GAZİOSMANPAŞA BİLİMSEL ARAŞTIRMA DERGİSİ (GBAD)
Gaziosmanpasa Journal of Scientific Research
ISSN: 2146-8168
http://dergipark.gov.tr/gbad
Araştırma Makalesi (Research Article)

Cilt/Volume : 13
Sayı/Number: 3
Yıl/Year: 2024
Sayfa/Pages: 123-135

Alınış tarihi (Received): 28.11.2024
Kabul tarihi (Accepted): 10.12.2024

Design of PID Controller with Set-point Filter Based on Time Response Specifications for Fractional Order Systems

Hasan SARMAN[1], **Tufan DOĞRUER**[1,*]

[1] *Tokat Gaziosmanpasa University, Electrical and Electronics Engineering, Tokat, Türkiye*
**Corresponding author: tufan.dogruer@gop.edu.tr*

ABSTRACT: This article addresses the development of a Proportional-Integral-Derivative (PID) controller design and set-point filter based on time response specifications for fractional-order systems. Fractional-order systems are challenging to control using traditional control methods due to the complexity of their dynamic behaviour. In this study, a set-point filter is integrated into the PID controller design to ensure the desired system performance and stability. During the design process, appropriate PID parameters and filter coefficients are determined by considering the system's time response specifications. The article first examines the mathematical models of fractional-order systems using integer-order approximation methods and then determines the PID controller parameters for these systems with an Improved-Grey Wolf Optimization-based optimization algorithm. The proposed approach aims to improve performance criteria such as overshoot, rise time, and settling time based on the desired system response. Simulation results demonstrate the effectiveness of the proposed method in improving system performance. This design approach offers significant potential to control complex systems commonly encountered in industrial applications.

Keywords – *Fractional order systems, PID controller, I-GWO algorithm, Optimization, Parameter tuning*

1. Introduction

Although fractional-order mathematics has advanced significantly in recent decades, the origins of the topic date back to ancient times. The first idea about this topic emerged in 1695 during correspondence between Leibnitz and L'Hospital. In the following years, this subject began to attract more attention from researchers. The mathematical modeling and simulation of systems and processes expressed by fractional-order integro-differential equations has led to the need for solving fractional-order differential equations. With a better understanding of fractional-order mathematics, its applications in many fields have also started to increase. As in many fields, the impact of fractional-order mathematics has begun to be felt in control theory as well. One of the first studies on this subject, the position control of large objects, was presented by Tustin in 1958 (Tustin et al., 1958). In 1960, Manabe presented a study on the application of fractional-order integration to control systems (Manabe, 1961). In the following years, the use of fractional-order integro-differential expressions in control applications and robotics has become more frequent.

After the concept of fractional-order mathematics emerged, some mathematicians made approximate definitions regarding fractional-order derivatives and integrals. The models of fractional-order control systems with high integer orders can be obtained with the help of certain approaches. In this context, approaches such as Continuous Fraction Expansion (CFE) (Podlubny et al., 2002), Matsuda method (Matsuda & Fujii, 1993), Oustaloup method (Oustaloup et al., 2000), Carlson method (Carlson & Halijak, 1964), and Charef method (Charef et al., 1992) can be used.

The concept of fractional order mathematics has also influenced controller design, which is one of the important topics in control theory. Controller design for fractional order systems is quite significant, and many studies have been conducted in this topic (Dogruer & Tan, 2018, 2019; Li & Gao, 2022; Luo et al., 2010; Zhao et al., 2005). Today, Proportional-Integral-Derivative (PID) controllers are among the most commonly used controller structures due to their numerous advantages. The reasons for this include their simple, reliable, and robust structures, their widespread familiarity, and the small number of parameters. Some of the most important classical design methods for calculating PID controller parameters include Ziegler-Nichols (Ziegler & Nichols, 1942), Åström-Hägglund (Åström & Hägglund, 1995), and Cohen-Coon (Cohen, 1953). Additionally, Refined Ziegler-Nichols (Hang et al., 1991), gain and phase margin-based methods (Cokmez et al., 2018; Ho et al., 1996), and frequency-domain design methods (Li & Gao, 2022; Meng et al., 2020) are also available. However, these methods may not always produce the desired results. Different controller parameters can be found to improve the output response of the control system. Therefore, optimization methods have been developed to determine the optimal control parameters. The aim of these methods is to identify the controller parameters that provide the best response. These parameter tuning methods can yield different results in different control systems, so there is no single method for tuning the best controller parameters.

At this point, optimization-based methods have become an important tool for maximizing the performance of control systems. In particular, nature-inspired methods such as Genetic Algorithms (GA) (Holland John, 1975), Particle Swarm Optimization (PSO) (Eberhart & Kennedy, 1995), and Artificial Bee Colony (ABC) (Karaboga & Basturk, 2007) algorithms are effectively used to determine control parameters. These methods go beyond traditional design approaches and have the ability to optimize multiple parameters simultaneously. One of these algorithms, the Grey Wolf Optimization (GWO) (Mirjalili et al., 2014) algorithm, has been effectively used in many engineering problems in recent years. However, the standard GWO algorithm can sometimes struggle to avoid local minima, and the solution quality may be lower than expected. Therefore, improvements made in the GWO algorithm are important for offering better performance and faster solution processes. The Improved-GWO (I-GWO) (Nadimi-Shahraki et al., 2021) aims to address these shortcomings. The I-GWO algorithm retains the advantages of the classical GWO while providing stronger exploration and exploitation capabilities. This improved version enables more effective results in the optimization of system parameters, offering benefits such as accelerated backtracking, better convergence rates, and reduced computation time. The GWO algorithm is used as a nature-based method to find the optimal values of parameters. By considering both linear and nonlinear behaviours of systems, it suggests more accurate and efficient solutions. The use of the I-GWO algorithm also offers a strong alternative for determining PID parameters. Such algorithms can produce different results in different control systems and can be customized to determine parameters suitable for each system's needs. As a result, the use of the I-GWO algorithm presents an important innovation in control system design, and the increasing prevalence of such optimization methods is expected. These types of improvements will allow for more efficient and effective results both practically and theoretically.

Models obtained using approximation methods are typically of high order, and the control of these systems is more difficult. In this paper, a second-order set-point filter, dependent on the PID controller parameters, is also used to achieve a more precise effect on the system's dynamics, thereby improving the system's response times. This approach allows high-order systems to become more manageable and makes the control processes more stable. In the study, simulation studies were conducted using Matsuda's fourth-order integer approximate model instead of the fractional-order plant. In the optimization algorithm, a multi-objective function based on the time response specifications, along with integral of time-absolute error (ITAE) and integral of time-square error (ITSE) integral performance criteria, was used to carry out the work.

The remainder of the paper is organized as follows: In the second section, the modelling of fractional-order systems and the PID controller design procedure based on the I-GWO-based are discussed, followed by an explanation of fractional-order control theory and its role in optimization processes. The third section tests the performance of the method through simulation studies. Finally, the fourth section discusses the results and evaluates the effectiveness of the methods.

2. Material and Methods

In this section, brief information about fractional order systems is given, as well as the procedure for determining PID controller parameters.

2.1. Fractional order systems

The history of fractional calculus dates back to the late 17th century. Beginning with Leibniz and L'Hospital, this field continued to develop through the works of renowned mathematicians such as Euler, Laplace, Fourier, Abel, and Laurent, laying the foundation for its current popularity. In the 18th century, significant contributions were made by Liouville, Grünwald, Letnikov, and Riemann (Machado et al., 2011). Although early studies were limited due to constrained computational capabilities, advancements in computer technology in recent years have turned fractional calculus into an area of interest for many researchers. In addition to being a branch of mathematics, it has found extensive applications in various disciplines, including signal processing, control theory, electrical circuits, bioengineering, and viscoelasticity.

Fractional order systems are systems that use derivative and integral operators with non-integer orders and are typically used in the modelling of dynamic systems (Sabatier et al., 2007). These types of systems are often defined by fractional order calculus. The general form of fractional derivative and integral operators can be expressed as follows:

$$ {}_aD_t^{\alpha} = \begin{cases} \dfrac{d^{\alpha}}{dt^{\alpha}} & ;\alpha > 0 \\ 1 & ;\alpha = 1 \\ \int_a^t (d\tau)^{-\alpha} & ;\alpha < 0 \end{cases} \qquad (1) $$

- α, represents the fractional order, which typically takes a value between 0 and 1.
- D denotes the fractional derivative operator.

The three important definitions used for fractional-order derivatives and integrals are as follows: the Riemann-Liouville definition, the Grünwald-Letnikov definition, and the Caputo definition (Petráš, 2011). The Riemann-Liouville definitions for fractional-order derivatives and integrals are presented in Equations 2 and 3 (Petráš, 2011). Γ(·) is the Gamma function, commonly used in fractional calculations.

$$ {}_aD_t^{\alpha} f(\mathrm{t}) = \frac{1}{\Gamma(\mathrm{n}-\alpha)} \frac{d^n}{dt^n} \int_{\alpha}^{t} \frac{f(\tau)}{(\mathrm{t}-\tau)^{\alpha-n+1}} d\tau \qquad ; n-1 < \alpha < n \tag{2} $$

$$ {}_aD_t^{-\alpha} f(\mathrm{t}) = \frac{1}{\Gamma(\alpha)} \int_{0}^{t} (t-\tau) f(\tau) d\tau \qquad ; 0 < \alpha < 1,\ t < 0 \tag{3} $$

The Laplace transform is a technique used to simplify the analysis and synthesis of equations. The Laplace transform of the Riemann-Liouville definition for fractional-order derivatives is represented in Equation 4 (Petráš, 2011).

$$ \mathcal{L}\left[{}_aD_t^{\alpha} f(t); s \right] = s^{\alpha} F(s) - \sum_{k=0}^{n-1} s^k \left[{}_aD_t^{\alpha-k-1} f(t) \right]_{t=0} \qquad ; n-1 \leq \alpha < n \tag{4} $$

The Laplace transform of the Riemann-Liouville definition for fractional-order integration is presented in Equation 5.

$$ \mathcal{L}\left[{}_aD_t^{-\alpha} f(t); s \right] = s^{-\alpha} F(s) \tag{5} $$

Fractional-order systems, described by differential equations with real-valued orders, can represent the plant, the controller, or both in a control system. A Fractional Order Control System (FOCS) is modelled by a fractional-order differential equation, where *r(t)* and *y(t)* denote input and output signals, a_k and b_k are constants, and α_k and β_k are real numbers.

$$ a_n D^{\alpha_n} y(t) + a_{n-1} D^{\alpha_{n-1}} y(t) + \ldots + a_0 D^{\alpha_0} y(t) = b_m D^{\beta_m} r(t) + b_{m-1} D^{\beta_{m-1}} r(t) + \ldots + b_0 D^{\beta_0} r(t) \tag{6} $$

Equation 7 represents the transfer function derived from Equation 6.

$$ G(s) = \frac{Y(s)}{R(s)} = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \ldots.. + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \ldots.. + a_0 s^{\alpha_0}} \tag{7} $$

For the Grünwald-Letnikov definition, and the Caputo definition, see reference (Podlubny, 1998).

2.2. PID controllers and tuning procedure

PI (Proportional-Integral) and PID controllers have played a pivotal role in the evolution of control engineering, remaining essential tools for almost seven decades. These controllers are highly valued for their simplicity, effectiveness, and versatility in a wide range of applications. The first breakthrough in controller design came in 1934 when a tuning rule was introduced specifically for setting the parameters of a PD (Proportional-Derivative) controller. Shortly after, in 1935, another tuning rule was developed, extending these concepts to PI and PID controllers, marking a significant milestone in control theory (O'Dwyer, 2012). As industries began to embrace automation, the demand for robust and efficient control systems grew. PI and PID controllers emerged as the standard due to their ability to maintain system stability, improve performance, and adapt to various dynamic environments. These controllers became indispensable in process control applications, where they regulate variables such as temperature, pressure, flow, and level with remarkable precision. Today, PI and PID controllers are estimated to be utilized in over 95% of industrial process control applications, solidifying their reputation as the backbone of modern control systems (Monje et al., 2010). Their wide adoption is attributed to their straightforward implementation, ease of tuning, and ability to handle both simple and complex systems effectively.

A typical feedback control system incorporating a controller with set-point filter, as shown in Figure 1, demonstrates the basic working principle. In such systems, the controller continuously monitors the error signal-defined as the difference between the desired set-point and the actual process output and adjusts the control input to minimize this error. Depending on the system requirements, the proportional, integral, and derivative components of the controller work together to achieve optimal control performance, balancing speed, accuracy, and stability.

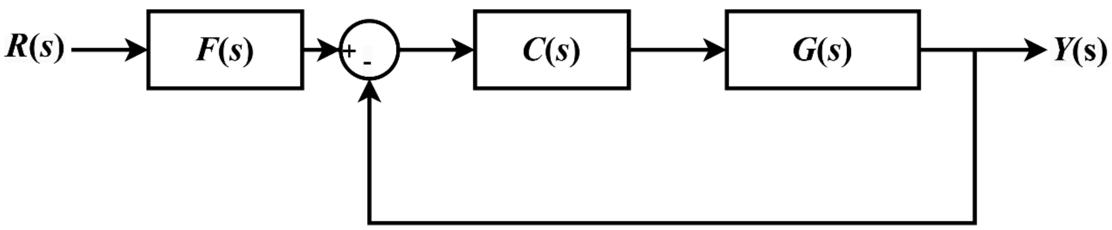

Figure 1. Unit feedback control system block diagram with set-point filter

Here, *G(s)* represents the system to be controlled, *R(s)* and *Y(s)* are the input and output signals, respectively, and *C(s)* represents the *PID* controller, with the equation given as follows.

$$C(s) = K_p + \frac{K_i}{s} + K_d s \tag{8}$$

K_p is the proportional gain, K_i the integral gain, and K_d the derivative gain. Proportional control speeds up the response but increases errors. Integral control eliminates steady-state error, while derivative control reduces overshoot and settling time, though it increases rise time and steady-state error. Combining these three effects yields a fast response with no steady-state error and limited maximum overshoot. In the study, a set-point filter is used in the control scheme to reduce the overshoot in the output, and its equation is as follows (Ajmeri, 2023). In the equation, T_i is the integral time constant, and T_d is the derivative time constant.

$$F(s) = \frac{1}{1 + T_i s + T_i T_d s^2} \tag{9}$$

The number of parameters to be determined in a PID controller is three. It is well-known that optimization-based algorithms yield successful results in determining the controller parameters. In this study, the I-GWO algorithm has been used to determine the parameters. The I-GWO is an optimization method based on the GWO algorithm, which is inspired by nature (Mirjalili et al., 2014). GWO models the behaviour of a wolf pack and aims to find the best solution in the solution space. The algorithm follows the paths of the leader wolves (the best solutions) and improves the solution by collaborating with other wolves. I-GWO enhances the performance of the GWO algorithm by making certain improvements, aiming to achieve faster and more accurate results. This improved version is particularly effective for solving complex problems such as the optimization of PID parameters.

In control systems, integral performance criteria such as integral of square error (ISE), integral of absolute error (IAE), ITSE, and ITAE are frequently used and appear in objective functions (Dogruer & Can, 2022). In the proposed method, a multi-objective objective function based on the time response characteristics, as given in Equation 10, is used (Bingul & Karahan, 2018). Here, M_p is the maximum overshoot, e_{ss} is the steady-state error, t_s is the settling time, t_r is the rise time, and β is the weighting factor. The β value is chosen to equally weight both components of the multi-objective function.

$$J_{MO} = (1-e^{-\beta})(M_p + e_{ss}) + e^{-\beta}(t_s - t_r) \tag{10}$$

The block diagram of the proposed control scheme is given in Figure 2.

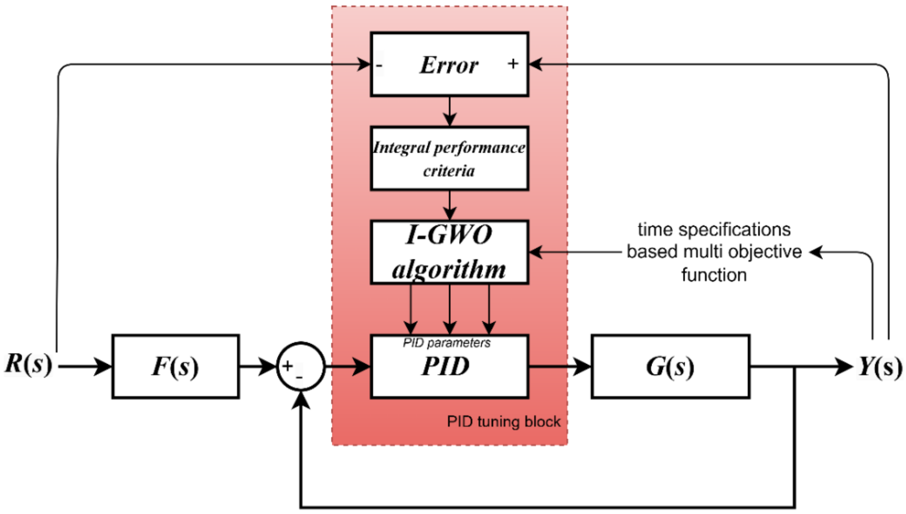

Figure 2. Proposed control scheme with I-GWO algorithm

The optimization process begins with the definition of the objective function and the initialization of the I-GWO algorithm. In the first iteration, the algorithm starts by assigning random values to the PID controller parameters. Time response characteristics are obtained from the closed-loop control system output in Figure 2 using a code and are transferred to the multi-objective function. Based on the value of the multi-objective function, the algorithm generates new values for the PID controller parameters. The time response characteristics of the output are then measured again, and based on the updated value of the multi-objective function, the algorithm updates the controller parameters. This process continues until the stopping criteria are met.

3. Simulation Study

In this section, two fractional-order plants selected from the literature are considered, and PID controller designs are implemented based on the I-GWO algorithm.

Example 1: The fractional-order system used in the literature by (Xue et al., 2006) is taken as an example below.

$$G(s)=\frac{N(s)}{D(s)}=\frac{1}{0.8s^{2.2}+0.5s^{0.9}+1} \tag{11}$$

The integer-order approximation of the fractional-order transfer function given by Equation 12 is expressed as a transfer function using Matsuda's 4th-order approximation method, as shown below. In Matsuda's integer-order approximation, the lower and upper bounds of the angular frequency are taken as 10^{-3} and 10^{3}, respectively.

$$G(s)=\frac{\begin{array}{l}s^{8}+6852s^{7}+2.835e06s^{6}+1.067e08s^{5}+7.922e08s^{4}+7.732e08s^{3}\\+1.581e08s^{2}+3.884e06s+1.75e04\end{array}}{\begin{array}{l}3.822s^{10}+2.53e04s^{9}+5.1e06s^{8}+1.575e08s^{7}+7.352e08s^{6}+9.989e08s^{5}\\+1.29e09s^{4}+8.724e08s^{3}+1.612e08s^{2}+3.899e06s+1.75e04\end{array}} \tag{12}$$

The lower and upper bounds for the K_p, K_i, and K_d parameters are set to [0, 100]. The I-GWO algorithm is run for 100 iterations to determine the controller parameters. The I-GWO algorithm is run 10 times, and the PID controller parameters are determined based on the multi-objective function. The figure shows the multi-objective function values according to the number of runs given in Figure 3.

When Figure 3 is examined, it can be seen that the smallest objective function value is obtained in the 3rd run. The convergence of the multi-objective function values for this run according to the number of iterations is presented in Figure 4. The figure also includes convergence curves for the ITAE and ITSE performance criteria. All the graphs exhibit a similar decreasing trend, indicating that as the number of iterations increases, the multi-objective function value decreases. In the first 10 iterations, there is a noticeable drop in all the graphs. This suggests that the algorithm achieves rapid convergence initially, meaning it quickly improves the function value. After the 10th iteration, the function values change more gradually and stabilize at a certain point. This implies that the optimization process reaches a kind of steady state, where the solution changes very little or has converged.

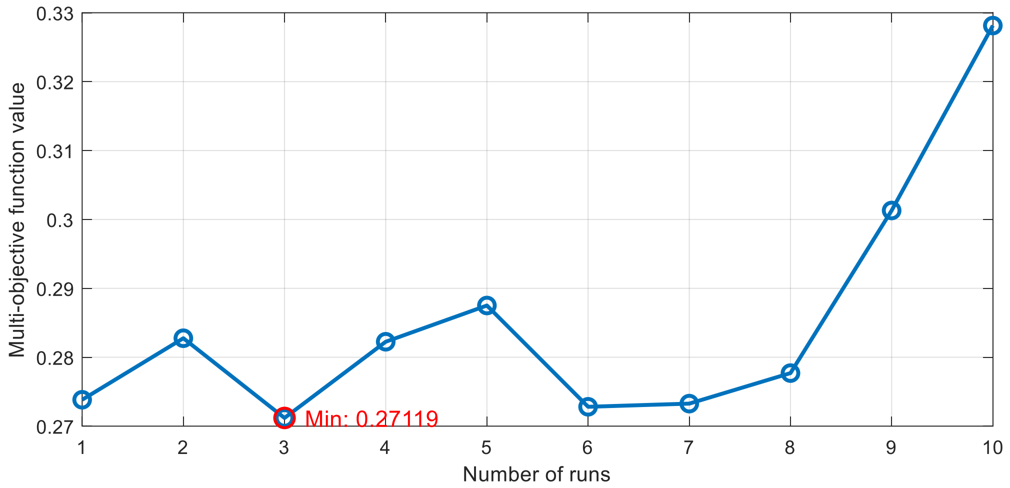

Figure 3. Multi-objective (MO) function values based on the number of runs

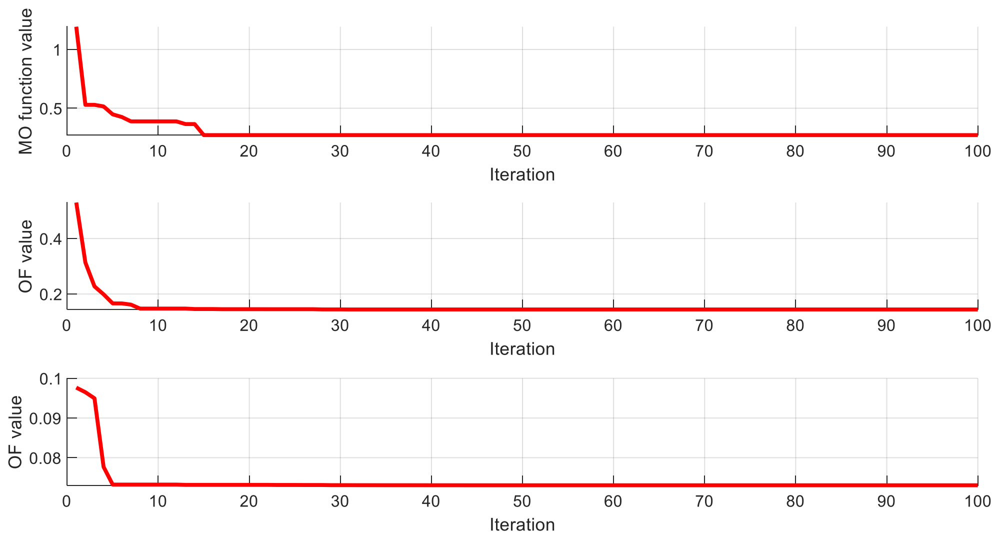

Figure 4. The convergence curves for Example 1 (top: MO, middle: ITAE, bottom: ITSE)

Upon the algorithm's termination, the best objective function value is achieved, and the corresponding PID controller parameters are identified. The determined controller parameters and fitness values are provided in Table 1. Based on the controller parameters, the set-point filter for multi-objective function is defined as follows:

$$F(s) = \frac{1}{0.09479s^2 + 0.56s + 1} \tag{13}$$

Table 1. PID controller parameters

		K_p	K_i	K_d	*OF*
Example 1	ITAE	45.1224	99.9998	9.63188	0.14419
	ITSE	41.9388	99.9997	8.15141	0.073057
	MO Function	52.6542	94.0652	8.92198	0.27119
Example 2	ITAE	58.9203	99.9879	10.2266	0.20727
	ITSE	53.4783	99.9999	6.91954	0.11061
	MO Function	70.3172	99.6728	7.47698	0.25306

The unit step responses of closed-loop systems obtained with the determined PID controller parameters are presented in Figure 5. The multi-objective function used to tune the system ensures that the system exhibits a rapid rise and steadily approaches the reference value. Notably, no overshoot is observed in the system, which suggests that the system has a good damping ratio. Additionally, reaching the reference value and becoming stable in approximately 1 second demonstrates that the system has a fast settling time. The absence of oscillations or fluctuations during the settling time clearly indicates that the system exhibits stable performance. It is clearly observed that the PID controller designed with the ITSE performance criterion causes oscillations and overshoot in the system. It is noteworthy that the PID controller designed with the ITAE performance criterion results in significantly less overshoot while also achieving a shorter settling time. Overall, the system's ability to quickly and stably follow the reference value without excessive oscillations suggests that the control design has been effectively implemented.

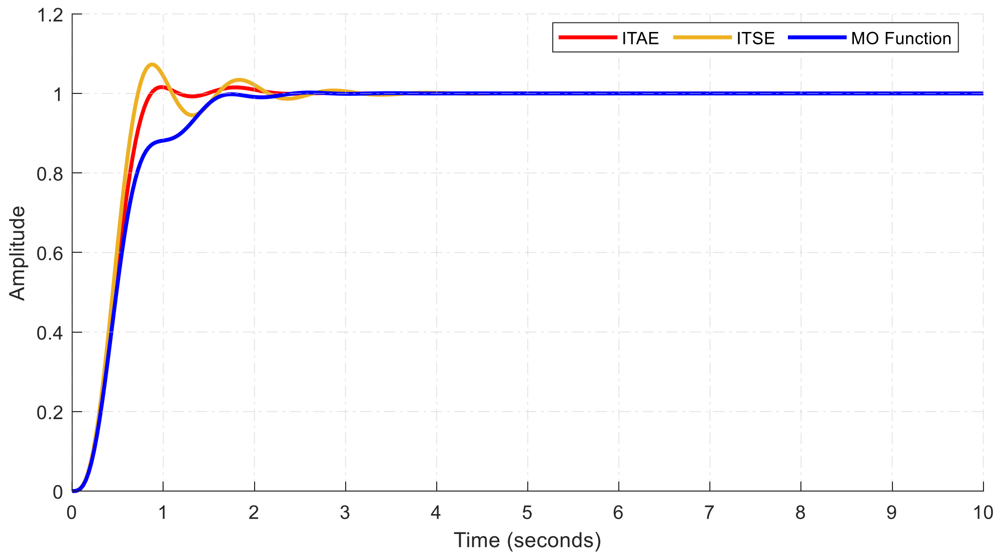

Figure 5. Step responses for Example 1

Additionally, the time response characteristics for both examples are presented in detail in Table 2.

Table 2. Time response specifications

		t_s (2%)	t_r	t_p	M_p (%)
Example 1	ITAE	0.8181	0.4806	0.9747	1.5984
	ITSE	2.0083	0.4140	0.8693	7.3030
	MO Function	1.5281	0.9554	2.3865	0
Example 2	ITAE	1.0944	0.6365	2.2177	0.3308
	ITSE	2.4817	0.5105	1.0699	6.9066
	MO Function	1.9410	1.2867	3.1748	0

Example 2: Let us consider below the FO-LTI plant model used in the literature by Monje et al (Monje et al., 2010).:

$$G(s) = \frac{N(s)}{D(s)} = \frac{1}{s^{2.3} + 3.2s^{1.4} + 2.4s^{0.9} + 1} \tag{14}$$

The fractional-order transfer function given by Equation 14 is expressed using Matsuda's 4th-order integer approximation method as follows. In Matsuda's integer-order approximation, the lower and upper bounds of the angular frequency are taken as 10^{-3} and 10^{3}, respectively.

$$G(s) = \frac{\begin{array}{l} s^{12} + 7629s^{11} + 8.2e06s^{10} + 2.564e09s^{9} + 1.244e11s^{8} + 1.961e12s^{7} + \\ 1.096e13s^{6} + 1.36e13s^{5} + 5.819e12s^{4} + 8.899e11s^{3} + 2.787e10s^{2} + \\ 2.844e08s + 9.185e05 \end{array}}{\begin{array}{l} 10.57s^{14} + 7.675e04s^{13} + 5.911e07s^{12} + 9.262e09s^{11} + 3.798e11s^{10} + \\ 5.481e12s^{9} + 3.122e13s^{8} + 7.987e13s^{7} + 8.621e13s^{6} + 4.021e13s^{5} + \\ 9.31e12s^{4} + 1.01e12s^{3} + 2.913e10s^{2} + 2.885e08s + 9.191e05 \end{array}} \tag{15}$$

As in Example 1, the lower and upper bounds for the K_p, K_i, and K_d parameters are set to [0, 100]. The I-GWO algorithm is executed for 100 iterations to optimize the controller parameters. For the plant defined in Equation 15, the optimization algorithm is run 10 times for each objective function. The values obtained after 10 runs of the multi-objective function are presented graphically in Figure 6. From the figure, it is clearly observed that the smallest objective function value was achieved in the 9th run. The obtained PID controller parameters are provided in Table 1. Furthermore, Figure 7 illustrates the convergence of the multi-objective function values during the 9th run as a function of the number of iterations. Additionally, the figure presents the convergence curves for the ITAE and ITSE performance criteria. All curves demonstrate a consistent decreasing trend, highlighting that the multi-objective function value progressively diminishes with an increasing number of iterations.

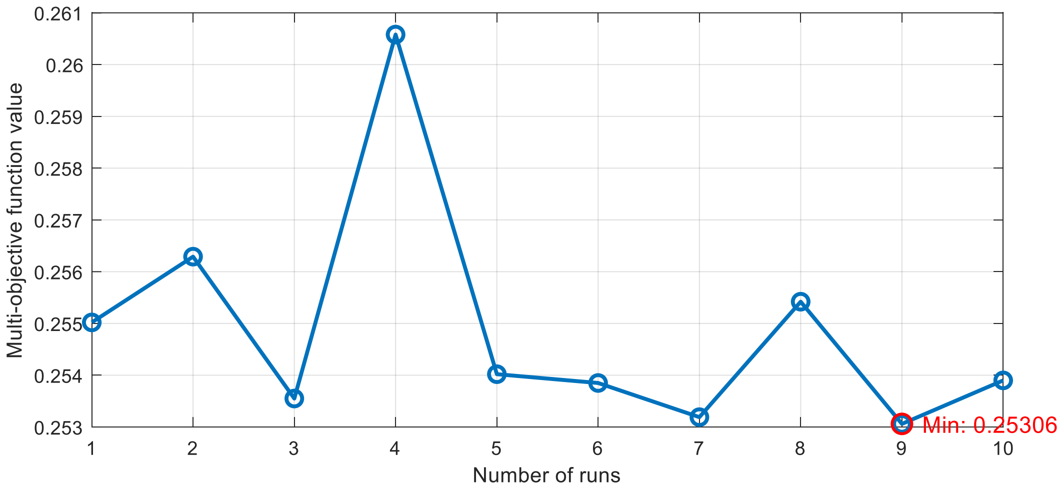

Figure 6. Multi-objective function values based on the number of runs for Example 2

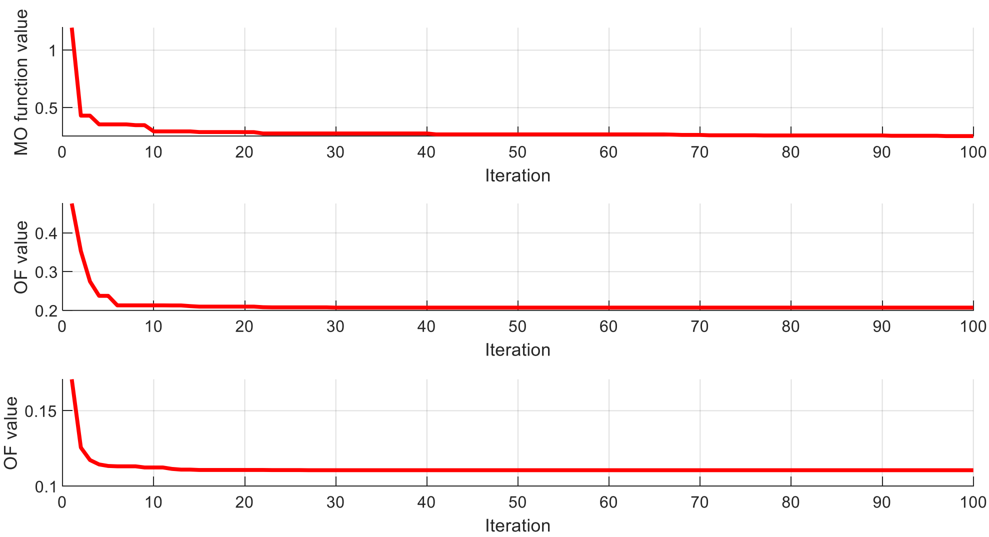

Figure 7. The convergence curves for Example 2 (top: MO, middle: ITAE, bottom: ITSE)

The closed-loop unit step responses of the systems obtained by applying the designed PID controllers to the system in Equation 15 are shown in Figure 8. The figure also includes the step response obtained using the PID controllers designed by (Monje et al., 2010). The Monje et al.'s method provides a slow transition without overshoot, but the long settling time may pose a disadvantage for applications requiring fast responses. The ITAE criterion yields a fast transient response with low overshoot and balanced performance but includes some oscillations. The ITSE criterion provides the fastest response but with higher overshoot and oscillations. The step response obtained according to the multi-objective function balances overshoot and response speed, providing an advantage. Therefore, the advantages and disadvantages of these methods should be evaluated based on application requirements. The figure offers valuable information for selecting an appropriate controller by clearly showing the performance of different control designs.

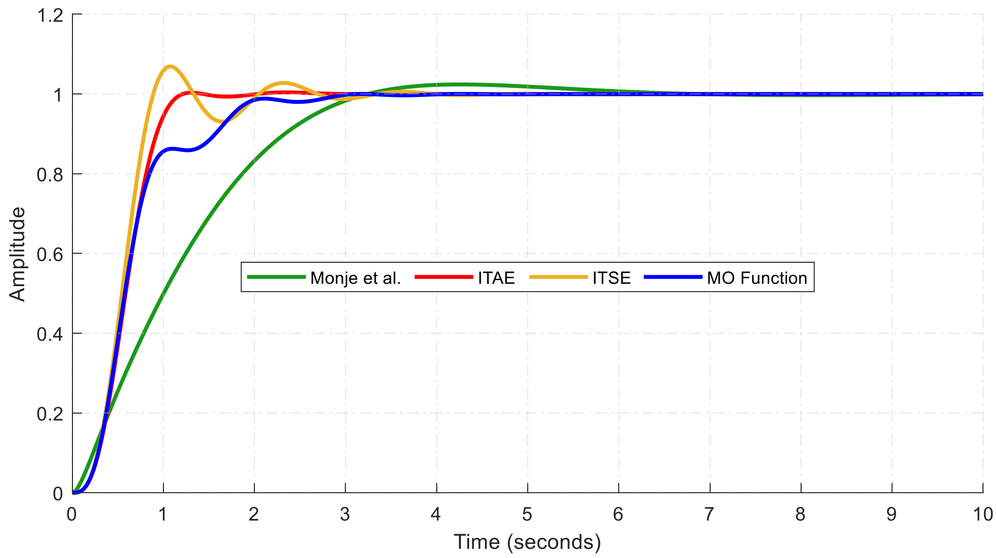

Figure 8. Step responses for Example 2

4. Conclusion

This study presents a controller design method for fractional-order systems. In the proposed approach, the parameters of a PID controller with a set-point filter are optimized using an I-GWO algorithm-based method. The algorithm is formulated with various objective functions, emphasizing the determination of controller parameters through a multi-objective function derived from time-domain response characteristics. To assess the performance of the proposed method, two different fractional-order systems were selected, and controller designs were implemented for each. The presence of the set-point filter in the proposed method has significantly contributed to achieving smooth responses without overshoot in the system outputs. The time responses of the controlled systems were analysed comparatively, highlighting the effectiveness of the proposed method.

5. References

Ajmeri, M. (2023). Analytical design of enhanced PID controller with set-point filter for unstable processes with time delay. *International Journal of Dynamics and Control*, *11*(2), 564-573.

Åström, K. J., & Hägglund, T. (1995). *PID controllers: theory, design, and tuning* (Vol. 2). Instrument society of America Research Triangle Park, NC.

Bingul, Z., & Karahan, O. (2018). A novel performance criterion approach to optimum design of PID controller using cuckoo search algorithm for AVR system. *Journal of the Franklin Institute*, *355*(13), 5534-5559.

Carlson, G., & Halijak, C. (1964). Approximation of fractional capacitors $(1/s)^{(1/n)}$ by a regular Newton process. *IEEE Transactions on Circuit Theory*, *11*(2), 210-213.

Charef, A., Sun, H., Tsao, Y., & Onaral, B. (1992). Fractal system as represented by singularity function. *IEEE Transactions on Automatic Control*, *37*(9), 1465-1470.

Cohen, G. (1953). Theoretical consideration of retarded control. *Trans. Asme*, *75*, 827-834.

Cokmez, E., Atiç, S., Peker, F., & Kaya, I. (2018). Fractional-order PI controller design for integrating processes based on gain and phase margin specifications. *IFAC-PapersOnLine*, *51*(4), 751-756.

Dogruer, T., & Can, M. S. (2022). Design and robustness analysis of fuzzy PID controller for automatic voltage regulator system using genetic algorithm. *Transactions of the Institute of Measurement and Control*, *44*(9), 1862-1873.

Dogruer, T., & Tan, N. (2018). Design of PI controller using optimization method in fractional order control systems. *IFAC-PapersOnLine*, *51*(4), 841-846.

Dogruer, T., & Tan, N. (2019). Lead and lag controller design in fractional-order control systems. *Measurement and Control*, *52*(7-8), 1017-1028.

Eberhart, R., & Kennedy, J. (1995). A new optimizer using particle swarm theory. MHS'95. Proceedings of the sixth international symposium on micro machine and human science,

Hang, C. C., Åström, K. J., & Ho, W. K. (1991). Refinements of the Ziegler–Nichols tuning formula. IEE Proceedings D (Control Theory and Applications),

Ho, W. K., Gan, O., Tay, E. B., & Ang, E. (1996). Performance and gain and phase margins of well-known PID tuning formulas. *IEEE Transactions on Control Systems Technology*, *4*(4), 473-477.

Holland John, H. (1975). Adaptation in natural and artificial systems. *Ann Arbor: University of Michigan Press*.

Karaboga, D., & Basturk, B. (2007). A powerful and efficient algorithm for numerical function optimization: artificial bee colony (ABC) algorithm. *Journal of Global Optimization*, *39*(3), 459-471. https://doi.org/10.1007/s10898-007-9149-x

Li, X., & Gao, L. (2022). A simple frequency-domain tuning method of fractional-order PID controllers for fractional-order delay systems. *International Journal of Control, Automation and Systems*, *20*(7), 2159-2168.

Luo, Y., Chen, Y. Q., Wang, C. Y., & Pi, Y. G. (2010). Tuning fractional order proportional integral controllers for fractional order systems. *Journal of Process Control*, *20*(7), 823-831.

Machado, J. T., Kiryakova, V., & Mainardi, F. (2011). Recent history of fractional calculus. *Communications in nonlinear science and numerical simulation*, *16*(3), 1140-1153.

Manabe, S. (1961). The noninteger integral and its application to control systems. *English Translation Journal Japan*, *6*, 83-87.

Matsuda, K., & Fujii, H. (1993). H (infinity) optimized wave-absorbing control-Analytical and experimental results. *Journal of Guidance, Control, and Dynamics*, *16*(6), 1146-1153.

Meng, F., Liu, S., Pang, A., & Liu, K. (2020). Fractional order PID parameter tuning for solar collector system based on frequency domain analysis. *IEEE Access*, *8*, 148980-148988.

Mirjalili, S., Mirjalili, S. M., & Lewis, A. (2014). Grey wolf optimizer. *Advances in engineering software*, *69*, 46-61.

Monje, C. A., Chen, Y., Vinagre, B. M., Xue, D., & Feliu-Batlle, V. (2010). *Fractional-order systems and controls: fundamentals and applications*. Springer Science & Business Media.

Nadimi-Shahraki, M. H., Taghian, S., & Mirjalili, S. (2021). An improved grey wolf optimizer for solving engineering problems. *Expert Systems with Applications*, *166*, 113917.

O'Dwyer, A. (2012). An overview of tuning rules for the PI and PID continuous-time control of time-delayed single-input, single-output (SISO) processes. *PID control in the third millennium: Lessons learned and new approaches*, 3-44.

Oustaloup, A., Levron, F., Mathieu, B., & Nanot, F. M. (2000). Frequency-band complex noninteger differentiator: characterization and synthesis. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, *47*(1), 25-39.

Petráš, I. (2011). *Fractional-order nonlinear systems: modeling, analysis and simulation*. Springer Science & Business Media.

Podlubny, I. (1998). *Fractional differential equations: an introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications*. elsevier.

Podlubny, I., Petráš, I., Vinagre, B. M., O'Leary, P., & Dorčák, Ľ. (2002). Analogue realizations of fractional-order controllers. *Nonlinear dynamics*, *29*, 281-296.

Sabatier, J., Agrawal, O. P., & Machado, J. T. (2007). *Advances in fractional calculus* (Vol. 4). Springer.

Tustin, A., Allanson, J., Layton, J., & Jakeways, R. (1958). The design of systems for automatic control of the position of massive objects. *Proceedings of the IEE-Part C: Monographs*, *105*(1S), 1-57.

Xue, D., Zhao, C., & Chen, Y. (2006). A modified approximation method of fractional order system. 2006 international conference on mechatronics and automation,

Zhao, C., Xue, D., & Chen, Y. (2005). A fractional order PID tuning algorithm for a class of fractional order plants. IEEE International Conference Mechatronics and Automation, 2005,

Ziegler, J. G., & Nichols, N. B. (1942). Optimum settings for automatic controllers. *Trans. Asme*, *64*(11).