Available online: May 22, 2018

Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat.
Volume 68, Number 1, Pages 441–450 (2019)
DOI: 10.31801/cfsuasmas.425508
ISSN 1303–5991 E-ISSN 2618-6470

http://communications.science.ankara.edu.tr/index.php?series=A1

# DETERMINING THE MOST VITAL ARCS ON THE SHORTEST PATH FOR FIRE TRUCKS IN TERRORIST ACTIONS THAT WILL CAUSE FIRE

ERTUĞRUL AYYILDIZ, GÖKHAN ÖZÇELİK, AND CEVRİYE TEMEL GENCER

Abstract. In case of fire, the supplying of the water requirements of the fire area is a vital issue. The water requirements must be satisfied as quickly as possible without encountering any obstacles. In the study, once a terrorist attack which will cause fire at certain area (node) is occurred, the situation in which the terrorists want to prevent the fire trucks' transportation to this area via the shortest path is considered. The main logic of the study is determining the risky arc(s) that will interdict and presenting a relatively safety paths for the fire trucks. Terrorists wants to maximize the shortest path of fire trucks depending on limited interdiction budget. In this context, the problem is considered within the framework of the Network Interdiction Problem (NIP), where there are two opposite sides as leader (terrorist) and follower (fire truck). As a result, the bi-level model of the problem is presented first, and the model is applied on a numerical explanatory example.

## 1. Introduction

Terrorism is one of the biggest problems in many countries. The governments must be take necessary precautions against any terrorist attack. In this study, a terror action which causes any fire and hardens the aid process for quenching is considered as a Network Interdiction problem (NIP) of which the main logic is based on game theory.

NIPs consist of two player, called as a leader and a follower, who have opposing objectives. The follower uses a network in order to optimize some objective functions such as moving a supply convoy through the network as quickly as possible or maximizing the amount of material transported through the network. The leader wants to restrict the follower's achievable objective values by interdicting arcs or

Received by the editors: October 17, 2017; Accepted: January 31, 2018.

2010 *Mathematics Subject Classification.* 90B10, 97M40.

*Key words and phrases.* Shortest path, network interdiction, mathematical modeling, terrorist actions.

nodes, for example, by attacking arcs or nodes to destroy them, to slow travel over arcs or to reduce arcs' or nodes' capacity [1].

In the literature, the first studies related to the network interdiction are usually carried out for determining the most important component(s) which are node(s) and arc(s). Wollmer [2] and Lubore & Scilia [3] studied to identify the most important arc(s) on the network. Wollmer [4], Ratliff et al. [5], Malik et al. [6], Ball et al. [7], Lin & Chern [8] and Jiang & Hu [9] aimed to find the most important arc(s) on the network in their work. In fact, almost all studies prior to R Kevin Wood [10] are specific to the application and are not extendable to more general contexts. Wood [10] developed a min–max formulation of maximum flow network interdiction problem (MFNIP) and then converted it to an integer-programming model. Therefore, he has led to many studies since then.

NIP is well studied in many scopes, namely maximizing the shortest path or minimizing the maximum flow on a network; nuclear smuggling interdiction; national defense; facility or sensor locations etc (see [4, 10–21]).

Generally, the studies is related to the NIP can be grouped into two main headings as Shortest Path NIP and Maximum Flow NIP. These studies are shown in Table 1.

Table 1. The studies related to the Maximum Network Flow NIP and the Shortest Path NIP

| Studies related to the Maximum Network Flow NIP | Studies related to the Shortest Path NIP |
|---|---|
| (1964)Wollmer [4] | (1977)Fulkerson and Harding [38] |
| (1970)McMasters and Mustin [22] | (1978)Golden [39]* |
| (1993)R.Kevin Wood [10] | (1982)Corley and David [40] |
| (1995)Chern and Lin [23] | (1989)Ball et al. [7] |
| (1995)Washburn and Wood [24] | (1989)Malik et al. [6] |
| (1998)Cormican et al. [11] | (1999)Wevley [41] |
| (2001)Bingol [25] | (2002)Israeli and Wood [1] |
| (2003)Burch et al. [26] | (2006)Khachiyan et al. [42]* |
| (2007)Royset and Wood [27] | (2008)Khachiyan et al. [43] |
| (2007)Smith and Lim [14] | (2008)Bayrak and Bailey [44] |
| (2009)Rocco and Ramirez-Marquez [28] | (2010)Ramirez-Marquez and Rocco [45]* |
| (2010)Altner et al. [29] | (2011)Yates and Lakshmanan [46]* |
| (2010)Zenklusen [20] | (2011)Cappanera and Scaparra [47] |
| (2011)Akgün et al. [16] | (2013)Yates and Sanjeevi [48]* |
| (2012)Lunday and Sherali [30] | (2014)Yates et al. [49] |
| (2013)Rad and Kakhki [31] | (2014)Yates and Chen [50]* |
| (2013)Shirdel [32] | (2014)Xiao et al. [51] |
| (2014)Sullivan and ColeSmith [33] | (2015)Borrero et al. [52] |
| (2015)Branch [34] | (2016)Song and Shen [53]* |
| (2015)Janjarassuk and Nakrachata-Amon [35] | (2016)Casas et al. [54] |
| (2017)Afshari Rad and Kakhki [36] | (2016)Borndörfer et al. [55] |
| (2017)Naoum-Sawaya and Ghaddar [37] | (2016)Sefair and Smith [56] |
| | (2016)Lozano and Smith [57] |
| | (2017)Sadeghi et al. [58]* |

*Some remarkable studies related to the Shortest Path NIP that consider the budget constraint.

Fulkerson & Harding [38] studied maximizing the shortest single source-sink path under a given budget constraint, with a linear cost function. Corley & David [40] modeled traditional shortest path problem in which there are two opposite sides as one interdictor and as one system operator. They tried to find the most important arc on the network with the help of algorithm they developed. Israeli & Wood [1] developed a bi-level mathematical model for the shortest path NIP. They formulated this model as a single-level mixed integer programming that could be directly solved. They also achieved more effective results by using two different decomposition methods. Song & Shen [53] considered the stochastic shortest path NIP. They studied the problem in which the interdictor tries to minimize the interdiction cost and the follower tries to use shortest path under uncertain arc lengths between certain two nodes.

In the handled problem, terrorists interdict some arc(s) (links or ways) depending on the amount of resources in order to make it difficult for fire trucks to reach this region from the shortest path after they attack at a certain area in a way that will cause fire. The sides engage in a two-step and sequential game process: the leader (terrorist) initially interdicts arc(s) to maximize the follower's (fire trucks') shortest path depending on his budget or resources, later; the follower try to find the shortest path using the uninterdicted arcs. In this game, it is assumed that the sides have sufficient information about each other. It is clear that the leader's main goal is to try to explicitly maximize the shortest path (achieved by follower) by interdicting the arc(s) (deleting/destroying the link(s) between the nodes).

The main goal of the study is to obtain the information of the interdicted arc(s) on the shortest path using the model's output. This information ensures to determine the risky arcs. It is clear that the interdicted arcs are on the shortest paths and terrorists tend to interdict these arcs. It is expected that if at least one arc is interdicted on a path, this path is called risky path. Hence, the interdicted arcs should not be included in the path while the paths that go to crime scene are determined.

The rest of the study is organized as follows. In the next section, the bi-level model for the problem is presented. In Section 3, an explanatory numerical example is carried out. Finally, the study is concluded in Section 4.

## 2. Methodology

In the present study, the shortest path NIP's model that was introduced by Israeli [59] is followed. The model is solved with the GAMS 23.5.1 (CPLEX solver) software with the help of benders decomposition technique. The mathematical model in which fire trucks are deterred from using of interdicted arc by increasing the effect (distance or time) of the arc when the terrorists interdict an arc is defined on the G = (N, A) network consisting of node set "N" and arc set "A" Israeli [59] :

**Indices:** $i \in N,$ nodes in $G$
$k \in A,$ arcs in $G$
$k \in FS(i),\ k \in RS(i),$ arc directed out of (into) node $i$

**Data:** $0 \leq c_k \leq \infty,$ nominal integer length of arc $k$
$0 \leq d_k \leq \infty,$ added integer delay if arc $k$ is interdicted
$r,$ vector of available interdiction resources
$R,$ matrix of interdiction-to-resource conversion

**Variables:** $x_k = 1$ if arc $k$ is interdicted by interdictor; else $x_k = 0$
$y_k = 1$ if arc $k$ is traversed by leader; else $y_k = 0$

**Formulation:** $\max\limits_{x \in X} \min\limits_{y} \sum\limits_{k \in A} (c_k + x_k d_k)\, y_k$

$$\text{s.t.} \sum_{k \in FS(i)} y_k - \sum_{k \in RS(i)} y_k = \begin{cases} 1, & i = S \\ 0, & i \in N - s - t \\ -1, & i = t \end{cases}$$

$y_k \geq 0,$ for any $k \in A$

where $X = \left\{ x \in \{0,1\}^{|A|} : rx \leq R \right\}$

**(a)** Node $s$ and node $t$ are source and terminal nodes, respectively.

**(b)** $FS(i)$ is set of arcs that come out from node $i$, $RS(i)$ is set of arcs that enter node $i$.

**(c)** Flow-balance constraints in variables $y$ route one unit of flow from $s$ to $t$, the inner minimization is a standard shortest-path model with arc lengths $c_k + x_k d_k$.

**(d)** $c_k$ is nominal length of arc $k$ and if arc $k$ is interdicted, $c_k + d_k$ states of the arc's length; $d_k$ is finite and may affect the solution of problem. (In this study, the all solutions are obtained considering "$d = 200$") ($d_k$ is penalty).

**(e)** $rx \leq R$, is side constraint related to the interdiction resources, so, $X$ represents a set of appropriate interdiction plans. (We assumed $X$ is not null set.) The terrorists have limited budget resources and each arc's interdiction cost is different.

**(f)** All data is assumed integral $d_k, c_k \geq 0$for any $k \in A$.

## 3. Application

In this section, an explanatory example is carried out on a sample network consisting of 10 nodes and 26 arcs. Node $k_1$ represents fire station, node $k_{10}$ represents the location where the fire is started by terrorists. In the network, the routes that can be traveled between the nodes are shown with arrows. The transportation is impossible provided there is no arrow between two nodes. Also, the arcs are completely identical to each other. Therefore, the traveled time on the arc also increases as the arc's length increases.The considered network is shown in Figure 1.

After the terrorists start the fire in $k_{10}$ location, they will attempt to maximize the time of arrival to fire location (fire trucks's shortest path) of fire trucks depending on the interdiction budget. It is assumed that terrorists can destroy arc($s$) to achieve this goal. The interdiction costs of arcs are shown in Table 2.

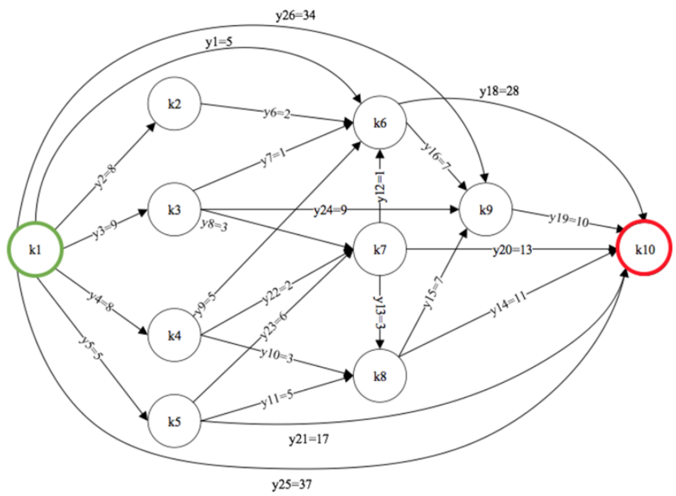


FIGURE 1. The considered network $G(10, 26)$

The interdiction plans are obtained for each interdiction budget levels of the terrorists considering these costs. For a better understanding of the interdiction plans, interdiction plans are examined for a few budget levels in the next section.

TABLE 2. The interdiction costs of the arcs

| **Arcs** | **Interdiction Cost (r)** | **Arcs** | **Interdiction Cost (r)** |
|---|---|---|---|
| $y_1$ | 4 | $y_{14}$ | 12 |
| $y_2$ | 5 | $y_{15}$ | 4 |
| $y_3$ | 7 | $y_{16}$ | 7 |
| $y_4$ | 11 | $y_{17}$ | 9 |
| $y_5$ | 12 | $y_{18}$ | 4 |
| $y_6$ | 4 | $y_{19}$ | 6 |
| $y_7$ | 5 | $y_{20}$ | 3 |
| $y_8$ | 6 | $y_{21}$ | 3 |
| $y_9$ | 3 | $y_{22}$ | 7 |
| $y_{10}$ | 3 | $y_{23}$ | 7 |
| $y_{11}$ | 8 | $y_{24}$ | 4 |
| $y_{12}$ | 2 | $y_{25}$ | 2 |
| $y_{13}$ | 8 | $y_{26}$ | 4 |

## 4. Discussion Conclusion

The model is solved to understand which arc(s) may be interdicted by terrorist who firstly starts a fire at the $k_{10}$location, later wants to interdict the arc(s) on path of the fire trucks that take the road from $k_1$location. The different interdiction plans ($R = 15, R = 20, R = 25$ and $R = 29$) for different interdiction budget scenarios are shown in Table 3.

Table 3. The different interdiction budget scenarios

| **Interdiction budget level** ($R$) | **15** | **20** |
|---|---|---|
| **Interdicted arc(s)** | $y_5, y_{10}$ | $y_1, y_{10}, y_{11}, y_{21}$ |
| **Selected path** | $y_1 \to y_{16} \to y_{19}$ | $y_4 \to y_{22} \to y_{20}$ |
| **Objective function value (OFV)** | 22 | 23 |
| **Interdiction budget level** ($R$) | **25** | **29** |
| **Interdicted arc(s)** | $y_{14}, y_{19}, y_{20}, y_{21}$ | $y_1, y_{14}, y_{19}, y_{20}, y_{21}$ |
| **Selected path** | $y_1 \to y_{18}$ | $y_{25}$ |
| **Objective function value (OFV)** | 33 | 37 |

Moreover, the interdiction plans are obtained for each interdiction level ($R \in \mathbb{Z}^+$) Firstly, starting from $R = 1$, the interdiction budget ($R$) is increased gradually (see Table 4). Should the optimal solution is an abnormally large number, there exists no optimal solution satisfying the follower's goal, and the objective value takes a big value according to the delay (penalty ($d$)). Namely, the fire trucks cannot arrive to the crime scene since all possible paths between $k_1$ and $k_{10}$ are closed by interdicting vital arc(s) on these paths. For this reason, the analysis is done up to ($R < 29$) (see Table 4-Figure 2).

Table 4. All interdiction solution

| **R** | **OFV** | **R** | **OFV** | **R** | **OFV** | **R** | **OFV** |
|---|---|---|---|---|---|---|---|
| **1** | 21 | **9** | 22 | **17** | 22 | **25** | 33 |
| **2** | 21 | **10** | 22 | **18** | 23 | **26** | 33 |
| **3** | 21 | **11** | 22 | **19** | 23 | **27** | 33 |
| **4** | 21 | **12** | 22 | **20** | 23 | **28** | 37 |
| **5** | 21 | **13** | 22 | **21** | 24 | **29** | 37 |
| **6** | 21 | **14** | 22 | **22** | 27 | | |
| **7** | 21 | **15** | 22 | **23** | 27 | | |
| **8** | 22 | **16** | 22 | **24** | 33 | | |

According to the results, the same objective function values are obtained for different interdiction budgets. (For example, $R = 8$ and $R = 17$) this situation is caused by discrete or binary interdiction variables (0 or 1). In this situation, it is

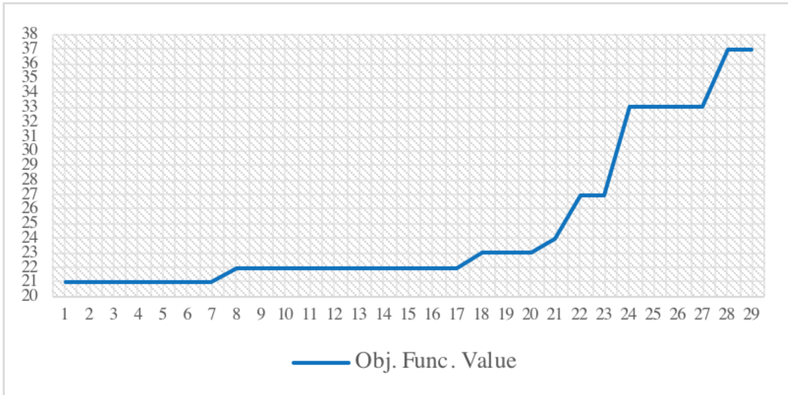


FIGURE 2. The graph of the objective function value - Interdiction budget levels

not reasonable to use more budget to obtain the same objective function value or to damage the follower at same proportion.

In this study, an approach, guiding for fire trucks in terrorist actions that cause fire, is presented to determine the risky arcs on the shortest path between the fire station and the fire location. For this purpose, the shortest path NIP's model that was proposed by Israeli [59] is followed. The model is solved using benders decomposition algorithm for a sample network consisting of 10 nodes and 26 arcs since it is bi-level. In the analysis, different interdiction plans are obtained for different budget levels. In this context, it is tried to understand which paths are more important or risky considering the interdicted arc(s). It is clear that the terrorists tend to interdict the arcs that are on the shortest path. Therefore, the authorities should determine risky arcs on the paths for the fire trucks or relief cars in a terror action. For this reason, it can be said that this study can help the authorities determine their security budgeting and security strategies in order to increase security level of cities.

## References


[1] Israeli, E. and Wood, R.K., Shortest- path network interdiction. *Networks*, 40(2), (2002), 97-111.

[2] Wollmer, R.D., Some methods for determining the most vital link in a railway network. RM-3321-ISA, The Rand Corporation, Santa Monica, California, 1963.

[3] Lubore, S.H. and Scilia, G.T., Determining the most vital link in a flow network, DTIC Document, 1971.

[4] Wollmer, R., Removing arcs from a network. *Operations Research,* 12(6), (1964), 934-940.

[5] Ratliff, H.D., Sicilia, G.T. and Lubore, S.H., Finding the n most vital links in flow networks. *Management Science*, 21(5), (1975), 531-539.

[6] Malik, K., Mittal, A.K. and Gupta, S.K., The k most vital arcs in the shortest path problem. *Operations Research Letters*, 8(4), (1989) , 223-227.
[7] Ball, M.O., Golden, B.L. and Vohra, R. V., Finding the most vital arcs in a network. *Operations Research Letters*, 8(2), (1989) , 73-76.
[8] Lin, K.-C. and Chern, M.-S., The fuzzy shortest path problem and its most vital arcs. *Fuzzy Sets and Systems*, 58(3), (1993), 343-353.
[9] Jiang, Y. and Hu, A., Finding the Most Vital Link with Respect to the Characteristic of Network Communication. *Journal of Networks*, (2011) 6(3).
[10] Wood, R.K., Deterministic network interdiction. *Mathematical and Computer Modelling*, 17(2), (1993), 1-18.
[11] Cormican, K.J., Morton, D.P., Stochastic network interdiction, Operations Research, 46(2), (1998), 184–197.
[12] Brown, G. et al., Defending critical infrastructure. *Interfaces*, 36(6), (2006), 530-544.
[13] Brown, G.G. et al., Interdicting a nuclear-weapons project. *Operations Research*, 57(4), (2009), 866-877.
[14] Lim, C. and Smith, J.C., Algorithms for discrete and continuous multicommodity flow network interdiction problems. *IIE Transactions*, 39(1), (2007), 15-26.
[15] Salmeron, J., Wood, K. and Baldick, R., Worst-case interdiction analysis of large-scale electric power grids. *IEEE Transactions on power systems*, 24(1), (2009), 96-104.
[16] Akgün, A., Tansel, B.A. and Wood, R.K., The multi-terminal maximum-flow network- interdiction problem. *European Journal of Operational Research*, 211(2), (2011), 241-251.
[17] Kennedy, K.T. et al., Nodal interdiction. *Mathematical and Computer Modelling*, 54(11), (2011), 3116-3125.
[18] Malaviya, A., Rainwater, C. and Sharkey, T., Multi-period network interdiction problems with applications to city-level drug enforcement. *IIE Transactions*, 44(5), (2012), 368-380.
[19] Granata, D., Steeger, G. and Rebennack, S., Network interdiction via a critical disruption path: branch-and-price algorithms. *Computers and Operations Research,* 40(11), (2013), 2689-2702.
[20] Zenklusen, R., Network flow interdiction on planar graphs. *Discrete Applied Mathematics*, 158(13), (2010), 1441-1455.
[21] Romich, A., Lan, G. and Smith, J.C., Algorithms for optimizing the placement of stationary monitors. *IIE Transactions*, 47(6), (2015), 556-576.
[22] McMasters, A.W. and Mustin, T.M., Optimal interdiction of a supply network. *Naval Research Logistics Quarterly*, 17(3), (1970), 261-268.
[23] Chern, M.-S. and Lin, K.-C., Interdicting the activities of a linear program A parametric analysis. *European Journal of Operational Research*, 86(3), (1995), 580-591.
[24] Washburn, A. and Wood, K., Two-Person Zero-Sum Games for Network Interdiction. Operations Research, 43(2), (1995) , 243-251.
[25] Bingöl, L., A Lagrangian heuristic for solving a network interdiction problem, Master Thesis, Naval Postgraduate School, Monterey, California, 2001.
[26] Burch, C., Carr, S., Krumke, M., Marathe, M., Phillips, C. and Sundberg, E. A., Decomposition-based approximation for network inhibition. In D. L. Woodruff (Eds.), Network Interdiction and Stochastic Integer Programming. Boston: Kluwer Academic Publishers, pp. 51-69, 2003.
[27] Royset, J.O. and Wood, R.K., Solving the bi-objective maximum-flow network-interdiction problem. *INFORMS Journal on Computing*, 19(2), (2007) , 175-184.
[28] Rocco, C.M. and Ramirez-Marquez, J.E., Deterministic network interdiction optimization via an evolutionary approach. *Reliability Engineering and System Safety*, 94(2), (2009) , 568-576.
[29] Altner, D.S., Ergun, A. and Uhan, N.A., The maximum flow network interdiction problem: valid inequalities, integrality gaps, and approximability. *Operations Research Letters*, 38(1), (2010) , 33-38.

[30] Lunday, B.J. and Sherali, H.D., Minimizing the maximum network flow: models and algorithms with resource synergy considerations. *Journal of the Operational Research Society*, 63(12), (2012), 1693-1707.
[31] Rad, M.A. and Kakhki, H.T., Maximum dynamic network flow interdiction problem: new formulation and solution procedures. *Computers and Industrial Engineering,* 65(4), (2013), 531-536.
[32] Shirdel, G.H., A Note on the Integrality Gap in the Nodal Interdiction Problem. *Journal of Sciences, Islamic Republic of Iran*, 24(3), (2013), 269-273.
[33] Sullivan, K.M. and Cole Smith, J., Exact algorithms for solving a Euclidean maximum flow network interdiction problem. Networks, 64(2), (2014), 109–124.
[34] Branch, K., Multi-Commodity Multi-Source-Sinks Network Flow. *Journal of Engineering and Applied Sciences*, 10(6), (2015), 118-122.
[35] Janjarassuk, U. and Nakrachata-Amon, T., A simulated annealing algorithm to the stochastic network interdiction problem. *In Industrial Engineering and Engineering Management (IEEM)*, 2015 IEEE International Conference on. IEEE, (2015), 230-233.
[36] Afshari Rad, M. and Kakhki, H.T., Two extended formulations for cardinality maximum flow network interdiction problem. *Networks*, 69(4), (2017), 367-377.
[37] Naoum-Sawaya, J. and Ghaddar, B., Cutting plane approach for the maximum flow interdiction problem. *Journal of the Operational Research Society*, (2017), 1-17.
[38] Fulkerson, D.R. and Harding, G.C., Maximizing the minimum source-sink path subject to a budget constraint. *Mathematical Programming,* 13(1), (1977), 116-118.
[39] Golden, B., A problem in network interdiction. *Naval Research Logistics Quarterly*, 25(4), (1978), 711-713.
[40] Corley, H.W. and David, Y.S., Most vital links and nodes in weighted networks. *Operations Research Letters*, 1(4), (1982), 157-160.
[41] Wevley, C., The quickest path network interdiction problem. Master Thesis, Naval Postgraduate School, Monterey, California, 1999.
[42] Khachiyan, L., Gurvich, V. and Zhao, J., Extending dijkstraâ-algorithm to maximize the shortest path by node-wise limited arc interdiction. *In International Computer Science Symposium in Russia*. Springer, (2006). 221-234.
[43] Khachiyan, L. et al., On short paths interdiction problems: Total and node-wise limited interdiction. *Theory of Computing Systems,* 43(2), (2008), 204-233.
[44] Bayrak, H. and Bailey, M.D., Shortest path network interdiction with asymmetric information. *Networks,* 52(3), (2008), 133-140.
[45] Ramirez-Marquez, J.E. and Rocco, C.M., A bi-objective approach for shortest-path network interdiction. *Computers and Industrial Engineering,* 59(2), (2010), 232–240.
[46] Yates, J. and Lakshmanan, K., A constrained binary knapsack approximation for shortest path network interdiction. *Computers and Industrial Engineering*, 61(4), (2011), 981–992.
[47] Cappanera, P. and Scaparra, M.P., Optimal allocation of protective resources in shortest-path networks. *Transportation Science,* 45(1), (2011), 64-80.
[48] Yates, J. and Sanjeevi, S., A length-based, multiple-resource formulation for shortest path network interdiction problems in the transportation sector. *International Journal of Critical Infrastructure Protection,* 6(2), (2013), 107-119.
[49] Yates, J., Wang, X. and Chen, N., Assessing the effectiveness of k-shortest path sets in problems of network interdiction. *Optimization and Engineering,* 15(3), (2014), 721-749.
[50] Yates, J. and Chen, N., A Spatial Segmentation Algorithm for Resource Allocation in an Integrated Spatial and Networked Environment. *Applied Spatial Analysis and Policy*, 7(4), (2014), 317-336.
[51] Xiao, K. et al., Stackelberg network interdiction game: nodal model and algorithm. *In Game Theory for Networks (GAMENETS)*, 2014 5th International Conference on. IEEE, (2014), 1-5.

[52] Borrero, J.S., Prokopyev, O.A. and Saura, D., Sequential shortest path interdiction with incomplete information. *Decision Analysis*, 13(1), (2015), 68-98.
[53] Song, Y. and Shen, S., Risk-Averse Shortest Path Interdiction. *INFORMS Journal on Computing,* 28(3), (2016), 527-539.
[54] Casas, I., Delmelle, E. and Yates, J., Geographic characteristics of a network interdiction problem. *GeoJournal,* 81(1), (2016), 37-53.
[55] Borndörfer, R., Sagnol, G. and Schwartz, S., An Extended Network Interdiction Problem for Optimal Toll Control. *Electronic Notes in Discrete Mathematics*, 52, (2016), 301-308.
[56] Sefair, J.A. and Smith, J.C., Dynamic shortest path interdiction. *Networks*, 68(4), (2016), 315-330.
[57] Lozano, L. and Smith, J.C., A Backward Sampling Framework for Interdiction Problems with Fortification. *INFORMS Journal on Computing*, 29(1), (2016), 123-139.
[58] Sadeghi, S., Seifi, A. and Azizi, E., Trilevel shortest path network interdiction with partial fortification. *Computers and Industrial Engineering*, 106, (2017), 400–411.
[59] Israeli, E., System Interdiction and Defense., DTIC Document, 1999.

*Current address*: Ertuğrul Ayyıldız: Karadeniz Technical University, Faculty of Engineering, Department of Industrial Engineering, Trabzon, TURKEY

*E-mail address*: `ertugrulayyildiz@ktu.edu.tr`

ORCID Address: `http://orcid.org/http://orcid.org/0000-0002-6358-7860`

*Current address*: Gökhan Özçelik: Karadeniz Technical University, Faculty of Engineering, Department of Industrial Engineering, Trabzon, TURKEY

*E-mail address*: `gozcelik@ktu.edu.tr`

ORCID Address: `http://orcid.org/0000-0001-5314-8576`

*Current address*: Cevriye Temel Gencer: Gazi University, Faculty of Engineering, Department of Industrial Engineering, Ankara, TURKEY

*E-mail address*: `ctemel@gazi.edu.tr`

ORCID Address: `http://orcid.org/0000-0002-3373-8306`