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Solution of Angular Parts of the Forbidden Beta Moment Matrix Elements for Rank 0, 1 and 2

Necla ÇAKMAK[1], İsmail ÇAYLI[1]

,[21] *Karabük University, Department of Physics, Karabük- Turkey*

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Abstract. In astrophysical environments, allowed Gamow-Teller (GT) transitions are important, particularly for *β*-decay rates in presupernova evolution of massive stars, since they contribute to the fine-tuning of the lepton-to baryon content of the stellar matter prior to and during the collapse of a heavy star. In environments where GT transitions are unfavored, first-forbidden transitions become important especially in medium heavy and heavy nuclei. Particularly in case of neutron-rich nuclei, first-forbidden transitions are favored primarily due to the phase-space amplification for these transitions. In this study, the angular part of the beta (β) moment matrix elements of the $0^+ \leftrightarrow 0^-$, $0^+ \leftrightarrow 1^-$ and $0^+ \leftrightarrow 2^-$ first forbidden beta decay transition have been solved directly without any assumption. In the calculation of the nuclear matrix elements have been considered the contribution coming from the spin-orbit term in the shell model potential.

Key Words*:* Nuclear Matrix Element, First Forbidden Beta Decay

1. INTRODUCTION

The physical processes containing the use of the beta moments are summarized as the calculation of the *logft* values for forbidden transitions, r-process and the double beta decay. It is well known that β decay processes are very important to understand the weak interaction processes and the nuclear structure. One of the basic problems in nuclear structure analysis and testing of nuclear models is the calculation of the nuclear matrix element. The first forbidden beta decays have a significant role in the validity of theories related to r-process and double beta decay (2νββ) Borzov (2006), Borzov (2003), Borzov (2003), Ejiri (2005), Schopper (1966), Barbero (1998), Civitarese (1996). The effects of the first forbidden transitions have been investigated in nucleosynthesis phenomena. The total β decay half-lives and delayed neutron emission probabilities has been performed by considering the Gamow-Teller and first forbidden transitions Borzov (2006).

The aim of the present study is to derivate angular part of the beta moment matrix elements for the double beta decay and the first forbidden beta decays. The *ft* values must be calculated for the investigation of the first forbidden beta decay where it is important to do an exact calculation of the

* Corresponding author. *Email address*: neclac@karabuk.edu.tr

transition probability. The relativistic and non-relativistic beta moment matrix elements have a significant role in the calculation of the transition probabilities. A reliable prediction of the transition probability provides an important contribution to the interpretation of the *ft* values.

The *ft* values are given by the following expressions:

$$(\mathrm{ft})_{\beta^{\pm}} = \frac{\mathrm{D}}{\left(\frac{g_A}{g_V}\right)^2 4\pi \mathrm{B}(\mathrm{I}_i \to \mathrm{I}_f, \beta\pm)}, \qquad D = \frac{2\pi^3\hbar^2 \ln 2}{g_v^2 m_e^5 c^4} = 6250\,\mathrm{sec}$$

where D and $\mathrm{B}(\mathrm{I}_i \to \mathrm{I}_f, \beta\pm)$ are a constant and the transition probability, respectively.

The nuclear matrix elements have two components in their very nature; angular and radial parts. In the present study we have focused on the angular part, which lets proper calculations of the *ft* values when we have accurate radial parts.

In references Civitarese *et al.* (1996), the matrix element of the relativistic beta moment has been calculated by assuming to be proportional to the matrix element of non-relativistic beta moment and the *ft* values have been given for the first forbidden beta decays. In this respect, the relativistic and the non-relativistic nuclear matrix elements have been derived directly without any assumption and the results have been by the parameter in the shell model potential. The numerical results are obtained using a simple proper implementation (in Fortran77) of the theoretical formalism explained is Section The log*ft* values for 0^- beta decays have been calculated in a previous study using nuclear matrix elements Cakmak *et al.* (2010).

2. FORMALISM

The first forbidden transitions (n=1) are given as follows by the matrix elements of the beta moments Bohr *et al.* (1969):

The matrix element of the beta moment for $\lambda^{\pi} = 0^-$ (rank 0):

The non-relativistic matrix element,

$$M(j_A, \kappa = 1, \lambda = 0) = g_A \sum_k t_-(k) r_k [Y_1(\hat{r}_k)\vec{\sigma}(k)]_0$$

The relativistic matrix element,

$$M(p_A, \lambda = 0) = \frac{1}{\sqrt{4\pi}} \frac{g_A}{c} \sum_k t_-(k) [\vec{\sigma}(k) \cdot \vec{v}_k$$

The matrix element of the beta moment for $\lambda^{\pi} = 1^-$ (rank 1):

The non-relativistic matrix element,

$$M(p_v, \lambda = 1, \mu) = g_v \sum_k t_-(k) r_k Y_{1\mu}(\hat{r}_k)$$

the relativistic matrix element,

$$M(j_v, \kappa = 0, \lambda = 1, \mu) = \frac{1}{\sqrt{4\pi}} \frac{g_v}{c} \sum_k t_-(k) [\vec{v}_k]_{1\mu}$$

the non-relativistic matrix element,

$$M(j_A, \kappa = 1, \lambda = 1, \mu) = g_A \sum_k t_-(k) r_k [Y_1(\hat{r}_k) \vec{\sigma}(k)]_{1\mu}$$

and

the non relativistic matrix element of the beta moment for $\lambda^\pi = 2^-$ (rank 2) is given

$$M(j_A, \kappa = 1, \lambda = 2, \mu) = g_A \sum_k t_-(k) r_k [Y_1(\hat{r}_k) \vec{\sigma}(k)]_{2\mu}$$

where g_v and g_A are vector and axial vector coupling constant, also $t_-(k)$, $Y_1(\hat{r}_k)$, $\vec{\sigma}_k$ and $\vec{v}_k$ are the isospin lowering, the spherical harmonic, the Pauli spin and the speed operators, respectively.

The moments that are independent of the position of the nucleons are coupled to the part of the lepton current that is constant over the nuclear volume. The leptonic matrix elements for these moments are completely the same as for the corresponding 0^+ and 1^+ moments since, for the parity-violating beta interaction, the coupling to the leptons is independent of the parity of the nuclear moments.

The 0^- and 1^- moments that are linear in r are coupled to the leptons through the derivative of the lepton wave functions and are thus multiplied by the factor *ik*, where k is the lepton wave number inside the nucleus. The dependence of k on the energy of the emitted leptons implies a deviation of the electron spectrum from that of allowed transitions Bohr et al. (1969).

The calculation of the corresponding nuclear beta matrix elements for these operators in a certain basis is important and the angular parts of these matrix elements have been calculated in this work. The matrix elements of the beta moments have solved by using the expressions in the Appendix.

The Woods-Saxon single particle basis has been used in the calculations. As known, the integral expressions with respect to the angular coordinates do not depend on the radial part of the mean field potential in case of the spherical symmetric potentials.

2.1. Solutions of The Matrix Element for $\lambda^{\pi} = 0^{-}$ Transitions

Analytic solution of the non-relativistic the matrix element

By using matrix elements of operators which depend on variables of two subsystems (see Appendix, Eq.(1)) is reproduced

$$< (l_p s_p) j_p m_p | M(j_A, \kappa = 1, \lambda = 0) | (l_n s_n) j_n m_n >=$$

$$= g_A \sum_k t_-(k)(-1)^{2\lambda} \Pi_{\lambda j_n} < j_n m_n \lambda\mu | j_p m_p > \begin{Bmatrix} 1 & 1 & \lambda \\ l_p & \frac{1}{2} & j_p \\ l_n & \frac{1}{2} & j_n \end{Bmatrix} < l_p || Y_1(\hat{r}_k) || l_n >< S_p || \vec{\sigma}(k) || S_n >,$$

$$\Pi_{\lambda j_n} = \Pi_{0 j_n} = \sqrt{(2\lambda+1)(2j_n+1)}, \qquad < l_p || Y_1(\hat{r}_k) || l_n > = \sqrt{\frac{3(2l_n+1)}{4\pi}}$$

$$< S_p || \vec{\sigma}(k) || S_n > = \sqrt{6}\ , \qquad \begin{Bmatrix} 1 & 1 & \lambda \\ l_p & \frac{1}{2} & j_p \\ l_n & \frac{1}{2} & j_n \end{Bmatrix} = \frac{(-1)^{l_n+j_n-\frac{1}{2}}}{\sqrt{3(2j_n+1)}}$$

and by using the notation, matrix elements of the spherical harmonic operator, matrix elements of the spin-angular momentum operator and explicit forms of the 9j symbols for spherical values of the arguments in Eqs. (3, 6, 7,9) in Appendix are given as follows

$$= g_A \sum_k t_-(k)(-1)^{l_n+j_n-\frac{1}{2}} \sqrt{\frac{6}{4\pi}(2l_n+1)} \begin{Bmatrix} \frac{1}{2} & l_n & j_n \\ l_p & \frac{1}{2} & 1 \end{Bmatrix} < j_n m_n 00 | j_p m_p >< l_n 010 | l_p 0 > .$$

The reduced nuclear matrix element is given according to Wigner-Eckart theorem (see Appendix Eq.(4)),

$$< (l_p s_p) j_p || M(j_A, \kappa = 1, \lambda = 0) || (l_n s_n) j_n >=$$

$$= g_A \sum_k t_-(k)(-1)^{l_n+j_n-\frac{1}{2}} \sqrt{\frac{6}{4\pi}(2j_n+1)(2l_n+1)} \begin{Bmatrix} \frac{1}{2} & l_n & j_n \\ l_p & \frac{1}{2} & 1 \end{Bmatrix} < l_n 010 | l_p 0 > .$$

Analytic solution of the relativistic the matrix element

The speed of particle and the total Hamiltonian is described in the following form

$$\vec{V}_k = \dot{\vec{r}} = \frac{i}{\hbar}[\vec{H}, \vec{r}]$$

$$\vec{H} = \frac{\hat{p}^2}{2m} + V_{Coul}(r) + V_{So}(\vec{l}.\vec{s}) + V_{central}(r)$$

where $V_{Coul}(r)$, $V_{So}(\vec{l}.\vec{s})$ and $V_{central}(r)$ are two body Coulomb and spin-orbit interaction potentials and the central potential, respectively.

$$[\vec{H}, \vec{r}] = -\frac{\hbar^2}{m}\vec{\nabla} + \frac{i}{\hbar}(\vec{r}\,x\,\vec{s})V_{So}, \qquad \vec{V}_k = -\frac{i\hbar}{m}\vec{\nabla} - \frac{V_{So}}{\hbar}(\vec{r}\,x\,\vec{s})$$

The central part of the mean field potential consists of the isoscalar and isovector terms

$$V_{central}(r) = -V_0 f(r)\left(1 - 2n\frac{N-Z}{A}t_z\right),$$

the spin-orbit term is defined as

$$V_{So}(\vec{l}.\vec{s}) = -\varepsilon_{ls}\frac{1}{r}\frac{dV_{central(r)}}{dr}$$

and the Coulomb part is given as

$$V_C(r) = e^2\frac{Z-1}{r}\left[\frac{3r}{2R_c} - \frac{1}{2}\left(\frac{r}{R_c}\right)^3\right] \quad (r \le R_c), \qquad V_C(r) = e^2\frac{Z-1}{r} \qquad (r > R_c)$$

$$f(r) = \frac{1}{1+e^{\frac{r-R_0}{a}}}, \qquad t_z = 1/2\,(neutrons), \qquad t_z = 1/2\,(protons).$$

The relativistic matrix element is rewritten

$$M(\rho_A, \lambda = 0) = -i\frac{g_A}{\sqrt{4\pi}}\sum_k t_-(k)\left[\frac{\hbar}{mc}(\vec{\sigma}_k \cdot \vec{\nabla}) - \frac{V_{so}}{\hbar c}(\vec{\sigma}_k \cdot \vec{r}_k)\right].$$

The first part of the nuclear matrix element is given by using matrix elements of operators which depend on variables of two subsystems (see Appendix, Eq.(2)),

$$< (l_p s_p)j_p m_p|\vec{\sigma}_k \cdot \vec{\nabla}|(l_n s_n)j_n m_n >=$$

$$= \delta_{j_p j_n}\delta_{m_p n_p}(-1)^{j_n+l_n+\frac{1}{2}}\begin{Bmatrix} l_p & l_n & 1 \\ \frac{1}{2} & \frac{1}{2} & j_n \end{Bmatrix} < l_p\|\vec{\nabla}\|l_n >< S_p\|\vec{\sigma}_k\|S_n >$$

and by using matrix elements of the operator $\vec{\nabla}$ and matrix elements of the spin angular momentum operator (see Appendix, Eq.(8)) is obtained

$$< l_p\|\vec{\nabla}\|l_n >= \left[\sqrt{l_n+1}A_{l_p l_n}\delta_{l_p l_n+1} - \sqrt{l_n}B_{l_p l_n}\delta_{l_p l_n-1}\right]$$

$$< (l_p s_p)j_p m_p|\vec{\sigma}_k\cdot\vec{\nabla}|(l_n s_n)j_n m_n > =$$

$$= \delta_{j_p j_n}\delta_{m_p m_n}(-1)^{j_n+l_n+\frac{1}{2}}\begin{Bmatrix} l_p & l_n & 1 \\ \frac{1}{2} & \frac{1}{2} & j_n \end{Bmatrix}\sqrt{6}\left[\sqrt{l_n+1}A_{l_p l_n}\delta_{l_p l_{n+1}} - \sqrt{l_n}B_{l_p l_n}\delta_{l_p l_{n+1}}\right]$$

and according to Wigner-Eckart theorem as follows

$$< (l_p s_p)j_p\|\vec{\sigma}_k\cdot\vec{\nabla}\|(l_n s_n)j_n >=$$

$$= \delta_{j_p j_n}\delta_{m_p m_n}(-1)^{j_n+l_n+\frac{1}{2}}\begin{Bmatrix} l_p & l_n & 1 \\ \frac{1}{2} & \frac{1}{2} & j_n \end{Bmatrix}\sqrt{6(2j_n+1)}\left[\sqrt{l_n+1}A_{l_p l_n}\delta_{l_p l_{n+1}} - \sqrt{l_n}B_{l_p l_n}\delta_{l_p l_{n-1}}\right].$$

The second part of the nuclear matrix element is given by using matrix elements of some scalar and vector products (see Appendix, Eq.(5)),

$$< (l_p s_p)j_p m_p|(\vec{\sigma}_k\cdot\vec{r}_k)|(l_n s_n)j_n m_n >=$$

$$= (-1)^{l_n+j_n+\frac{1}{2}}\ \delta_{j_p j_n}\delta_{m_p m_n}\sqrt{6(2l_n+1)} < l_n 010|l_p 0 > \begin{Bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ j_n & l_n & l_p \end{Bmatrix}$$

and the reduced nuclear matrix element is written according to Wigner-Eckart theorem,

$$< (l_p s_p)j_p\|(\vec{\sigma}\cdot\vec{r})\|(l_n s_n)j_n > =$$

$$= (-1)^{l_n+j_n+\frac{1}{2}}\ \sqrt{6(2l_n+1)(2j_n+1)}\ \begin{Bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ j_n & l_n & l_p \end{Bmatrix} < l_n 010|l_p 0 > .$$

The general form is given as follows

$$< (l_p s_p)j_p\|M(\rho_A,\lambda=0)\|(l_n s_n)j_n >=$$

$$= -i\frac{g_A}{\sqrt{4\pi}}\sum_k t_-(k)\left\{\frac{\hbar}{mc}\sqrt{6(2j_n+1)}(-1)^{j_n+l_n+\frac{1}{2}}\begin{Bmatrix} l_p & l_n & 1 \\ \frac{1}{2} & \frac{1}{2} & j_n \end{Bmatrix}\right.$$

$$\left(\sqrt{l_n+1}A_{l_nl_p}\delta_{l_pl_{n+1}} - \sqrt{l_n}B_{l_pl_n}\delta_{l_pl_{n-1}}\right) -$$

$$\left.-\frac{V_{so}}{\hbar c}(-1)^{l_n+j_n+\frac{1}{2}}\sqrt{6(2l_n+1)(2j_n+1)} < l_n010|l_p0 > \begin{Bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ j_n & l_n & l_p \end{Bmatrix}\right\}.$$

2.2 Solution of The Matrix Element for $\lambda^\pi = 1^-$ Transitions

Analytic solution of the non-relativistic the matrix element

By using matrix elements of operators which depend on variables of two subsystems (see Appendix, Eq.(2)) is solved [6]

$$< (l_ps_p)j_pm_p|M(\rho_v,\lambda=1,\mu)|(l_ns_n)j_nm_n > =$$

$$= g_v\sum_k t_-(k)\sqrt{\frac{18}{4\pi}(2j_n+1)(2l_n+1)} < j_nm_n1\mu|j_pm_p >< l_n010|l_p0 > \begin{Bmatrix} 1 & 1 & \lambda \\ l_p & \frac{1}{2} & j_p \\ l_n & \frac{1}{2} & j_n \end{Bmatrix}$$

$$= g_v\sum_k t_-(k)(-1)^{j_n+l_p-\frac{1}{2}}\sqrt{\frac{3}{4\pi}(2j_n+1)(2l_n+1)} < l_n010|l_p0 >< j_nm_n1\mu|j_pm_p > \begin{Bmatrix} j_p & j_n & 1 \\ l_n & l_p & \frac{1}{2} \end{Bmatrix}$$

and the reduced matrix element is given according to Wigner-Eckart theorem,

$$< (l_ps_p)j_p||M(\rho_v,\lambda=1)||(l_ns_n)j_n >=$$

$$= g_v\Sigma_k t_-(k)(-1)^{j_n+l_p-\frac{1}{2}}\sqrt{\frac{3}{4\pi}(2j_n+1)(2j_n+1)(2l_n+1)} < l_n010|l_p0 > \begin{Bmatrix} l_n & \frac{1}{2} & j_n \\ j_p & 1 & l_p \end{Bmatrix}.$$

Analytic solution of the relativistic matrix element

The relativistic matrix element is rewritten by making some operations

$$M(j_v, \mathrm{K}=0, \lambda=1, \mu) = \frac{1}{\sqrt{4\pi}}\frac{g_V}{c}\sum_k t_-(k)\left\{-\frac{i\hbar}{m}\vec{\nabla}_{1\mu} - \frac{V_{so}(\vec{l}\cdot\vec{s})}{\hbar}[\vec{r}\times\vec{s}]_{1\mu}\right\}.$$

The first part of the reduced nuclear matrix element is reproduced by using matrix elements of the operator $\vec{\nabla}$ (see Appendix Eq.(8)),

$$< (l_p s_p) j_p \| \vec{\nabla}_{1\mu} \| (l_n s_n) j_n > =$$

$$= (-1)^{j_n + l_p - \frac{1}{2}} \sqrt{(2j_n+1)(2j_p+1)} \begin{Bmatrix} j_p & j_n & 1 \\ l_n & l_p & \frac{1}{2} \end{Bmatrix} \left[\sqrt{l_n+1} A_{l_p l_n} \delta_{l_p l_{n+1}} - \sqrt{l_n} B_{l_p l_n}\right]$$

the second part of the nuclear matrix element is reproduced by using matrix elements of some scalar and vector products (see Appendix Eq.(12)),

$$< (l_p s_p) j_p \| r_k [\vec{n}\times\vec{s}]_{1\mu} \| (l_n s_n) j_n > =$$

$$= (-1)^{l_p + j_n + \frac{1}{2}} \delta_{s_p s_n} \sqrt{(2l_n+1)(2j_n+1)(2j_p+1)} < l_n 010 | l_p 0 > \begin{Bmatrix} j_p & l_p & \frac{1}{2} \\ l_n & j_n & 1 \end{Bmatrix}.$$

$$.\frac{1}{2}\left[(j_p - l_p)(j_p + l_p + 1) - (j_n - l_n)(j_n + l_n + 1)\right]$$

and the reduced matrix element is given as follows

$$< (l_p s_p) j_p \| M(j_v, \mathrm{K}=0, \lambda=1) \| (l_n s_n) j_n > =$$

$$= i\frac{g_V}{\sqrt{4\pi}}\sum_k t_-(k) \left\{\frac{\hbar}{mc} < (l_p s_p) j_p \| \vec{\nabla}_{1\mu} \| (l_n s_n) j_n > + \frac{1}{\hbar c} < (l_p s_p) j_p \left\| V_{so}(\vec{l}\cdot\vec{s}) r_k \frac{[\vec{n}\times\vec{s}]_{1\mu}}{i} \right\| (l_n s_n) j_n >\right\}$$

.

Analytic solution of the non-relativistic matrix element

By using matrix elements of operators which depend on variables of two subsystems in the same approach is reproduced

$$< (l_p s_p) j_p m_p | M(j_A, \mathrm{K}=1, \lambda=1, \mu) | (l_n s_n) j_n m_n > =$$

$$= g_A \sum_k t_-(k)\sqrt{3(2j_n+1)} < j_n m_n 1\mu | j_p m_p > \begin{Bmatrix} 1 & 1 & 1 \\ l_p & \frac{1}{2} & j_p \\ l_n & \frac{1}{2} & j_n \end{Bmatrix} < l_p \| Y_1(\hat{r}_k) \| l_n >< s_p \| \vec{\sigma}_k \| s_n >$$

$$= g_A \sum_k t_-(k) \sqrt{\frac{54}{4\pi}(2l_n+1)(2j_n+1)} < j_n m_n 1\mu | j_p m_p > \begin{Bmatrix} 1 & 1 & 1 \\ l_p & \frac{1}{2} & j_p \\ l_n & \frac{1}{2} & j_n \end{Bmatrix} < l_n 010 | l_p 0 >$$

and is given as follows according to Wigner-Eckart theorem

$$< (l_p s_p) j_p \| M(j_A, \kappa = 1, \lambda = 1) \| (l_n s_n) j_n >=$$

$$= g_A \sum_k t_-(k) \sqrt{\frac{54}{4\pi}(2l_n+1)(2j_n+1)(2j_p+1)} < j_n m_n 1\mu | j_p m_p > \begin{Bmatrix} 1 & 1 & 1 \\ l_p & \frac{1}{2} & j_p \\ l_n & \frac{1}{2} & j_n \end{Bmatrix} < l_n 010 | l_p 0 >.$$

1.1. Solution of The non-Relativistic Matrix Element for $\lambda^\pi = 2^-$ Transition

By using matrix elements of operators which depend on variables of two subsystems is solved

$$< (l_p s_p) j_p m_p | M(j_A, \kappa = 1, \lambda = 2, \mu) | (l_n s_n) j_n m_n > =$$

$$= g_A \sum_k t_-(k) (-1)^4 \sqrt{5(2j_n+1)} < j_n m_n 2\mu | j_p m_p > \begin{Bmatrix} 1 & 1 & 2 \\ l_p & \frac{1}{2} & j_p \\ l_n & \frac{1}{2} & j_n \end{Bmatrix} < l_p \| Y_1(\hat{r}_k) \| l_n >< S_p \| \vec{\sigma}(k) \| S_n >$$

$$= g_A \sum_k t_-(k) \sqrt{\frac{90}{4\pi}(2j_n+1)(2l_n+1)} \begin{Bmatrix} 1 & 1 & 2 \\ l_p & \frac{1}{2} & j_p \\ l_n & \frac{1}{2} & j_n \end{Bmatrix} < j_n m_n 2\mu | j_p m_p > < l_n 010 | l_p 0 >.$$

The reduced matrix element is given as follows

$$< (l_p s_p) j_p \| M(j_A, \kappa = 1, \lambda = 2, \mu) \| (l_n s_n) j_n >=$$

$$= g_A \sum_k t_-(k) \sqrt{\frac{90}{4\pi}(2j_n+1)(2l_n+1)(2j_p+1)} < l_n 010|l_p 0 > \begin{Bmatrix} 1 & 1 & 2 \\ l_p & \frac{1}{2} & j_p \\ l_n & \frac{1}{2} & j_n \end{Bmatrix}.$$

3. CONCLUSION

The existence of the nucleosynthesis process and its probability depends on the neutron capture and transition probabilities. The relativistic and non-relativistic beta moment matrix elements have a significant role in the calculation of the transition probabilities. A reliable prediction of the transition probability provides an important contribution to the interpretation of the *ft* values. Moreover, the consideration of the contributions coming from forbidden transitions ensures a more reliable investigation of the r-process and double beta decay.

The nuclear matrix elements must be calculated for the consideration of these contributions. Therefore, this study will be an important reference for scientists studying related to the double beta decay, the first forbidden beta decay, r-process. The angular part of transitions reduced nuclear matrix elements are given for some states in the table. The reduced relativistic and non-relativistic matrix elements may obtain by using a simple proper implementation or may calculate directly for some nuclei states. (You could contact for proper implementation.)

Table 1. The angular part of transitions reduced nuclear matrix elements for $\lambda^\pi = 0^-, \lambda^\pi = 1^-$ and $\lambda^\pi = 2^-$. The reduced nuclear matrix elements are given in unit of g_A and g_v.

$\lambda^\pi = I^\pi$	Nuclear Matrix Elements	Single Particle Transition		
		$1s_{1/2} \rightarrow 1p_{1/2}$	$1d_{5/2} \rightarrow 1f_{5/2}$	$1d_{5/2} \rightarrow 1f_{5/2}$
		$l_i = 1, j_i = 1/2$ $l_f = 1, j_f = 1/2$	$l_i = 3, j_i = 5/2$ $l_f = 2, j_f = 3/2$	$l_i = 3, j_i = 5/2$ $l_i = 2, j_i = 5/2$
$\lambda^\pi = 0^-$	$M(j_A, K=1, \lambda=0)/g_A$	0.398	–	1.296
	$M(p_A, \lambda=0)/ig_A$	0.026	–	0.125

$\lambda^{\pi} = 1^{-}$	$M\,(p_v, \lambda = 1, \mu)/g_v$	-0.398	1.424	-2.245
	$M\,(j_v, K = 0, \lambda = 1, \mu)/ig_v$	-3.647	0.045	0.124
	$M\,(j_A, K = 1, \lambda = 1, \mu)/g_A$	0.563	-0.534	1.179
$\lambda^{\pi} = 2^{-}$	$M\,(j_A, K = 1, \lambda = 2, \mu)/g_A$	–	-0.650	1.522

Appendix

As follows are taken from Ref. Varshalovich et al. (1988).

Matrix elements of operators which depend on variables of two subsystems,

$$< n_1'j_1'n_2'j_2'j'm' \left| \left(\overrightarrow{P_a}\,(1) \otimes \overrightarrow{Q_a}(2) \right)_{c\gamma} \right| n_1j_1n_2j_2jm >=$$

$$= (-1)^{2c} \Pi_{cj} C_{jmc\gamma}^{j'\,m'} \begin{Bmatrix} a & b & c \\ j_1' & j_2' & j' \\ j_1 & j_2 & j \end{Bmatrix} < n_1'j_1' \| \overrightarrow{P_a}\,(1) \| n_1j_1 >< n_2'j_2' \| \overrightarrow{Q_a}\,(2) \| n_2j_2 >$$

$$< n_1'j_1'n_2'j_2'j_{m'}' \left| \hat{P}_a(1).\hat{Q}(2) \right| n_1j_1n_2j_2j_m >=$$

$$= \delta_{j'\,j} m_{m'\,m} (-1)^{j+j_1+j_2'} \begin{Bmatrix} j_1' & j_1 & \alpha \\ j_2 & j_2' & j \end{Bmatrix} < n_1'j_1' \| \overrightarrow{P_a}\,(1) \| n_1j_1 >< n_2'j_2' \| \hat{Q}_a\,(2) \| n_2j_2 > \quad (3)$$

The notain,

$$\pi_{abc\ldots} = \sqrt{(2a+1)(2b+1)(2c+1)\ldots.}$$

The Wigner-Eckart theorem,

$$< n'j'm' \left| \widehat{M}_{kx} \right| njm >= (-1)^{j'-m'} \begin{pmatrix} j' & k & j \\ -m' & x & m \end{pmatrix} < n'j' \| \widehat{M} \| n_j >=$$

$$= (-1)^{2k} C_{jmkx}^{j'\,m'} \frac{< n'j' \| M \| nj}{\sqrt{2j'+1}}$$

Matrix elements of some scalar and vector products,

$< l's'J'M'|\hat{n}.\vec{s}|lsJM >=$

$= (-1)^{l+s+j}\delta_{J\,J'}\delta_{M\,M'}\delta_{s\,s'}\sqrt{s(s+1)(2s+1)(2l+1)}C_{l010}^{l'\,0}\begin{Bmatrix}1 & s & s\\ j & l & l'\end{Bmatrix}$

Matrix elements of the spherical harmonic operator $\ddot{Y}_{Lv} \equiv Y_{Lv}(\vartheta,\varphi)$,

$< l'\|\vec{Y}_L\|l \geq \sqrt{\frac{(2L+1)(2l+1)}{4\pi}}\; C_{l0L0}^{l'\,0}$

(7)

Matrix elements of the spin angular momentum operator $\hat{S} \equiv \hat{S}_1$,

$< S'\|\vec{S}_1\|S >= \delta_{ss'}\sqrt{s(s+1)(2s+1)}$

Matrix elements of the operator $\ddot{\Delta}(r,\vartheta,\varphi) \equiv \ddot{\Delta}_1(r,\vartheta,\varphi)$,

$< n'l'\|\vec{\nabla}_1\|nl >= \sqrt{l+1}A_{n'l'\,nl}\delta_{l'\,l+1} - \sqrt{l}B_{n'l'\,nl}\delta_{l'\,l-1},$

$$A_{n'l'\,nl} = \int_0^\infty \psi^*_{n'l'}(r)\left(\frac{\partial}{\partial r} - \frac{l}{r}\right)\psi_{nl(r)r^2}dr$$

$$B_{n'l'\,nl} = \int_0^\infty \psi^*_{n'l'}(r)\left(\frac{\partial}{\partial r} - \frac{l+1}{r}\right)\psi_{nl(r)r^2}dr$$

Explicit forms of the 9_j symbols for spherical values of the arguments,

$$\begin{Bmatrix}g & g & 0\\ e & d & c\\ b & a & c\end{Bmatrix} = \frac{(-1)^{b+d+c+g}}{\sqrt{(2c+1)(2g+1)}}\begin{Bmatrix}a & b & c\\ e & d & g\end{Bmatrix}$$

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