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DELAY EQUALIZATION FOR ANALOG FILTERS WITH PARTICLE SWARM OPTIMIZATION

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Received: 16.12.2024

Accepted: 17.01.2025

ABSTRACT

In this paper, delay equalization of active analog filters is performed by Particle Swarm Optimization (PSO). First and second-order active analog allpass filter blocks are used for the design. In order to obtain a stable delay in the transmission band of the filter, first and second-order active allpass filter blocks were cascaded at the output of the filter to obtain a delay value close to the ideal. The added allpass filter blocks increase the total delay, but the equalization of the ripple in the group delay is more critical. Three parameters are used in the design of these allpass filter blocks. PSO was used to determine these parameters. The PSO algorithm was preferred due to its rapid and successful results and ease of implementation. During the testing phase, the delay values of the 5th-order Chebyshev lowpass filter were examined before and after the delay equalization process. The fluctuation in the delay value was successfully reduced by %84.11. This result demonstrated the success of the algorithm used in determining the parameters and the delay equalization process.

Keywords: Active Analog Filters, Delay Equalization, Particle Swarm Optimization (PSO).

**ANALOG FİLTRELERDE GECİKME EŞİTLEME İŞLEMİNİN
PARÇACIK SÜRÜ OPTİMİZASYONU YÖNTEMİ İLE
GERÇEKLEŞTİRİLMESİ**

ÖZ

Bu çalışmada aktif analog filtrelerin gecikme eşitleme işlemi Parçacık Sürü Optimizasyonu (PSO) ile gerçekleştirilmiştir. Tasarım yapılırken birinci ve ikinci dereceden aktif analog tüm-geçiren filtre blokları kullanılmıştır. Filtrenin iletim bandında sabit bir gecikme elde etmek için, filtrenin çıkışına birinci ve ikinci dereceden aktif tüm-geçiren filtre blokları kaskad bağlanarak ideale yakın gecikme değeri elde edilmiştir. Eklenen tüm-geçiren filtre blokları toplam gecikmeyi artırır, ancak grup gecikmesindeki dalgalanmaların eşitlenmesi daha kritik bir durumdur. Tüm-geçiren filtre bloklarının tasarımında kullanılan üç parametre bulunmaktadır. Parametreler belirlenirken PSO kullanılmıştır. PSO algoritması; hızlı ve başarılı sonuçlar üretmesi, kolay uygulanabilir olması nedeniyle tercih edilmiştir. Test aşamasında 5. Dereceden Chebyshev alçak geçiren filtrenin gecikme değerleri, gecikme eşitleme işleminden önce ve sonra incelenmiştir. Gecikme değerindeki dalgalanma başarılı bir şekilde %84.11 azaltılmıştır. Bu sonuç, parametrelerin belirlenmesinde kullanılan algoritmanın ve gecikme eşitleme işleminin başarılı olduğunu göstermiştir.

Anahtar Kelimeler: Aktif Analog Filtreler, Gecikme Eşitleme, Parçacık Sürü Optimizasyonu.

1. INTRODUCTION

Analog filters are electrical circuits that employ a filtering process to remove the frequency components of a signal that are not desired, while allowing the desired components to pass through. Analog filters used in the processing of continuous-time signals are divided into two as active and passive according to the elements used. Active filters have been used for many years due to a number of advantages they offer, including low power consumption, ease of use in integrated designs and low noise. Analog filters, which have design method approaches such as Butterworth, Chebyshev and Bessel, are used in audio and video processing, communication systems, biomedical devices and many other fields. Analog filter theory is a long established theory with a long history. There are many sources

containing detailed information on the design and analysis of analog filters. Today, there are many studies in the literature on analog filters, which are widely used in different fields. These studies focus on new and rational solutions to the problems encountered in analog filter design. One of these problems is the delay equalization problem. Electrical filter naturally causes a delay in the signal it processes. As an ideal, it is desirable that these delays are constant over the passband of the filter. Delay equalization is mostly performed by using allpass filters. There is some work to find the design parameters of allpass filters with optimization.

Iterative methods have been used to equalize the group delays of filters in related works. Vrbata et al. used an iterative procedure to design a group delay equalizer. They used allpass blocks while performing delay equalization (Vrbata et al., 2002). With the progress in the literature, various optimization methods have been preferred instead of iterative methods. Zaplatilek et al. used optimization to design group delay equalizers for analog filters. By calculating f_0 , f_r and Q values with the Differential Evolution algorithm, reduction of up to 40.34% in the delay distortion value after the delay equalization process (Zaplatilek et al., 2007). Zhao et al. used optimization in group delay equalizer design based on the mathematical model they created. In the model they developed on MATLAB, they used Genetic Algorithms to determine the relationship between the transfer function parameters and the values of the circuit elements (Zhao et al., 2010). By combining different optimization methods, it is possible to achieve a uniform response in the passband of the filter. For example, Ziska & Vrbata presented a new approach combining Differential Evolution (DE) and modified Remez algorithms. First, the DE algorithm is used to estimate the group delay equalizer. The modified Remez algorithm is then applied. In this way, it is possible to achieve a small delay distortion in the group delay value (Ziska & Vrbata, 2006). In the following years, delay equalization was performed using different optimization algorithms. Doğan & Yüksel used Vortex Search Algorithm in their study. They reduced the group delay by 40.3% by calculating the parameters of the circuits used for delay equalization with the vortex search algorithm (Doğan & Yüksel, 2015). In addition to delay equalization, there are studies that use optimization algorithms for circuit design. In his study, Kuyu designed electrical

circuits using various algorithms such as Genetic Algorithms (GA), Differential Evolution algorithm (DE), Particle Swarm Optimization Algorithm (PSO), Ant Colony Algorithm (ACO), Vortex Search Algorithm (VS). Sample designs have shown that PSO gives more successful results (Kuyu, 2016). PSO algorithm provides fast and sufficient results in multidimensional problems. Tamer & Karakuzu demonstrated this with the simulation results they obtained in their study. PSO has important advantages such as not requiring gradient calculation and differential equation solution, and requiring few parameters (Tamer & Karakuzu, 2006). Compared to other known optimization algorithms such as Genetic Algorithms (GA) and Differential Evolution algorithm (DE), PSO has better convergence speed. For instance, Özsağlam & Çunkaş compared these three algorithms using eight different test functions available in the literature and showed that PSO has better convergence speed (Özsağlam & Çunkaş, 2008). There are also studies in the literature that bring a different perspective to delay equalizer design. For instance, Şengül considers and designs the delay equalizer as a single block, and then obtains this block with first and second-order structures. With the help of the Hurwitz polynomial, the delay equalizer is designed as a single block and then the resulting polynomial is divided into first and second-order sections. This provides simplicity in the design process (Şengül, 2015).

In this paper, a delay equalizer for analog filters is designed. The delay equalizer is constructed with first and second-order active allpass filters. PSO was used to calculate the design parameters of the allpass filters. PSO is chosen because it gives fast and successful results as well as working with adjustable parameters. The delay and frequency values of the filters to be used for delay equalization were used as input for the optimization process, and the design parameters of the cascade connected first and second-order allpass filters used in the delay equalizer design were calculated. A different approach was used in the calculations. Delay values are maintained as elements of an array, new delay values are calculated with simple mathematical equations and the optimization is terminated according to the value of the difference between the maximum and minimum value of the array. With this approach, the design process, which is a very complex process, is simplified. A

literature review revealed no studies addressing the use of PSO in the design of delay equalizers for analog filters. Therefore, this paper contributes to the literature. In addition, the success of the method is evaluated by conducting performance analyses after the delay equalization process.

2. METHODOLOGY

Filters cause a time domain delay in the signals they process. In this case, the problem is that the signal at a certain frequency is delayed more or less than the signal at another frequency. In the ideal case, it is desired that these delays are constant. In other words, the group delay of the filters should not change as the frequency changes, it should always remain at a constant value. Alternatively, this can be said that the filter phase response to changing frequency should be as linear as possible. To achieve this, we need to use delay equalization circuits. The delay equalization process aims to ensure that the signal has as constant a delay value as possible in the desired frequency range. Allpass filters are used for delay equalization. Allpass filters are filters that change only the phase of the signal without changing its amplitude (Winder, 2002). In the following, the first and second-order active allpass filter structures used in delay equalization and the optimization method used in this paper are discussed in detail.

2.1. The Definition of Ideal Delay

The ideal delay should be as shown in Figure 1 (Schaumann et al., 2001).

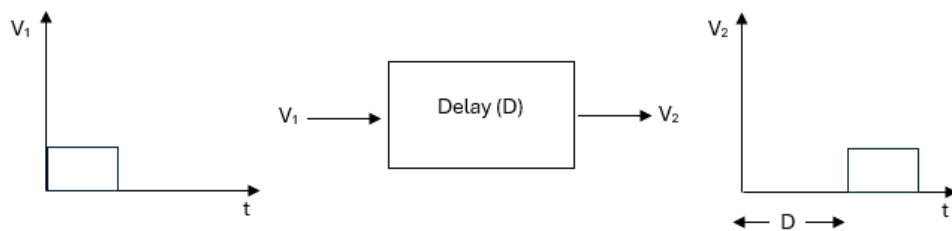


Figure 1. Ideal delay form (Schaumann et al., 2001)

The equation of the system can be written as (Schaumann et al., 2001).

$$v_2(t) = v_1(t - D) \quad (1)$$

The transfer function of the system presented in Figure 1 is given below.

$$T(s) = \frac{v_2(s)}{v_1(s)} \quad (2)$$

To calculate the group delays using the transfer functions of the filters, the derivative of the phase of the transfer function with respect to frequency is taken. Equation 3 stands for this.

$$D(\omega) = -\frac{d\theta(\omega)}{d\omega} \quad (3)$$

2.2. First and Second-order Delay Equalizer

It was explained in the previous sections that the delay in the filters is expected to be normal and in this case it is desired that the delay is equal. It was explained that an ideal delay should have a constant value independent of frequency.

When delay equalization is performed, allpass filters are mostly used. As shown in Figure 2, allpass filters perform this operation by cascade connection to the filters for equalizing the delay (Schaumann et al., 2001).



Figure 2. Cascade connection of a filter and allpass filter

$T_F(s)$ corresponds to any filter and $T_E(s)$ corresponds to the allpass filter to be used in delay equalization. Cascade connected allpass filter serves to keep the delay as

constant as possible with minimum ripple in the desired passband range. Delay equalization is possible with two different structures, first-order and second-order. As the complexity of the filter to be delay equalized increases, higher order delay equalizers can be designed by connecting these structures successively. It should be noted that each additional filter stage will increase the total delay. Therefore, it is optimal to solve the ripple value allowed in the group delay by adding the minimum number of allpass filters.

2.2.1. First-order Equalizer

The equations of the allpass filter used in first-order equalizers are as follows, where α_0 is denotes numerator and denominator coefficient of the first order allpass structure (Williams, 2014).

$$T(s) = \frac{s - \alpha_0}{s + \alpha_0} \quad (4)$$

It can be seen that when the absolute value is determined, the numerator and denominator are equal, independent of the frequency and therefore the transfer function is equal to 1. In other words, the allpass filter directly transfers the signal applied from the input to the output without changing the amplitude. However, the allpass filter provides a phase shift. The phase shift is shown in Equation 5.

$$\beta(\omega) = -2 \tan^{-1} \frac{\omega}{\alpha_0} \quad (5)$$

It has been stated in the previous sections that the derivative of the phase shift must be taken to obtain the group delay. In this case, the equation giving the group delay of the first-order allpass circuit is given by:

$$T_d = \frac{2\alpha_0}{\alpha_0^2 + \omega^2} \quad (6)$$

The circuit diagram of the first-order equalizer is shown in Figure 3 (Su, 2012).

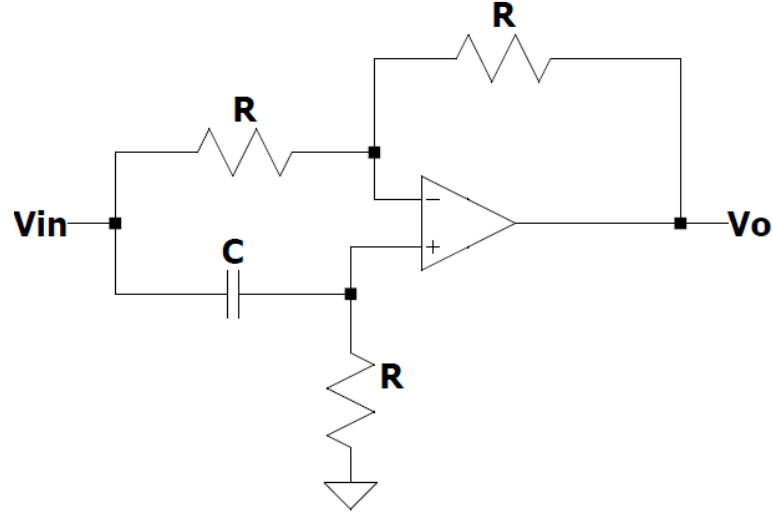


Figure 3. First-order active allpass circuit

The value α_0 in the transfer function of the filter is used to calculate the R and C values. The related equation is given by (Winder, 2002):

$$\alpha_0 = \frac{1}{RC} \quad (7)$$

2.2.2. Second-order Equalizer

The complexity of the problem increases as the order of the circuit to be delay equalized increases. In such cases, first-order delay equalizers may not be sufficient. In cases where first-order delay equalizers are not sufficient, second-order equalizers and complex structures obtained by cascading them are used. The equation giving the group delay of the second-order equalizer is given by, where Q is denotes quality factor and ω_0 is denotes pole frequency (Williams, 2014):

$$T_d = \frac{2Q\omega_0(\omega^2 + \omega_0^2)}{Q^2(\omega^2 - \omega_0^2) + \omega^2\omega_0^2} \quad (8)$$

The circuit diagram of the second-order equalizer is shown in Figure 4 (Schaumann et al., 2001). This circuit is based on General Impedance Converter (GIC) circuit. C value can be selected optionally according to the designer's desire. The R value is calculated by means of the equation given by:

$$RC = \frac{1}{\omega_0} \quad (9)$$

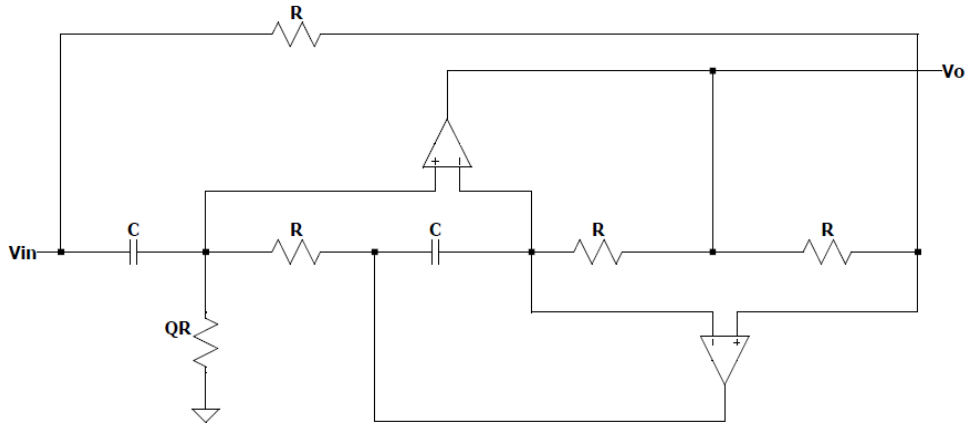


Figure 4. Second-order active allpass circuit

2.3. The Method of Optimization

Optimization is the most efficient adjustment of parameters or variables to achieve the best performance or outcome of a given system, process or problem (Clerc, 2010). In this paper, the system consists of a filter circuit whose delay is to be equalized and a cascade allpass filter structure. According to the desired delay distortion value (ΔD), the order of the allpass filter changes.

In this paper, the PSO algorithm is used. This theory was first proposed by Kennedy and Eberhart in 1995. The PSO algorithm has an approach that is motivated by nature and can solve complex optimization problems in this way. In nature, acting as an individual is often less successful than acting as a swarm. Therefore, in PSO,

multiple particles are used to optimize the solution space and to obtain the optimum result. In the beginning, each particle is at random speed and position. Each particle tries to find the global optimum by continuously updating its best position and at the same time the best position of the swarm (Kennedy & Eberhart, 1995).

As each particle creating the swarm seeks the optimum value, some bounds should be determined according to the structure of the problem. In delay equalizer design, these limits are used to find α_0 , ω_0 and Q values. One of the most important factors in determining the limits is the delay distortion value. In this process, the velocities and positions of the particles are constantly updated to stay within the bounds. Each particle determines its own best position and then the global best position is found. If the conditions are sufficient, the iteration is terminated. Otherwise the iteration continues.

In the PSO algorithm, each particle is initially assigned a random velocity (v) and position (x). In each iteration, the velocity and position are updated to find the local best position (p_{best}) and global best position (g_{best}). Equations of motion are given as follows:

$$v_n^{i+1} = wv_n^i + c_1r_1(p_{best} - x_n^i) + c_2r_2(g_{best} - x_n^i) \quad (10)$$

$$x_n^{i+1} = x_n + v_n^{i+1} \quad (11)$$

where v represents velocity of particle, x represents position of particle, w represents inertia weight, c_1 and c_2 represents cognitive and social learning parameters of swarm and r_1 and r_2 are random values in the range [0,1]. The superscript i denotes the iteration and the subscript n denotes the n^{th} particle (Clerc, 2010). The iteration is terminated when the desired condition is achieved. The W value is effective in maintaining the present velocity of the particle and should be chosen in the range [0,1]. A high W value gives the particle the ability to search a large area during the exploration process. As W decreases, the local search capability increases. Li et al. studied the effect of adaptively increasing the inertia weight on PSO performance.

They suggested that the initial high W value decreases as the iteration progresses (Li et al., 2019). Choudhary et al. used a linearly decreasing W strategy in their study. They showed that this method has a positive effect on PSO performance (Choudhary et al., 2023). The c_1 value is critical for the particles to focus on the cognitive best solution. A high c_1 value allows particles to perform a more localized search. c_2 value is decisive for particles to focus on the global best solution in the swarm. A high c_2 value increases the swarm effect in the solution, leading to faster convergence and avoidance of local minima, but decreases the exploration power (Clerc, 2010). The determination of c_1 and c_2 values is specific to the problem. There are papers that try to find the optimum c_1 and c_2 values using various methods. Talita et al. conducted a test to find the optimum parameter values in their study (Talita et al., 2021). Rehman et al. tested the PSO algorithm for different values of c_1 and c_2 . They analyzed the effect of acceleration coefficients in PSO via different scenarios (Rehman et al., 2020). The total number of particles in the swarm is also an important parameter affecting the optimization. A higher number of particles allows for a larger solution space to be searched but requires more computational power and time. Similarly, as the number of iterations increases, more computation is required. Finally, the boundary values of the variables whose optimum value is to be calculated should also be given as input to the algorithm. Setting the bounds incorrectly will cause the algorithm to produce invalid solutions. In this paper, the parameters are set to make an optimal contribution to the solution of the problem. The value of w is initially set to 0.9 to perform a wide search and then dynamically defined to decrease to 0.5 depending on the iteration for faster convergence. c_1 and c_2 are set to 1.5 and 2.0 respectively to provide a more stable model and focus on the global best solution. In the analysis, it was observed that the model is stable. The number of particles and the number of iterations are set to 100 and 1000, respectively, to provide a wide search space and high accuracy. In this paper, the optimization process is performed by means of a program written using Python coding language. Firstly, the frequency values of any filter for which delay equalization is desired and the delay values corresponding to the frequency values are maintained in a different array. Delay distortion value and normalization

frequency value are given to the program as input. Afterwards, the frequency and delay values are normalized. Equation 6 for the first-order equalizer and Equation 8 for the second-order equalizer are used to calculate new delay values for each normalized frequency value.

The α_0 value used in the first-order equalizer is found by optimization. The optimization function defined in the program calculates new delay values for each α_0 value and adds the new delay values to the existing delay values. Afterwards, it checks whether the difference between the maximum delay value and the minimum delay value is smaller than the delay distortion value. In case it is smaller, the software program terminates and outputs the optimum α_0 value. Otherwise the software program continues to run.

In the second-order equalizer, two different variables need to be obtained by optimization. As can be seen from Equation 8, two different variables, ω_0 and Q , affect the delay value of the filter. The optimization function defined in the program calculates new delay values for each ω_0 and Q value and adds the new delay values to the existing delay values. Afterwards, it checks whether the difference between the maximum delay value and the minimum delay value is smaller than the delay distortion value. In case it is smaller, the software program is terminated and the optimum ω_0 and Q values are given as output. Otherwise the software program continues to run.

Combining the two software programs into a single software program provides a more efficient solution. The software program first aims to perform delay equalization by using a first-order equalizer. If the equalization process cannot be performed with a first-order equalizer, the delay equalization process is completed by using one or more second-order equalizers connected in cascade. Due to its algorithm, the software program aims to reach the solution with the minimum number of allpass filters. The flowchart showing the algorithm of the software program is given in Figure 5.

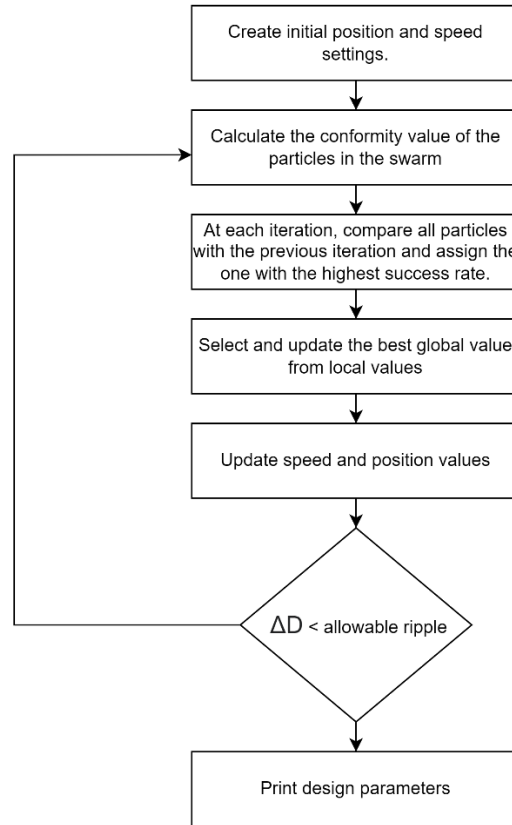


Figure 5. Algorithm of the software program

3. SIMULATIONS

In this paper, a delay equalizer is designed to equalize the group delay of a 5th order lowpass circuit using the Chebyshev approximation. The Chebyshev lowpass filter has a cut-off frequency of $f_0 = 5$ kHz and a passband ripple of 0.5-dB. Figure 6 shows the normalized delay and frequency values of the lowpass filter.

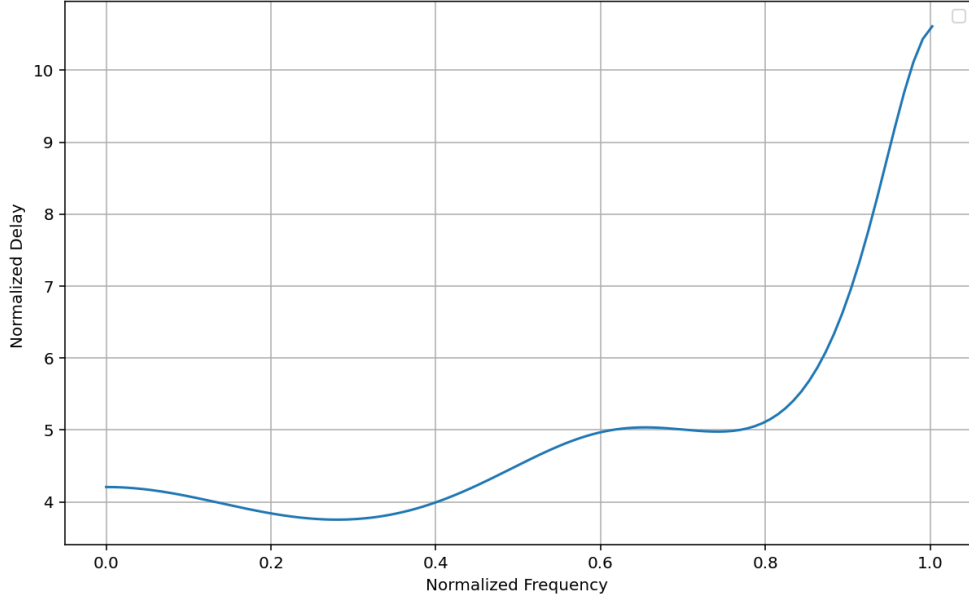


Figure 6. Lowpass filter normalized delay

As shown in Figure 6, there are variable delay values in the passband of the lowpass filter. Currently, the delay distortion value is 218 microseconds (μs). Therefore, delay equalization process is necessary. An optimization program will be used for this. Delay equalization is performed using one first-order and two second-order circuits connected cascade. The values obtained as a result of the optimization process are presented in Table 1.

Table 1. Values obtained by optimization

Filter	α_0	ω_0	Q
First-order Allpass Filter	0.29	-	-
Secon-order Allpass Filter 1	-	0.45	0.91
Secon-order Allpass Filter 2	-	0.82	1.81

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The values of α_0 , ω_0 and Q given in Table 1 are obtained by using PSO algorithm. The circuit diagram designed using the values in Table 1 is given in Figure 7. The upper three sections are the Chebyshev filter and the lower three are the allpass equalizer. A buffer is added between the cascade connected allpass sections for isolation. The delay values of each cascaded circuit stage are given in Figure 8, Figure 9 and Figure 10 respectively.

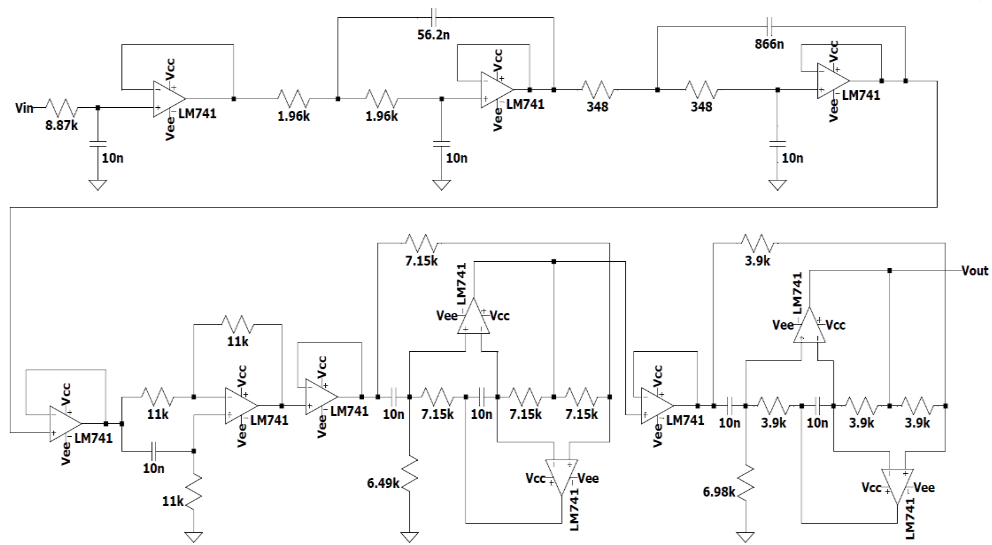


Figure 7. Circuit of delay equalizer for 5th order Chebyshev lowpass filter.

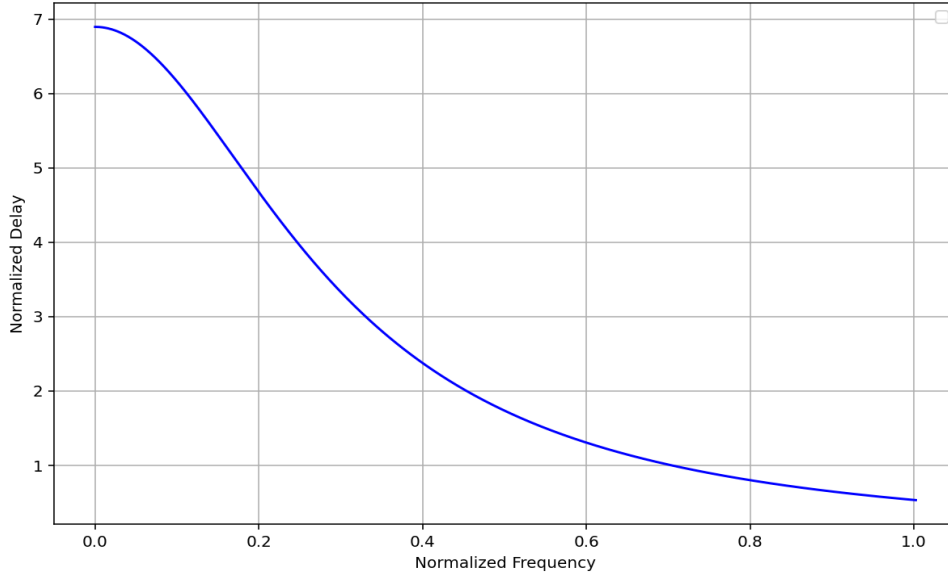


Figure 8. First-order allpass stage delay.

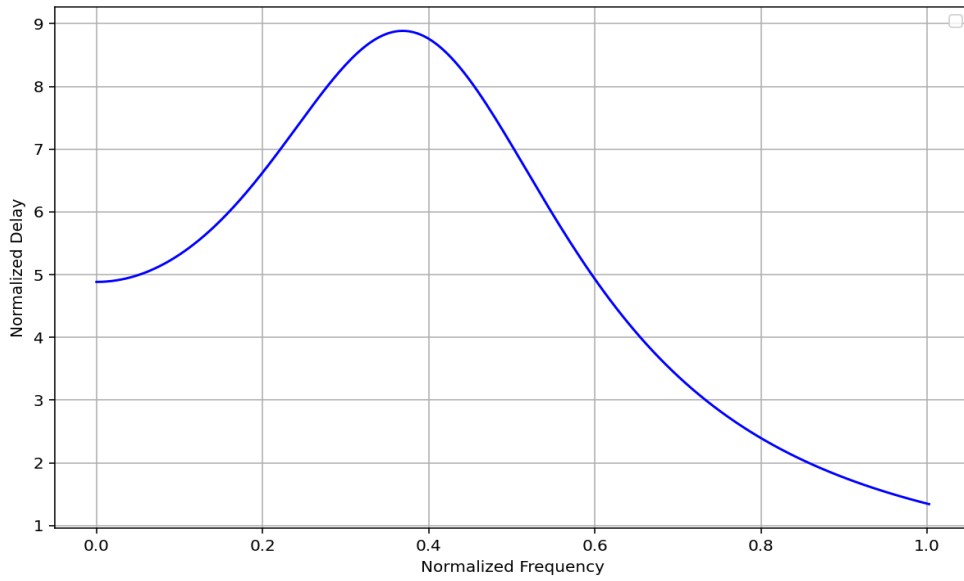


Figure 9. Second-order allpass stage-1 delay

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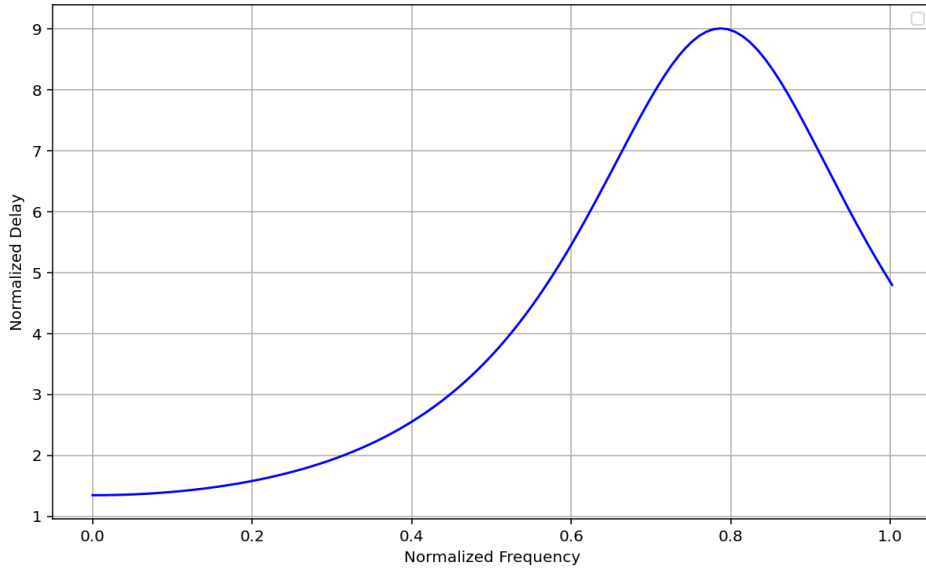


Figure 10. *Second-order allpass stage-2 delay.*

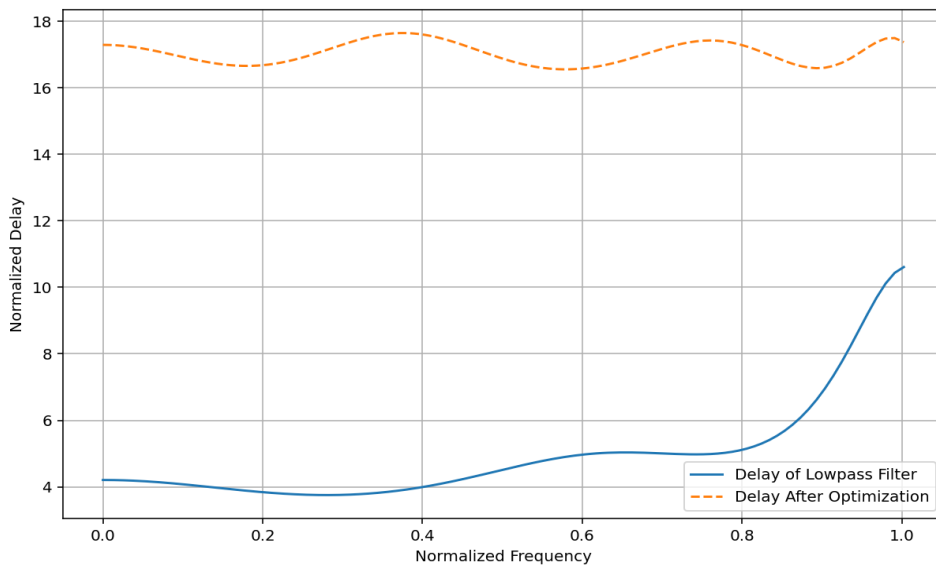


Figure 11. *Group delay after equalization.*

The graph showing the group delay of the new circuit obtained with allpass filters cascaded to the lowpass filter is given in Figure 11.

The delay distortion value, which was 218 μs in the initial condition, became 34.8 μs after the delay equalization process. While the normalized average delay value was 4.29 in the initial condition, after equalization the normalized average delay value became approximately 17.12. It is expected that there will be some increase in the total delay value. The important point is that the delay variation in the passband without equalization is about 160%, while after equalization it is 6.37%. The delay variation value is the percentage ratio of the delay distortion value to the average delay value. This shows that the delay equalization process has been performed quite successfully. The PSO algorithm code was implemented using the Python programming language. The code was executed in the Anaconda Navigator environment using the Spyder 6.0.1 IDE. Circuits were designed and simulated using LTSpice software program. The analysis was performed on Windows 11 operating system with 11th Gen Intel(R) Core(TM) i5-11400H @ 2.70GHz processor in 154 seconds. The performance analysis obtained after the delay equalization process is given in Table 2.

Table 2. Values obtained after delay equalization

Normalized values	Before equalization	After equalization	Change rate
Average delay	4.29	17.12	299.07% increased
Delay distortion	6.86	1.09	84.11% decreased
Delay variation	160%	6.37%	-

4. CONCLUSION

The calculation of the design parameters of the first and second-order allpass filters used for delay equalization by classical methods is difficult and takes time. In this paper, PSO algorithm is used to calculate the related parameters. The delay and frequency values are maintained in an array, the delay distortion value and the equations giving the delay of the allpass filters are given as input to the software program and thus the problem is simplified as keeping the ripple in the desired range

after new values are added to the elements of an array using Equation 6 and Equation 8. In this way, a quick and efficient optimization process was made possible. As a result of the optimization, the values given in Table 1 were obtained and a successful equalization process was performed using these values as shown in Figure 11. The paper has the advantage of being easily adaptable to different frequency ranges and system requirements. The permissible delay distortion value can be optionally determined and delay equalization can be performed using the minimum possible number and order of allpass filter circuits. The methods and findings presented in this paper can be applied in various fields such as signal processing, communication systems and control systems.

ACKNOWLEDGEMENT

The author(s) declare(s) no conflict of interest.

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