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Gamow-Teller Strength Distributions for Some Magic Nuclei

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Abstract. The total Gamow-Teller strengths and their energy distributions for ^{96}Zr, $^{96}Sr, ^{54}Ca, ^{28}O, ^{24}O$ and ^{14}C have been obtained within the framework of Random Phase Approximation (RPA). The effective interaction potential has been described by considering the commutativity of the Gamow-Teller operator with the central part of the nuclear Hamiltonian.

Keywords: β decay, Shell model, Magic nuclei

1. INTRODUCTION

It is well known that charge exchange spin-spin transitions are very important to understand the basic astrophysical and nuclear processes such as the initial step of the hydrogen fusion reaction leading to nucleosynthesis, the electron capture reactions leading to stellar collapse and supernova formation [1]. The first experimental analysis of the Gamow-Teller resonance was done for $^{90}Zr(p,n)$ reaction at 35 MeV [2]. The (p,n) charge exchange reaction is one of the most efficient ways in the experimental identification of Gamow-Teller resonance in heavy nuclei at intermediate energies [3-6]. Furthermore, it was also shown that the $(^{3}He,t)$ reaction is another efficient way in the experimental investigation of the Gamow-Teller excitations for the bombarding energies exceeding 100 MeV/nucleon [7,8]. These excitations were extracted by using the $^{208}Pb(^{3}He,t)\ ^{208}Bi$ reaction at different energies [9-11]. $(^{6}Li,{}^{6}He)$ reaction is also important to study the collective spin-isospin excitations [12]. The Gamow-Teller excitations were searched for $^{48}Ca \to {}^{48}Sc$, $^{90}Zr \to {}^{90}Nb$ and $^{208}Pb \to {}^{208}Bi$ transitions by using $(^{6}Li,{}^{6}He)$ reaction at different energies [13-16]. The total transition strength of the Gamow-Teller excitations is given by a model-independent sum rule $S_{\beta-} - S_{\beta+} = 3(N-Z)$, which should be nearly exhausted by the β^{-} transition strength summed over all Gamow-Teller states in the daughter nucleus (Z+1,N-1) formed after the (p,n) reactions. Surprisingly, only a half of the GT sum rule value was identified from (p,n) measurements in the 1980's on targets throughout the periodic table [17]. This difficulty is known as the quenching problem of the GT strength. Wakasa et al. accurately measured the $^{90}Zr(p,n)$ spectra at 295 MeV [18]. They

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successfully identified the GT strength in the continuum region through multipole decomposition (MD) analysis which extracted the $\Delta L = 0$ component from the cross sections. They obtained a GT quenching factor, defined as $Q = (S_{\beta-} - S_{\beta+})/3(N - Z)$, of 0.90 ± 0.05, where the error is due to the uncertainty of the MD analysis. To reduce the systematic uncertainties, K.Yako et al. performed consistent analyses on both the (p,n) and (n,p) data. They measured $^{90}Zr(n,p)$ reaction at 293 MeV and obtained a reliable GT quenching factor ($Q = 0.88 \pm 0.06$) [19]. The Gamow-Teller resonance (GTR) for double magic nuclei has been searched within the framework of different theoretical models. Especially, there have been different attempts to study GTR distribution in ^{208}Bi [20-25]. The calculations in Ref [21,22,24] have been performed in a self-consistent way. The phonon damping model has been used to calculate the strength distribution of the Gamow-Teller resonance in ^{90}Nb [26] and the results which are in reasonable agreement with the experimental data have been obtained. The GTR for ^{48}Ca, ^{90}Zr and ^{208}Pb has been searched within the random phase approximation (RPA) [27-31]. There are many nuclei having typical for classical magic nuclei resonance like behavior of magicity parameters such as $E(2_1^+)$ energy, $E(4_1^+)/E(2_1^+)$ ratio and quadrupole deformation parameter (β_2) and specific features in nucleon separation energies [32].

In the present work, the GT strength distribution for some new magic nuclei has been calculated within the framework of the RPA. The nucleon-nucleon effective interaction has been defined according to the method developed by Pyatov and Salamov [33-39] in which the effective interaction constant is determined in a self-consistent way and taken out to be a free parameter. The aim of the present study is to understand Gamow Teller states for new double magic nuclei and to provide a motivation for the experimental investigations in order to measure the Gamow-Teller states for these nuclides.

2. THEORETICAL FORMALISM

The conserved quantities such as linear momentum, angular momentum, particle number are a consequence of the invariance of the total nuclear Hamiltonian under symmetry transformations, but the Hamiltonians with the broken symmetry are often handled in constructing the nuclear model or in approximate solution of the problem. For instance, the study of the 1^- states in even-even nuclei (electric dipole excitations) is related to the translational invariance of total Hamiltonian.

The restoration of the rotational invariance in coordinate space is important in the investigation of the 1^+ states in even-even nuclei (magnetic dipole excitations). Unlike the rotational invariance in coordinate space, the rotational invariance in isospin space is not an exact symmetry of the total Hamiltonian. However, the rotational invariance in isospin space is an exact symmetry of the nuclear part of the total Hamiltonian. In other words, the nuclear part of the total Hamiltonian commutes with the isospin operator. This commutativity is violated in the mean field approximation and its restoration plays an important role in understanding the isobaric analogue excitations. Furthermore, the central term in the nuclear part of the total Hamiltonian commutes with the Gamow-Teller operator. The violation of this commutativity due to the mean field approximation is an important problem in the study of the GT excitations. Hence, this violation should be restored to perform a reliable investigation about these excitations.

In the present work, the mean field potential is described in the following form:

$$V_{mean} = V_{central}(r) + V_{ls}(r)(\vec{l}\vec{s}) + V_c(r)\left(\frac{1}{2} - t_z\right). \qquad (1)$$

The central part of the mean field potential consists of the isoscalar and isovector terms:

$$V_{central}(r) = -V_0 f(r)\left(1 - 2\eta\frac{N-Z}{A}t_z\right),$$

The spin-orbit term is defined as

$$V_{ls}(r) = -\xi_{ls}\frac{1}{r}\frac{dV_{central}(r)}{dr},$$

and the Coulomb part is given as

$$V_c(r) = e^2\frac{Z-1}{r}\frac{3r}{2R_c} - \frac{1}{2}\left(\frac{r}{R_c}\right)^3, (r \leq R_c),$$

$$V_c(r) = e^2\frac{Z-1}{r}, (r > R_c),$$

$$f(r) = \frac{1}{1 + e^{\frac{r-R_0}{a}}},$$

$$t_z = 1/2, (neutrons), \qquad t_z = -1/2, (protons).$$

The spin-isospin transition (Gamow-Teller) operator is defined as:

$$G_\mu^+ = \sum_{i=1}^{A} \sigma_\mu(i) t + (i),$$

$$G_\mu^- = (-1)^\mu \sum_{i=1}^{A} \sigma_{-\mu}(i) t - (i), \tag{2}$$

$$G_\mu^- = (G_\mu^+)^\dagger.$$

Here, $\sigma_\mu(i)$ is the Pauli operator in the spherical basis $(\mu = 0, \pm 1)$. t (i) and t+(i) are the isospin lowering and raising operators, respectively. The commutation condition between the total nuclear Hamiltonian and Gamow-Teller (GT) operator can be described as follows:

$$[H, G_\mu^\pm] = [V_c + V_{\vec{l}\vec{s}} G_\mu^\pm], \tag{3}$$

where V_c , and $V_{\vec{l}\vec{s}}$ are Coulomb, and spin-orbit interaction potentials, respectively. Let us consider a system of nucleons in a spherical symmetric average field. In this case, the corresponding single particle Hamiltonian of the system is given by

$$H_{sp} = \sum_{jm} \varepsilon_j(\tau) a_{jm}^\dagger(\tau) a_{jm}(\tau), (\tau = n\ , p) \tag{4}$$

where $\varepsilon_j(\tau)$ is the single particle (sp) energy of the nucleons with angular momentum $j(\tau)$, and the $a_{jm}^+(\tau)$ $(a_{jm}(\tau))$ is the particle creation (annihilation) operator. The commutation of the Hamiltonian in Eq.(4) with GT operator is different from the expression in Eq.(3):

$$[H_{sp}, G_\mu^\pm] \neq [V_c + V_{\vec{l}\vec{s}}, G_\mu^\pm],$$

or

$$[H_{sp} - (V_c + V_{\vec{l}\vec{s}}), G_\mu^\pm] \neq 0 \tag{5}$$

According to Pyatov Method, the nucleon-nucleon residual interaction giving the GT excitations in the neighbor odd-odd nuclei is chosen in the following form:

$$h_{GT} = \sum_{\rho=\pm} \frac{1}{2\gamma_\rho} \sum_{\mu=0,\pm1} [H_{sp} - V_c - V_{\vec{l}\vec{s}}, G_\mu^\rho]^\dagger [H_{sp} - V_c - V_{\vec{l}\vec{s}}, G_\mu^\rho] \tag{6}$$

This effective interaction is considered in such a way that the broken commutation relation between the total Hamiltonian operator and GT operator is restored. The strength parameter of the residual interaction is found from the following condition

$$\left[H_{sp} + h_{GT} - V_c - V_{\vec{l}\vec{s}}, G_\mu^\rho\right] = 0 \tag{7}$$

and taken out to be a free parameter.

$$\gamma_\rho = \frac{1}{2}\langle 0\left|\left[\left[H_{sp} - (V_c + V_{ls}), G_\mu^\rho\right], G_\mu^\rho\right]\right|0\rangle. \tag{8}$$

Thus, the total Hamiltonian giving the GT 1^+ states in intermediate nuclei can be defined as follows:

$$H = H_{sp} + h_{GT}. \tag{9}$$

In RPA, the m^{th} excited 1^+states in odd-odd nuclei are considered as the phonon excitations and described by:

$$\left|m >= Q_m^\dagger(\mu)\right|0 >= \sum_{np}[\psi_{np}^m A_{np}^\dagger(\mu) + (-1)^\mu \varphi_{np}^m A_{np}(\mu)]|0 >, \tag{10}$$

where $Q_m^\dagger(\mu)$ is the RPA phonon creation operator, $|0>$ is the phonon vacuum which corresponds to the ground state of an even-even nucleus and fulfills $Q_m(\mu)|0>= 0$ for all m. The ψ_{np}^m and φ_{np}^m are quasi boson amplitudes. Assuming that the phonon operators obey the commutation relations given below

$$< 0|[Q_m(\mu), Q_{m'}^\dagger(\mu')]|0 >= \delta_{mm'}\delta_{\mu\mu'} ,$$

we obtain the following ortho-normalization condition for amplitudes ψ_{np}^m and φ_{np}^m :

$$\sum_{np}\left[\psi_{np}^m\psi_{np}^{m'} - \varphi_{np}^m\varphi_{np}^{m'}\right] = \delta_{mm'}. \tag{11}$$

The energies and wave functions of the GT 1^+ states have been obtained from RPA equation of motion:

(12)

$$[H, Q_m^{\dagger}(\mu)]|0>= \omega_m Q_m^{\dagger}(\mu)|0>,$$

where ω_m is the energy of the GT 1^+ states occurring in neighboring odd-odd nuclei. For the Gamow-Teller beta strength function, we have

$$B_{GT}^{(\pm)}(\omega_m) = \sum_{\mu} \left|\langle 1_m^+, \mu \middle| G_\mu^{\pm} \middle| 0_{g.s.}^+ \rangle\right|^2, \tag{13}$$

These strength functions are related to each other by the Ikeda sum rule:

$$\sum_m B_{GT}^{(-)}(\omega_m) - \sum_m B_{GT}^{(+)}(\omega_m) = 3(N - Z). \tag{14}$$

3. RESULTS AND DISCUSSION

The energy distribution of the GT strengths for the parent nuclei $^{96}Zr, ^{96}Sr, ^{54}Ca, ^{28}O, ^{24}O$ and ^{14}C are calculated in this section. The single particle levels have been obtained by using Woods-Saxon potential with Chepurnov parametrization [40]. The nucleon-nucleon residual interaction potential has been included in such a way that the broken SU(4) symmetry in the mean field approximation has been restored.

The calculated values of the total GT strength for the nuclei under consideration are given in Table 1. Also, the energy spectra of these total strengths are shown in Figure 1 and 2. The distributions for the β^+ decay of $^{28}O, ^{24}O$ and ^{14}C are not presented due to the negligible contributions of these decays to the total GT strength. It can be explained as follows: the β^+ decay of neutron excess nuclei is originated from the fact that SU(4) symmetry is not an exact symmetry of total Hamiltonian. In other words, the β^+ decay probability would be zero if SU(4) was an exact symmetry of Hamiltonian. However, the SU(4) symmetry violation in mean field approximation leads to an increase in β^+ decay probability. Hence, the restoration of this symmetry violation is important in the investigation of GT transitions.

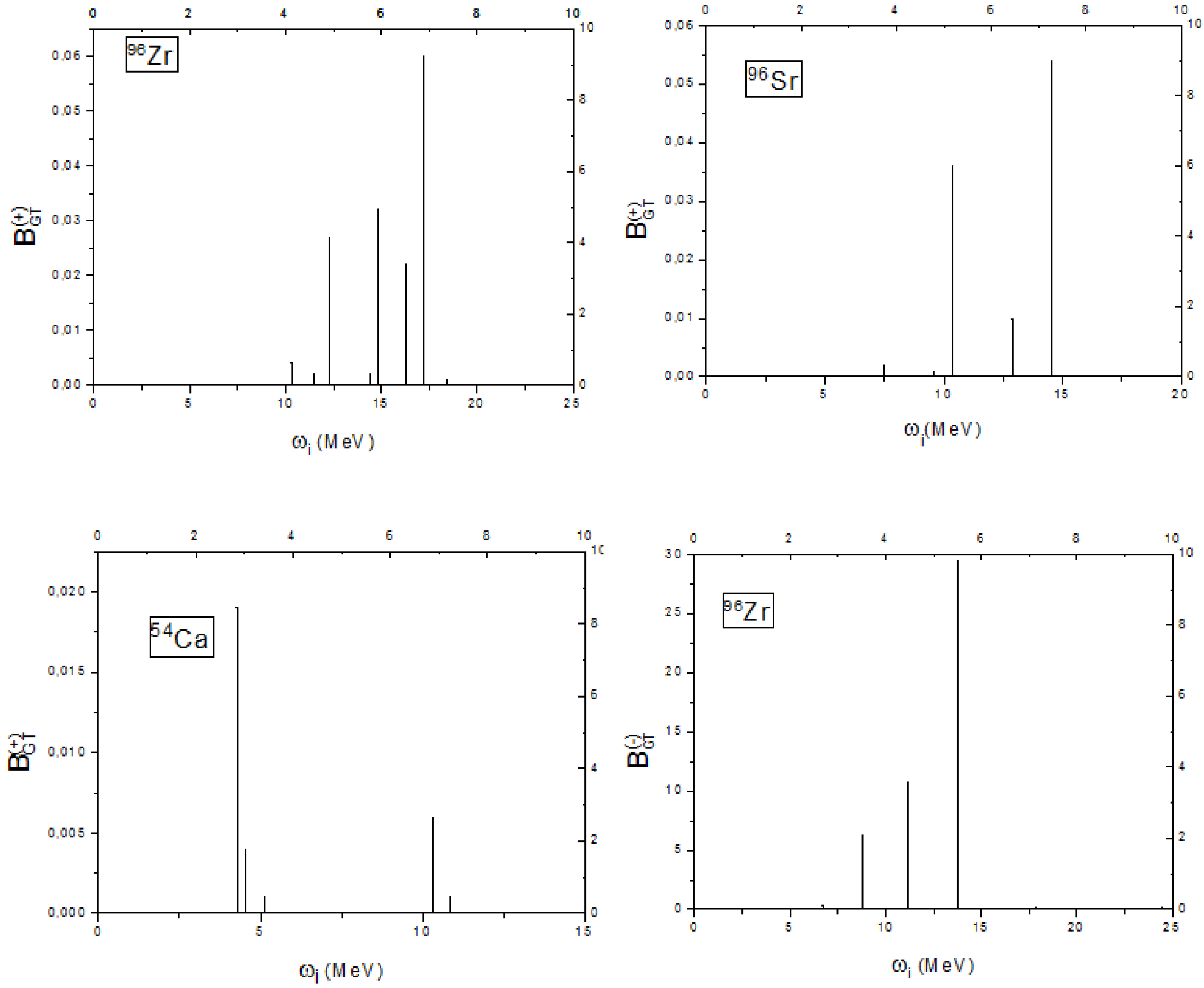

Figure 1. The energy distribution of β^{+} transitions.

While the spectrums for the β^{-} decay of ^{54}Ca, ^{28}O, ^{24}O and ^{14}C show a single peak, those for the β^{-} decay of ^{96}Zr, ^{96}Sr exhibit more fragmentation over the final states in their neighbor nuclei. There would be one degenerate state including 100% of the total strength if SU(4) was an exact symmetry. As known, the spectrum of the excited states for heavy mass nuclei usually consists of three energy regions: i) low energy region, ii) GT region, and iii) isovector spin monopole resonance (IVSMR) region [29]. The spectrum for ^{96}Zr is divided into low energy region ($\omega_m < 10MeV$), GT region ($10MeV < \omega_i < 15MeV$) and isovector spin monopole resonance region ($\omega_i < 15MeV$). The low energy region, GT region and isovector spin monopole resonance region includes 13.72%, 83.87% and 2.41% of total β^{-} strength. The total strength for ^{96}Sr divided between GT region ($9.5MeV < \omega_i < 14.5MeV$) and isovector spin monopole resonance region ($\omega_i > 14.5MeV$) The contributions of GT and IVSMR regions to the total strength have been determined as 90.41% and 9.59%, respectively. The contributions coming from the GT region for other remaining nuclei are included by one excited state. Also, the total strength for light mass nuclei shifts to lower energies.

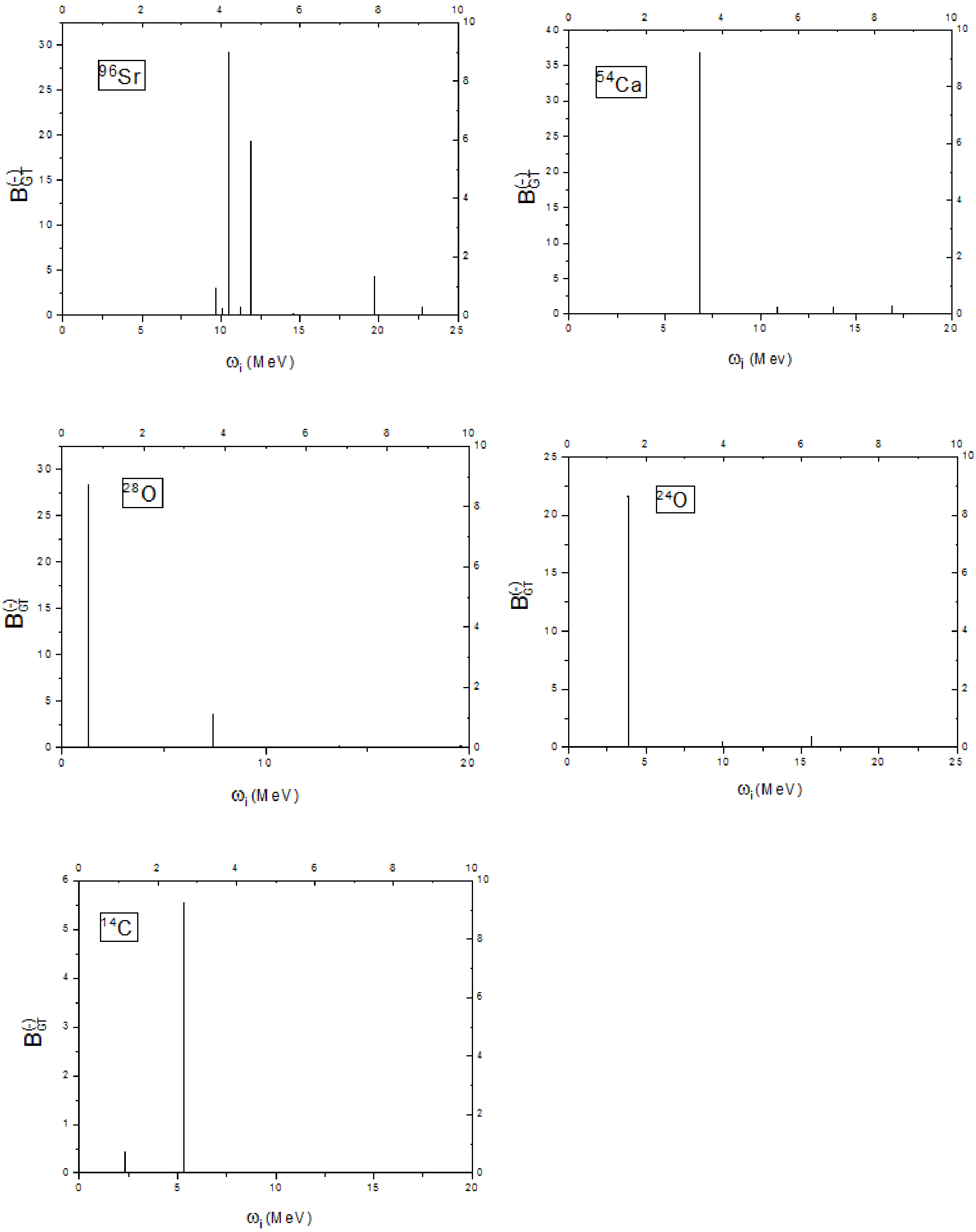

Figure 2. The energy distribution of β^- transitions.

Table 1. The total GT strength values.

No	Nuclei	S^-	S^+	$S^- - S^+$	3(N-Z)
1	^{96}Zr	47.73	0.15	47.58	48
2	^{96}Sr	58.89	0.10	58.79	60
3	^{54}Ca	40.67	0.03	40.64	42
4	^{28}O	32.87	0.002	32.87	36
5	^{24}O	23.06	0.001	23.06	24
6	^{14}C	6.00	0.004	6.00	6

4. CONCLUSION

The GT strength distribution has been studied for some new magic nuclei within a self-consistent method. The excited states in neighbor odd-odd nuclei have been obtained using a mathematical formalism which is free of the effective interaction parameter. The use of this method leads to a remarkable reduction in the β^+ decay probability. Thus, the β^+ transitions for ^{28}O, ^{24}O and ^{14}C have no considerable contribution to the total GT strength. In the light mass nuclei, while the spectrum for (N-1, Z+1) nuclei has a single peak, the β^- spectrum for heavier nuclei shows more fragmentation.

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