on "Mathematics Teacher Knowledge, Beliefs and Identity" of the biannual Congress of the European Society for Research in Mathematics Education. Dr. Aslan-Tutak has participated in various international and national projects, and her research interests are teacher education, mathematical knowledge for teaching, area-specialized mentoring, professional development to improve in-service teacher practices, classroom-based assessment, and interdisciplinary mathematics education.

Second Author: Ozan Güven is a senior student at FMV Işık Erenköy Science High School. He has been interested in mathematics from a young age. He took part in various national and international competitions. He has a particular interest in advanced mathematics, which led him to pursue a mathematical modelling project in advanced calculus.

Conflict of Interest

There is no conflict of interest.

Funding

No funding was received.

Ethical Standards

We have carried out the research within the framework of the Helsinki Declaration. The research does not include any harmful implementation, and the researchers do not obtain any special or sensitive information. There was no data collection with humans.

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