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Bir Otomobil Yağı Fabrikası için Karma Tamsayılı Doğrusal Programlama Yöntemi ile Bütünleşik Üretim Planlaması

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Özet

Şirketlerin ana problemlerinden biri üretimdeki verimsizlikler olup bu yüzden şirketlerin amaçlarına uygun olarak en öncü isteklerinden biri az kaynakların kullanım verimliliğini en ekonomik şekilde artırarak üretimi iyileştirmektir. Bu yüzden literatürde çok çeşitli yöneylem araçları ortaya çıkmıştır. Bu teknikler arasında doğrusal programlama izlenebilirlikteki ve elde edilen çözümün parametrelerdeki değişikliklerden nasıl etkilendiğini anlamaya yönelik duyarlılık analizindeki gücü ile büyük bir öneme sahiptir. Şu anda şirketler, artan rekabetten dolayı ücretlendirme politikalarındaki değiştirme özgürlüklerini büyük ölçüde kaybetmişlerdir. Bu yüzden karlılıklarını artırabilmek için maliyetleri en aza indirgeyen yolları seçmektedirler. Sonuç olarak verimli ve etikli bir üretim planlama yöntemi olan doğrusal programlamanın önemi gün geçtikçe artmaktadır. Bu çalışma bir otomobil yağı fabrikası için bütünleşik üretim planlamada karma tamsayılı doğrusal programın modellenmesini ve bunun sağladığı faydaları ortaya koymaktadır.

Anahtar kelimeler: Bütünleşik üretim planlama; karma tamsayılı doğrusal programlama

Aggregate Production Planning Model based on Mixed Integer Linear Programming for a Lubricant Factory

Abstract

One of the main problems of the firms is the inefficiency of the production; hence, one of the primary objectives of them is to improve the production by the increased utilization of the scarce recourses the most economically in accordance with the goals of the firms. Therefore, a variety of operational research tools have been evolved in the literature. Among those techniques, linear programming is of great importance with its power of tractability as well as the sensitivity analysis it enables the planner to investigate the robustness of the solution it generates. At present, firms have been lost their freedoms of changing the prices to a large extent because of the increasing competition, hence, they choose the way of minimizing the costs to be able to improve their profitability. As a result, the importance of the linear programming which is the effective production planning method has been growing day by day. This study models and demonstrates the benefits of the mixed integer linear programming in aggregate production planning for a lubricant factory.

***Keywords*:** Aggregate production planning; mixed integer linear programming

1. Introduction

The detailed description of production is provided in [1]. One of the main problems of the enterprises is increasing the produced goods and services with efficient and productive combination of scarce resources. In economics, production can be defined as generating all kinds of benefits, i.e., providing useful goods and services. The production of useful goods and services can't be limited with only the production of goods; but it also includes all the activities between the producer and the consumer. In general, production is regarded as the procurement and the combination of physical units rather than service. Therefore, it can be defined as the process or method of converting a group of input to a certain output. Especially in industrial organizations, it represents changing the structure shapes of raw materials or intermediate goods [1]. Among three production models, [2], in Walrassian model, production process is described as a set of input-output relations based on cost minimization with respect to market-determined prices where there is no internal social organization analysis of the firm. In contrast, neo-Hobbesian model deals with firms as a social organization where the key to understand the internal structure of the firm is the concept of malfeasance. Marxian model focuses on the ownership and control of the means of production where for the analysis of market equilibration and competition it is essential to consider the ownership of the means of production, and the command over the production process which this ownership permits.

"Production" can be regarded as widely acceptable where production is terming for activities to make a change on a physical asset that will increase its value, through the use of machine, people, material where raw materials and half products are transformed into products, or a service is introduced while production management is combining machine, material and labor resources to ensure production of certain amount of products in desired quality and in desired period with the lowest cost possible [3]. Kobu [4] indicates three objectives of production management: (i) meeting the demands of customers in best possible way in terms of amount, price, time and quality, (ii) keeping stock levels as low as possible / having higher inventory turnover rate, (iii) increasing the utilization of resources (labor and machines).

Primary functions of production management are planning and programming for how to perform production activities [5]. Production planning can be described as acquisition of resources and raw materials in addition to the planning production activities, that are necessary to transform raw materials into finished products in order to meet customer demand in the most efficient and economical way possible [6]. The goal of production planning is indicated by [6] as making planning decision through the optimization of trade-off between economic objectives (minimization of cost or maximization of profit) and less tangible objective of customer satisfaction. In production planning, managers of manufacturing firms seeks the best specifications for aggregate levels of production, inventory and work force to meet demands [7].

Aggregate production planning (APP) determines the best way to meet demand in the intermediate future planning horizon (3 to 18 months ahead) by adjusting controllable variables such as regular and overtime production rates, inventory levels, labor levels, subcontracting, backordering rates and so on [8]. The APP model can be described as a mid-term planning tool analyzing the relationship between the offer and the demand to set production levels to satisfy demands [9]. There have been considerable interests to APP from both practitioners and academics [10]. This is due to mainly APP models' ability proving control over production and inventory costs [11].

In addition to single objective aggregate models, in the literature there are also more realistic aggregate models for some real life situations with multiple objectives [12, 13]. Saad [14] provides six categories to classify conventional models for solving APP problems: linear programming [15]; linear decision rule [16]; transportation method [17]; management coefficient approach [18]; search decision rule [19]; and simulation [20]. One of the most widely used modeling approach for APP is mathematical programming, which can be defined as a branch of mathematics dealing with techniques for maximizing or minimizing objective(s) subject to (non)linear constraints with discrete and or continuous variables [21]. Mixed integer linear programs (MILP) are more convenient to model APP problems where some of the decision variables in the mathematical models are required to have discrete values; i.e. whole or integer numbers [22]. In the literature, there are several approaches proposed to deal with APP modeled as MILP. Among them, there are deterministic approach [23], resource based approach [11], multi-criteria approach [24], hybrid model with MILP and simulation model [25], and probabilistic linear program [26].

This paper proposes a MILP for an APP model with multi-lines, multi-products with limited resources for a lubricant company, that produce several products in specific lines each of which has a certain level of production rate in terms of units per hour and dedicated to certain category of products with specified available hours for production in the company. The objective is to determine the optimum production levels with limited resources to minimize the overall total cost of production (unit cost, shortages cost and setup cost).

The paper is organized as follows. The problem is described in Section 2, and in Section 3, a model is introduced. In Section 4, numerical results of an example and its sensitivity to the parameters are discussed. Section 5 is the conclusion part summarizing the importance of the study and future research directions.

2. Problem Description

In this study, the production planning of a lubricant factory that has 76 product types, 7 lines, 4 product families (bottle, tin, barrel, maintenance) for which there are available hours for production (operating capacities) was analyzed through a mixed integer linear program. The purpose is to determine the least costly production levels based on unit production cost, shortage cost and setup cost, for each product type with raw materials available accounting for operating capacities and demand requirements.

The aggregate production model proposed by this study is deterministic in general and for which the mathematical formulation is provided in subsequent section followed by numerical analysis through an illustrative example.

3. Model Formulation

The APP problem for the lubricant factory is formulated as a mixed integer liner programming model. In the generalized form of the model, the company is assumed to have *f* production families using *l* production lines to produce *p* product types with *r* resources.

The Figure 1 represents the compact representation of purposed mathematical model. In Figure 1, (0) represents the objective function corresponds to the total cost from the production accounting for the cost of unit production, setup cost and shortages cost. The constraints (1) are production balance relations considering shortages and demand for each product. The constraints (2) are limits on available resources. Operating capacities based lines at which each product is processed and their production rates for each product family are given by constraints (3). Constraints (4) are production indicators, i.e. production can be performed for each product provided that setup is made for that product. Constraints (5) are variable type constraints indicating non-negative production and shortages levels for each product while constraints (6) indicate binary variables depending on whether production is performed or not for each product.

4. Numerical Results

The problem described in Section 2 and proposed model described in Section 3 were illustrated for *l*=7 production lines with *p*=76 products, *f*=4 product families and *r*=11 raw materials with the specified parameters for the data provided in Appendix 1.

The mixed integer linear programming model purposed was solved using one of the most often used algebraic modeling language AMPL (A Mathematical Programming Language) with CPLEX v11.2.0. as the solver. Figure 2 represents how AMPL works.

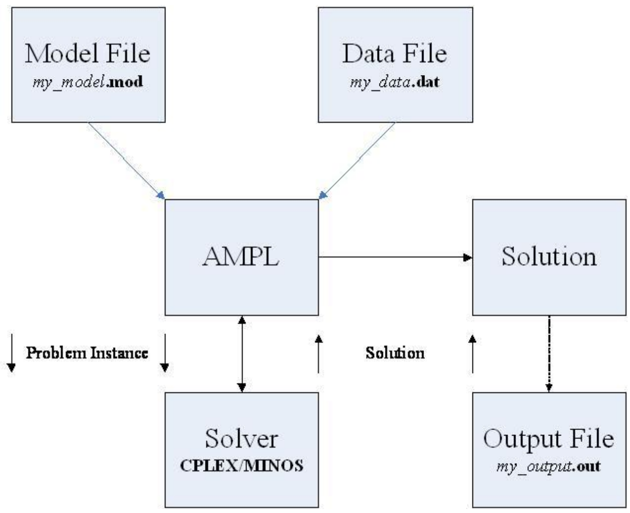

Figure 2. AMPL Framework

AMPL is a way to use generic modeling terms to represent optimization problems translating optimization problems into terms that an optimization solver such as CPLEX, MINOS, and others can understand. It is just one of the algebraic modeling languages similar to GAMS, LINGO, etc. It separates the model (my_model.mod file) from the data (my_data.dat file). It reads the model from .mod file and data from the .dat file and puts them together into format that the solver understands.

Then, it hands over this problem instance to the solver, which in turn, solves the instance, and hands back the solution to AMPL, that can write into an output file (my_output.out).

The results of the problem instance and corresponding sensitivity analysis are provided in Table 1 and Table 2 respectively.

Notation

Subscripts

i	Product
j	Resource
k	Line
m	Family

Sets

P	Set of products to produce = {1,…, p}
R	Set of resources for production = {1,…, r}
L	Set of production lines = {1,…, l}
F	Set of product families = {1,…, f}
ζ^m	Set of line $k \in L$ that are used for the product family m

Parameters

c_i	Unit production cost of product $i \in P$
b_i	Unit shortage cost of product $i \in P$
s_i	Production setup cost for product $i \in P$
d_i	Demand of product $i \in P$
a_j	Available resource $j \in R$
u_{ij}	Amount of resource $j \in R$ used for each unit of product $i \in P$
h_k	Production rate for line $k \in L$
o_m	Operating capacity (available hours) for product family $m \in F$
z_{ik}	Indicator whether product $i \in P$ is processed in line $k \in L$

$$= \begin{cases} 1 & \text{if product } i \in P \text{ is processed in line } k \in L \\ 0 & \text{otherwise} \end{cases}$$

Continuous variables

x_i	Amount of product $i \in P$ to be produced
w_i	Stock-out amount for product $i \in P$

Binary variable

y_i

$$= \begin{cases} 1 & \text{if production is performed for product } i \in P \\ 0 & \text{otherwise} \end{cases}$$

Mathematical Model

Min
$$\sum_{i \in P} (c_i x_i + b_i w_i + s_i y_i) \quad (0)$$

s.t.
$$x_i + w_i = d_i \quad ,\forall i \in P \quad (1)$$

$$\sum_{i \in P} u_{ij} x_i \le a_j \quad ,\forall j \in R \quad (2)$$

$$\sum_{i \in P} \sum_{k \in L : k \in \zeta^m} \frac{x_i z_{ik}}{h_k} \le o_m \quad ,\forall m \in F \quad (3)$$

$$x_i \le M y_i \quad ,\forall i \in P \quad (4)$$

$$x_i, w_i \ge 0 \quad ,\forall i \in P \quad (5)$$

$$y_i \quad binary \quad ,\forall i \in P \quad (6)$$

Figure 1. Proposed MILP Model for the APP Problem

Table 1. Results of Illustrative Example

	x	w	y		x	w	y
1	1371	-2.27374e-13	1	39	384	-1.13687e-13	1
2	247	0	1	40	12	0	1
3	2	0	1	41	8	0	1
4	67	0	1	42	293	0	1
5	164	0	1	43	3	0	1
6	5	0	1	44	302	0	1
7	1110	-2.27374e-13	1	45	3	0	1
8	1141	-2.27374e-13	1	46	265	0	1
9	0	3	0	47	27	0	1
10	1798	-4.54747e-13	1	48	1	0	1
11	763	-1.13687e-13	1	49	214	0	1
12	704	-1.13687e-13	1	50	369	0	1
13	1	0	1	51	0	0	0
14	0	1	0	52	3413	-9.09495e-13	1
15	83	0	1	53	2	0	1
16	4	0	1	54	221	0	1
17	8	0	1	55	262	0	1
18	100	0	1	56	2	0	1
19	82	0	1	57	132	0	1
20	7	0	1	58	160	0	1
21	0	2	0	59	2	0	1
22	8	0	1	60	55	0	1
23	271	0	1	61	87	0	1
24	618	-1.13687e-13	1	62	6	0	1
25	18	0	1	63	26	0	1
26	16	0	1	64	72	0	1
27	21	0	1	65	2	0	1
28	1268	-2.27374e-13	1	66	2070	-4.54747e-13	1
29	49	0	1	67	245	0	1
30	40	0	1	68	10	0	1
31	1327	-2.27374e-13	1	69	5	0	1
32	699	-1.13687e-13	1	70	1	0	1
33	233	0	1	71	4366	-9.09495e-13	1
34	2700	-4.54747e-13	1	72	0	0	0
35	6	37	1	73	1197	-2.27374e-13	1
36	1276	-2.27374e-13	1	74	0	820	0
37	39	0	1	75	0	1454	0
38	268	0	1	76	0	819	0

Table 2. Sensitivity Analysis of Resource Utilization

	available resource	available resource slack	line	availability	availability slack
1	0	109047,00	bottle	0	15,37
2	0	21369,00	maintenance	o	41,34
3	0	673,45	others [4]	0	33,70
4	0	6339,93	others [5]	0	39,42
5	0	14281,70			
6	0	24144,90			
7	0	92190,30			
8	0	37250,00			
9	0	22340,20			
10	0	22558,10			
11	0	305083,00			

Table 2 illustrates that the raw materials are not used in complete in the company. That is, the company currently incurs unnecessary costs associated with raw materials that were purchased but not used in the production. Hence, the mathematical model constructed can easily point out the requirements in exact providing opportunity to decrease the cost of raw materials, hence, to increase profitability of the company. For instance, raw material 11, OMY48201, has the greatest inefficiency of 305083, which is the unused amount, indicated by available_resource.slack column of Table 2 and row 11. Note that, as all the slacks associated with resources come to positive, the respected reduced costs are zero as they are expected from complementary slackness theorem.

Furthermore, the sensitivity analysis conducted indicates that currently utilizations of bottle line workers, maintenance line workers are low as they have a lot of idle time provided in Table 3.

Table 3. Efficiency of Production Lines

Line	Idle Time (hrs.)	Utilization (%)
Bottle	15,37	63,84
Maintenance	41,34	2,73
Others-1	33,70	20,71
Others-2	39,42	7,25

The mathematical model constructed above highlights the potential improvement in the company. For instance, the utilization of workers at production lines tabulated in Table3, can be improved by allocating them in different works at their idle times. (Note that, the poor utilization rates probably result from fictitious data-due to the company's privacy policy-, though, without loss of generality, the linear programming framework presented in this study provides potential improvement areas as discussed above)

5. Conclusion and Discussion

In this study, one of the major problems of Opet Lubricant Factory: weekly aggregated production plan and its analysis were addressed through mixed integer linear program where the current production plan in the firm is not regular; therefore, the orders of the raw materials are randomly issued.

As a result, the firm faces to high level of inventory costs. Moreover, the production is not well organized and planned with the ignorance of the elements' costs, i.e. setup, unit production or shortages costs. All of them cause increased production cost with low utilization of both material and workers.

This study presents a linear programming framework to minimize above cost components along with the optimal production quantity amount for each product. It enables planners to improve the efficiency of the production highlighting potential improvements areas through sensitivity analysis the mathematical model proposed can provide. The power of the framework developed points out the potential opportunities such as allocation of resources and work-force to enhance the operations in the firm, hence increase the utilization of resources and workforce, consistent with [27] and [28].

Although this model is deterministic as all parameters are known by certainty, it can be further extended taking into uncertainties [29], and incorporated with strategic plans of firms [30] in future studies.

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Appendix-1: AMPL Data File

param:	c	b	s	d	:=
1	3	15	125	1371	
2	9	50	125	247	
3	462	1924	60	2	
4	3	7	125	67	
5	9	25	125	164	
6	462	1414	60	5	
7	2	8	125	1110	
8	6	25	125	1141	
9	273	1699	60	3	
10	2	4	125	1798	
11	4	10	125	763	
12	6	13	125	704	
13	286	636	60	1	
14	6	18	125	1	
15	25	68	100	83	
16	26	83	100	4	
17	260	865	60	8	
18	1	6	125	100	
19	8	23	125	82	
20	33	88	100	7	
21	28	78	100	2	
22	326	1123	60	8	
23	6	15	125	271	
24	27	58	100	618	
25	30	62	100	18	
26	25	57	100	16	
27	298	682	60	21	
28	24	54	100	1268	
29	25	52	100	49	
30	268	560	60	40	
31	2	4	125	1327	
32	5	9	125	699	
33	7	12	125	233	
34	21	44	100	2700	
35	25	49	100	43	
36	26	49	100	1276	
37	237	459	60	39	
38	4	8	125	268	
39	17	40	100	384	
40	16	35	100	12	
41	225	493	60	8	
42	18	34	100	293	
43	254	438	60	3	
44	18	38	100	302	
45	251	464	60	3	
46	26	50	100	265	
47	23	51	100	27	
48	265	631	60	1	
49	3	7	125	214	
50	2	5	125	369	
51	238	948	60	0	
52	1	3	125	3413	
53	249	556	60	2	
54	4	9	125	221	
55	19	41	100	262	
56	217	465	60	2	
57	4	9	125	132	
58	23	41	100	160	
59	267	479	60	2	
60	5	13	125	55	
61	24	65	100	87	
62	270	631	60	6	
63	5	13	125	26	
64	25	64	100	72	
65	250	697	60	2	
66	2	5	125	2070	
67	4	13	125	245	
68	31	80	100	10	
69	282	555	60	5	
70	402	640	60	1	
71	1	2	40	4366	
72	476	1284	60	0	
73	2	4	40	1197	
74	1	0	25	820	
75	1	0	25	1454	
76	1	0	25	819;	

param a:=

1	111393
2	21665
3	71591
4	8688
5	16220
6	24682
7	93188
8	37843
9	27495
10	113806
11	344002;

1	1600
2	1200
3	1000
4	900
5	150
6	3600
7	4800;

param o:=

1	42.5
2	42.5
3	42.5
4	42.5;

param h:=

param u:	1	2	3	4	5	6	7	8	9	10	11:=
1	0	0	0	0.847	0	0	0	0	0	0	0
2	0	0	0	3.388	0	0	0	0	0	0	0
3	0	0	0	175	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0,1045	0,59228	0	0
8	0	0	0	0	0	0	0	0,4181	2,36912	0	0
9	0	0	0	0	0	0	0	216000	122,4	0	0
10	0	0	0	0	0	0	0	0	0	0,605696	0,1344
11	0	0	0	0	0	0	0	0	0	1,817088	0,4032
12	0	0	0	0	0	0	0	0	0	2,422784	0,5376
13	0	0	0	0	0	0	0	0	0	125,06	27,75
14	0,4633	0	0	0	0	0	0	0	2,253161	0,5346	0
15	2,08	0	0	0	0	0	0	0	10,1152	2,4	0
16	2.275	0	0	0	0	0	0	0	11,0635	2.625	0
17	23,4	0	0	0	0	0	0	0	113.796	27	0
18	0	0	0	0	0.878	0	0	0	0	0	0
19	0	0	0	0	3.512	0	0	0	0	0	0
20	0	0	0	0	17,5	0	0	0	0	0	0
21	0	0	0	0	16	0	0	0	0	0	0
22	0	0	0	0	180	0	0	0	0	0	0
23	0,4633	0	0	0	0	0	0	0	0	0,613008	2,162279
24	2,08	0	0	0	0	0	0	0	0	2.752	9,7072
25	2.275	0	0	0	0	0	0	0	0	3,01	10,61725
26	2,08	0	0	0	0	0	0	0	0	2.752	9,7072
27	23,4	0	0	0	0	0	0	0	0	30,96	109.206
28	0	0	0.984	0	0	0	0	0	0	5.576	8.112
29	0	0	0.984	0	0	0	0	0	0	5.576	8.112
30	0	0	11,07	0	0	0	0	0	0	62,73	91,26
31	0	0	0,055104	0	0	0	0	0	0	0,643776	0,11648
32	0	0	0,165312	0	0	0	0	0	0	1,931328	0,34944
33	0	0	0,220416	0	0	0	0	0	0	2,575104	0,46592
34	0	0	0.984	0	0	0	0	0	0	11.496	2,08
35	0	0	10762,5	0	0	0	0	0	0	12,57375	2.275
36	0	0	0.984	0	0	0	0	0	0	11.496	2,08
37	0	0	11,07	0	0	0	0	0	0	129,33	23,4
38	0	0	0	0	0	0	0	0	0	2,623015	0
39	0	0	0	0	0	0	0	0	0	15,7008	0
40	0	0	0	0	0	0	0	0	0	15,7008	0
41	0	0	0	0	0	0	0	0	0	176.634	0
42	0	0	0	0	0	0	0	0	0	0	12
43	0	0	0	0	0	0	0	0	0	0	144
44	0	0	0	0	0	0	0	0	0	0	0
45	0	0	0	0	0	0	0	0	0	0	0
46	0	0.976	0	0	0	0	0	0	0	12.544	2,4
47	0	0.976	0	0	0	0	0	0	0	12.544	2,4
48	0	11	0	0	0	0	0	0	0	141,12	27
49	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0,053321	0	0	0	0	0	0	0,678428	0,06936
51	0	0	11,3775	0	0	0	0	0	0	144,7625	14,8
52	0	0	0	0	0	0,14048	0	0	0	0,71118	0
53	0	0	0	0	0	28,8	0	0	0	145,8	0
54	0	0	0	0	0	0	0	0	0	1,62603	0
55	0	0	0	0	0	0	0	0	0	9.744	0
56	0	0	0	0	0	0	0	0	0	109,62	0
57	0	0	0	0	0	0	0	0	0	0,4352	0
58	0	0	0	0	0	0	0	0	0	2,56	0

59	0	0	0	0	0	0	0	0	0	29,6	0
60	0	0	0	0	0	0	0	0	0	1,691445	0
61	0	0	0	0	0	0	0	0	0	9.968	0
62	0	0	0	0	0	0	0	0	0	115.255	0
63	0	0	0	0	0	0	0	0	0	0	0
64	0	0	0	0	0	0	0	0	0	0	0
65	0	0	0	0	0	0	0	0	0	0	0
66	0	0	0	0	0	0	0,2476	0	0	0	0
67	0	0	0	0	0	0	0,74279	0	0	0	0
68	0	0	0	0	0	0	4.935	0	0	0	0
69	0	0	0	0	0	0	50,76	0	0	0	0
70	0	0	0	0	0	0	0	0	0	0	0
71	0	0	0	0	0	0	0	0	0	0	0
72	0	0	0	0	0	0	0	0	0	0	0
73	0	0	0	0	0	0	0	0	0	0	0
74	0	0	0	0	0	0,5137314	0	0	0	0	0
75	0	0	0	0	0	0,2655717	0	0	0	0	0
76	0	0	0	0	0	0,5568927	0	0	0	0	0;

param z:	1	2	3	4	5	6	7:=
1	1	0	0	0	0	0	0
2	0	1	0	0	0	0	0
3	0	0	0	0	1	0	0
4	1	0	0	0	0	0	0
5	0	1	0	0	0	0	0
6	0	0	0	0	1	0	0
7	1	0	0	0	0	0	0
8	0	1	0	0	0	0	0
9	0	0	0	0	1	0	0
10	1	10	0	0	0	0	0
11	0	0	1	0	0	0	0
12	0	1	0	0	0	0	0
13	0	0	0	0	1	0	0
14	0	1	0	0	0	0	0
15	0	0	0	1	0	0	0
16	0	0	0	1	0	0	0
17	0	0	0	0	1	0	0
18	1	0	0	0	0	0	0
19	0	1	0	0	0	0	0
20	0	0	0	1	0	0	0
21	0	0	0	1	0	0	0
22	0	0	0	0	1	0	0
23	0	1	0	0	0	0	0
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26	0	0	0	1	0	0	0
27	0	0	0	0	1	0	0
28	0	0	0	1	0	0	0
29	0	0	0	1	0	0	0
30	0	0	0	0	1	0	0
31	1	0	0	0	0	0	0
32	0	0	1	0	0	0	0
33	0	1	0	0	0	0	0
34	0	0	0	1	0	0	0
35	0	0	0	1	0	0	0
36	0	0	0	1	0	0	0
37	0	0	0	0	1	0	0
38	0	0	1	0	0	0	0
39	0	0	0	1	0	0	0
40	0	0	0	1	0	0	0
41	0	0	0	0	1	0	0
42	0	0	0	1	0	0	0
43	0	0	0	0	1	0	0
44	0	0	0	1	1	0	0
45	0	0	0	0	1	0	0
46	0	0	0	1	0	0	0
47	0	0	0	1	0	0	0
48	0	0	0	0	1	0	0
49	1	0	0	0	0	0	0
50	1	0	0	0	0	0	0
51	0	0	0	0	1	0	0
52	1	0	0	0	0	0	0
53	0	0	0	0	1	0	0
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55	0	0	0	1	0	0	0
56	0	0	0	0	1	0	0
57	0	0	1	0	0	0	0
58	0	0	0	1	0	0	0
59	0	0	0	0	1	0	0
60	0	0	1	0	0	0	0
61	0	0	0	1	0	0	0
62	0	0	0	0	1	0	0
63	0	0	0	0	0	0	0
64	0	0	0	1	0	0	0
65	0	0	0	0	1	0	0
66	1	0	0	0	0	0	0
67	0	0	1	0	0	0	0
68	0	0	0	1	0	0	0
69	0	0	0	0	1	0	0
70	0	0	0	0	1	0	0
71	0	0	0	0	0	0	1
72	0	0	0	0	1	0	0
73	0	0	0	0	0	0	1
74	0	0	0	0	0	1	0
75	0	0	0	0	0	1	0
76	0	0	0	0	0	1	0;