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RESEARCH ARTICLE

Transfer Matrix Formalism for Two-Dimensional (2D) Superconducting Material

Şevval Taşdemir[1*]

[1] Kocaeli University, Faculty of Arts and Sciences, Department of Physics, 41380, Kocaeli, Türkiye

ABSTRACT

This study introduces a fundamental transfer matrix formalism for superconductors. The transfer matrix is constructed by solving Maxwell's equations under the TM (Transfer Magnetic) mode configuration and including boundary conditions at the interface of the superconductors. This matrix enables an investigation of the scattering properties of electromagnetic wave interacting with superconducting surfaces. Then, reflection coefficient (R) and transmission coefficient (T) are derived from elements of the transfer matrix. This formalism provides a basis for understanding the interaction of electromagnetic waves with the surface of superconductors for advanced studies such as coherent perfect absorption (CPA), spectral singularities, and PT symmetry. The results highlight the influence of the London penetration depth on the reflection coefficient (R), the transmission coefficient (T) and transfer matrix. Additionally, surface currents resulting from TM mode configuration and Meissner effect, are also expressed in terms of the London penetration depth. In this context, we establish a foundation for studying the potential applications of superconductors.

Keywords: Transfer matrix; Superconductors; London penetration depth; TM mode configuration

1. INTRODUCTION

Superconductors are a significant class of materials that conduct an electric current without resistance and energy loss at a certain critical temperature T_c (Tinkham 1974). In conventional conductors, electric current is carried by individual electrons, whereas in superconductors, the current is carried by pairs of electrons known as Cooper pairs. Above the critical temperature, Cooper pairing breaks down, and superconductivity disappears. Furthermore, superconductors exclude magnetic fields below the critical temperature; this phenomenon is known as the Meissner effect (Tinkham & Lobb 1989). Examining the intriguing magnetic and electrical properties and analyzing interaction with the electromagnetic waves of superconductors play a crucial role in developing innovative technologies in various fields. The London equations, presented in London (1964), describe the phenomenon of superconductivity within a classical framework.

$$\frac{d\vec{J}_s}{dt} = \frac{n_s e^2}{m}\vec{E}, \tag{1}$$

$$\nabla \times \vec{J}_s = -\frac{n_s e^2}{m}\vec{B}. \tag{2}$$

Here, $\vec{J}_s$ is the super liquid component of current density, n_s is the liquid charge density, and m is the charge mass (The London equations are explained in detail in the Supplementary Materials section). Equations (1) and (2) give relationships of current density with electric and magnetic fields, respectively. Typically, the current has two components: normal and superfluid. The normal component of current is neglected in the case of superconductivity. In this regime, the superconducting state predominates, and the electric current is carried by the supercurrent, which is composed of cooper pairs. However, during phase transitions, as mentinoned in Schmidt (2013), the normal current must be considered. The normal current corresponds to the conventional flow of charge carriers, which experiences resistance as a result of scattering mechanisms. In contrast, the supercurrent is characterized by the movement of Cooper pairs that flow without resistance.

$$\vec{J} = \vec{J}_s + \vec{J}_n. \tag{3}$$

London equations are important to understand the electromagnetic behaviour of superconductors at low temperatures. These equations combine Maxwell's equations with the hydrodynamic model of superconductors, explaining properties such as supercurrents and the Meissner effect through the electromagnetic behaviour of the superconducting state. The application of these equations often depends on the choice of the electromagnetic mode configurations, such as the TM and TE modes, which provide different insights into the interaction of electromagnetic waves with superconducting materials. The

Corresponding Author: Ş Taşdemir **E-mail:** sevval.tasdemir@kocaeli.edu.tr

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TM mode configuration is particularly crucial for analyzing the magnetic field penetration and current distributions within the superconductor, as it directly couples to the magnetic field component perpendicular to the surface. In contrast, the TE mode configuration primarily addresses the behaviour of the electric fields and provides information on phenomena such as surface impedance and electromagnetic wave propagation.

This study focuses on the TM mode configuration because it directly engages with the magnetic field component perpendicular to the surface, facilitating an application of the London equations. Moreover, TM modes are particularly advantageous for examining the role of the London penetration depth and boundary conditions which are essential to understanding the electromagnetic response of superconductors.

The transfer matrix method is highly effective for investigating surface interactions with electromagnetic waves. As demonstrated in several studies (Sarısaman & Taş 2018, 2019b,a; Oktay et al. 2020; Sarısaman et al. 2024), this method is widely used to explore material properties such as coherent perfect absorbers (CPA), spectral singularity points, and PT-symmetric. This paper aims to establish a fundamental framework for the investigation of these properties in superconductors by using the transfer matrix method.

Firstly, Maxwell equations are solved using the London equations under the TM mode configuration and obtained boundary conditions on the surface of the superconductor slab (SC). The transfer matrix is formed by considering boundary conditions and R and T coefficients are formulated. Finally, surface currents are obtained in the direction of the electric field components as a result of electromagnetic wave interaction with the surface of SC slab under the TM mode configuration. We predict that our results will significantly contribute to the understanding of surface behaviour in superconductors.

2. SOLUTIONS OF MAXWELL EQUATIONS

Maxwell's equations play an important role in analyzing the interaction of electromagnetic waves with SC slab system. As shown in Figure 1, the solutions of Maxwell equations must be obtained for regions I, II, and III. Then, by incorporating the London penetration depth into these solutions, we can derive the boundary conditions for the surface of SC slab. We consider a linear and homogeneous SC slab with thickness L positioned in the xz-plane as shown in Figure 1. The electromagnetic wave is sent with incident angle θ to the SC slab. Here, θ denotes the angle between the incident light and the surface normal. The Maxwell's equations are expressed as,

$$\vec{\nabla}\cdot\vec{D} = \rho(z), \tag{4}$$
$$\vec{\nabla}\cdot\vec{B} = 0, \tag{5}$$
$$\vec{\nabla}\times\vec{E} = -\partial_t\vec{B}, \tag{6}$$
$$\vec{\nabla}\times\vec{H} = \vec{J}(z) + \partial_t\vec{D}. \tag{7}$$

Here, magnetic field $\vec{B}$, electric field $\vec{E}$ and current density $\vec{J}$ depend on position and time. We use the notations $c = 1/\sqrt{\mu_0\varepsilon_0}$ for speed of light in vacuum, $k = \omega/c$ for wave vector. Also notice that, $\vec{H} = \vec{B}/\mu$ and $\vec{D} = \varepsilon\vec{E}$, μ and ε are magnetic and electric permittivity, respectively. Assuming that $\mu \approx \mu_0$ due to the Meissner effect inside of superconductor slab and $\varepsilon = n^2\varepsilon_0$, where n is the refractive index. μ_0 and ε_0 are magnetic and electric permittivity in vacuum, respectively. Using the time-harmonic oscillation approximation, fields can be separated into time and position components as $\phi(\vec{r},t) = e^{-i\omega t}\phi(\vec{r})$ where ϕ parameter indicate $\vec{E}, \vec{B}, \vec{J}, \vec{D}, \vec{H}$ fields. Then, the time-independent Maxwell equations are obtained as follows,

$$\vec{\nabla}\times\vec{E} = i\omega B(\vec{r}), \tag{8}$$
$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} - ik\frac{n^2}{c}E(\vec{r}). \tag{9}$$

In the TM mode configuration, the components of $\vec{E}$ are in x and z-direction, the component of $\vec{B}$ field is only in y-direction as follows,

$$E(\vec{r}) = E_x(x,z)\vec{i} + E_z(x,z)\vec{k}, \tag{10}$$
$$B(\vec{r}) = B(x,z)\vec{j}. \tag{11}$$

$E_x(x,z)$ and $B(x,z)$ denote spatial plane waves that propagate in the x-direction.

$$B_y(x,z) = B_y(z)e^{ikx}, \tag{12}$$
$$E_x(x,z) = E_x(z)e^{ikx}, \tag{13}$$
$$E_z(x,z) = E_z(z)e^{ikx}. \tag{14}$$

Considering that the current density $\vec{J}$ is zero outside a superconductor, we take the curl of Equation (9) which is written

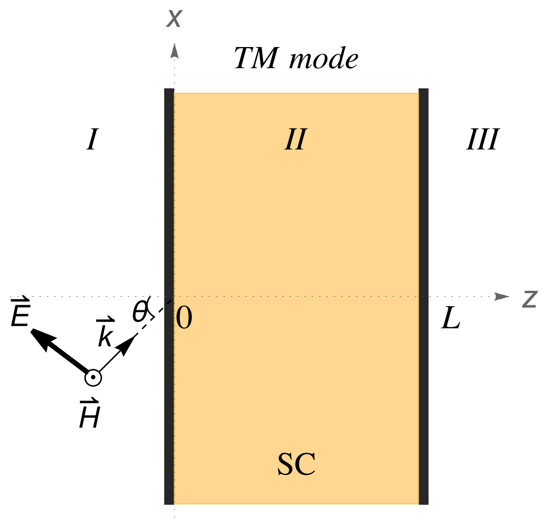

Figure 1. The figure represents an electromagnetic wave model which is sent by θ angle to the surface normal with the SC slab under the TM mode configuration. Electromagnetic wave propagate in x-direction and SC Slab which has L thickness is positioned along the z-axis.

as,

$$\vec{\nabla} \times \vec{\nabla} \times \vec{B}(\vec{r}) = -ik\frac{n^2}{c}\vec{\nabla} \times \vec{E}(r). \tag{15}$$

The components of the electric field in Equation (10) and the magnetic field the Equation (11) are placed in Equation (15). Thus, the 1-dimensional Helmholtz equation, which describes the motion of the waves due to the magnetic field, is obtained.

$$\nabla^2 B_y(z) = -k^2 n^2 B_y(z). \tag{16}$$

The Equation (16) depends on the refractive index, the wave number and the magnetic field. The solution to 1-dimensional Helmholtz equation is given by $B_y(z) = B_1 e^{iknz} + B_2 e^{-iknz}$.

2.1. Meissner Effect

Below the critical temperature T_c, the superconductors do not allow the electric and magnetic fields to exist their inside. As a result, surface currents are generated due to the exclusion of the magnetic field within the material. The electric field is zero ($\vec{E} = 0$) in SC slab system. This situation is named the Meissner effect and is expressed by the following equation:

$$\nabla \times \vec{B} = \mu_0 \vec{J}. \tag{17}$$

Taking the curl of the Equation (17) yields the wave equation for the interior of the SC slab.

$$\vec{\nabla} \times \vec{\nabla} \times \vec{B} = \mu_0(\vec{\nabla} \times \vec{J}). \tag{18}$$

The London Equation (2) is replaced in Equation (18) and it gives the 1-dimensional Helmholtz equation for region II.

$$\nabla^2 B_y(z) = \mu_0 \left(\frac{n_s e^2}{m}\right) B_y(z). \tag{19}$$

This expression $\lambda = \sqrt{m/\mu_0 n_s e^2}$ is London penetration depth that characterizes the extent the magnetic field can penetrate into a superconductor. It varies depending on the critical temperature T_c and the intrinsic properties of the material. Substituting λ into Equation (19), the Helmholtz equation becomes:

$$\nabla^2 B_y(z) - \frac{1}{\lambda^2} B_y(z) = 0. \tag{20}$$

The solution of the Equation (20) is given by $B_y(z) = A_1 e^{\frac{z}{\lambda}} + A_2 e^{-\frac{z}{\lambda}}$ and depends on London penetration depth in z-direction As a result, the wave modes both inside and outside of the SC slab are as follows:

$$B_y(z) = \begin{cases} A_1 e^{iknz} + A_2 e^{-iknz}, & z < 0, \\ B_1 e^{\frac{z}{\lambda}} + B_2 e^{-\frac{z}{\lambda}} & 0 < z < L, \\ C_1 e^{iknz} + C_2 e^{-iknz} & z > L. \end{cases} \tag{21}$$

Here, A_i, B_i, and C_i represent the amplitudes of the waves in regions I, II, and III, respectively.

2.2. Super Currents

The TM mode configuration leads to the emergency of surface currents in superconductors as a result of the Meissner effect. These currents occur as a result of the exclusion of the magnetic field inside the superconductors. The Equation (17) gives rise to the surface currents that are related to the magnetic field.

$$\partial_x B_y(x, z) = \mu_0 J_z, \tag{22}$$

$$-\partial_z B_y(x, z) = \mu_0 J_x. \tag{23}$$

The current density has two components in x and z- direction. We obtain these components by taking the partial derivatives of the Equation (12) according to x and z.

$$J_x = -\frac{1}{\mu_0} B'_y(z) e^{ikx}, \tag{24}$$

$$J_z = \frac{ik}{\mu_0} B_y(z) e^{ikx}. \tag{25}$$

J_x is real and J_z is the imaginary part of the current density that is described as,

$$\vec{J} = J_x \vec{i} + J_z \vec{k}. \tag{26}$$

$B'_y(z)$ is obtained by taking the derivative of Equation (21) as follows:

$$B'_y(z) = ikn \begin{cases} A_1 e^{iknz} - A_2 e^{-iknz} & z < 0 \\ -\frac{i}{\lambda kn} B_1 e^{\frac{z}{\lambda}} + \frac{i}{\lambda kn} B_2 e^{-\frac{z}{\lambda}} & 0 < z < L \\ C_1 e^{iknz} - C_2 e^{-iknz} & z > L. \end{cases} \tag{27}$$

The electric field and the surface currents are proportional to each other ($\vec{E} \propto \vec{J}$), and the components of $\vec{E}$ and $\vec{J}$ are in the same directions. Firstly, London Equation (1) express the electric field in terms of the current density.

$$\vec{E}(\vec{r}, t) = \left(\frac{m}{n_s e^2}\right) \frac{d}{dt} \vec{J}(\vec{r}, t). \tag{28}$$

The current density is given by $\vec{J}(\vec{r}, t) = e^{-i\omega t} \vec{J}(\vec{r})$ and depends on time and position. By replacing the current density $\vec{J}(\vec{r}, t)$ into the Equation (28), the time-independent version of the electric field $\vec{E}$ is obtained.

$$\vec{E}(\vec{r}) = \left(\frac{i\omega m}{n_s e^2}\right) \vec{J}(\vec{r}). \tag{29}$$

Taking into account the components of the current density J_x and J_z, the components of the electric fields are determined in terms of the London penetration depth λ.

$$E_x \vec{i} + E_z \vec{k} = \left(\frac{i\omega m}{n_s e^2}\right) (J_x \vec{i} + J_z \vec{k}). \tag{30}$$

E_x and E_z are imaginary and real parts of the electric field, respectively.

$$E_x = -\left(\frac{i\omega m}{n_s e^2}\right) J_x = i\lambda^2 \omega B'_y(z) e^{ikx}, \tag{31}$$

$$E_z = -\left(\frac{i\omega m}{n_s e^2}\right) J_z = \lambda^2 \omega k B_y(z) e^{ikx}. \tag{32}$$

Table 1. The components of $\vec{E}$, $\vec{D}$ and $\vec{J}$ fields inside and outside of SC slab are demonstrated in TM mode configuration in the table. $B_y(z)$ and $B'_y(z)$ are defined in Equations (21) and (27), respectively.

E	D	J
$E_x = i\lambda^2\omega B'_y(z)e^{ikx}$	$D_x = i\lambda^2\omega\varepsilon_0 n^2 B'_y(z)e^{ikx}$	$J_x = -\frac{B'_y(z)}{\mu_0}e^{ikx}$
$E_y = 0$	$D_y = 0$	$J_y = 0$
$E_z = \lambda^2\omega k B_y(z)e^{ikx}$	$D_z = \lambda^2\omega k\varepsilon_0 n^2 B_y(z)e^{ikx}$	$J_z = ik\frac{B_y(z)}{\mu_0}e^{ikx}$

Table 2. This table shows the components of the H and B filed, where $B_y(z)$ is defined in Equation (21).

B	H
$B_x = 0$	$H_x = 0$
$B_y = B_y(z)e^{ikx}$	$H_y = \frac{B_y(z)}{\mu_0}e^{ikx}$
$B_z = 0$	$H_z = 0$

Using these field components in Table 1 and Table 2, boundary conditions can be calculated. Table 3 presents these boundary conditions, which are crucial for obtaining the transfer matrix. The transfer matrix gives the relationship with each other incoming and outgoing waves interacting with the SC slab. As shown in Table 3, applying boundary conditions (1) and (3), and performing term by-term addition and subtraction, the following set of equations is obtained:

$$B_1 = \frac{1}{2}\left[(1+\sigma)A_1 + (1-i\sigma)A_2\right], \tag{33}$$

$$B_2 = \frac{1}{2}\left[(1-i\sigma)A_1 + (1+i\sigma A_2\right]. \tag{34}$$

Notice that, $\sigma = i\lambda kn$. Similarly, by applying the same operations of the term by-term addition and subtraction to boundary conditions (2) and (4), we obtained the new set of equations which provide B_1 and B_2 as follows:

$$B_1 = \frac{1}{2}\left[(1+i\sigma)C_1 e^{iknL} + (1-i\sigma)C_2 e^{-iknL}\right]e^{\frac{-L}{\lambda}}, \tag{35}$$

$$B_2 = \frac{1}{2}\left[(1-i\sigma)C_1 e^{iknL} + (1+i\sigma)C_2 e^{-iknL}\right]e^{\frac{L}{\lambda}}. \tag{36}$$

Equations (33) and (35) are equal to Equations (34) and (36), respectively.

$$aA_1 + bA_2 = [aC_1 e^{iknL} + bC_2 e^{-iknL}]e^{\frac{-L}{\lambda}}, \tag{37}$$

$$bA_1 + aA_2 = [bC_1 e^{iknL} + aC_2 e^{-iknL}]e^{\frac{L}{\lambda}}. \tag{38}$$

It should be noted that, $a = 1 + i\sigma$ and $b = 1 - i\sigma$. Finally, Equations (37) and (38) are added and subtract term by term:

$$C_1 = \frac{1}{a^2 - b^2}\left[(a^2 e^{\frac{L}{\lambda}} - b^2 e^{\frac{-L}{\lambda}})A_1 + ab(e^{\frac{L}{\lambda}} - e^{\frac{-L}{\lambda}})A_2\right]e^{-iknL}, \tag{39}$$

$$C_2 = \frac{1}{a^2 - b^2}\left[ab(e^{\frac{-L}{\lambda}} - e^{\frac{L}{\lambda}})A_1 + (a^2 e^{\frac{-L}{\lambda}} - b^2 e^{\frac{L}{\lambda}})A_2\right]e^{iknL}. \tag{40}$$

These equations form the elements of the transfer matrix. As a result of this, the relationship between the incoming and outgoing on the SC slab in Figure 2, is determined with a transfer matrix.

Table 3. Boundary conditions for SC slab are shown in the table. We define the quantity $\sigma = i\lambda kn$.

Boundary conditions
1) $A_1 + A_2 = B_1 + B_2$
2) $B_1 e^{\frac{L}{\lambda}} + B_2 e^{-\frac{L}{\lambda}} = C_1 e^{iknL} + C_2 e^{-iknL}$
3) $\sigma(A_1 - A_2) = B_1 - B_2$
4) $B_1 e^{\frac{L}{\lambda}} - B_2 e^{-\frac{L}{\lambda}} = \sigma(C_1 e^{iknL} - C_2 e^{-iknL})$

$$\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \mathbf{M}\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} \tag{41}$$

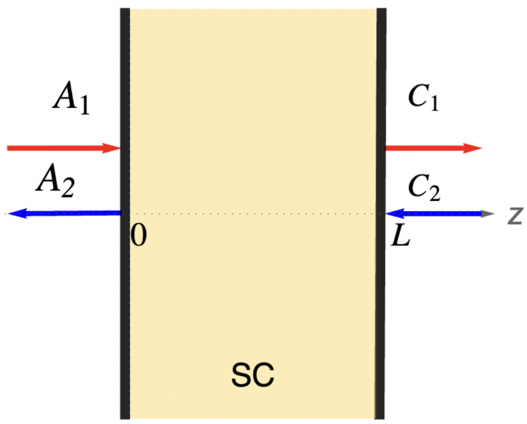

Figure 2. Figure shows the amplitudes of left and right waves in regions I and III, respectively.

3. TRANSFER MATRIX

The transfer matrix is a crucial tool used to determine the transmitted and reflected waves at the surface boundary of the materials. It plays an important role to analyses superconducting behavior on the surface of the material. Using the Equations (39) and (40), transfer matrix is constructed as follows:

$$\mathbf{M} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \tag{42}$$

The transfer matrix (42) contains all the information about the transmitted and reflected waves on the SC slab. The elements of the transfer matrix is defined as follows,

$$M_{11} = [e^{-iknL}(a^2 e^{L/\lambda} - b^2 e^{-L/\lambda})]\gamma, \tag{43}$$

$$M_{12} = ab[e^{-iknL}(e^{L/\lambda} - e^{-L/\lambda})]\gamma, \tag{44}$$

$$M_{21} = ab[e^{iknL}(e^{-L/\lambda} - e^{L/\lambda})]\gamma, \tag{45}$$

$$M_{22} = [e^{iknL}(a^2 e^{-L/\lambda} - b^2 e^{L/\lambda})]\gamma. \tag{46}$$

Here, $\gamma = \frac{1}{a^2-b^2}$ and coefficients A_1 and A_2 represent the right and left wave amplitudes in the I-region, while coefficients C_1 and C_2 represent the right and left wave amplitudes in the III-region in Figure 2. Notice that, $\det|\mathbf{M}| = 1$ and $T^l = T^r$. The left and right transmission coefficients $T^{l,r}$ and reflection $R^{l,r}$ coefficients are given by Mostafazadeh (2009).

$$T^l = T^r = T = \frac{1}{M_{22}}, \qquad R^r = \frac{M_{12}}{M_{22}}, \qquad R^l = -\frac{M_{21}}{M_{22}}. \quad (47)$$

We can define R and T coefficients for SC slab by using the Equation (47) as follows:

$$T = \frac{4i\sigma}{\chi} e^{-iknL}, \quad (48)$$

$$R^l = \frac{(1+\sigma^2)\psi}{\chi}, \qquad R^r = \frac{(1+\sigma^2)\psi}{\chi} e^{-2iknL}. \quad (49)$$

We denote that, $\psi = e^{L/\lambda} - e^{-L/\lambda}$, $\gamma = 1/4i\sigma$ and $\chi = a^2 e^{-L/\lambda} - b^2 e^{L/\lambda}$. Equation (48) is a general formalism for analysing the surface of SC slab in terms of R and T coefficients. This formalisation provides a foundation for investigating applications such as PT-symmetry, CPA, and spectral singularity for superconductors, as seen in works Sarısaman et al. (2024), Mostafazadeh (2009), and Mostafazadeh & Sarısaman (2015).

4. CONCLUSION

In this study, to understand the electromagnetic interaction with superconductors, we solved the Maxwell equations and determined these fields $\vec{E}, \vec{D}, \vec{B}$ and $\vec{H}$ both inside and outside the material. By using these fields, we derived the boundary conditions for the interface of the SC slab. In this way, we constructed a general transfer matrix from boundary conditions for superconductors in the TM mode configuration. This matrix allows for the analysis of the interactions between the superconducting surface and the electromagnetic wave. In this context, the reflection coefficient (R) and transmission coefficient (T) are formulated by London penetration depth λ. Furthermore, the obtained transfer matrix forms a crucial basis for advanced studies such as CPA, PT-symmetry and spectral singularity. On the other hand, by considering the Meissner effect, we solved Maxwell's fourth Equation (7) and observed the emergence of the surface currents related to the magnetic field. The components of the current generated on the surface (xz- plane) are perpendicular to the magnetic field. Also, the surface current $\vec{J}$ is expressed in terms of the London penetration depth λ. Consequently, this study provides a foundation for research on the interactions of superconductors with electromagnetic waves.

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SUPPLEMENTARY

London Equations

To obtain the London equations, we determine the current density as seen in this study Saif (1992),

$$\vec{j}_n = \sigma \vec{E}. \quad (50)$$

Here, σ is the electrical conductivity, and $\vec{E}$ is the electrical fields. Also, the current $\vec{j}_n$ is proportional to the electric field $\vec{E}$. The force acting on the electrons is defined by the Coulomb force.

$$\vec{F} = m\vec{a}, \quad (51)$$

$$\vec{F} = e\vec{E}, \quad (52)$$

$$e\vec{E} = m\frac{\partial \vec{v}}{\partial t}. \quad (53)$$

Equation (50) is written for superconductors as follows,

$$\vec{j}_s = evn_s. \quad (54)$$

Here, n_s is the density of superconducting electrons. Taking the time derivative of Equation (54) gives first London Equation (1), as seen in Saif (1992).

$$\frac{\partial \vec{j}_s}{\partial t} = e\frac{\partial v}{\partial t} n_s = e\left(\frac{e\vec{E}}{m}\right) n_s = \frac{e^2 n_s}{m}\vec{E}. \quad (55)$$

Taking the curl of Equation (55),

$$\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{j}_s) = \left(\frac{e^2 n_s}{m}\right)(\vec{\nabla} \times \vec{E}). \quad (56)$$

The curl of the electric field is as follows:

$$\vec{\nabla} \times \vec{E} = \left(\frac{m}{e^2 n_s}\right)\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{j}_s). \quad (57)$$

This expression gives the relationship of super current density with electric field. Replacing third equation of Maxwell Equation (6) into Equation (57):

$$\left(\frac{m}{e^2 n_s}\right)\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{j}_s) = -\frac{\partial}{\partial t}\vec{B}. \quad (58)$$

Equation (58) gives the relationship the super current with magnetic field and second London Equation (2).

$$\frac{\partial}{\partial t}\left(\vec{\nabla} \times \vec{j}_s + \left(\frac{e^2 n_s}{m}\right)\vec{B}\right) = 0. \quad (59)$$

$$\vec{\nabla} \times \vec{j}_s = -\left(\frac{e^2 n_s}{m}\right) \vec{B}. \qquad (60)$$

LIST OF AUTHOR ORCIDS

Şevval Taşdemir https://orcid.org/0000-0002-4106-7872

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