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FIXED POINT THEOREMS FOR SINGLE VALUED α-ψ-MAPPINGS IN FUZZY METRIC SPACES

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Abstract. This article is the forward result of α-admissible and (α,ψ)-contractive mappings in fuzzy metric spaces. We establish new theorem for such contractions to find fixed point in fuzzy metric space. Our Theorem is generalizations of some interesting outputs in metric spaces. Moreover, an example and application to ordinary differential equations are also elaborated to verify the result of the theorem.

1. Introduction

Nature of work on fixed point theory are versatile and always have been source of motivation for further research. The basic core of this theory is Banach contraction principle. Gregori and Sapena [8] have explained Banach contraction principle in complete fuzzy metric space. The basic idea behind this extension of metric spaces is fuzzy sets. The fuzzy set was innovated by Zadeh [7] in 1965. Kramosil and Michalek [3] prefaced the concept of fuzzy metric space using continuous t-norm and probabilistic metric space in 1975. A modification of this result was done by George and Veeramani [2] which is used more frequently to find fixed point in fuzzy metric space. We also consider the fuzzy metric space in the accordance with George and Veeramani [2]. In 2012, Samet et. al. [9] gave the (α,ψ)-contractive and α-admissible mapping to evaluate fixed point theorem in complete metric spaces. Many researchers worked on this types of contractive mappings [5-7]. Salimi [10] modified the belief of (α,ψ)-contractive and α-admissible mapping to get fixed point theorem in complete metric space. Mihet [4] generated the concept of η-contraction mapping which generalize the fuzzy contraction introduced by Gregori and Sapena [8]. In 2014, Hong [11] introduced (α,ψ)- contraction for set-valued mapping in fuzzy metric space. Many other authors contributed their research to give direction in the field of fuzzy metric space [13, 14, 15, 16, 17, 18]. This above work intensified

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our efforts to prove our result using (α, ψ)-contraction mapping and α-admissible mapping in fuzzy metric space.

2. Preliminaries

Definition 2.1. [6] *A binary operation* $* : [0,1] \times [0,1] \to [0,1]$ *is continuous* t*-norm if* $*$ *satisfies the following conditions:*

(i) $*$ *is commutative and associative;*
(ii) $*$ *is continuous;*
(iii) $a * 1 = a$ *for all* $a \in [0,1]$*;*
(iv) $a * b \leq c * d$ *whenever* $a \leq c$ *and* $b \leq d$ *for all* $a, b, c, d \in [0,1]$.

Definition 2.2. [2] *A 3-tuple* $(\mathcal{K}, M, *)$ *is said to be a fuzzy metric space if* $\mathcal{K}$ *is an arbitrary set, '$*$' is a continuous* t*-norm,* M *is fuzzy set on* $\mathcal{K}^2 \times [0, \infty)$ *satisfying the following conditions for all* $x, y, z \in \mathcal{K}$ *and* $s, t > 0$

(F1) $M(x, y, t) > 0$*;*
(F2) $M(x, y, t) = 1$ *for all* $t > 0$ *iff* $x = y$*;*
(F3) $M(x, y, t) = M(y, x, t)$*;*
(F4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$*;*
(F5) $M(x, y, t, \cdot) : [0, \infty) \to [0, 1]$ *is left continuous.*

The function $M(x, y, t)$ *denotes the degree of nearness between* x *and* y *with respect to* t*. We refer to these spaces as GV-spaces.*

Lemma 2.3 ([2])**.** *In fuzzy metric space* $(\mathcal{K}, M, *)$*,* $M(x, y, \cdot)$ *is non-decreasing for all* $x, y \in \mathcal{K}$.

Definition 2.4 ([2])**.** *Let* $(\mathcal{K}, M, *)$ *be a fuzzy metric space. Then,*

(a) a sequence $\{x_n\}$ *in* $\mathcal{K}$ *is said to be convergent to a point* $x \in \mathcal{K}$ *if for all* $t > 0$*,* $\lim_{n \to \infty} M(x_n, x, t) = 1$.
(b) a sequence $\{x_n\}$ *in* $\mathcal{K}$ *is called Cauchy sequence if for* $t > 0$ *and for each* $\epsilon \in (0, 1)$ *there exists* $n_0 \in N$ *such that* $M(x_n, y_m, t) > 1 - \epsilon$ *for all* $n, m \geq n_0$.
(c) A fuzzy metric space in which every Cauchy sequence is convergent is called a complete fuzzy metric space.

In 2015, Saha et al. [12] introduced a new fuzzy contraction mapping and prove that such mappings have fixed point in complete fuzzy metric space-spaces. This contraction is called α-ψ - fuzzy contraction.

Definition 2.5. [12] *Let* $(\mathcal{K}, M, *)$ *be a fuzzy metric space and, let* $f : \mathcal{K} \to \mathcal{K}$ *and* $\alpha : \mathcal{K} \times \mathcal{K} \times (0, \infty) \to (0, \infty)$ *be two mappings. Then the map* f *is said to be* α*-admissible if*

$$\alpha(x, y, t) \leq 1 \quad \Rightarrow \quad \alpha(fx, fy, t) \leq 1, \ \forall \ t > 0 \text{ and } x, y \in \mathcal{K}.$$

Definition 2.6. [12] *Let $(\mathcal{K}, M, *)$ be a fuzzy metric space and, let $f : \mathcal{K} \to \mathcal{K}$ and $\alpha, \eta : \mathcal{K} \times \mathcal{K} \times (0, \infty) \to (0, \infty)$ be two mappings. Then the map f is said to be α-admissible mapping apropos to η if*

$$\alpha(x, y, t) \leq \eta(x, y, t) \quad \Rightarrow \quad \alpha(fx, fy, t) \leq \eta(fx, fy, t), \ \forall\ t > 0 \text{ and } x, y \in \mathcal{K}.$$

Also, if we take $\eta(x, y, t) = 1$, then this definition reduces to Definition 2.4 and if we take $\alpha(x, y, t) = 1$, then we say that f is an η-subadmissible mapping.

Definition 2.7. [11] *Let Ψ be a family of function $\psi : [0, 1] \to [0, 1]$ is continuous, non-decreasing, and $\psi(s) > s$ for each $s \in [0, 1)$.*

Lemma 2.8. [11] *Let $\psi \in \Psi$. For every $r > 0$, $\psi(r) > r$ if and only if $\lim_{n\to\infty} \psi^n(r) = 1$ uniformly for $r \in [0, 1)$, where ψ^n is the n^{th} iterate of ψ.*

3. Main Results

A series of results on α-admissible mapping gave the idea of present theorem in fuzzy metric space. The idea has originated from the Hong [11] and Samet et al.[9] problems in metric spaces.

Theorem 3.1. *Let $(\mathcal{K}, M, *)$ be a complete fuzzy metric space and let f be an α-admissible mappings apropos to η such that*

$$\alpha(x, y, t) \leq \eta(x, y, t) \Rightarrow M(fx, fy, t) \geq \psi(N(x, y, t)), \tag{1}$$

where $N(x, y, t) = \min\{M(x, y, t), M(y, fy, t), M(x, fy, t) * M(y, fy, t)\}$.
Suppose that the following conditions hold:

- *(i) There exists $x_0 \in \mathcal{K}$ such that $\alpha(x_0, fx_0, t) \leq \eta(x_0, fx_0, t)$ for each $t > 0$,*
- *(ii) f is continuous or for any sequence $\{x_n\} \subseteq \mathcal{K}$ converging to $x \in \mathcal{K}$ and $\alpha(x_n, x_{n+1}, t) \leq \eta(x_n, x_{n+1}, t)$ for all $x \in \mathcal{K}$ and $t > 0$ where $\alpha(x_n, x, t) \leq \eta(x_n, x, t)$.*

Then there exists a fixed point of f.

Proof. Let $x_0 \in \mathcal{K}$ such that $\alpha(x_0, fx_0, t) \leq \eta(x_0, fx_0, t)$.
Define a sequence $\{x_n\}$ in $\mathcal{K}$ $x_n = f^n x_0 = fx_{n-1}$ for all $n \in N$. If $x_{n+1} = x_n$ for some $n \in N$, then existence of fixed point is apparent.
Now we suppose that $x_n \neq x_{n+1}$ for all $n \in N$. Since f is an α-admissible mapping with respect to η and $\alpha(x_0, fx_0, t) \leq \eta(x_0, fx_0, t)$. We interpret that

$$\alpha(x_1, x_2, t) = \alpha(fx_0, f^2x_0, t) \leq \eta(fx_0, f^2x_0, t) = \eta(x_1, x_2, t)$$

Inductively, we get

$$\alpha(x_n, x_{n+1}, t) \leq \eta(x_n, x_{n+1}, t) \quad \text{for all } n \in N \cup \{0\}.$$

Now, by contractive condition (1) with $x = x_{n-1}$, $y = x_n$, we have

$$M(fx_{n-1}, fx_n, t) \geq \psi(N(x_{n-1}, x_n, t)), \tag{2}$$

by means of above equation, we have

$$M(x_n, x_{n+1}, t) \geq \psi(N(x_{n-1}, x_n, t)) \quad \text{for all } n \in N \text{ and } t > 0.$$

Also, $N(x_{n-1}, x_n, t) = \min\{M(x_{n-1}, x_n, t),$

$$M(x_n, fx_n, t), M(x_{n-1}, fx_n, t) * M(x_n, fx_n, t)\}$$

$$\begin{aligned} N(x_{n-1}, x_n, t) &= \min(x_{n-1}, x_n, t), M(x_n, x_{n+1}, t)\} \\ &= M(x_{n-1}, x_n, t) \end{aligned}$$

As if $N(x_{n-1}, fx_n, t) = M(x_n, x_{n+1}, t)$,

$$M(x_n, x_{n+1}, t) \geq \psi(N(x_{n-1}, x_n, t)) = \psi(M(x_n, x_{n+1}, t)) > M(x_n, x_{n+1}, t),$$

a contradiction confirms that $N(x_{n-1}, x_n t) = M(x_{n-1}, x_n t)$,.

$$\Rightarrow M(x_n, x_{n+1}, t) \geq \psi(M(x_{n-1}, x_n, t)).$$

Also, $M(x_n, x_{n+1}, t) \geq \psi(M(x_{n-1}, x_n, t)) \geq \cdots \geq \psi^n(M(x_0, x_1, t))$.

Now, for all $m > n$, $m, n \in N$ and $t > 0$, we can write

$$\begin{aligned} M(x_n, x_m, t) &\geq M(x_n, x_{n+1}, t_{n+1}) * M(x_{n+1}, x_{n+2}, t_{n+2}) * \cdots * M(x_{m-1}, x_m, t_m) \\ &\geq \prod_{i=n}^{m-1} M(x_i, x_{i+1}, t_{i+1}) \\ &\geq \prod_{i=n}^{m-1} \psi^i(M(x_0, x_1, t_i)) \end{aligned}$$

$t_i > 0$, $i = n+1, n+2, \cdots, m$ and $\sum_{i=n+1}^{m} t_i = t$, after considering lemma 2.8, we can assume that $\psi^i(M(x_0, x_1, t_i)) > 1 - \frac{1}{2^i}$ where i is large enough.

As we know that the series $\sum_{i=1}^{\infty} 1/2^i$is convergent and therefore $\prod_{i=1}^{\infty}(1 - 1/2^i)$ is convergent, too.

Eventually $\lim_{n\to\infty} \prod_{i=1}^{\infty}(1 - 1/2^i) = 1$. We analyse that $\{x_n\}$is an Cauchy sequence. Hence $(\mathcal{K}, M, *)$ is a complete fuzzy metric space, therefore there exists $p \in \mathcal{K}$ such that $x_n \to p$.

Now, if suppose that $f$is a continuous function, then

$$fp = \lim_{n\to\infty} fx_n = \lim_{n\to\infty} x_{n+1} = p.$$

So p is a fixed point.

On the other hand, since

$$\alpha(x_n, x_{n+1}, t) \leq \eta(x_{n-1}, x_{n+1}, t) \quad \text{for all} \quad n \in N \cup \{0\} \text{ and } x_n \to p \text{ as } n \to \infty.$$

$$\alpha(x_n, p, t) \leq \eta(x_n, p, t)$$

Now, by condition (1), we get

$$M(fp, x_{n+1}, t) \geq \psi(N(p, x_n, t)) > N(p, x_n, t)$$

where $N(p, x_n, t) = \min\{M(p, x_n, t), M(p, fx_n, t)\}$.

Letting $n \to \infty$ in the above inequality, and hence $M(fp, p, t) \geq \psi(M(fp, p, t)) \geq M(fp, p, t)$, we get $M(fp, p, t) = 1$, this implies $fp = p$. □

Corollary 3.2. *Let $(\mathcal{K}, M, *)$ be a complete fuzzy metric space. let f be an α-admissible mappings with apropos $\eta = 1$ such that*

(i) For $\psi \in \Psi$, $x, y \in \mathcal{K}$ and $t > 0$

$$\alpha(x, y, t) \leq 1 \Rightarrow M(fx, fy, t) \geq \psi(N(x, y, t)),$$

(ii) there exists $x_0 \in \mathcal{K}$ such that $\alpha(x_0, fx_0, t) \leq 1$ for each $t > 0$,

(iii) for any sequence $\{x_n\} \subseteq \mathcal{K}$ converging $x \in \mathcal{K}$ and $\alpha(x_n, x_{n+1}, t) \leq 1$ for all $n \in N$ and $t > 0$ we have $\alpha(x, fx, t) \leq 1$ for all $n \in N$ and $t > 0$.

where $N(x, y, t) = \min\{M(x, y, t), M(y, fy, t), M(x, fy, t) * M(y, fy, t)\}$.
Then f has a fixed point.

Example 3.3. *Let $\mathcal{K} = [0, \infty)$ and $(\mathcal{K}, M, *)$ be endowed with fuzzy metric space*

$$M(x, y, t) = \frac{t}{t + |x - y|}$$

for all $x, y \in \mathcal{K}$ and $t > 0$.
Let the single-valued mapping $f : \mathcal{K} \to \mathcal{K}$ be defined by

$$fx = \left\{ \begin{array}{l} \frac{x}{3}, x \in [0, 1] \\ x^3 + 1, x \in (1, \infty) \end{array} \right\}$$

Define: $\alpha : \mathcal{K} \times \mathcal{K} \times [0, \infty) \to \mathcal{K}$ and $\psi \in \Psi$ by $\alpha \equiv 1$,

$$\alpha(x, y, t) = \left\{ \begin{array}{l} 1, x, y \in [0, 1] \\ 10, x \in (1, \infty) \end{array} \right\}, t > 0$$

and $\psi(s) = \frac{s}{3}$ for $s \in [0, 1]$, respectively. Then assurance of fixed point for f.

Corollary 3.4. *Let $(\mathcal{K}, M, *)$ be a complete fuzzy metric space. Let f be an η-subadmissible set valued mappings with respect to $\alpha = 1$ such that*

(i) For $\psi \in \Psi$, $x, y \in \mathcal{K}$ and $t > 0$,

$$\eta(x, y, t) \geq 1 \Rightarrow M(fx, fy, t) \geq \psi(N(x, y, t)),$$

(ii) there exists $x_0 \in \mathcal{K}$ such that $\eta(x_0, fx_0, t) \geq 1$ for each $t > 0$,

(iii) for any sequence $\{x_n\} \subseteq \mathcal{K}$ converging $x \in \mathcal{K}$ and $\eta(x_n, x_{n+1}, t) \geq 1$ for all $n \in N$ and $t > 0$ we have $\eta(x, fx, t) \geq 1$ for all $n \in N$ and $t > 0$.

where $N(x, y, t) = \min\{M(x, y, t), M(y, fy, t), M(x, fy, t) * M(y, fy, t)\}$.
Then f has a fixed point.

Example 3.5. *Let $\mathcal{K} = [0,\infty)$ and $(\mathcal{K}, M, *)$ be endowed with fuzzy metric space*

$$M(x,y,t) = \frac{t}{t + |x-y|}$$

for all $x, y \in \mathcal{K}$ and $t > 0$.
Let the single-valued mapping $f : \mathcal{K} \to \mathcal{K}$ be defined by

$$fx = \left\{ \begin{array}{c} \frac{x^2+2}{x+1}, x \in [0,1] \\ cosx, x \in (1,\infty) \end{array} \right\}$$

Define: $\eta : \mathcal{K} \times \mathcal{K} \times (0,\infty) \to \mathcal{K}$ and $\psi \in \Psi$ by $\alpha \equiv 1$,

$$\eta(x,y,t) = \left\{ \begin{array}{c} 2, x, y \in [0,1] \\ 1/5, x \in (1,\infty) \end{array} \right\}, t > 0$$

and $\psi(s) = \frac{s}{5}$ for $s \in [0,1]$, respectively. Then assurance of fixed point for f.

Proof. Clearly, $(\mathcal{K}, M, *)$ is a complete fuzzy metric space. We show that f is an η-subadmissible mapping.

Let $x, y \in \mathcal{K}$ and $t > 0$; if $\eta(x,y,t) \geq 1$,then $x, y \in [0,1]$. On the other hand, for all $x \in [0,1]$, we have $fx \geq 1$. It follows that $\eta(fx, fy, t) \geq 1$.

If, $x \in (1,\infty)$, then $\eta(fx, fy, t) \leq 1$, which is a contradiction. Hence $x \notin (1,\infty)$.

Now, if $\{x_n\}$ is a sequence in $\mathcal{K}$ such that $\eta(x_n, x_{n+1}, t) \geq 1$, for all $n \in N$ and $x_n \to x$ as $n \to \infty$, then $\{x_n\} \subset [0,1]$ and hence $x \in [0,1]$. This implies that $\eta(x_n, x, t) \geq 1$ for all $n \in N$and $t > 0$.

Let $\eta(x,y,t) \geq 1$. Then $x, y \in [0,1]$. We get

$$M(fx, fy, t) = \frac{t}{t + |\frac{x^2}{x+1} - \frac{y^2}{y+1}|} \geq \frac{t}{t + |x-y|} > \frac{1}{5}(\frac{t}{t+|x-y|}) \geq \psi(M(x,y,t))$$

implies $M(fx, fy, t) \geq \psi(M(x,y,t))$.

Then all the conditions of above corollary are satisfied. Hence, f has a fixed point. □

4. Application: A solution of ordinary differential equation

We consider the two-point boundary value problem of second order differential equation:

Example 4.1.

$$\frac{d^2x}{dt^2} = (f(t, x(t)), t \in [0,1] \text{ with } x(0) = x(1) = 0. \tag{3}$$

where $f : [0,1] \times \mathcal{R} \to \mathcal{R}$ is a continuous function.

Let $\mathcal{K} = C(I)(I = [0,1])$ be the space of all continuous functions defined on I and

let $d(x,y) = \sup |x(t) - y(t)|$ for all $x, y \in \mathcal{K}$. Then, $(\mathcal{K}, M, *)$ is a complete fuzzy metric space.

The boundary value problem (3) can be written as the integral equation

$$x(t) = \int_0^1 G(t,s) f(s, x(s)) ds$$

for all $t \in I$,
where $G(t,s)$ *is the Green function given by*

$$G(t,s) = \begin{cases} t(1-s), & 0 \le t \le s \le 1, \\ s(1-t), & 0 \le s \le t \le 1 \end{cases}$$

Define the operator $S : \mathcal{K} \to \mathcal{K}$ *by*

$$Sx(t) = \int_0^1 G(t,s) f(s, x(s)) ds$$

for all $t \in I$.

Then, the problem (3)is equivalent to find $x^* \in \mathcal{K}$ *that is a fixed point of* S.
To solve this problem we assumed the following condition:

(i) there exists a function $\hbar : \mathcal{K}^2 \to \mathcal{R}$ *such that if* $\hbar(x,y) \ge 0$ *for all* $x, y \in \mathcal{K}$, *then we have*

$$0 \le f(t, x(t)) - f(t, y(t)) \le 384\psi(|x(t) - y(t)|)$$

where $\hbar : R^+ \to R^+$ *is nondecreasing and right upper continuous function with* $\hbar(0) = 0$ *and* $\hbar(t) < t^2$ *for all* $t > 0$;
(ii) there exists $x_0 \in \mathcal{K}$ *such that* $\hbar(x_0, Sx_0) \ge 0$;
(iii) for all $x, y \in \mathcal{K}$, *if* $\hbar(x,y) \ge 0$ *implies* $\hbar(Sx, Sy) \ge 0$;
(iv) If for any sequence $\{x_n\} \subseteq \mathcal{K}$ *converging to* $x \in \mathcal{K}$ *and* $\hbar(x_n, x_{n+1}) \ge 0$ *for all* $n \in N_0$ *then* $\hbar(x_n, x) > 0$ *for all* $n \in N_0$ *and* $\hbar(x,x) \ge 0$;
(v) If for any sequence $\{x_n\} \subseteq \mathcal{K}$ *such that* $\hbar(x_n, x_{n+1}) \ge 0$ *for all* $n \in N_0$, *then* $\hbar(x_m, x_n) \ge 0$ *for all* $m, n \in N$ *with* $m < n$.

Consider $x, y \in \mathcal{K}$ *such that* $\hbar(x(t), y(t)) \geq 0$ *for all* $t \in I$. *From condition (i) we have*

$$\begin{aligned}[Sx(t) - Sy(t)]^2 &= \left(\int_0^1 G(t,s)[f(s,x(s)) - f(s,y(s))]ds\right)^2 \\ &\leq \int_0^1 G(t,s)^2 ds \int_0^1 [f(s,x(s)) - f(s,y(s))]^2 ds \\ &\leq 48 \int_0^1 G(t,s)^2 ds \int_0^1 \xi(|x(s) - y(s)|)ds \\ &\leq 48 \left(\sup_{t\in I} \int_0^1 G(t,s)^2 ds\right) \int_0^1 \xi(|x(s) - y(s)|)ds \\ &\leq 48 \left(\sup_{t\in I} \int_0^1 G(t,s)^2 ds\right) \xi(d(x,y)) \\ &\leq \xi(d(x,y)).\end{aligned}$$

we have

$$\left(\sup_{t\in I} |Sx(t) - Sy(t)|\right)^2 \leq \xi(d(x,y)) \tag{4}$$

Clearly $\sup_{t\in I} \int_0^1 G(t,s)^2 ds = 1/48$.

By (4), we get $\frac{t}{t+(|Sx(t)-Sy(t)|)^2} \geq \frac{t}{t+\xi(d(x,y))}$.

Putting $\psi(t) = t$ *and* $\xi(t) = t$, *we obtain*

$M(Sx(t), Sy(t), r) \geq \psi(M(x(t), y(t), r))$ *for all* $x, y \in \mathcal{K}$ *such that* $\hbar(x(t), y(t)) \geq 0$ *for all* $t \in I$.

Define the function $\alpha : \mathcal{K} \times \mathcal{K} \times [0,\infty) \to [0,\infty)$ *by*

$$\alpha(x,y,r) = \begin{cases} 1, & \text{if } \hbar(x(t),y(t)) \geq 0, \\ 0, & \text{otherwise} \end{cases}$$

Then for all $x, y \in \mathcal{K}$, *we deduce* $M(Sx(t), Sy(t), r) \geq \psi(M(x(t), y(t), r))$.

It easily verifies that all the hypotheses of Corollary(3.2) are satisfied. Therefore S *has a fixed point in* $\mathcal{K}$.

References

[1] Doric, D., Common fixed points for generalized (ψ, ϕ) weak contraction, *Applied Mathematics Letters,* 22, (2009) 1896–1900.

[2] George, A. and Veeramani, P., On some results in fuzzy metric spaces, *Fuzzy Sets Syst.,* 64, (1994) 395–399.

[3] Kramosil, I. and Michalek, J., Fuzzy metric and statistical metric spaces, *Kybernetica,* 11 (5), (1975) 336-344.
[4] Mihet, D., Fuzzy ψ-contractive mapping in non-archimedean fuzzy metric spaces, *Fuzzy sets Syst.,* 159 , (2008) 739-744.
[5] Murthy, P.P., Tas, K. and Patel, U.D., Common fixed point theorems for generalized (ψ, ϕ) weak contraction condition in complete metric spaces, *Journal of Inequalities and Applications Applied Mathematics,* 139, (2015) 14 pages.
[6] Schweizer, B., and Sklar, A., Probabilistic metric spaces, *Elsvier, North Holand,* 1983.
[7] Zadeh, L.A., Fuzzy sets, *Inform. and Control* 8, (1965) 338–353.
[8] Gregori, V., Sapena, A., On Fixed Point Theorems in fuzzy metric space, *Fuzzy Sets and Systems,* 125, (2002) 245–252.
[9] Samet, B., Vetro, C., Vetro, P., Fixed point theorem for $\alpha - \psi$ contractive type mappings, *Nonlinear Anal.,* 75, (2012) 2154 – 2165.
[10] Salimi, P., Latif, A., Hussian, N., Modified $\alpha - \psi$ Contractive Mappings with Applications, *Fixed Point Theory Appl.,* Article ID 151 (2013).
[11] Hong, S., Fixed Points for Modified Fuzzy ψ-Contractive Set-valued Mappings in fuzzy metric space, *Fixed Point Theory and Applications,* 1(12), (2014) 11-pages. Doi:10.1186/1687-1812-2014-12.
[12] Saha, P., Choudhury, B.S., Das, P., A new contractive mapping principle in fuzzy metric spaces, *Boll. Unione Mat. Ital.,* DOI 10.1007/s40574-015-0044-y.
[13] Saini, R.K., Gupta, V., Singh, S.B., Fuzzy Version of Some Fixed Points Theorems On Expansion Type Maps in Fuzzy Metric Space, *Thai Journal of Mathematics,* 5(2), (2007) 245–252.
[14] Gupta, V., Saini, R.K., Mani, N., Tripathi, A.K., Fixed point theorems by using control function in fuzzy metric spaces, *Cogent Mathematics,* 2(1), (2015).
[15] Gupta, V., Kanwar, A., V-Fuzzy metric space and related fixed point theorems, *Fixed Point Theory and Applications,* 51,(2016) DOI: 10.1186/s13663-016-0536-1.
[16] Gupta, V., Verma, M. and Gulati, N., Unique fixed point results for sequence of self mappings in generalized fuzzy metric spaces, *Journal of Uncertain Systems,* 10(2), (2016) 108–113.
[17] Gupta, V., Verma, M., Khan, M.S., Common Fixed Point in Generalized Fuzzy Metric Spaces, *The Journal of Fuzzy Mathematics, Los Angeles,* 25(3), (2017) 533-541.
[18] Gupta, V., Saini, R.K., Kanwar, A., Some coupled fixed point results on modified intuitionistic fuzzy metric spaces and application to integral type contraction, *Iranian Journal of Fuzzy Systems,* 14(5), (2017) 123–137.

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