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Fuzzy Implicative Ideals in KU-algebras

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Abstract – In this paper, we consider KU-implicative ideal (briefly implicative ideal) in KU-algebras. The notion of fuzzy implicative ideals in KU-algebras are introduced, several appropriate examples are provided and their some properties are investigated. The image and the inverse image of fuzzy implicative ideals in KU-algebras are defined and how the image and the inverse image of fuzzy implicative ideals in KU-algebras become fuzzy implicative ideals are studied. Moreover, the Cartesian product of fuzzy implicative ideals in Cartesian product of KU-algebras are given.

***Keywords* –** Fuzzy implicative ideal ,image (inverse image) of fuzzy implicative ideals, Cartesian product of fuzzy implicative ideals.

1. Introduction

BCK-algebras form an important class of logical algebras introduced by Iseki [11,12,13] and was extensively investigated by several researchers. It is an important way to research the algebras by its ideals. The notions of ideals in BCK-algebras and positive implicative ideals in BCK-algebras (i.e. Iseki's implicative ideals) were introduced by Iseki [11,12,13]. The notions of commutative ideals in BCK-algebras and implicative ideals in BCK-algebras were introduced by [18-24]. Zadeh [33] introduced the notion of fuzzy sets. At present this concept has been applied to many mathematical branches, such as group, functional analysis, probability theory, topology, and so on. In 1991, Xi [32] applied this concept to BCK-algebras, and he introduced the notion of fuzzy sub-algebras (ideals) of the BCK-algebras with respect to minimum, and since then Y.B. Jun et al studied fuzzy ideals (cf.[10], [14], [15]), and moreover several fuzzy structures in BCC-algebras are considered (cf [2-9]). Prabpayak and Leerawat [29,30] introduced a new algebraic structure which is called KU-algebra. They gave the concept of homomorphisms of KU-algebras and investigated some related properties. Mostafa et al. [25 - 28] introduced the notion of fuzzy KU-ideals of KU-algebras and then they investigated several basic properties which are related to fuzzy KU-ideals. The idea of implicative ideal was introduced by Meng et al.

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[22,23], they established the concepts of implicative ideals and commutative ideals in BCI-algebras and investigated some of their properties. Mostafa et al. [26,27] introduced the notion of implicative ideals and commutative ideals of KU-algebras and investigated of their properties.

In this paper, the notion of fuzzy implicative ideals of KU - algebras is introduced and then the several basic properties are investigated. How the image and the pre-image of fuzzy implicative ideal under homomorphism of KU-algebras become fuzzy of implicative ideal are studied. Moreover, the product of fuzzy implicative ideal to product fuzzy implicative ideal is established.

2. Preliminaries

Definition 2.1. [29,30] Algebra (X,*,0) of type (2,0) is said to be a KU-algebra, if it satisfies the following conditions:
(ku_1) $(x*y)*[(y*z))*(x*z)]=0$,
(ku_2) $x*0=0$,
(ku_3) $0*x=x$,
(ku_4) $x*y=0$ and $y*x=0$ implies $x=y$,
(ku_5) $x*x=0$, For all $x,y,z\in X$.

On a KU-algebra $(X,*,0)$ we can define a binary relation $\leq$ on X by putting

$$x\leq y\Leftrightarrow y*x=0$$

Thus a KU-algebra X satisfies the conditions:

$(ku_{1'})$ $(y*z)*(x*z)\leq(x*y)$
$(ku_{2'})$ $0\leq x$
$(ku_{3'})$ $x\leq y, y\leq x$ implies $x=y$,
$(ku_{4'})$ $y*x\leq x$.

Theorem 2.2. [25] In a KU-algebra X, the following axioms are satisfied:
For all $x,y,z\in X$,
(1) $x\leq y$ imply $y*z\leq x*z$,
(2) $x*(y*z)=y*(x*z)$, for all $x,y,z\in X$,
(3) $((y*x)*x)\leq y$.
(4) $((y*x)*x)*x))=(y*x)$

Proof. No. (4) Since $\overbrace{(y*z)*(x*z)\leq(x*y)}^{KU_1'}$ implies $x*((y*z)*z)\leq(x*y)$, put x=0, we have

$$0*((y*z)*z)\leq(0*y)\Rightarrow(y*x)*x\leq y, we\ have\ \overbrace{(y*x)\leq((y*x)*x)*x)}^{by(1)}-\tilde{1}$$

But, $(y*x)*[((y*x)*x)*x)] = [(y*x)*x)]*[(y*x)*x)] = 0$
$i.e((y*x)*x)*x) \le (y*x) ---------- \tilde{2}$

From $\tilde{1}$, $\tilde{2}$,we have $((y*x)*x)*x)) = (y*x)$.

We will refer to X is a KU-algebra unless otherwise indicated.

Definition 2.3. [29,30] Let I be a non empty subset of a KU-algebra X. Then I is said to be an ideal of X, if
(I_0) $0 \in I$
(I_1) $\forall y, z \in X$, if $(y*z) \in I$ and $y \in I$, imply $z \in I$.

Definition 2.4. [25] Let I be a non empty subset of a KU-algebra X. Then I is said to be an KU- ideal of X, if
(F_0) $0 \in I$
(F_{KU}) $\forall x, y, z \in X$, if $x*(y*z) \in I$ and $y \in I$, imply $x*z \in I$.

Definition 2.5 [27] A KU-algebra X is said to be implicative if it satisfies the identity

$$x = (x * y) * x \text{ for all } x, y \in X.$$

For the properties of KU-algebras, we refer the reader to [12 - 16].

3. Fuzzy Implicative Ideals

We now review some fuzzy logic concepts

Definition 3.1. [33] Let X be a non-empty set, a fuzzy subset μ in X is a function

$$f : X \to [0,1].$$

Definition 2.11. [25] Let X be a KU-algebra, a fuzzy set μ in X is called a fuzzy ideal of X if it satisfies the following conditions:
(F_0) $\mu(0) \ge \mu(x)$ for all $x \in X$.
(FI) $\forall x, y \in X$, $\mu(y) \ge \min\{\mu(x*y), \mu(x)\}$.

Definition 3.2. [25] Let μ be a fuzzy set in a set X. For t $\in$ [0, 1], the set

$$\mu_t = \{x \in X \mid \mu(x) \ge t\}$$

is called upper level cut (level subset) of μ.

Definition 3.3. A non empty subset μ of a KU-algebra X is called a fuzzy implicative ideal of X, if $\forall x, y, z \in X$,

(F_0) $\mu(0) \geq \mu(x)$
(F_1) $\mu((x*y)*x) \geq \min\{\mu(z*((x*y)*x)), \mu(z)\}$.

Example 3.4. Let $X = \{0,1,2,3,4\}$ in which the operation $*$ is given by the table

*	0	1	2	3	4
0	0	1	2	3	4
1	0	0	1	3	4
2	0	0	0	3	4
3	0	0	0	0	4
4	0	0	0	0	0

Then $(X,*,0)$ is a KU-Algebra. Define a fuzzy set $\mu : X \to [0,1]$ by

$$\mu(0) = t_0,\ \mu(1) = \mu(2) = t_1,\ \mu(3) = t_2,\ \mu(4) = t_3$$

where $t_0, t_1, t_2, t_3 \in [0,1]$ with $t_0 > t_1 > t_2 > t_3$.

Routine calculation gives that μ is a fuzzy implicative ideal of KU-algebra X.

Lemma 3.5. If μ is a fuzzy implicative ideal of KU - algebra X and if $x \leq z$, then $\mu(x) \geq \mu(z)$.

Proof. if $x \leq z$, then $z * x = 0$, this together with $0 * x = x$, $x * x = x * 0 = 0$ and $\mu(0) \geq \mu(z)$. Put $y = 0$ in (F_1), we get

$$\mu((x*0)*x) \geq \min\{\mu(z*((x*0)*x)), \mu(z)\}$$
$$\mu(0*x) \geq \min\{\mu(z*(0*x)), \mu(z)\} = \min\{\mu(z*x), \mu(z)\}$$
$$\mu(x) \geq \min\{\mu(0), \mu(z)\} = \mu(z)$$

Lemma 3.6. Let μ be a fuzzy implicative ideal of KU-algebra X, if the inequality

$z * x \leq y$ hold in X, Then $\mu(x) \geq \min\{\mu(y), \mu(z)\}$.

Proof. Assume that the inequality $y * x \leq z$ holds in X, then

$$\mu(z*x) \geq \mu(y)$$

by (**Lemma 3.5**). Put x =y in (F_2), we have $\mu((x*x)*x) \geq \min\{\mu(z*((x*x)*x)), \mu(z)\}$ i.e. $\mu(x) \geq \min\{\mu(z*x), \mu(z)\}$, but $\mu(z*x) \geq \mu(y)$, then $\mu(x) \geq \min\{\mu(y), \mu(z)\}$, this completes the proof.

Proposition 3.7. The intersection of any set of fuzzy implicative ideals of KU-algebra X is also fuzzy implicative ideal.

Proof. Let $\{\mu_i\}$ be a family of fuzzy implicative-ideals of KU-algebra X, then for any $x, y, z \in X$,

$$(\cap\mu_i)(0) = \inf(\mu_i(0)) \geq \inf(\mu_i(x)) = (\cap\mu i)(x)$$

and

$$\begin{aligned}(\cap\mu_i)((x * y)*x) &= \inf(\mu_i((x * y)*x)) \geq \inf(\min\{\mu_i(z*((x * y)*x)), \mu_i(z)\}) \\ &= \min\{\inf(\mu_i(z*((x * y)*x)), \inf(\mu_i(z)\} \\ &= \min\{(\cap\mu_i)(z*((x * y)*x)), (\cap\mu_i(z)\}.\end{aligned}$$

This completes the proof.

Theorem 3.8. A fuzzy subset μ of KU - algebra X is a fuzzy implicative-ideal of X if and only if, for every t ∈ [0,1], μ_t is either empty or an implicative ideal of X.

Proof. Assume that μ is a fuzzy implicative - ideal of X, by (F1), we have μ (0) ≥ μ (x) for all x ∈ X therefore μ (0) ≥ μ (x) ≥ t for x ∈ μt and so 0 ∈ μ_t.

Let z* ((x * y) * x)∈ μt and z ∈ μt, then μ(z*((x * y) * x)) ≥ t and μ (z) ≥ t, since μ is a fuzzy implicative - ideal it follows that μ((x * y)*x) ≥ min {μ(z* ((x * y) * x)), μ (z)} ≥ t and therefore (x * y)*x ∈ μ_t. Hence μ_t is an KU-ideal of X.

Conversely, we only need to show that (F_1) and (F_2) are true. If (F_1) is false then there exist x\`∈ X such that μ (0) < μ(x\`). If we take t\`= (μ (x\`) μ (0))/2, then μ(0) < t\` and 0 ≤ t\` < μ (x\`) ≤ 1, thus x\`∈ μ and μ ≠ ϕ As μ is an KU–implicative ideal of X, we have 0 ∈ $\mu_{t`}$ and so μ (0) ≥ t\`. This is a contradiction.

Now, assume (F_2) is not true,then there exist x\`, y\` and z\` such that,

$$\mu((x`* y`)* x`) < \min\{\mu(z` *(x`* y`)* x`), \mu(z`)\}$$

Putting t\`={ μ ((x\`* y\`)* x\`)+min{μ (z\` *(x\`* y\`)* x\`), μ (z\`)}} /2, then μ ((x\`* y\`)* x\`) < t\` and 0 ≤ t\` < min {μ (z\` *(x\`* y\`)* x\`), μ (z\`)}} /2 ≤1 , hence μ (z\` *(x\`* y\`)* x\`) > t\` and μ (z\`) > t\`,which imply that (x\`* y\`)* x\`) ∈ μ (t\`) and z\`∈ $\mu_{t`}$, since μ_t is an implicative-ideal, it follows that (x\`* y\`)* x\`) ∈ $\mu_{t`}$ and that μ (x\`* y\`)* x\`) ≥ t\`, this is also a contradiction. Hence μ is a fuzzy implicative ideal of X.

Corollary 3.9. If a fuzzy subset μ of KU-algebra X is a fuzzy implicative-ideal, then for every t ∈ Im (μ), μ_t is an implicative-ideal of X.

Definition 3. 10. [32] Let *f* be a mapping from the set *X* to a set *Y*. If μ is a fuzzy subset of *X*, then the fuzzy subset *B* of *Y* defined by

$$f(\mu)(y) = B(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu(x), & \text{if } f^{-1}(y) = \{x \in X, f(x) = y\} \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

is said to be the image of μ under *f*.

Similarly if β is a fuzzy subset of Y, then the fuzzy subset μ = β o f in X (i.e the fuzzy subset defined by μ (x) = β (f (x)) for all x ∈ X) is called the primage of β under f.

Theorem 3.11. An onto homomorphic preimage of a fuzzy implicative-ideal is also a fuzzy implicative-ideal.

Proof. Let f : X → X\` be an into homomorphism of KU-algebras, β a fuzzy implicative-ideal of X\` and μ the preimage of β under f, then β (f (x)) = μ (x), for all x ∈ X.

Let x ∈ X, then μ (0) = β (f (0)) ≥ β (f (x)) = μ (x). Now let x, y, z ∈ X then

$$\begin{aligned}\mu((x*y)*x) &= \beta(f(x*y)*x)) = \beta((f(x)*f(y))*f(x)) \\ &\geq \min\{\beta(f(z)*(f(x)*f(y))*f(x))), \beta(f(z))\} \\ &= \min\{\beta(f(z*((x*y)*x)), \beta(f(z))\} \\ &= \min\{\mu(z*((x*y)*x)), \mu(z)\}.\end{aligned}$$

The proof is completed.

Definition 3.12. [31] A fuzzy subset μ of X has sup property if for any subset T of X, there exist $t_0 \in T$ such that $\mu(t_0) = \sup_{t \in T} \mu(t)$.

Theorem 3.13. Let X → Y be a homomorphism between KU-algebras X and Y. For every fuzzy implicative ideal μ in X, f (μ) is a fuzzy implicative-ideal of Y.

Proof. By definition

$$B(y') = f(\mu)(y') = \sup_{x \in f^{-1}(y')} \mu(x) \text{ for all } y' \in Y \text{ and } \sup \phi = 0$$

We have to prove that $B((x' * y') * x') \geq \min\{B(z' * (x' * y') * x'), B(z')\}$, ∀ x\`, y\`, z\` ∈ Y.

Let f : X → Y be an onto a homomorphism of KU - algebras, μ a fuzzy implicative - ideal of X with sup property and β the image of μ under f, since μ is a fuzzy implicative - ideal of X, we have μ(0) ≥ μ(x) for all x∈ X. Note that 0 ∈ f^{-1}(0\`), where 0, 0\` are the zero of X and Y respectively. Thus, $B(0') = \sup_{t \in f^{-1}(0')} \mu(t) = \mu(0) \geq \mu(x)$, for all $x \in X$, which implies that $B(0') \geq \sup_{t \in f^{-1}(x')} \mu(t) = B(x')$, for any $x' \in Y$. For any $x', y', z' \in Y$, let

$$x_0 \in f^{-1}(x') \quad , \quad y_0 \in f^{-1}(y') \quad , \quad z_0 \in f^{-1}(z')$$

be such that

$$\mu(z_0 * ((x_0 * y_0) * x_0)) = \sup_{t \in f^{-1}(z_0 * ((x_0 * y_0) * x_0))} \mu(t) \quad , \quad \mu(z_0) = \sup_{t \in f^{-1}(z')} \mu(t)$$

and

$$\mu(z_0 * ((x_0 * y_0) * x_0)) = B\{f(z_0 * ((x_0 * y_0) * x_0)\} = B(z' * ((x' * y') * x'))$$

$$= \sup_{z_0*((x_0*y_0)*x_0)\in f^{-1}(z'*(x'*y')*x')} \mu(z_0*((x_0*y_0)*x_0)) = \sup_{t\in f^{-1}((z'*(x'*y')*x')} \mu(t).$$

Then

$$B((x'*y)*x') = \sup_{t\in f^{-1}(x'*y)*x')} \mu(t) = \mu((x_0*y_0)*x_0) \geq \min\{\mu(z_0*(x_0*y_0)*x_0)), \mu(z_0)\} =$$

$$\min\left\{ \sup_{t\in f^{-1}(z'*((x'*y')*x'))} \mu(t), \quad \sup_{t\in f^{-1}(z')} \mu(t) \right\} = \min\{B(z'*((x'*y')*x')), \ B(z')\}.$$

Hence B is a fuzzy implicative -ideal of Y.

4. Cartesian Product of Fuzzy Implicative-ideal

Definition 4.1. [1] A fuzzy μ is called a fuzzy relation on any set S, if μ is a fuzzy subset

$$\mu : S \times S \to [0,1]$$

Definition 4.2. [1] If μ is a fuzzy relation a set S and β is a fuzzy subset of S, then μ is fuzzy relation on β if $\mu(x, y) \leq \min\{\beta(x), \beta(y)\}$, $\forall\ x, y \in S$.

Definition 4.3. [1] Let μ and β be fuzzy subset of a set S, the Cartesian product of μ and β is define by $(\mu \times \beta)(x, y) = \min\{\mu(x), \beta(y)\}$, $\forall\ x, y \in S$.

Lemma 4.4. [1] let μ and β be fuzzy subset of a set S then,
(i) $\mu \times \beta$ is a fuzzy relation on S.
(ii) $(\mu \times \beta)_t = \mu_t \times \beta_t$ for all $t \in [0,1]$.

Definition 4.5. [1] If β is a fuzzy subset of a set S, the strongest fuzzy relation on S, that is, a fuzzy relation on β is μ_β given by $\mu_\beta(x, y) = \min\{\beta(x), \beta(y)\}$, $\forall\ x, y \in S$.

Lemma 4.6. [1] For a given fuzzy subset S, let μ_β be the strongest fuzzy relation on S the for $t \in [0,1]$, we have $(\mu_\beta)_t = \beta_t \times \beta_t$.

Proposition 4.7. For a given fuzzy subset β of KU- algebra X, let μ_β be the strongest fuzzy relation on X. If μ_β is a fuzzy implicative ideal of $X \times X$, then $\beta(x) \leq \beta(0)$ for all $x \in X$.

Proof. Since μ_β is a fuzzy implicative ideal of $X \times X$, it follows from (F_1) that

$$\mu_\beta(x, x) = \min\{\beta(x), \beta(x)\} \leq (0, 0) = \min\{\beta(0), \beta(0)\}$$

where $(0, 0) \in X \times X$, then $\beta(x) \leq \beta(0)$.

Remark 4.8. Let X and Y be KU-algebras, we define $*$ on $X \times Y$ for every $(x, y), (u, v) \in X$ x Y, $(x, y) * (u, v) = (x * u, y * v)$, then clearly $(x * y, *, (0, 0))$ is a KU-algebra.

Theorem 4.9. let μ and β be a fuzzy implicative-ideals of KU - algebra X, $\mu \times \beta$ is a fuzzy implicative-ideal of $X \times X$.

Proof. for any $(x, y) \in X \times X$, we have,

$(\mu \times \beta)(0, 0) = \min\{\mu(0), \beta(0)\} \geq \min\{\mu(x), \beta(x)\} = (\mu \times \beta)(x, y)$.

Now let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$, then,

$$\begin{aligned}
&(\mu \times \beta)((x_1 * y_1) * x_1), ((x_2 * y_2) * x_2)) \\
=\ & \min\{\mu((x_1 * y_1) * x_1)), \beta((x_2 * y_2) * x_2))\} \\
\geq\ & \min\{\min\{\mu(z_1 * (x_1 * y_1) * x_1))), \mu(z_1)\}, \min\{\beta((x_2 * y_2) * x_2)), \beta(z_2)\}\} \\
=\ & \min\{\min\{\mu(z_1 * (x_1 * y_1) * x_1)), \beta(z_2 * (x_2 * y_2) * x_2))\}, \min\{\mu(z_1), \beta(z_2)\}\} \\
=\ & \min\{(\mu \times \beta)(z_1 * (x_1 * y_1) * x_1), z_2 * (x_2 * y_2) * x_2)), (\mu \times \beta)(z_1, z_2)\}.
\end{aligned}$$

Hence $\mu \times \beta$ is a fuzzy implicative ideal of $X \times X$.

Analogous to [28], we have a similar results for implicative-ideal, which can be proved in similar manner, we state the results without proof.

Theorem 4.10. let μ and β be a fuzzy subset of KU-algebra X,such that $\mu \times \beta$ is fuzzy implicative -ideal of $X \times X$, then
(i) either $\mu(x) \leq \mu(0)$ or $\beta(x) \leq \beta(0)$ for all $x \in X$,
(ii) if $\mu(x) \leq \mu(0)$ for all $x \in X$, then either $\mu(x) \leq \beta(0)$ or $\beta(x) \leq \beta(0)$,
(iii) if $\beta(x) \leq \beta(0)$ for all $x \in X$, then either $\mu(x) \leq \mu(0)$ or $\beta(x) \leq \mu(0)$,
(v) either μ or β is a fuzzy implicative - ideal of X.

Theorem 4.11. let β be a fuzzy subset of KU-algebra X and let μ_β be the strongest fuzzy relation on X, then β is a fuzzy implicative ideal of X if and only if μ_β is a fuzzy implicative-ideal of $X \times X$.

Proof. Assume that β is a fuzzy implicative-ideal X, we note from (F_1) that

$$\mu_\beta(0, 0) = \min\{\beta(0), \beta(0)\} \geq \min\{\beta(x), \beta(y)\} = \mu_\beta(x, y)$$

Now, for any $(x_1,x_2), (y_1,y_2),(z_1,z_2) \in X \times X$, we have from (F_2)

$$\begin{aligned}
&\mu_\beta((x_1 * y_1) * x_1, (x_2 * y_2) * x_2) \\
=\ & \min\{\beta((x_1 * y_1) * x_1), \beta((x_2 * y_2) * x_2)\} \\
\geq\ & \min\{\min\{\beta(z_1 * ((x_1 * y_1) * x_1)), \beta(z_1)\}, \min\{\beta(z_2 * ((x_2 * y_2) * x_2)), \beta(z_2)\}\} \\
=\ & \min\{\min\{\beta(z_1 * ((x_1 * y_1) * x_1)), \beta(z_2 * ((x_2 * y_2) * x_2))\}, \min\{\beta(z_1), \beta(z_2)\}\} \\
=\ & \min\{\mu_\beta(z_1 * ((x_1 * y_1) * x_1), z_2 * ((x_2 * y_2) * x_2)), \mu_\beta(z_1, z_2)\}.
\end{aligned}$$

Hence μ_β is a fuzzy implicative-ideal of $X \times X$.

Conversely: for all $(x, y) \in X \times X$, we haveMin $\{\beta(0), \beta(0)\} = \mu_\beta(x, y) = \min\{\beta(x), \beta(y)\}$ It follows that $\beta(0) \geq \beta(x)$ for all $x \in X$, which prove (F_1).

Now, let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$, then

$$\begin{aligned}
&\min\{\beta(((x_1 * y_1) * x_1), \beta((x_2 * y_2) * x_2)\} = \mu_\beta((x_1 * y_1) * x_1), (x_2 * y_2) * x_2)) \\
\geq\ & \min\{\mu_\beta((z_1, z_2) * ((x_1, x_2) * (y_1, y_2)) * (x_1, x_2)), \mu_\beta(z_1,z_2))\}
\end{aligned}$$

$= \min\{\mu_\beta(z_1 * ((x_1 * y_1) * x_1), z_2 * ((x_2 * y_2) * x_2)), \mu_\beta(z_1, z_2)\}$
$= \min\{\min\{\beta(z_1 * ((x_1 * y_1) * x_1)), \beta(z_2 * ((x_2 * y_2) * x_2))\}, \min\{\beta(z_1), \beta(z_2)\}\}$
$= \min\{\min\{\beta(z_1 * ((x_1 * y_1) * x_1)), \beta(z_1)\}, \min\{\beta(z_2 * ((x_2 * y_2) * x_2)), \beta(z_2)\}\}$

In particular, if we take $x_2 = y_2 = z_2 = 0$, then, $\beta((x_1 * y_1) * x_1) \geq \min\{\beta(z_1 * ((x_1 * y_1) * x_1)), \beta(z_1)\}$.

This prove (F_1) and completes the proof.

5. Conclusion

we have studied the fuzzy of implicative ideal in KU-algebras. Also we discussed few results of fuzzy of implicative ideal in KU-algebras under homomorphism, the image and the pre- image of fuzzy implicative ideal under homomorphism of KU-algebras are defined. How the image and the pre-image of fuzzy implicative ideal under homomorphism of KU-algebras become fuzzy of implicative ideal are studied. Moreover, the product of fuzzy implicative ideal to product fuzzy implicative ideal is established. Furthermore, the main purpose of our future work is to investigate the foldedness of other types of fuzzy ideals with special properties such as a bipolar intuitionistic (interval value) fuzzy n-fold of implicative ideals in some algebras.

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Conflicts of Interest

State any potential conflicts of interest here or “The author declare no conflict of interest”.

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